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ABSTRACT

Plates reinforced by ribs or joists are common elements in lightweight building structures, as well as in other engineering structures. However, these structures are not well suited for the common simplified prediction models such as Statistical Energy Analysis (SEA) or the European calculation standard EN 12354. There are several reasons for this: One is that the overall vibration field is not diffuse due to the orthogonal behaviour introduced by the ribs. Another reason is that the modal density is not well behaving due to the periodicity introduced by the ribs. In the present paper these two aspects of the ribbed plate are investigated, focusing on the latter, using a modal model of a ribbed plate. Some suggestions are given regarding how to proceed towards a simplified prediction model for ribbed plates.

1. INTRODUCTION

Plates reinforced by ribs or joists are common elements in lightweight building structures, as well as in other engineering structures (vehicles, ships). However, these structures are not well suited for the conventional prediction models based on SEA. There are several reasons for this: the overall vibration field is not diffuse due to the orthogonal behaviour introduced by the ribs; the modal density is not a smooth function due to the periodicity introduced by the ribs; the highly oriented spatial attenuation that is common in these structures, etc. (see more below).

Multi-story buildings built with lightweight framing systems are of major interest for several reasons. For instance: the production costs are usually lower, especially in a prefabricated production approach; the low weight implies simpler and cheaper foundations and transportation; and in case of wood frames it is said to be an environmental friendly material.

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Through the years, the building industry has learnt that building with lightweight structures in combination with high acoustic quality demands is associated with a large risk. This is due to 1) the large variability together with 2) the lack of relevant prediction models. Recently a Swedish state-of-the-art report concerning acoustics in wooden buildings has been published [1]. Among other things this state-of-the-art report concludes that it is crucial to have reliable prediction tools in the early stage of a building project.

Flanking transmission means transmission of sound through the flanking partitions, as opposed to direct transmission. It is often one of the main acoustic problems for lightweight constructions – one reason being that, unlike the direct transmission path, it can hardly be measured in a laboratory. For homogeneous structures, simplified prediction models for flanking transmission have been available for a long time, mainly using power methods and SEA [2,3]. The available prediction models are less applicable to lightweight constructions.

In Statistical Energy Analysis (SEA) [4,3] the studied structure is divided into several energy components, *subsystems*, where the vibrational energy is assumed to be evenly distributed. Once the structural acoustic coupling between the elements is known, the calculation can be quickly performed. Closely related to SEA is the prediction standard EN 12354 [5]. It is based on different simplified models, especially those of Gerretsen [2,6] that mainly use measured data as input data. The relationship between SEA and EN 12354 are discussed by Nightingale et al. [7], and it is revealed that many of the basic assumptions and limitations are shared with SEA. Papers that attempt to treat flanking transmission in lightweight structures are made by Nightingale [8], using EN 12354, and Craik et al. [9], using SEA. However, in these papers the physical facts for the considered structure was not considered, and the quality and generality of the results are thus limited.

SEA is a powerful prediction technique since it yields very simple results with low computational effort. However, as indicated above there are several drawbacks with SEA of today. Avoiding these problems will be an important step in the fields of structural and building acoustics. The important problems for traditional SEA for the structures under consideration are listed here and described: **Non-Diffuseness.** In rib stiffened plate structures the overall vibration field is not diffuse due to the orthogonal behaviour introduced by the ribs. Thus, the basic SEA assumption about diffuseness is then not fulfilled and the vibration field has a complicated relationship to the structural transmission loss and the SEA coupling loss factor [10]. **Periodicity and modal density.** In its present state of development the SEA-technique has severe difficulties in handling spatially periodic structures [11], especially the pass band – stop band phenomena. For a finite structure the result is that the modes group together in pass bands. The modal density is then not a smooth function of the frequency – it is rather an on/of type of function. Moreover, due to the periodicity not being perfect there will be so-called Anderson localisation effects [12,13,14]. Keane and Price [15] and Tso and Hansen [16] indicate that SEA can be made to work for periodic structures, if special care is taken. However, the results are not directly applicable, as it is based on 1D theory. **Spatial attenuation.** There will be a highly directional spatial attenuation in the considered structures, with high attenuation perpendicular to the ribs, see Sjökvist [17, 18]. This implies that there is no simple connection between the energy density and the structural transmission factor. See also Sjökvist and Hammer [19]. **Sound radiation.** The Sound radiation property of a structure with an irregular vibration field is very complicated. The *radiation efficiency* σ is commonly used to determine the coupling loss factor between structure

