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Studies on Mathematics Education and Society

# LANDSCAPES OF INVESTIGATION

CONTRIBUTIONS  
TO CRITICAL  
MATHEMATICS  
EDUCATION

EDITED BY MIRIAM GODOY PENTEADO  
AND OLE SKOVSMOSE



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# II. Inclusive Landscapes of Investigation

*Ole Skovsmose*

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By means of two examples, one concerning polygons and one concerning erosions of democracy, I characterise the conception of inclusive landscapes of investigation. These are teaching-learning environments that are accessible for everybody, and invite dialogue across differences. This brings me to refer to universal design, which provides a broader perspective on the construction of inclusive environments. Finally, I relate the concept of critique to the characteristics of inclusive landscapes of investigation.

Inclusive mathematics education tries to provide learning environments for all groups of students. Inspired by the work of the Épura research group, the idea that inclusive mathematics education could provide learning environments evolved: environments where all students, independent of particular differences, can *learn together*.<sup>1</sup> This leads me to the idea of forming inclusive landscapes of investigation.

In literature one can find two different interpretations of inclusive mathematics education, which I refer to as the specific and the general. According to the specific interpretation, inclusive mathematics education concerns students with disabilities such as, for instance, blind or deaf students. The book *Inclusive Mathematics Education: Research Results from Brazil and Germany*, edited by David Kollosche et al. (2019), addresses

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1 The Épura research group was founded in 2008, and is associated to Unesp in Rio Claro. Épura members are, first of all, Master's and PhD students and researchers working with inclusive education inspired by critical mathematics education. The group is coordinated by Miriam Godoy Penteadó and Ole Skovsmose.

problems related to this interpretation. According to the general interpretation, inclusive mathematics education concerns students learning together across cultural, economic and political differences, as well as across differences with respect to learning capacities. The book *Diversity in Mathematics Education: Towards Inclusive Practices*, edited by Bishop et al. (2015), addresses this general interpretation. In the following, I have both the specific and the general interpretation in mind.

In this chapter I discuss the notion of *inclusive landscapes of investigation*,<sup>2</sup> which are landscapes intended to be accessible for different groups of students, whatever the differences concerning their abilities or social diversities. When the notion of landscapes of investigation was first developed, I did not have inclusive mathematics education in mind. However, now I want to extend the discussion of landscapes by incorporating concerns about inclusion.

I am going to present two examples, which will serve as references for the following discussion: the first, *Polygons*, relates to the specific interpretation of inclusive mathematics education, while the second, *Erosion of Democracy*, relates to the general interpretation. After the examples, I will outline a general characteristic of inclusive landscapes of investigation. As a conclusion, I will address the notion of critique, and in this way relate the discussion to the concerns of critical mathematics education.

## 1. Polygons

The landscape *Polygons* was developed in an inclusive setting in Brazil, where deaf and hearing students were learning together. The example is described by Amanda Moura (2020) and by Amanda Moura and Miriam Godoy Penteadó (2019).<sup>3</sup>

In Brazil it has become common practice to integrate children with different diagnoses of disability into the regular school system and not to let them remain in specialised institutions. In the city of Rio Claro in the São Paulo State, one finds “inclusive schools” that receive students

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2 For a general presentation of landscapes of investigation, see Chapter 1, “Entering Landscapes of Investigation”.

3 See also Chapter 12 by Moura and Penteadó.

with disabilities. Here one finds classrooms with, for instance, both deaf and hearing students, as was the case in the classroom where Moura conducted her study.

One investigation in which the students (around ten years old) were engaged concerned the classification of geometric figures. The students were presented with a huge number of figures cut out of cardboard and were asked to group the figures according to characteristics they found relevant. Some figures had curved edges, some had straight edges and some simply looked strange. How could they sort things out?

This activity led to the question: What is a polygon? Another question quickly emerged: What sign in Libras should they use for the word “polygon”? (Libras is the Brazilian sign language used by deaf people). During the process, deaf and hearing students worked together in groups, and in order to facilitate the communication, an interpreter who could speak Libras was around. However, there was also direct communication between hearing and deaf students. For years the students had been in the same class, and the hearing students had learned some Libras.

