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## **The Devil Dwells in the Tails**

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## The devil dwells in the tails

### A quantile regression approach to firm growth

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Bernd Ebersberger · Morten B. Jensen

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**Abstract** This paper explores firm growth rate distribution in a Gibrat's Law context. It is novel in two respects. First, rather than limiting the analysis to a focus on the conditional mean, we investigate the entire shape of the distribution. Second, we show that differences in the firm growth rate process between large and small firms are highly circumstantial and depend on the industry dynamics. The data used include more than 9,000 Danish manufacturing, services and construction firms. We provide robust evidence indicating that firm growth studies should concentrate less on explaining means and instead focus on other parts of the firm growth rate distribution.

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## 1 Introduction

It has been customary to analyze firm growth and test Gibrat's Law using the OLS (ordinary least squares) estimation method. This paper argues that this method is not the most appropriate and that quantile regression is a better approach, since it enables the researcher to consider the entire distribution of firm growth patterns, whereas OLS considers only the distribution mean. Recent studies, reviewed below, find evidence that firm growth rate distributions tend to exhibit fat tails, which are not picked up by traditional OLS analysis. In this paper, we study firm growth using quantile regression and find that this approach provides new and valuable insights into the dynamics of firms growth.

Using data on more than 9,000 Danish firms, we look for specific industry characteristics that influence the dependencies of the firm growth process. Explaining firm growth using a quantile regression approach with estimates from every 5<sup>th</sup> quantile, we investigate the full shape of the firm growth rate distribution and its dependence on industry scale and firm specific effects. Interacting firm size with industry scale effects, we estimate differences in the firm growth rate distribution between large and small firms taking account of the industry dynamics faced by these firm. In other words, this study tries to say something about the central moments of the firm growth rate distribution and its dependencies, and also to reveal the devil dwelling in its tails.

Gibrat's Law (1931) was the first formal model of the dynamics of firm size. Gibrat based his model on empirical data that suggested that increments to firms size growth were proportional to their current size. Although there are several studies that support Gibrat's Law (see e.g. Hart and Prais (1956) and Simon and Bonini (1958)), numerous others have questioned it. First, Hymer and Pashigian (1962) and Mansfield (1962) found that the level of variance in growth rates is negatively correlated with firm size. Second, many regression studies have suggested that firm growth is negatively correlated with firm size (see e.g. Evans (1987a, b), Hall (1987), Dunne et al. (1989), Dunne and Hughes (1994), Hart and Oulton (1996) and Reichstein and Dahl (2004)).<sup>1</sup>

Stanley et al. (1996) provide a different approach to investigating Gibrat's law, proposing that the shape of the empirical growth rate distribution is peaked and has fat tails resembling an exponential (Laplace) distribution

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<sup>1</sup>However, this empirical finding has been attributed to sample attrition/selection bias. Exits are not included in these studies, and the sample includes predominantly small firms, producing a bias in the size variable in favor of small firms. Harhoff et al. (1998), however, indicated that the negative correlation persists even when controlling for sample attrition. For reviews of Gibrat's Law, see e.g. Sutton (1997) and Lotti et al. (2003).

rather than the Gaussian distribution assumed by Gibrat's Law. They proposed a revised Gibrat model in which the growth rate of firms depends not only on current size, but also on previous size, leading to an exponential-like growth rate distribution. Similar empirical patterns were found by Bottazzi et al. (2001, 2002) who highlighted a tent-shaped pattern of growth rate distributions. Using a simulation approach, Bottazzi and Secchi (2003) were able to reproduce these patterns by revising an Ijiri and Simon (1977) type of model. Two mechanisms reproduce the empirical pattern: cumulative and self-reinforcing mechanisms in the way firms search for new solutions to opportunities as argued by Arthur (1994); and the presence of firm specific capabilities discussed by Penrose (1959), Barney (1991, 2001), Foss (1997), Dosi et al. (2000), Eisenhardt and Martin (2000) among others. The studies by Stanley et al. and Bottazzi and colleagues investigate growth rate distributions assuming they are symmetric; however, using the same data source as our study relies on, Reichstein and Jensen (2005) found that growth rate distributions may exhibit significant skewness.

