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Encounter Probability of Individual Wave Height

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Abstract

Some coastal and offshore structures, e.g. offshore platforms and vertical wall breakwaters in deep water, are often designed according to a design individual wave height.

The conventional method for the determination of the design individual wave height is first to obtain the design significant wave height corresponding to a certain exceedence probability within a structure lifetime (encounter probability), based on the statistical analysis of long-term extreme significant wave height. Then the design individual wave height is calculated as the expected maximum individual wave height associated with the design significant wave height, with the assumption that the individual wave heights follow the Rayleigh distribution.

However, the exceedence probability of such a design individual wave height within the structure lifetime is unknown.

The paper presents a method for the determination of the design individual wave height corresponding to an exceedence probability within the structure lifetime, given the long-term extreme significant wave height. The method can also be applied for estimation of the number of relatively large waves for fatigue analysis of constructions.

1. Introduction

Some coastal and offshore structures, e.g. offshore platforms and vertical wall breakwaters in deep water, are often designed according to a design individual wave height.

The determination of the design individual wave height is based on a long-term wave measurement or hindcast. Most often the data set consist of maximum

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significant wave heights for the most severe storms within a number of years. An extreme significant wave height is the peak value of the significant wave height in a storm which exceeds a predefined threshold.

The conventional method for the determination of the design individual wave height is first to obtain the design significant wave height corresponding to a certain exceedence probability within a structure lifetime (encounter probability), based on the statistical analysis of the extreme data set. Then the design individual wave height is calculated as the expected maximum individual wave height associated with the design significant wave height (sea state), with the assumption that the individual wave heights follow the Rayleigh distribution.

For example, if the design level for the design significant wave height is a return period of 100 years (T=100 years), then by extreme analysis we find out that $H_s^{100\ years} = 10.2\ m$. With the assumption that the individual wave heights follow the Rayleigh distribution, the expected maximum individual wave height associated with the design significant wave height is

$$(H_{max})_{mean} \approx \left(\sqrt{\frac{\ln N}{2}} + \frac{0.577}{\sqrt{8 \ln N}} \right) H_s^{100\ years} \tag{1}$$

where N is the number of individual waves related to $H_s^{100\ years}$. In engineering practice it is normally assumed that N is in the order of 1000 for the stationary peak of the storm, which corresponds to $(H_{max})_{mean} = 19.7\ m$.

However, the return period of such design individual wave height remains unknown.

The paper presents a method for determination of the return period of a design individual wave height. The encounter probability of the design individual wave height, i.e. exceedence probability within structure lifetime L, is

$$p = 1 - \exp\left(-\frac{L}{T}\right) \tag{2}$$

The only input of the method is an extreme data set (minimum input). The accuracy of the result can be improved if more information are available, e.g. the duration of the storms, joint distribution of wave height and period etc.

The method can also be applied for estimation of the number of relatively large waves for fatigue analysis of constructions.

2. Distribution of individual wave height in a storm: $F_L(H)$

Because the structures are located in deep water, it is assumed that, with a given significant wave height H_s, the individual wave height H follows the Rayleigh distribution:

$$F_R(H) = 1 - \exp\left(-2\left(\frac{H}{H_s}\right)^2\right) \tag{3}$$

However, the significant wave height is varying throughout a storm. Based on some prototype records it is assumed that H_s grows and decays linearly between the threshold of significant wave height, $H_{s,t}$, and the peak significant wave height $H_{s,p}$. For the convenience, an equivalent storm history is used, cf. Fig. 1.

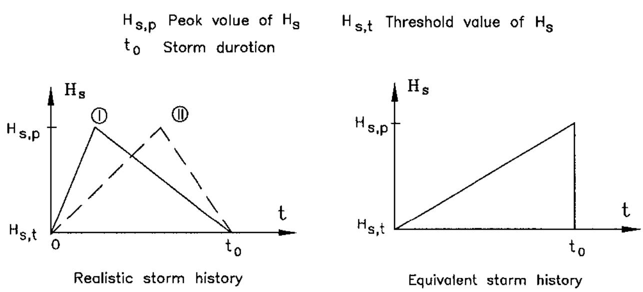

Fig. 1. Realistic storm history and the equivalent storm history (Burcharth et al. 1992).