and room. However, for the structures considered, the radiation efficiency is not easily determined. **Double layers.** Lightweight structures are often of double plate construction. If the two plates of a double structure are loosely coupled, there should be a subsystem describing each plate, and if they are rigidly coupled then the subsystem should include the entire structure. The question is how to treat the structures in between these two extremes. In [20, 21, 22] the wave characteristic of a double-plate structure is analysed as a whole system, using a wave based model. **Coupling at the joint.** Brunskog [23] describes the basic theory of the structural transmission loss of a junction, when the surrounding media is periodic in nature using wave theory with high resolution. The primary conclusion is that the periodic nature of the wave media surrounding the discontinuity clearly affects the amount of power transmitted through a junction – it is thus not correct to use a theory assuming the surrounding structure to be uniform. In the past this has not been taken into account in the SEA modeling of flanking transmission.

In the long run, the purpose is to develop SEA models for structural and flanking transmission in lightweight and rib stiffened structures. The basic idea is to re-examine the basic assumptions in SEA and elaborate on them so that they do not violate the important physical facts for the considered structures. In the present paper the focus will mainly be on the modal density and in part how to treat the periodic and orthotropic effects. A full SEA model will not be presented herein, but just a few hints will be offered on how to proceed.

2. METHOD

A. Some notes on SEA modeling

For the SEA model being under development the idea is that the modes of the stiffened plate will be divided in different SEA subsystems, depending on the main direction of the mode, see Figure 1. The modes that are representing waves travelling parallel to the beams are treated as one SEA subsystem and the modes travelling perpendicular to the beams are treated as another subsystem. This is in analogous to the modelling of a sound field in box-shaped rooms with most absorption on one surface, see Nilsson [24,25]. The periodic effects can then be incorporated in the subsystem for perpendicular modes, whereas the non-periodic effects need to be included in the subsystem for parallel modes. Also note that both plate and the beams are included in the same SEA subsystem.

This modeling will probably solve a few problems, mainly the problems with incorporating periodic effects, the problem of orthotropic energy transport and perhaps to some extend non-diffuseness. However, new problems and difficulties will appear: How should the division between the modes be done (which criteria)? How can a coupling loss factor between the subsystems be found? How could a theoretical estimate of the modal density be found? Moreover, many of the problems mentioned in the introduction still exist.

B. Modal model

The purpose of the modal model presented here is to examine some of the parameters and aspects in a SEA model. Thus, we are considering a very simplified model consisting of a thin plate and thin beams in pure bending, without the effects of beam rotation, in-plane waves, thick plate/beam theory, etc. To derive the results, use is made of Hamilton's principle, by minimizing the difference between kinetic and potential energy in the structure in an arbitrary time period. This is inspired by ideas and models by H. Chung, see ref. [10].
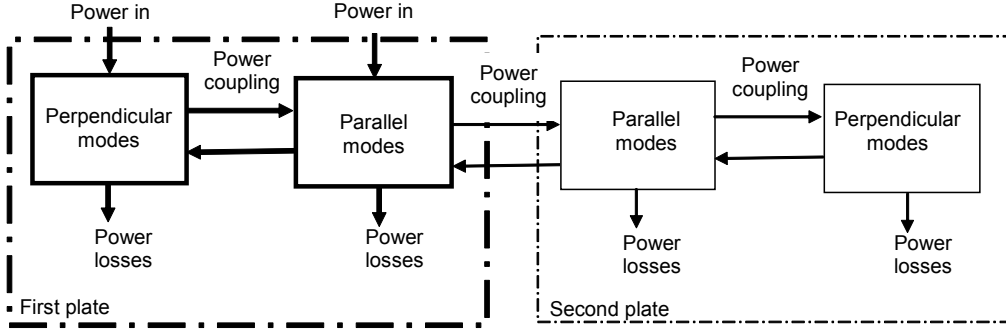
Figure 1: The proposed SEA subsystems. The bold system to the left is the excited rib stiffened plate. This is the one present if just one plate is considered. The right thin lined part is describing the case if a second rib stiffened plate is coupled. Each rib stiffened plate is described by two subsystems, describing the perpendicular and parallel modes.