This latter approach to firm growth argues that Gaussian statistics are unfit for studying firm growth, a statement endorsed by McKelvey and Andriani (2005), who argue that *managers live in the world of extremes; researchers using statistics report findings about averages* (McKelvey and Andriani 2005, pp. 224–225). Using Gaussian statistics, such as regression analysis, is misleading and does not uncover details that are of particular importance. Instead of limiting our analysis to a central moment of a given distribution we should look at the full shape of the distribution. This is in line with Koenker and Bassett's (1978) argument that Gaussian errors are inappropriate in many situations. Linear quantile regression is much more appropriate in this context.

This paper uses quantile regression to explain firm growth. To the best of our knowledge very few previous contributions have adopted this method to study firm growth, for instance, the study by Lotti et al. (2003). Among other things, Lotti et al. found that small firms grow faster than large firms in specific industries, and that this pattern is consistent across the 10<sup>th</sup>, 25<sup>th</sup>, 50<sup>th</sup>, 75<sup>th</sup> and 90<sup>th</sup> quantiles. Only a few working papers, e.g. Coad and Rao (2006) and Coad (2006), have studied this phenomenon using this method.

Despite the numerous publications on Gibrat's Law and the relationship between firm growth and firm size, few studies have attempted to empirically explore whether the correlation between the two is circumstantial—that industry specific circumstances dictate differences in the growth processes of small and large firms. Reichstein and Dahl (2004) argued that observed heteroskedasticity from an OLS regression of firm growth against firm size, to some extent might be explained by the different effects of firm size on firm growth, across industry borders. Studies on the shape of the firm growth rate distribution have been carried out at industry level, and argue for differences across industries. However, these studies fail, statistically, to explain how and to what extent the firm growth rate process differs across industry borders.

This paper is organized as follows. Section 2 describes the model used to study firm growth rate distributions and how industry scale and firm specific

effects shape different parts of the distribution. Section 3 briefly presents the data and discusses the quantile regression approach. The results of the quantile regressions are reported in Section 4. Section 5 summarizes the results.

## 2 The model

Our analysis is based on the model developed by Davies and Geroski (1997). It investigates the determinants of changes in market shares. Davies and Geroski draw heavily on the Gibrat's Law literature, but augment the model by including both industry scale and firm specific effects. Additionally, their model includes a number of interaction effects between firm size and industry level variables. This provides the opportunity to distinguish between a common effect of firm size on firm performance and an effect that is circumstantial with reference to the industry dynamics.

The model tested in the present paper can be represented by the following equation:

$$FGR_{ij} = \vec{\alpha}_j + \vec{\lambda}_j LFS_{ij} + \vec{\beta} \vec{x}_{ij} + \varepsilon_{ij} \quad (1)$$

where  $FGR_{ij}$  and  $LFS_{ij}$  are the growth rate, and the logarithm of the size of firm  $i$  operating in industry  $j$ , respectively.  $\varepsilon_{ij}$  is the traditional independently identically distributed error term with zero mean and  $\sigma_{ij}$  variance.  $\vec{\alpha}_j$  and  $\vec{\lambda}_j$  are vectors of industry scale variables, and  $\vec{\beta}$  is a vector of the parameter estimates attached to a vector of the firm specific variables,  $\vec{x}_{ij}$ . Specifically the vectors can be represented by:

$$\vec{\alpha}_j = \alpha(RSG_j, HFD_j, ISB_j, MES_j, GRS_j, \vec{\psi}_j) \quad (2)$$

$$\vec{\lambda}_j = \lambda(RSG_j, HFD_j, ISB_j, MES_j, GRS_j) \quad (3)$$

$$\vec{x}_{ij} = (LFS_{ij}, LAE_{ij}) \quad (4)$$

The firm level vector,  $\vec{x}_{ij}$ , holds the logarithm of the firm size variable. However, it also contains a variable measuring the logarithm of firm age. This particular variable has been used in much of the firm growth literature (see e.g. Evans 1987a, b; Dunne and Hughes 1994 and Jovanovic 1982).