If it is further assumed that the average wave period within a storm is constant and independent of H_s, then the distribution of individual wave height in a storm is

$$F_S(H) = \int_{H_{s,t}}^{H_{s,p}} F_R(H) \frac{1}{H_{s,p} - H_{s,t}} \, \mathrm{dH}_s$$

$$= 1 - \frac{1}{H_{s,p} - H_{s,t}} \int_{H_{s,t}}^{H_{s,p}} \exp\left(-2\left(\frac{H}{H_s}\right)^2\right) \mathrm{dH}_s \tag{4}$$

Note that $F_S(H)$ is independent of storm duration.

Fig. 2. shows an example of the difference between the individual wave height distribution in a storm and the Rayleigh distributions corresponding to $H_{s,t}$ and $H_{s,p}$, respectively.

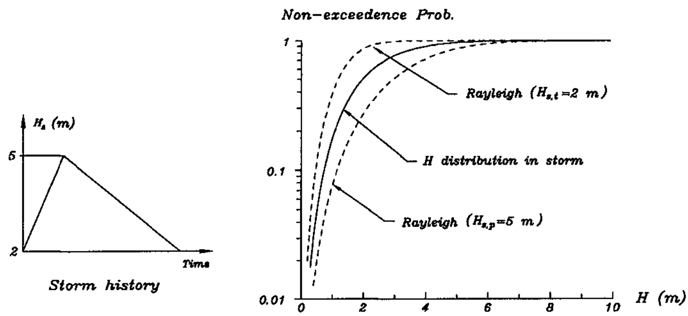

Fig. 2. Example of individual wave height distribution in a storm.

3. Long-term distribution of significant wave height: $F(H_s)$

The long-term distribution of significant wave height is obtained by the statistical analysis of the extreme data set. The general procedure is:

1) Choice of the extreme data set based on long-term wave height measurement/hindcast
2) Choice of several theoretical distributions as the candidates for the extreme wave height distribution
3) Fitting of the extreme wave heights to the candidates by a fitting method.
4) Choice of the distribution based on the comparison of the fitting goodness among the candidates

For more details please refer to Burcharth et al. (1994). The obtained long-term distribution of H_s gives information on the occurrence probability of storms over the threshold $H_{s,t}$ (which can be converted to the number of the storms in the structure lifetime), and the corresponding peak value $H_{s,p}$ in the storms.

4. Long-term distribution of individual wave height: $F_L(H)$

If we keep the assumption that the average wave period is constant and independent of the significant wave height, the long-term distribution of individual wave height can be expressed as (cf. Fig. 3)

$$F_L(H) = \int_{H_{s,t}}^{\infty} F_S(H)\, f(H_s)\, \mathrm{dH_s} \tag{5}$$

where $f(H_s)$ is the density function of the long-term significant wave height distribution, obtained by the extreme analysis, and $F_S(H)$ is the distribution of individual wave height in a storm.

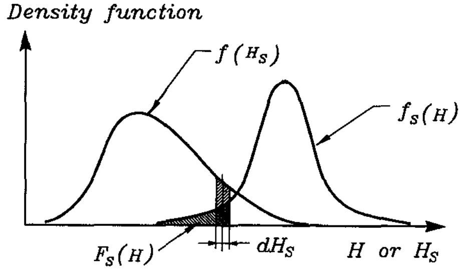

Fig. 3 Illustration of the integral in eq (5).

5. Return period and encounter probability of design individual wave height

According to the definition of return period, the return period of the individual wave height H is

$$T = \frac{1}{\lambda' \,(\, 1 \;-\; F_L(H))} \tag{6}$$

where λ' is the number of individual waves related to extreme storm, i.e. all H_s in the storm $\geq H_{s,t}$, and $F_L(H)$ is the long-term distribution of individual wave height.

The only unknown in eq (6) is λ'. Obviously the value of λ' depends on the threshold level $H_{s,t}$. The lower $H_{s,t}$, the larger λ'.