One possibility in sign language is to do a spelling out of the letters, making them with the hands: P-O-L-Y-G-O-N. However, such spelling is a last resort, as spelling out too many words makes communication in Libras slow and awkward. It is more efficient to try to decide upon a single sign for the word “polygon”. Libras is a language in construction, and many concepts do not have a particular sign. With respect to mathematics, there is no well-defined extension of the regular sign system, and nothing called “mathematics in Libras”. This makes it relevant to negotiate signs for particular concepts, such as “polygon”. But which sign should we use?

The discussion led to some mathematical clarifications of the notion of polygon. One is reminded of the process described by Lakatos (1976), where the sequence of proofs and refutations leads to further clarifications of the notion for polyhedron. In a similar way, the complexity of the notion of polygon was revealed through the discussion about which sign to use for this concept. One problem in choosing a particular sign was trying not to assume that one is dealing with a particular polygon, like a rectangle for instance. The sign should refer to the general properties of a polygon, and not to a particular group of

polygons. In the end, the students decided upon a sign, namely the sign for “many” followed by the sign for “lines”, repeated with the hands moving in different directions showing a “P”.

*Polygons* turned into an inclusive landscape of investigation where the participation of both deaf and hearing students was not only possible, but necessary. The students had to judge the adequacy of the suggested signs, considering what signs already exist in Libras as well as the significant mathematical properties of a polygon. An important feature of an inclusive landscape of investigation is that different groups of students can come to work together. An inclusive landscape of investigations facilitates meetings across differences, as was the case with the landscape *Polygons*.

## 2. Erosions of Democracy

The Weimar Republic was formed in 1919 and destroyed in 1933, when Adolf Hitler came to power. During that period a disastrous erosion of democracy took place. One could consider to what extent similar erosions have taken place elsewhere in the world. I find that a broader discussion of democracy is vitally important today, where non-democratic movements and authoritarian discourses seem to be gaining more and more influence. I see this as being a challenge also to mathematics education. As an illustration, let us consider *Erosions of Democracy* as a possible landscape of investigation, and to what extent it can become inclusive, considering the general interpretation of inclusive education.

This landscape is a thought experiment, which as far as I am aware has not been tried out previously. However, I have presented this thought experiment on different occasions, such as at the *Segundo Colóquio de Pesquisa em Educação Matemática Crítica* (Second Colloquium in Research in Critical Mathematics Education), Rio Claro, 2018, and at the *Primeiro Encontro Mato-Grossense de Professores que Ensinam Matemática* (First Meeting for Mathematics Teachers in Mato Grosso), Tangará da Serra, 2018. After these presentations I received many comments and suggestions which inspired me to carry out a further elaboration of the thought experiment.

Previously (Skovsmose, 1994), I have highlighted that democracy refers to at least four sets of ideals that concern: procedures for voting; fair distribution of welfare; equal opportunities and obligations; and rights to express oneself. Each of these ideals can be eroded. In many discussions about democracy, the existence of procedures for voting have been considered as definitional for a democracy. My point in mentioning the four sets of ideals is that a democracy only gets established through a variety of components, the right to vote being only one of them.

Voting is an act through which one allows somebody else to speak in one's name. The investigation of voting procedures is a mathematical issue, which can be challenging also for university students in mathematics (see Steiner, 1988; and Obraztsova and Elkind, 2012). However, the landscape *Erosions of Democracy* can focus on some specific issues such as the problem of "a tyranny of the majority". As an illustration of what this could mean, one can consider a small community of seven people, four coming from the north and three from the south. Every decision in this community is based on voting. The four from the north vote together. It has to be decided who is going to do the manual work. With four votes against three, it is decided that people from the south have to do it. Who is going to pay taxes? With four votes against three, it is decided that people from the south will have to pay. In the end, it is voted that the three people from the south should serve as slaves in the small community. The possibility of a tyranny of the majority shows that while voting constitutes part of democratic procedures, it far from ensures democracy. But how are we to eliminate the possibility of a "tyranny of the majority"? This question may provide an entrance to further investigations of procedures for voting.