The industry scale vectors, ( $\vec{\alpha}_j$  and  $\vec{\lambda}_j$ ), contain four common variables. First, we include a measure of regional specialization growth,  $RSG_j$ . This variable accounts for firm growth attributable to the dynamics of the local region. Firms located in regions where there is a high demand for final goods may exhibit significantly higher performance than other firms (Krugman 1991). Second, the Herfindahl index ( $HFD_j$ ) is included to control for industry concentration. Schumpeter (1942) argued that a high level of concentration would produce profits leading to a higher level of innovative activity and research and development (R&D) expenditure. Third we include a measure for industry turbulence using the Hymer and Pashigian approach based on changes in market shares ( $ISB_j$ ). Fourth, we include a measure of minimum efficient scale

**Table 1** Variable descriptions

Abbreviation	Description
$FGR_{ij}$	Firm sales growth 1994–1996 ( $\log(FS_{ijt}) - \log(FS_{ijt-1})$ )
$LFS_{ij}$	Logarithm of firm size in terms of 1994 sales in thousands of Danish Kroner ( $\log(FS_{ij})$ )
$LAE_{ij}$	Logarithm of firm age ( $\log(1994 - \text{establishment year})$ )
$RSG_j$	Regional specialization growth index calculated by the growth in the revealed comparative advantage index ( $RCA_j$ ) Balassa (1965) from 1994 to 1996.
$HFD_j$	The Herfindahl concentration index calculated by the sum of the squared share of sales across the industry. $\left( \sum_{i=1}^n \left[ \frac{FS_{ijt}}{\sum_{i=1}^n FS_{ijt}} \right]^2 \right)$ .
$ISB_j$	The instability index is measured using the Hymer and Pashigian approach summing the absolute changes in market shares by the three digit industry codes $\left( \sum_{i=1}^n \left  \frac{FS_{ijt}}{\sum_{i=1}^n FS_{ijt}} - \frac{FS_{ijt-1}}{\sum_{i=1}^n FS_{ijt-1}} \right  \right)$ .
$MES_j$	Industry minimum efficient scale of production measured by medium sized firms in the industry, based on employment statistics.
$GRS_j$	Growth of the industry measured by the differences in the logarithms of industry sales for 1994 to 1996, using a three digit level of aggregation.

( $MES_j$ ) of the industry. This controls for entry barriers in the industry, but also measures the extent to which incumbents can disregard external competitive pressures. A measure of the general growth of the industry ( $GRS_j$ ) is included to account for differences in growth trends across industry borders. We also control for firm age using the logarithm of firm age ( $LAE_{ij}$ ).

Unlike Davies and Geroski, we do not interact all industry scale effect variables with firm size, so the two industry scale vectors (Eqs. 2 & 3) are not identical.  $\vec{\alpha}_j$  differs from  $\vec{\lambda}_j$  in containing a vector,  $\vec{\psi}_j$ , with nine industry dummies. They control for potential variations in firm growth rates attributable to industry differences, which the other industry variables do not capture. We assume the effect of firm size to be similar across industries once we account for the moderating effect of size caused by market concentration, entry barriers, growth and turbulence.  $\vec{\psi}_j$  is therefore not included in  $\vec{\lambda}_j$ . Table 1 contains a more detailed description of the variables included in the model.

### 3 Data and method

#### 3.1 The data

The data for the analysis are drawn from the NewBiz database published by Dansk Markeds Information A/S. The database contains all Danish limited liability companies, partnerships and limited partnerships and holds information on e.g. number of employees, industry classification, year of birth, geographical location and various financial variables. It contains information from 1993–1997 and is updated quarterly. Using 1993 and 1997 data is problematic as it leads to a substantial loss of observations, because the financial variables are imperfect in the first and last years. Consequently, we rely on 1994 and