Smith (1988) investigated by field measure the relation between $H_{s,t}$ and P, occurrence probability of the event $H_s \geq H_{s,t}$. The threshold $H_{s,t}$ is represented by the number of extreme storms per year, λ. The definition of λ and P is illustrated by Fig. 4.

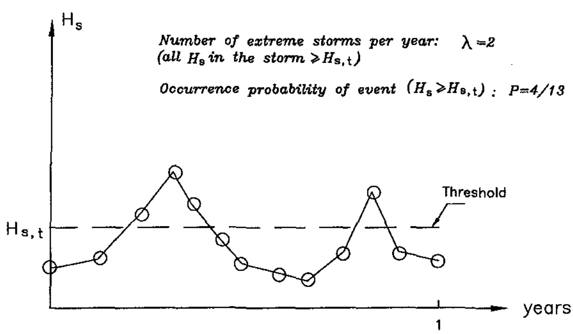

Fig. 4. Illustration of definition of λ and P.

The locations of the field measurement are given in Table 1. They represent a wide geographical spread and variety of wave conditions. Each location includes 58,440 significant wave heights by hindcast study three hours apart from January 1, 1956 to December 31, 1975 (20 years).

Table 1. Locations investigated by Smith (1988)

Site	mean wave height in 20 years (m)
Atlantic City, New Jersey	0.65
Nagshead, North Carolina	0.65
Daytona Beach, Florida	0.67
Newport, Oregon	2.76
Half-Moon Bay, California	2.14

A linear regression analysis for all locations gives

$$P = 0.003\ \lambda \tag{7}$$

with a correlation coefficient of 0.97. λ' can be calculated based on eq (7), as will be shown in the next example. Then the return period and encounter probability of the individual wave height can be calculated by eqs (6) and (2), respectively.

6. Example

Extreme data set

An extreme wave data set for the 15 most severe storms in a period of 20 years for a deep water location in Mediterranean Sea is given in Table 2. The data set is obtained by hindcast study. The threshold level for identifying the extreme storms is $H_{s,t} = 3\ m$.

Table 2. Extreme wave data set.

Rank i	Peak $H_{s,p}$ metres	Peak period T_P seconds	Wave direction degrees
1	9.32	14.0	143
2	8.11	14.1	139
3	7.19	13.4	123
4	7.06	10.8	123
5	6.37	11.9	143
6	6.15	11.1	185
7	6.03	12.3	135
8	5.72	10.5	176
9	4.92	10.7	150
10	4.90	10.6	129
11	4.78	11.8	161
12	4.67	9.9	120
13	4.64	9.2	122
14	4.19	10.5	137
15	3.06	11.1	154

Long-term distribution of significant wave height

The number of extreme storm per year is $\lambda = 15/20$. By fitting the peak significant wave height to the Weibull distribution

$$F(H_s) = 1 - \exp\left[-\left(\frac{H_s - H_{s,t}}{A}\right)^k\right] \tag{8}$$

where $H_{s,t} = 3\ m$, we obtain the Weibull distribution parameters $A = 3.24$ and $k = 1.83$.

By inserting the definition of return period T

$$T = \frac{1}{\lambda\,(\,1 \;-\; F(H_s)\,)} \tag{9}$$

into eq (8), we obtain

$$H_s = A\left(-ln(\frac{1}{\lambda T})\right)^{\frac{1}{k}} + H_{s,t} \tag{10}$$

The fitting is depicted in Fig. 5.

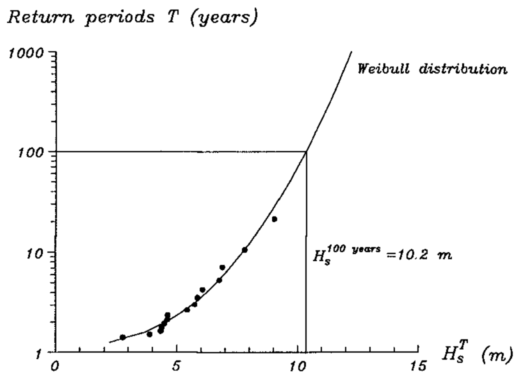

Fig. 5. Fitting of extreme data set to Weibull distribution.