One can imagine functioning democracies in both rich and poor countries. Much more difficult to imagine is a functioning democracy in a country with extreme differences between rich and poor. Decisions in a democracy are expressions of a shared will, and if extreme differences are maintained, one must suspect that the shared will has become subsumed by particular interests. There are different ways of mathematically describing distributions of welfare; one is referred to as the Gini Coefficient. However, a more elementary mathematical technique can also illustrate a distribution of welfare. As part of *Erosions of Democracy*, the students can investigate the distribution of welfare



in imagined countries with a population of, say, 100 people each. The distributions of welfare in such imagined mini-countries can be the same as the distributions in real countries, and in this way the students can explore principal features of economic inequalities the world over.<sup>4</sup>

In a democracy, one assumes equity with respect to the right to vote, but equity relates to any relationship with the law. For instance, the students can investigate to what extent differences with respect to juridical procedures can be related to a people's political position, economic situation or ethnicity. A specific issue concerns the way in which the police and the penitentiary system are operating. With reference to Brazil, statistics show that in 2019 the police in Rio de Janeiro killed 1810 people.<sup>5</sup> One can follow up and clarify the ethnic distribution behind such a number. One can also consider the number of people put in jail in Brazil, or in any country for that matter, and see to what extent one can identify any ethical biases. Where it is possible to identify marked biases, one might question to what extent everybody is treated equitably with respect to the law. This is an important issue with respect to *Erosions of Democracy*.

The classic issue of freedom of speech concerns possibilities for expressing oneself, as well as the right to articulate opinions about political, religious, cultural or any issues for that matter. In a democracy, the general obligation of the press is to give a voice to everybody, and the students could consider to what extent such rights are observed or ignored by the various media outlets. There are different ways of addressing such an issue, and a starting point can be taken from the way Reginaldo Britto investigates the visibility of white and black children in different magazines (see his chapter "Media and Racism" in this book). The same investigative procedure can be used for revealing different degrees of visibility of various politicians in different media outlets depending on their ethnicity, gender or political position. An investigation of visibility can show to what extent a voice is given to everybody in an equal way, or if certain media outlets operate with particular biases.

In my imagination, *Erosions of Democracy* may become inclusive by the way it calls for contributions from students with different experiences

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4 Inspiration for such an approach can be found in Smith (2011).

5 See *BBC News: Rio Violence: Police Killings Reach Record High in 2019*. <https://www.bbc.com/news/world-latin-america-51220364>

and backgrounds. In Denmark—reflecting a general European trend—one finds a growing animosity towards groups of people who are considered immigrants and are described as being “foreigners”. This trend causes an erosion of democracy—for instance, with respect to equity in a variety of situations. In order to address such erosion, it is important to bring together students with different experiences and backgrounds. I see such interactions between those labelled as “foreign students” and those claimed to be “not-foreign students” as an important resource for addressing erosions of democracy. Exploring the landscape *Erosions of Democracy* does not presuppose any homogeneity of the students. Rather, the exploration of the landscapes benefits from diversities. In this sense, I think of it as an inclusive landscape of investigation.

### 3. Features of Inclusive Landscapes of Investigation

Like any landscape of investigation, inclusive landscapes provide space for investigations. Such landscapes are not organised by sequences of problems to be solved, or by exercises to be answered. Rather, they provide invitations for students to engage in inquiry processes. Questions can be raised, and answers can be suggested, leading to new questions.