The normal (vertical) deflection of the plate is denoted $w(x, y)$ and the deflection of the beams by $w_b(x, j)$, where $j = 1, 2, …, J$. Simply supported boundary conditions are assumed at the extreme ends, and the lengths of the plate are denoted l_x and l_y for the x- and y-directions, respectively. The deflections of the plate and the J beams are expressed in terms of Fourier sine sums, as

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} c_{mn} \varphi_m(x) \psi_n(y), \quad w_b(x, j) = \sum_{m=1}^{\infty} c_{mj}^b \varphi_m(x), \tag{1}$$

respectively, where the Fourier sine mode shapes are given by

$$\varphi_m(x) = \sqrt{\frac{2}{l_x}} \sin k_m x, \ \psi_n(y) = \sqrt{\frac{2}{l_y}} \sin \kappa_n y, \ k_m = \frac{\pi m}{l_x}, \ \kappa_n = \frac{\pi n}{l_y}. \tag{2}$$

Note that these modes are not the real modes of the coupled problem, just the modes of the un-coupled problem. The sine modes form an orthonormal basis, which will be used in the following. However, in order to solve the problem at hand by numerical methods, we need to truncate the series. Thus, the sums are truncated to N and M sine modes, respectively.

In order to make use of matrix notations and algebra, the so-far unknown amplitude coefficients are organised as coefficient vectors

$$\mathbf{c} = \begin{bmatrix} c_{11} \\ c_{12} \\ \vdots \\ c_{MN} \end{bmatrix}, \quad \mathbf{c}_b = \begin{bmatrix} c_{11}^b \\ c_{12}^b \\ \vdots \\ c_{MJ}^b \end{bmatrix} \tag{3}$$

where it should be noted that the vectors have different size. The coupling between the plate and the beams is expressed as equal deflection, $w(x, y_j) = w_b(x, j)$. In terms of our sine Fourier expansion (1), this coupling condition is expressed as

$$c_{mj}^b = \sum_n c_{mn} \psi_n(y_j).$$
(4)

which is a coupling between the two sets of unknown constants, only using the known sine modal functions. Let the matrix $\mathbf{J}$ be defined such that $\mathbf{c}_b = \mathbf{Jc}$ expresses the condition in equation (4), where

$$\mathbf{J} = \begin{bmatrix} \ddots & & 0 \\ & \mathbf{J'} & \\ 0 & & \ddots \end{bmatrix}, \quad \mathbf{J'} = \begin{bmatrix} \psi_1(y_1) & \psi_2(y_1) & \cdots & \psi_N(y_1) \\ \psi_1(y_2) & \psi_2(y_2) & \cdots & \psi_N(y_2) \\ \vdots & & & \\ \psi_1(y_S) & \psi_2(y_S) & \cdots & \psi_N(y_S) \end{bmatrix}$$
(5)

The kinetic and potential energies can now be expressed in terms of the Fourier series. I order to minimize the Hamilton's principle, we take the partial derivatives with respect to c_{mn} and set them equal to zero. As a result we obtain a system of $M \times N$ equations in the same number of unknowns. Minimising the functional of the Hamilton's principle, the coefficient vector of the forced problem can be derived and similarly the eigenvalues can be computed from the eigenvalue problem. In the present study, it is the latter we are mostly interested in, that is

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\mathbf{c} = \mathbf{0} \Rightarrow \mathbf{M}^{-1}\mathbf{Kc}_i = \omega_i^2 \mathbf{c}_i,$$
(6)

where diagonal stiffness and mass matrixes for the plate and the beams have been defined and introduced

$$\mathbf{K}_p = \begin{bmatrix} \ddots & & 0 \\ & D'\left(k_m^2 + \kappa_n^2\right)^2 & \\ 0 & & \ddots \end{bmatrix}, \quad \mathbf{K}_b = \begin{bmatrix} \ddots & & 0 \\ & EIk_m^4 & \\ 0 & & \ddots \end{bmatrix}$$

$$\mathbf{M}_p = \begin{bmatrix} \ddots & & 0 \\ & m_p'' & \\ 0 & & \ddots \end{bmatrix}, \quad \mathbf{M}_b = \begin{bmatrix} \ddots & & 0 \\ & m_b' & \\ 0 & & \ddots \end{bmatrix}$$
(7)

and

$$\mathbf{K} = \mathbf{K}_p + \mathbf{J}^T \mathbf{K}_b \mathbf{J}, \quad \mathbf{M} = \mathbf{M}_p + \mathbf{J}^T \mathbf{M}_b \mathbf{J}.$$
(8)

C. Mode shapes and a modal expression for the forced problem
By solving the eigenvalue problem (6) the actual mode shapes of the structure can be determined. These can then be used to express a solution to the forced problem. From this eigenvalue problem we obtain the eigenvalues ω_i^2 and the eigenvectors $\mathbf{c}_i$. The actual mode shape of the i'th mode then is

$$\vartheta_i(x,y) = \sum_{n,m=1}^{N,M} c_{n,m}^{(i)} \varphi_m(x) \psi_n(y) = \mathbf{c}_i^T \mathbf{q}(x,y) \qquad (9)$$

where the vector $\mathbf{q}(x,y)$ is defined from this expression. It is now possible to use the ordinary modal expressions for the forced problem using the real modes, see e.g. [26].