If the design level for the design significant wave height is a return period of 100 years (T=100 years), we get $H_s^{100\ years} = 10.2\ m$.

By eq (1) with $N = 1000$, the expected maximum individual wave height associated with the design significant wave height is $(H_{max})_{mean} = 19.7\ m$. In the following we will try to obtain the return period and encounter probability of $(H_{max})_{mean}$.

Long-term distribution of individual wave height: $F_L(H)$

The long-term distribution of individual wave height is calculated by eq (5) and shown in Fig. 6.

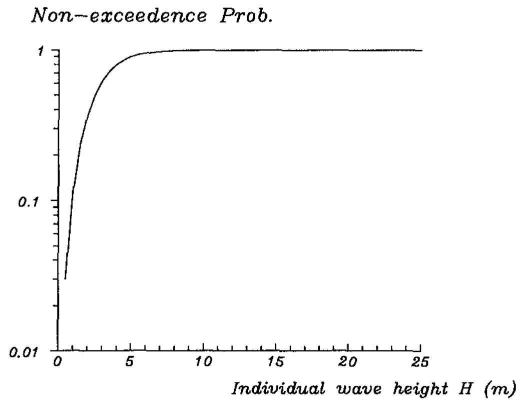

Fig. 6. Long-term distribution of individual wave height.

Return period and encounter probability of individual wave height

By eq (7) we get the occurrence probability of the event $(H_s \geq H_{s,t})$

$$P = 0.003\lambda = 0.0025$$

If we assume that the average wave period is $\overline{T} = 12$ s (average wave period in Table 2), the number of individual waves per year is

$$365 \times 24 \times 60 \times 60/\overline{T} = 2,628,000$$

and the number of individual waves related to the extreme storm is

$$\lambda' = 2,628,000 \times P = 6750$$

The return period of individual wave height is calculated by eq (6) and shown in Fig. 7.

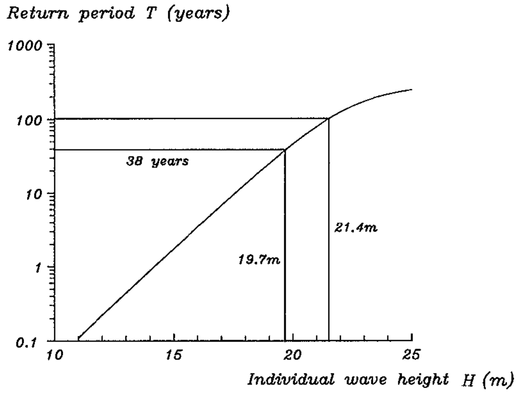

Fig. 7. Return period of individual wave height.

It can be seen from Fig. 8 that the return period of $(H_{max})_{mean} = 19.7\ m$ is 38 years. By Fig. 8 we can also choose a design individual wave height corresponding to a certain return period, e.g. $H^{100\ years} = 21.4\ m$.

Fig. 8 gives the encounter probability of individual wave height within a structure lifetime of 25 years.

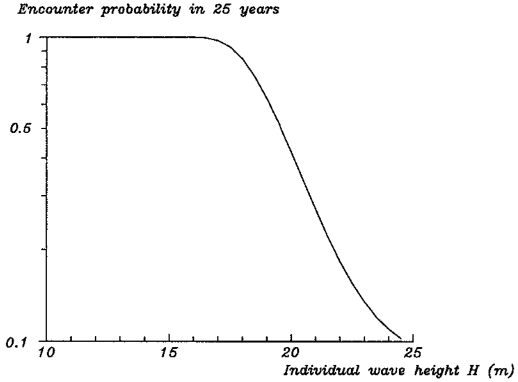

Fig. 8. Encounter probability of individual wave height.

7. Conclusions

- A method for estimation of return period and encounter probability of individual wave height has been developed. The method is based on extremely limited wave information (extreme data set). Improvement of the estimate can be expected if more information, e.g. the storm duration, is available.

- The method can also be applied for estimation of the number of relatively large waves for fatigue analysis of constructions.

8. Acknowledgement

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