The discussion of *Polygons* and *Erosions of Democracy* brings me to highlight the following two features of *inclusive landscapes of investigation*:

1. Inclusive landscapes of investigation provide learning milieus that are *accessible to everybody*. Differences among students do not cause specific conditions for entering and moving around in such landscapes. One might meet a variety of challenges that acknowledge diversities amongst students. The very conceptions of students being “normal” or “not-normal”, “foreigner” or “not-foreigner”, having “abilities” or “disabilities” lose significance in an inclusive learning environment.
2. Inclusive landscapes of investigation *invite dialogues across differences*. Establishing conditions for dialogue is a general feature of a landscape of investigation. In inclusive landscapes, such dialogue is also supposed to take place across differences, whatever kind of differences one might have in mind. There

could be differences with respect to cultural backgrounds, religious convictions, nationalities or economic conditions, as well as with respect to abilities.<sup>6</sup>

“Inclusive landscapes of investigation” is far from being a well-defined label, and I am not trying to provide the notion with more specificity than what was just outlined. However, I want to condense this specificity into one statement: *inclusive landscapes of investigation invite meetings amongst and across differences*.

The notion of *universal design* was coined by Ronald Mace, who was an architect preoccupied with designing environments accessible for everybody, independent of their physical conditions. In 1963, Selwyn Goldsmith (1997) published *Designing for the Disabled*, which highlighted the idea of ensuring the free movement of all, including people with disabilities, such as people in wheelchairs. It was such a concern that Mace captured with the expression “universal design”.

One can think of inclusive landscapes of investigation as an example of universal design (see Marcelly, 2015, for a presentation of this idea). This is an important comparison, although with some limitations. With reference to architecture, “moving around” is a rather well-defined physical property, while the possibility of “moving around” in a landscape of investigation is not a simple quality of the landscape as such. It is also a quality of the way the landscapes are acted out in educational practice.

In the case of *Polygons*, the activities could have become non-inclusive if, for instance, the group organisation of the students had been different by allocating hearing students to one group and deaf students to another. Naturally, being allocated to the same group is not a sufficient condition for establishing inclusive education. It is, for instance, also important to create conditions for the students to communicate with each other and to explore issues together. *Erosions of Democracy* could become non-inclusive if the investigations were differentiated according to the mathematical abilities of the students. It could transform into non-inclusivity as well if political, economic or cultural differences were to define the groupings of the students. Again, more conditions

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6 A similar characteristic can also be found in Skovsmose (2019). Here one also finds further references to people who have developed the notion, such as Roncato (2015).

are necessary for establishing inclusive learning environments, such as developing a shared interest in acknowledging and understanding different worldviews.

#### 4. Dialogue across Differences

I see critical activities as being rooted in dialogues.<sup>7</sup> This is the reason that critical mathematics education holds a particular interest in creating landscapes of investigation, which provide conditions for establishing dialogue between students and teachers, and amongst students themselves.

The concern for creating inclusive landscapes of investigation is in line with this overall idea. However, through inclusive landscapes, one provides conditions for establishing dialogues *across differences*. Such dialogues are important extra resources for critical activities. I will illustrate this point by again referring to *Polygons* and *Erosions of Democracy*.

Critique has an epistemic as well as a socio-political dimension.<sup>8</sup> The epistemic dimension is illustrated by the landscape *Polygons*, while *Erosion of Democracy* illustrates the possibility of developing a socio-political critique.

The landscape *Polygons* was accessible for both hearing and deaf students; in fact, the presence of both groups of students was crucial for conducting a critical epidemic investigation of the notion of polygon. When one considers the properties of a polygon that could define the sign in Libras, it is important that the perspectives of both hearing and deaf students are articulated, and that everybody engages in a dialogue where ideas are not just stated, but discussed, dissected and developed.<sup>9</sup> A dialogue across differences is an important resource for critical reflections of an epistemic nature.

The landscape *Erosions of Democracy* is inclusive to the extent that everybody is invited to contribute to its exploration. In fact, the presence of a diversity of perspectives is important for addressing questions like: To what extent is systemic poverty an obstruction for a functioning

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7 For an elaboration of this point, see Chapter 1.