**Table 2** Descriptive statistics on non-interactive terms in the model (N=9105)

Variable	1 <sup>st</sup> qrtl.	Mean	Median	3 <sup>rd</sup> qrtl.	S.D.
<i>FGR<sub>ij</sub></i>	-0.086	0.052	0.080	0.262	0.525
<i>LFS<sub>ij</sub></i>	7.510	8.904	8.500	10.210	1.978
<i>LAE<sub>ij</sub></i>	2.200	2.611	2.710	3.140	0.508
<i>RSG<sub>j</sub></i>	-0.057	0.004	-0.005	0.065	0.112
<i>HFD<sub>j</sub></i>	0.003	0.028	0.008	0.023	0.062
<i>ISB<sub>j</sub></i>	0.048	0.105	0.097	0.161	0.072
<i>MES<sub>j</sub></i>	4.000	11.690	6.000	9.000	19.450
<i>GRS<sub>j</sub></i>	0.107	0.129	0.138	0.166	0.062
<i>Industry dummies</i>					
Supplier dominated	0.000	0.087	0.000	0.000	0.281
Scale intensive	0.000	0.074	0.000	0.000	0.262
Specialized suppliers	0.000	0.040	0.000	0.000	0.195
Science based	0.000	0.025	0.000	0.000	0.157
Construction	0.000	0.149	0.000	0.000	0.356
Wholesale trade	0.000	0.149	0.000	0.000	0.356
Specialized services	0.000	0.225	0.000	0.000	0.418
Scale intensive services	0.000	0.039	0.000	0.000	0.193
ICT intensive services	0.000	0.182	0.000	0.000	0.386

1996 data for our analysis. Table 2 summarizes the variables included in the model and Table 3 presents the correlation matrix with the industry dummies excluded.

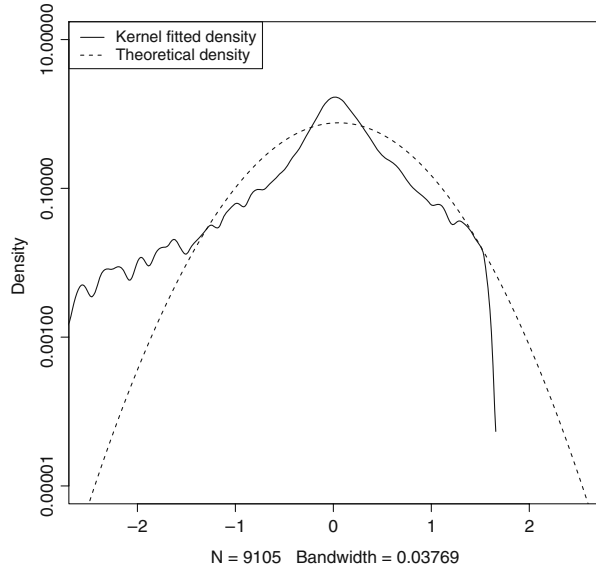
Figure 1 depicts the distribution of the normalized dependent variable ( $FGR_{ij}$ ) using the normal kernel function to estimate the true shape and the distribution. Silverman's plug-in estimate is used to set the bin of the estimation. Also, the associated Gaussian distribution with mean zero and standard deviation values of the data are added as a reference curve. This reveals that the empirical distribution is more peaked than the Gaussian shape often assumed. This suggests that the distribution will have fatter/heavier tails giving support to recent studies on the shape of the firm growth rate distribution (see e.g. Stanley et al. 1996; Bottazzi et al. 2001, 2002 and Reichstein and Jensen 2005). In these data it seems only to be the case in the lower tail.

