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Model predictive control of heating in a low energy single-family house

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Abstract.
This paper presents a control scheme that uses a Model Predictive Control (MPC) approach to manage the heating system of a low-energy single-family house. The house is equipped with an air-to-water Heat Pump (HP) and individually controlled hydronic underfloor heating circuits for each of its 11 heating zones. The MPC scheme is designed to maintain individual room comfort levels in each zone, while incorporating weather forecasts and following a heating reference to allow for load shifting for periods with low energy prices and high Photovoltaic (PV) production calculated by an upper level in a hierarchical control scheme. The focus of the design has been the model structure that allows for fast solutions to the MPC optimisation problem, while still capturing the high complexity, non-linear dynamics of the building. The controller is tested on a high-fidelity simulation model of the house, achieving disturbance rejection and system stabilisation. The rapid solving time makes repeated experiments and longer simulations feasible.

1 Introduction

In the European Union, the residential sector accounted for 28% of energy consumption in 2020 [1], with 63.6% used for heating [2]. Reducing this consumption and using renewable sources is essential. However, the electrical grids are increasingly being strained by the use of HPs, power production from intermittent renewable energy sources, and more. To incentivise off-peak demand, grid operators vary tariffs during the day [3]. Varying energy tariffs create cost challenges for HPs, but also opportunities for savings, e.g., by using a price-aware controller to shift loads and store or discharge heat based on low or high-cost periods [4, 5, 6]. MPC has been studied extensively as one way of utilising the variable electricity price to reduce the cost of residential heating by predicting the future behaviour of the building and penalising the consumption of power based on its price [7, 8]. However, challenges for MPC schemes for heat management remain. Modelling may be difficult due to the complex non-linear dynamics and result in models unsuitable for real-time optimisation in the MPC control laws due to their slow computation times [8].

This paper concerns the control of a single family nearly zero-emission building. It uses an electrical air-to-water HP connected to hydronic underfloor heating, and a PV system to generate electricity. The house has 11 rooms divided on 2 floors and individual floor heating circuits for each room; each flow-controlled by an ON/OFF valve.

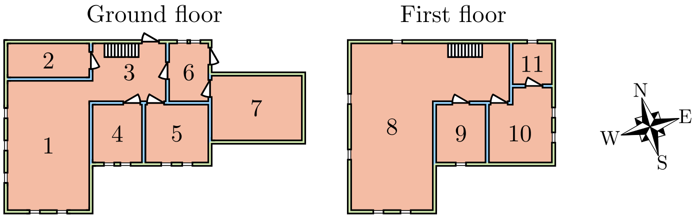

Figure 1. Approximate floor plan of the house. Total heated gross area: 230 m^2.

The house in question has previously been used as a testbed for design of control laws for residential heating as part of OPSYS 2.0 [9, 10]. As part of this, a high-fidelity, high-complexity digital twin of the house has been created in Dymola [11]. This model consists of more than 22500 equations, and is thus unsuitable for direct use in a control scheme. Previous work done in [10] has already created an MPC that achieves energy flexibility by, calculating a reference for the heat consumption for the entire house based on dynamics, prices etc., but uses a simple lower control level for distributing the energy to the different zones. This paper concerns the development of an alternative lower control level consisting of a MPC, which achieves reference tracking for room temperatures in all 11 zones by selecting optimal valve positions and a central forward temperature from the HP, while tracking the heating reference from the upper level and thus achieving load shifting. The MPC formulation also allows direct usage of forecasts in determining the optimal inputs.

High system complexity, non-linear dynamics such as bilinear terms between temperatures and mass flows, and many binary inputs in the form of the 11 valves can result in a model structure that would make timely solutions of the MPC optimisation infeasible [8]. Thus, the focus of this paper is the design of a highly computationally efficient optimisation problem in MPC formulation.

2 Method

Table 1 shows a description of the variables used throughout this section.

Table 1. Variable overview.