D. Modal density

In the numerical simulations, the number of modes up to a certain frequency is determined from the eigenvalues ω_i^2. Then, by counting the number of modes in a frequency band, and repeating this for several nominally equal structures, an estimate of the modal density is found. As a comparison the theoretical modal density of an un-stiffened plate will be used [26],

$$\frac{dN}{d\omega} = \frac{l_x l_y}{4\pi} \sqrt{\frac{m''}{B'}} \qquad (10)$$

E. Simulations

In the simulations, the following material data were used: Truncation of the sums, $N=M=100$. Flexural rigidity of the plate $D' = 2800 \cdot (1+i \cdot 0.02)$ Nm^2. Young's modulus times the moment of inertia of the beams $EI = 1.35 \cdot 10^6 \cdot (1+ i \cdot 0.02)$ Nm^2. Width and length of the structure are l_x=4 m and l_y=6 m, respectively (when simulating several nominally equal structures, these are made slightly random but keeping the area unchanged). Mass density per unit area of the plate, $m_p'' = 10.8$ kg/m^2, and mass density per unit length of the beams, $m_b' = 6.75$ kg/m. The beams are located periodically at a distance of 0.5 m apart, starting 0.25 m from the edge.

3. RESULTS

Figure 2 shows the modal density of the entire structure, estimated from several simulations. Both estimates with a constant bandwidth of 10 Hz and a 1/6 octave bandwidth is shown. As a comparison is the theoretical modal density for a thin plate shown.
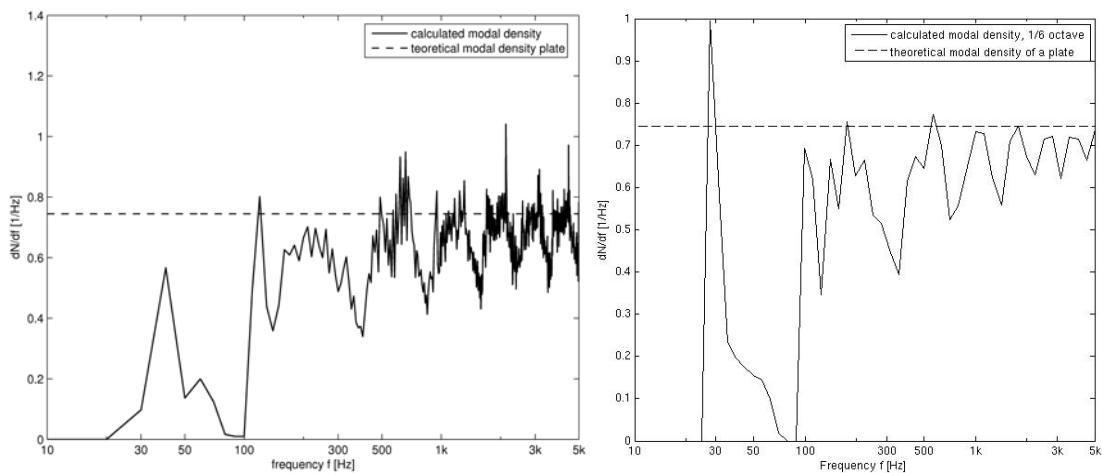