**Table 3** Correlation matrix of continuous variables (N=9105)

	<i>FGR<sub>ij</sub></i>	<i>LFS<sub>ij</sub></i>	<i>LAE<sub>ij</sub></i>	<i>RSG<sub>j</sub></i>	<i>HFD<sub>j</sub></i>	<i>ISB<sub>j</sub></i>	<i>MES<sub>j</sub></i>
<i>FGR<sub>ij</sub></i>							
<i>LFS<sub>ij</sub></i>	-0.008						
<i>LAE<sub>ij</sub></i>	<b>-0.024</b>	<b>0.201</b>					
<i>RSG<sub>j</sub></i>	<b>0.047</b>	-0.004	<b>-0.055</b>				
<i>HFD<sub>j</sub></i>	-0.001	<b>0.019</b>	-0.016	0.015			
<i>ISB<sub>j</sub></i>	0.013	<b>-0.036</b>	-0.016	<b>0.018</b>	<b>0.324</b>		
<i>MES<sub>j</sub></i>	<b>0.028</b>	<b>0.306</b>	<b>0.089</b>	<b>-0.019</b>	<b>0.096</b>	<b>-0.125</b>	
<i>GRS<sub>j</sub></i>	<b>0.027</b>	<b>-0.078</b>	<b>-0.081</b>	<b>0.198</b>	<b>-0.219</b>	<b>-0.069</b>	<b>-0.075</b>

Note: figures in bold are significant at the 5% level

**Fig. 1** Kernel density plot of the empirical firm growth rate distribution and associated Gaussian distribution (log-scale)



### 3.2 Method

The shape of the firm growth rate distribution calls for a different approach than the traditional OLS regression method. OLS assumes the dependent variable to be Gaussian distributed. Quantile regression represents an alternative analytical tool which does not assume any particular distributional form of the dependent variable. Compared to OLS regression, quantile regression provides a more complete story of the relationship between variables, because it does not restrict itself to analysis of the mean (Koenker 2005). We apply the linear quantile regression method introduced by Koenker and Basset (1978) to investigate the factors influencing firm growth rates. This approach has two major advantages. First, it reveals differences in the relationships between the dependent and independent variables at different points in the conditional distribution of the dependent variable. Rather than focusing on a specific moment in the distribution, linear quantile regression provides a method of analysis suited to studying all defined values of the dependent variable. It hence enables us to investigate the dependencies of the tails, as well as the central values of a given dependent variable distribution.

Second, the quantile regression coefficient estimates are more robust than the least squares regressions where the mean value of the dependent variable is predicted. This is especially true in the presence of outliers and for distributions of error terms that deviate from normality (see Buchinsky 1998; Koenker and Hallock 2001). These are important when studying a dependent variable that is not Gaussian.

Koenker and Basset (1978) suggest studying either how one specific quantile of particular interest is linearly correlated with a set of explanatory variables,



or studying how the linear correlation changes across a number of quantiles. The latter of these approaches should provide an understanding of the entire shape of the distribution and how it may be influenced by the explanatory variables.

Consider the linear regression model  $y_i = \vec{x}_i \vec{\beta} + \epsilon_i$  for  $i = 1, \dots, n$  where  $\vec{x}_i$  and  $\vec{\beta}$  are  $k$  vectors of explanatory variables and their estimated coefficients.  $y_i$  and  $\epsilon_i$  are the dependent variable and the iid distributed error term, respectively. The OLS estimator is found by minimizing the sum of the squared residuals:

$$\min_{\mu \in R^k} \sum_{i=1}^n (y_i - \vec{x}_i \vec{\beta})^2 \quad (5)$$

The quantile regression estimator on the other hand is the vector  $\vec{\beta}$  that minimizes:

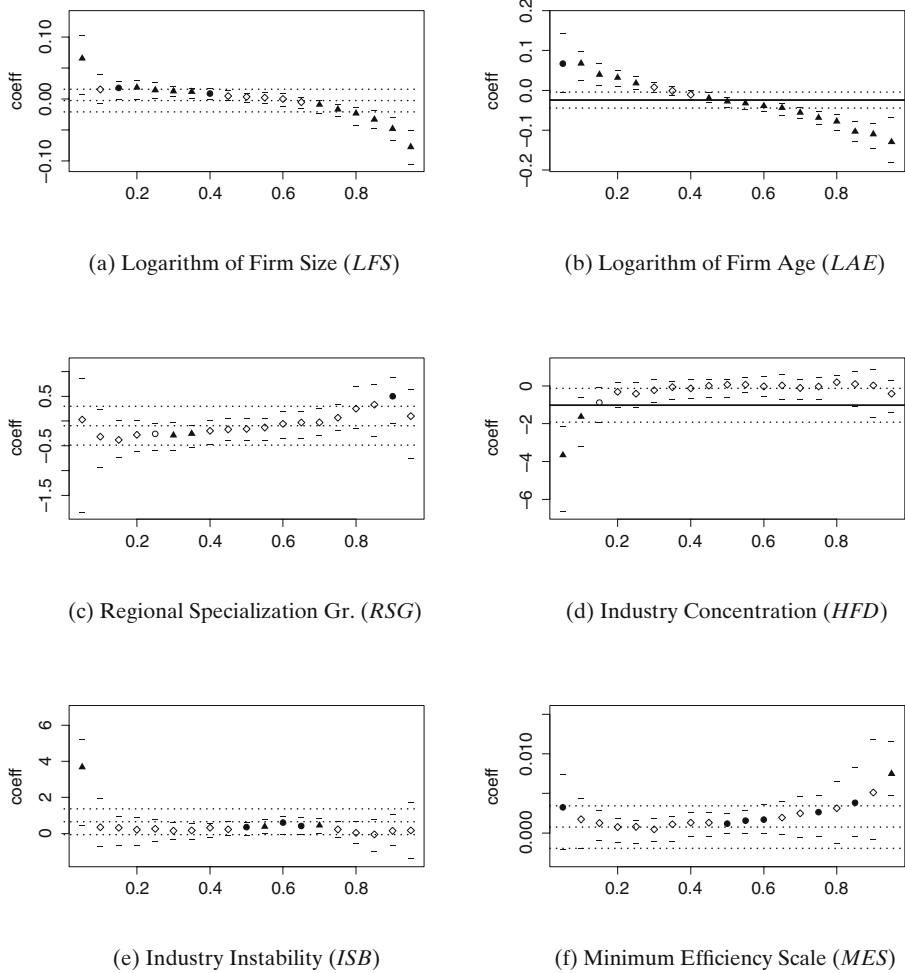
$$\min_{\vec{\beta} \in R^k} \left[ \sum_{i \in \{i: y_i \geq \vec{x}_i \vec{\beta}\}} \tau |y_i - \vec{x}_i \vec{\beta}| + \sum_{i \in \{i: y_i < \vec{x}_i \vec{\beta}\}} (1 - \tau) |y_i - \vec{x}_i \vec{\beta}| \right] \quad (6)$$

$\tau$  is the quantile defined as  $Q_{Y|X}(\tau|x) = \inf\{y : F_{X|Y}(y|x) \geq \tau\}$  in which  $\tau$  is bounded between zero and one, and  $y$  is a random sample from a random variable,  $Y$ , which have the distribution function  $F(F(y) = P(Y \leq y))$ .

Equation 6 is the objective function and represents an asymmetric linear loss function. For  $\tau = 0.5$ , however, it becomes the absolute loss function determining the median regression. One of the strengths of the quantile approach is that  $\tau$  may vary within its bounded interval ( $0 < \tau < 1$ ) representing different quantiles. Doing so reveals the conditional distribution of  $y$  given  $\vec{x}$ . The coefficient estimate for the exogenous variable is interpreted in much the same fashion as the OLS regression coefficients. The quantile coefficients may be interpreted as the marginal change in the dependent variable due to a marginal change in the exogenous variable, conditional on being in the  $\tau$ -th quantile of the distribution. Changing estimated coefficients with varying quantiles is indicative of heteroskedasticity issues (Koenker 2005). We normalized the continuous variables to avoid any bias attributed to multicollinearity between the regressors and the interaction effects.