8 See my discussion of critique in Chapter 1.

9 For a further discussion of how negotiations of sign contribute to the shared learning of deaf and hearing students, see Sales, Penteado and Moura (2015).

democracy? To what extent are different groups of people treated differently with respect to legal procedures? Can such differences be related to people's political opinions? To their economic status? To their ethnicity? Which specific examples can one refer to? For addressing such questions and other issues concerning possible erosions of democracy, dialogues across differences are essential. It is important that different voices are not only heard, but also engaged in a dialogue. With respect to the thought experiment *Erosion of Democracy*, we can only speculate about the interaction among the students, but in order to critically address democratic issues, I find it is important to establish dialogues between different worldviews.

I find that dialogues across differences provide important extra resources for critical activities of both epistemic and socio-political natures. For this reason, it is important to explore possibilities for establishing inclusive landscapes of investigation. It is also my hope that such critical activities might support acknowledgements of diversities without falling into dominant discursive stereotypes.

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## References

- Biotto Filho, D. (2015). *Quem não sonhou em ser um jogador de futebol? Trabalho com projetos para reelaborar foregrounds*. Doctoral dissertation. Rio Claro, SP: Universidade Estadual Paulista (Unesp).
- Bishop, A. Tan, H. & Barkatsas, T. N. (Eds) (2015). *Diversity in mathematics education: Towards inclusive practices*. Springer.
- Faustino, A. C. (2018). *Como você chegou a esse resultado? O processo de dialogar nas aulas de matemática dos anos iniciais do Ensino Fundamental*. Doctoral dissertation. Rio Claro, SP: Universidade Estadual Paulista (Unesp).
- Goldsmith, S. (1997). *Designing for the disabled*. Architectural Press.

- Kollosche, D., Marcone, R., Knigge, M., Penteado, M. G. & Skovsmose, O. (Eds) (2019). *Inclusive mathematics education: State-of-the-art research from Brazil and Germany*. Springer.
- Lakatos, I. (1976). *Proofs and refutations*. Cambridge University Press.
- Marcelly, L. (2015). *Do improviso às possibilidades de ensino: Um estudo de caso de uma professora de matemática com estudantes cegos*. Doctoral dissertation. Rio Claro, SP: Universidade Estadual Paulista (Unesp).
- Moura, A. Q. (2020). *Cenários para investigação e escola inclusiva: Possibilidades de diálogo entre surdos e ouvintes em aulas de matemática*. Doctoral dissertation. Rio Claro, SP: Universidade Estadual Paulista (Unesp).
- Moura, A. Q. & Penteado, M. G. (2019). The role of the interpreter of Brazilian sign language in the dialogue among deaf and hearing students in mathematics classes. In M. Knigge, D. Kollosche, R. Marcone, M. G. Penteado & O. Skovsmose (Eds), *Inclusive mathematics education: Research results from Brazil and Germany* (pp. 253–270). Springer.
- Obraztsova, S. & Elkind, E. (2012). Optimal manipulation of voting rules. In Conitzer, Winikoff, Padgham & van der Hoek (Eds), *Proceedings of the 11th international conference on autonomous agents and multiagent systems (AAMAS 2012)*.
- Roncato, C. R. (2015). *Cenários investigativos de aprendizagem matemática: Atividades para a autonomia de um aprendiz com múltipla deficiência sensorial*. Master's thesis. São Paulo, SP: Universidade Anhanguera de São Paulo.
- Sales, E. R., Penteado, M. G. & Moura, A. Q. (2015). A Negociação de Sinais em Libras como Possibilidade de Ensino e de Aprendizagem de Geometria. *Bolema*, 29(53), 1268–1286.
- Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Kluwer Academic Publishers.
- Skovsmose, O. (2011). *An invitation to critical mathematics education*. Sense Publishers.
- Skovsmose, O. (2019). Inclusions, meetings, landscapes. In D. Kollosche, R. Marcone, M. Knigge, M. G. Penteado & O. Skovsmose (Eds), *Inclusive mathematics education: Research results from Brazil and Germany* (pp. 71–84). Springer.
- Smith, D. J. (2011). *If the world were a village: A book about the world's people*. Second edition. Kids Can Press.
- Steiner, H. G. (1988). Mathematization of voting systems: Some classroom experiences. *International Journal of Mathematical Education in Science and Technology*, 19(2), 199–213.