Figure 2: Modal density. The solid line is the estimate from repeated simulations. The dashed line (- - -) is the theoretical modal density for a plate (12). Left: constant 10 Hz bandwidth; right: 1/6 octave bandwidth.
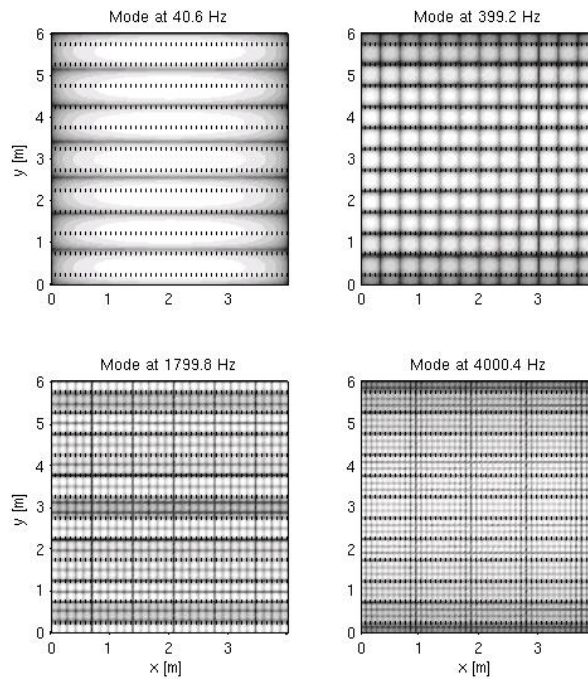
Figure 3: A few real mode shapes. Dashed lines (---) indicates the rib stiffeners.
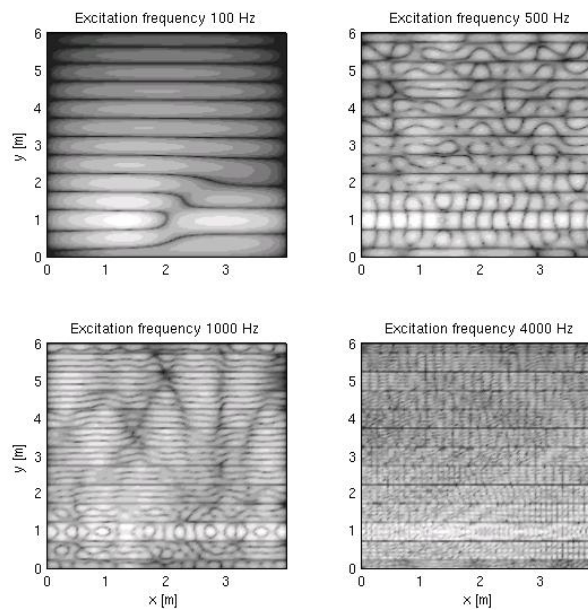

Figure 4: A few forced vibration fields, excited in $x = 1.32$ m, $y = 1$ m.

Figure 3 shows four samples of real mode shapes of the entire structure. The rib stiffeners are indicated in the figure as dashed lines. Figure 4 shows forced vibrations field, excited with a point force. Third octave bands results are shown.

4. ANALYSIS AND DISCUSION

In Figure 2 it is clearly seen that the modal density of the rib stiffened plate is affected by the periodically attached stiffeners: The modal density is in general lower than the theoretical value for the unstiffened plate, and it has undulation behavior, the undulations having equal width in

the parameter *kl*. This is a typical behavior of a periodic structure, c.f. [17,18]. However, in this 2D case the pass band – stop band behavior is not as distinct as it would have been in a 1D case (as studied in ref. [17,18]): In an 1D case the modal density would have been zero in the stop bands, but in the present case there is just an undulation roughly between 0.8 and 0.4 modes per Hz. The reason for the periodicity to not be so clearly seen in a 2d case is that there exist different directions of pass band propagation at different frequencies, see e.g. [27].

In the SEA model under development, two different modal densities for the two subsystems will be used. Then, for the perpendicular modes the periodic effects of can be incorporated in a similar way as in refs. [17,18], where the modal density in a one-dimensional structure was modified so that a high modal density was found for the pass bands and almost zero modal density was used in the stop bands.

In Figure 3 a few of the real vibration modes can be seen. The modes are quite clearly built up by several sine modes. The pattern is slightly different along the two main directions: If excluding the 40.6 Hz mode, one can notice that in the direction along the beams there is not much change in vibration amplitude, whereas perpendicular to the beams there is a slow global amplitude variation, in two cases with lower vibrations closer to the edge. Forced results are shown in Figure 4. Similar results have been presented before, see e.g. [20]. In a plate without beams, there would instead have been a more radial attenuation from the point of excitation, and the attenuation would have been weaker.

One may note that several steps are needed in order to proceed towards a practically working SEA model. The present paper just indicates on the possibilities. One important problem that has to be solved is how to get general results concerning e.g. the modal density. Moreover, a practical method or principle to divide the modes in different mode groups has to be developed. Furthermore, several of the problematic aspects of SEA mentioned in the introduction, partly including non-diffuseness and spatial attenuation, still has to be considered.

4. CONCLUSIONS

The paper indicates that it might be possible to incorporate periodic effects of rib stiffened plates in SEA modeling if 1) a subdivision of the modes is made, and 2) a prober modal density is used for the subsystems. It has moreover been shown that the modal density of a (total) rib stiffened plate is affected by the periodicity, it having an undulation behavior typically associated with pass band – stop band phenomena.

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