Variable	Description		Variable	Description	
m	Mass	kg	s	Solar scaling factor	$\mathbb{R}$
T	Temperature	K or °C	c	Specific heat capacity	J kg K^{-1}
T_{out}	HP outflow temperature	K or °C	P	Power	W
T_{in}	HP inflow temperature	K or °C	Q	Heat	J
T_{f}	Temperature of a floor	K or °C	I_{s}	Solar intensity	$\mathbb{R}$
T_{r}	Temperature of a room	K or °C	V	Cost function	$\mathbb{R}$
T_{a}	Ambient temperature	K or °C	Q, R, S	Weights	$\mathbb{R}$
a, b	Heat flow coefficients	$\mathbb{R}$	H_{p}	Prediction horizon	$\mathbb{N}$
P_{h}	Heat supplied by the HP	W	γ	Slack variables	$\mathbb{R}$
q_{nom}	Nominal mass flow	kg s^{-1}	q	Mass flow	kg s^{-1}
v	Valve position	$\{0, 1\}$	P_{PV}	Power from PV	W
p	Electricity price	DKK/Wh	i	Room index	$\{1, 2, \ldots, 11\}$

2.1 Modelling

For modelling of the dynamics of the house, data generated from the high-fidelity Dymola digital twin has been used for parameter estimation, which uses additional measurements not normally present in the real world system, notably the heat from each hydronic underfloor circuit P_i. For a full description of the modelling process see [12, p. 15-57]. The model use the Mixed Logical Dynamical (MLD) model format [13], resulting in efficiently solvable Mixed Integer Quadratic Programming (MIQP) problems for the control law.

The model of the house's thermal dynamics, from the heat flow from the underfloor heating to the rooms and walls of the house are modelled using a Resistor-Capacitor (RC) model as seen in many previous models from other studies [14, 15]. The model has three components as shown in Figure 2.

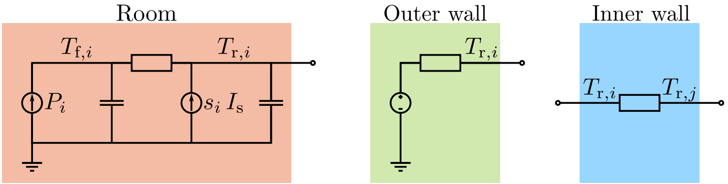

Figure 2. Electrical equivalent diagrams for the three components with states and inputs. Component colours match that of Figure 1.

The different components are connected based on the physical connections of the different rooms in the house, which results in a linear dynamic model after zero-order hold discretization with a sample time of 15 minutes. This RC model is augmented with two models for the heat in the building, both based on:

$$Q = m\, c\, \Delta T \quad \Rightarrow \quad P = q\, c\, \Delta T \tag{1}$$

Two main assumptions are made in this section. First, assuming the mass flows in the heating circuits are independent of the number of open circuits and that the return flow temperature $T_{\mathrm{in},i}$ is proportional to the corresponding floor temperature $T_{\mathrm{f},i}$, we approximate P_i as:

$$P_i = c\, q_i\, b_i\, (T_\mathrm{out} - T_{\mathrm{f},i}) \tag{2}$$

Secondly, the heat output of the HP, P_h, is approximated using equation (1) to allow for tracking the reference from the upper level to achieve load shifting and thus energy flexibility:

$$P_\mathrm{h} = \sum_i q_i\, c\, (T_\mathrm{out} - T_\mathrm{in}) = c \sum_i q_i\, T_\mathrm{out} - c \sum_i q_i\, T_\mathrm{in} \tag{3}$$

With the same assumption concerning $T_{\mathrm{in},i}$ and $T_{\mathrm{f},i}$, we model the total return flow temperature T_in as a flow-weighted average of the individual $T_{\mathrm{in},i}$s:

$$T_\mathrm{in} = \frac{1}{\sum_i q_i} \sum_i q_i T_{\mathrm{in},i} = \frac{1}{\sum_i q_i} \sum_i a_i\, q_i\, T_{\mathrm{f},i} \quad \Rightarrow \quad P_\mathrm{h} = c \sum_i q_i\, T_\mathrm{out} - c \sum_i a_i\, q_i\, T_{\mathrm{f},i} \tag{4}$$

Next, notice that equation (2) and (4) are linear except for the bilinear terms: $q_i\, T_\mathrm{out}$ and $q_i\, T_{\mathrm{f},i}$. However, the bilinear terms can be converted into a more desirable form by substituting $q_i = q_{\mathrm{nom},i}\, v_i$. This results in the terms now consisting of real-valued inputs multiplied with binary-valued inputs and real-valued states multiplied with binary-valued inputs. Thus, the lower level's model can be written as a MLD model:

$$\begin{aligned} x_{k+1} &= A\, x_k + E\, d + D_1\, z_k, \qquad & x &= \begin{bmatrix} T_{\mathrm{f},i} & T_{\mathrm{r},i} \end{bmatrix}^\mathrm{T} \in \mathbb{R}^{22}, \quad y = \begin{bmatrix} T_{\mathrm{r},i} & P_\mathrm{h} \end{bmatrix}^\mathrm{T} \in \mathbb{R}^{11} \\ y_k &= C\, x_k + D_2\, z_k, & d &= \begin{bmatrix} I_s & T_\mathrm{a} \end{bmatrix}^\mathrm{T} \in \mathbb{R}^{2} \end{aligned} \tag{5}$$

with auxiliary variables defined as:

$$z = \left[\begin{array}{ccc|ccc} T_{\text{f},1}\, v_1 & \cdots & T_{\text{f},11}\, v_{11} & T_{\text{out}}\, v_1 & \cdots & T_{\text{out}}\, v_{11} \end{array}\right]^{\text{T}} = \left[\begin{array}{c|c} z_{\text{f},i} & z_{\text{out},i} \end{array}\right]^{\text{T}} \in \mathbb{R}^{22} \tag{6}$$

2.2 Control overview

The upper layer achieves energy flexibility by, calculating a reference for the heat consumption for the entire house based on dynamics, prices etc. The lower layer attempts to follow that heating reference while maintaining comfort levels, see Figure 3.

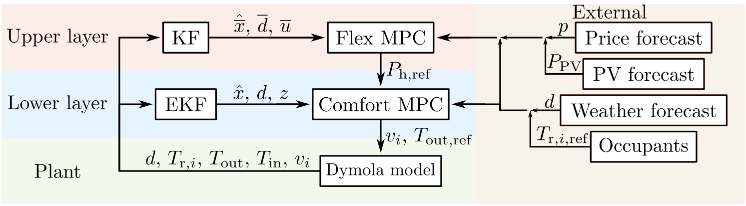

Figure 3. Simplified block diagram of how the different control modules are connected with each other, forecast data, and the Dymola model. Upper layer: T_{s}: 1 h, H_{p}: 24 h. Lower layer: T_{s}: 15 m, H_{p}: 1 h. A Kalman Filter (KF) and an Extended Kalman Filter (EKF) are used for estimating the unmeasured floor temperatures [12, p. 63-68].

2.3 Lower control level

The lower control level has the following cost function:

$$V_k = \sum_{\iota=1}^{H_{\text{p}}} \left\| y_{k+\iota} - y_{\text{ref},k+\iota} \right\|_{Q_\iota}^2 + \sum_{\iota=1}^{H_{\text{p}}} \left\| T_{\text{out},k+\iota} \right\|_{R_\iota}^2 + S\, \|\gamma\|_1 \tag{7}$$

The first term is for reference tracking for $T_{\text{r},i}$ and P_{h}. References for $T_{\text{r},i}$ come from the inhabitants. The reference for P_{h} is calculated by the upper level. The second is to reduce T_{out}, which increases the efficiency of the HP. The third penalises violation of softened terminal constraints to enforce stability. To handle the auxiliary variables from equation (6), a series of linear constraints are introduced as per [16]. Thus, e.g. the relation $z_{\text{f},1} = T_{\text{f},1}\, v_1$ is encoded as four constraints using the product rule from [16]. After lifting with a prediction horizon of 1 hour the control law results in a MIQP problem with a convex cost function, and mixed integer linear inequality constraints. This guarantees a globally optimal solution to the control problem. To limit computational complexity, the optimiser only searches for one set of optimal valve positions for the prediction horizon, that is $v_{i,k+1} = v_{i,k+2} = v_{i,k+3} = v_{i,k+4}$ in the optimisation problem.

2.4 Upper control level

The upper control level follows [10]. The same RC model structure is used as for the multi-zone model. Only one room connected to an outer wall is used with a sample time of 1 h and a quadratic HP model is used. Note, the controller only allows buying electricity from the grid and not selling.

3 Results

To test the control scheme, a 3-month simulation has been run, using the Dymola model as the plant, using weather, PV production, and electricity price data from winter. The control scheme is implemented in MATLAB using YALMIP [17] to interface to Gurobi's MIQP solver. All simulations have been run on a 6 core Intel i7-10750H running at an average 4.5 GHz during the test. The 3-month simulation takes approximately 2 hours to run, with 14.4% of the calculation time being spent on the lower level, 16.2% on the upper level, 0.02% spent on Kalman filters and saving data, and the remainder on running the Dymola model. Each layer of the control scheme takes an average 0.13 seconds to update. Figure 4 shows relevant plots for the lower control layer.