## 4 Results

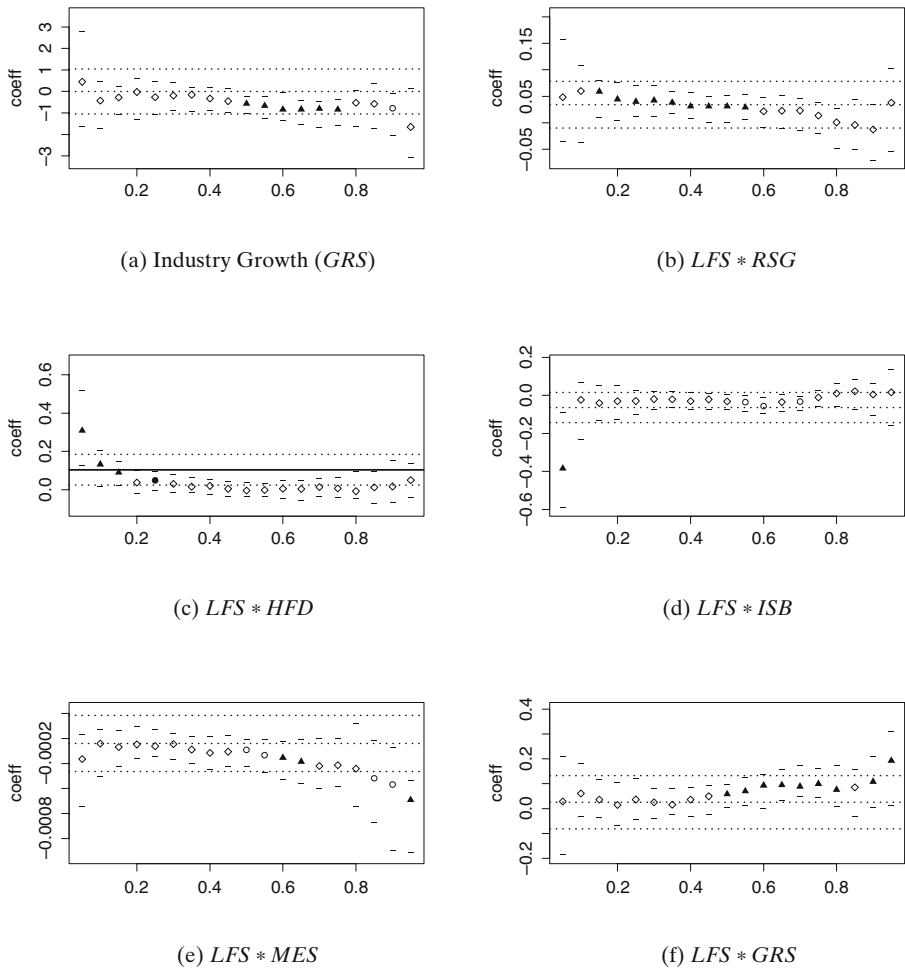
In this section, we empirically investigate how the firm growth rate distribution is shaped by firm characteristics and industry circumstances. In particular, we explore to what extent the effect of firm size is dictated by the dynamics of the industry. The results of the quantile regressions are represented by the Figs. 2a through 3f. The horizontal axis of the diagrams represents the quantiles. The vertical axis represents the estimated coefficients. We estimate quantile regressions for every 5<sup>th</sup> quantile starting with the 5<sup>th</sup> and ending with the 95<sup>th</sup>.



**Fig. 2** Quantile regression results (a–f)

This amounts to 19 quantile regressions and 19 quantile regression coefficients for each of the explanatory variables. These are represented graphically by the three different indicators depending on the level of significance. Hollow circles indicate insignificant estimates. Solid circles are significant at the 10% level and a solid triangular shape indicates significance at the 5% level. Whiskers on each side of the dots indicate the upper and lower 10% confidence interval.

We also estimated the corresponding OLS regression represented by three horizontal lines. The middle line represents the estimated coefficient while the dotted lines on either side represent the confidence interval at the 10% level. A solid line represents a significant estimate while a dotted line represents an insignificant OLS coefficient at the 10% level. The significant OLS coefficients,



**Fig. 3** Quantile regression results (continued) (a–f)

therefore, are those associated with the logarithm of firm age (*LAE*), industry concentration level (*HFD*), and the interaction term for the logarithm of firm size and industry concentration.<sup>2</sup>

Including both quantile and OLS regression results reveals substantial differences between the results of the two types of regressions. While the results of the OLS regression suggest that firm size is not significant in explaining firm growth, the quantile regression results suggest the reverse that firm size