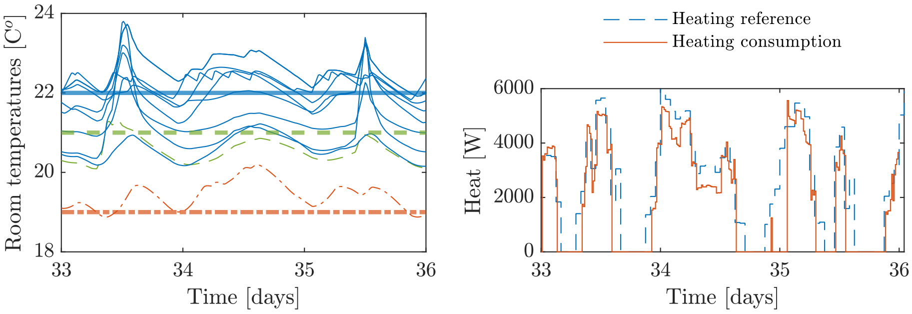

Figure 4. Left: Room temperatures and references for 3 days of the simulation. Line style and colour match the rooms respective references but with a thinner line. Right: Heating reference as calculated by the upper layer, and the consumption.

Note, the stable response, good P_h reference tracking, and the controller reducing T_r in anticipation of increased solar heating on midday on the 33rd and 35th day. Figure 5 shows relevant plots for the upper control layer.

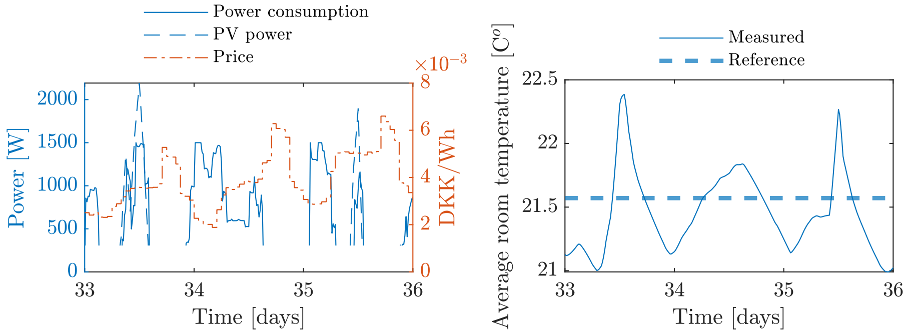

Figure 5. Left: Power and price. Right: Average room temperature reference tracking. Notice here, how the control scheme avoids using electric power in expensive periods and utilises the free PV power when available.

4 Discussion

The presented MLD model structure enables the implementation of a fast-computing MPC using MIQP formulations, while simultaneously enabling reference tracking for individual rooms, and accounting for the dynamics of the house, weather, and following a reference for the heating of the house calculated by the upper level in the control scheme. The model structure also allows estimation of the return temperature using the floor temperature states, which in future work could be beneficial in the upper level of the control scheme as it influences the efficiency of the HP.

The control scheme achieves fast computation times on a consumer laptop, making it feasible to implement the control algorithm on real buildings, perform longer efficiency studies, and study the effects of electric and thermal storage. The MPCs used in the study had poorly tuned parameters and relied on perfect forecasts and HP models. Therefore, rigorous testing and implementation on a real system are necessary to determine the model's potential. However, challenges with the implementation of these control laws on real building automation systems will have to be addressed, as it requires addressing issues such as difficult interfacing with HPs, parameter estimation on real life data, and ensuring forecast accuracy [10]. Further research is needed to assess the effectiveness and potential of the model structure in real-world scenarios.

5 Conclusion

This study presents a hierarchical MPC scheme for the management of the heating system in low-energy single-family houses developed using a digital twin. The lower layer achieves individual temperature reference tracking in rooms and is able to follow a heating reference output from the upper layer. The MLD formulation of the proposed model combined with the low amount of states results in a MIQP, which achieves fast and efficient solve times for the controller. However, parameter estimation and testing on real buildings are still necessary to evaluate the real performance of the control structure.

Credits author statement
Nielsen, **Larsen:** Conceptualization, Methology, Software, Writing, Formal analysis. **Thorsteinsson**, **Bendtsen:** Conceptualization, Review & Editing, Supervision.

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