<sup>2</sup>The standard errors of the OLS regression estimates have been corrected for bias in terms of heteroskedasticity. We followed Long and Erwins's (2000) recommendations using MacKinnon and White's (1985) method.

is significant in explaining both the lower and the upper quantiles of the firm growth rate distribution. Size has a positive effect on growth for shrinking firms, and a negative effect on growth for the fastest-growing firms. Thus, size appears to have a moderating effect on growth. This is consistent with the observation that variance in firms' growth rates decreases with size. Similar results were obtained for the effect of firm's age on growth. The OLS estimate for firm age is significant. But the significance is even more pronounced in the tails of the distribution suggested by the quantile regression results, indicating that age, in particular, has a limiting effect on growth in the upper quantiles, though to a lesser extent in the lower quantiles of the growth rate distribution.

There are also some interesting differences between the OLS and quantile regression results for the industry variables. The OLS results suggest that the concentration level alone is significant. The quantile regressions provide a much richer picture. With regard to concentration level, the quantile regressions indicate concentration to be negatively significant when regressing against the lower quantiles. This suggests that only the growth of shrinking firms is responsive to concentration. In contrast, fast growing firms do not seem to be responsive to concentration, which is why the OLS estimate becomes significantly negative. Firms operating in industries characterized by high minimum efficiency scale enjoy both advantages and disadvantages. There is a difference in the skewness of the firm growth rate distribution between firms operating in high and low efficiency scale industries.

Studying the results in terms of the interaction effects it is clear that the effect of firm size on firm growth is in part dictated by industry circumstances. Interacting the logarithm of firm size with the regional specialization growth variable and the industry growth variable, produces significant and positive estimates in the lower ranges from the 5<sup>th</sup> and 55<sup>th</sup> quantiles. These patterns suggest that firm growth rate distribution tends to have a less fat lower tail if the large firm is located in a region which increases its specialization in the particular industry. Large firms benefit to a greater extent from increases in specialization in a regionally bounded area. Increased specialization in a region may be considered to be a safety net for large firms that are experiencing low growth rates, providing a lower bound to their growth performance.

Similar patterns can be observed for large firms in concentrated industries. Concentration seems to put a lower bound on the growth rate of large firms. Shrinking firms grow faster if they are large players in a concentrated market.

Finally the quantile regressions suggest that firm size has a positive impact on firm growth rates when the firm is located in an industry that is experiencing high growth rates. This is particular true when regressing against the upper quantiles. All quantiles except the 90<sup>th</sup>, that are above the 45<sup>th</sup>, show significant positive estimates, see Fig. 3f, indicating that large firms operating in high growth industries tend to exhibit firm growth rate distributions with fatter upper tails. Firms are hence more likely to exhibit extreme growth rates if they are of considerable size and operates in a growing market.

## 5 Concluding remarks

The aim of this paper was to study the factors that influence the growth of firms. We applied an alternative regression method enabling a more in depth study of this phenomenon, considering the entire distribution of firm growth and not just the mean.

We found considerable differences between the OLS regression and the quantile regression results. Firm size is insignificant in the OLS regression, but the quantile regression reveals that firm size has a significant impact on firm growth for a considerable part of the distribution. Similar results were found for the interactions between firm size and growth in regional specialization, and between firm size and industry growth. In the other direction, the OLS regression shows significant impacts of industry concentration, and the interaction between industry concentration and firm size. These findings are largely rejected in the quantile regression, which show that this applies only to a few quantiles at the left hand end of the distribution. We suspect that OLS regression findings are driven by a few extreme outliers, which clearly influences the mean. Our study shows that results can literally be turned around, when a more detailed method is applied.

For future research, we recommend that studies of firm performance should consider applying the quantile regression approach, which will reveal more details in relation to the patterns, which are overlooked in conventional OLS analysis. This recommendation is endorsed by Cantner and Krüger (2007), who argue that this particular method reveals patterns that are particularly interesting for evolutionary economics.

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