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# Distributed Cooperative Control of Nonlinear and Non-identical Multi-agent Systems

Ali Bidram [a], Frank L. Lewis [a], Ali Davoudi [a], and Josep M. Guerrero [b]

[a] The University of Texas at Arlington Research Institute and Department of Electrical Engineering, The University of Texas at Arlington, TX, e-mail: ali.bidram@mavs.uta.edu, lewis@uta.edu, davoudi@uta.edu

[b] Department of Energy Technology, Aalborg University, 9220 Aalborg East, Denmark, e-mail: joz@et.aau.dk

***Abstract*— This paper exploits input-output feedback linearization technique to implement distributed cooperative control of multi-agent systems with nonlinear and non-identical dynamics. Feedback linearization transforms the synchronization problem for a nonlinear and heterogeneous multi-agent system to the synchronization problem for an identical linear multi-agent system. The controller for each agent is designed to be fully distributed, such that each agent only requires its own information and the information of its neighbors. The proposed control method is exploited to implement the secondary voltage control for electric power microgrids. The effectiveness of the proposed control is verified by simulating a microgrid test system.**

***Index Terms*— Distributed cooperative control, Input-output feedback linearization, Multi-agent systems, Microgrid.**

## I. Introduction

Multi-agent systems, inspired by the natural phenomena such as swarming in insects and flocking in birds, have received much attention due to their flexibility and computational efficiency. In these phenomena, the coordination and synchronization process necessitates that each agent exchange information with other agents according to some restricted communication protocols [1]-[3].Cooperative control schemes for synchronization of multi-agent systems are mainly categorized in to regulator synchronization problems and tracking synchronization problem. In the regulator synchronization problem, also called leaderless consensus, all agents synchronize to a common value that is not prescribed or controllable. In the tracking synchronization problem all agents synchronize to a leader node that acts as a command generator [4]-[8].

In this paper, the tracking synchronization problem for nonlinear and heterogeneous multi-agent systems is of concern. Cooperative control schemes for multi-agent systems with non-identical and nonlinear dynamics are sparse in the literature [9]-[13].This paper exploits input-output feedback linearization to transform the nonlinear heterogeneous dynamics of the agents to linear dynamics. Using feedback linearization, the synchronization problem for nonlinear and non-identical multi-agent systems is transformed to the synchronization problem for linear and identical multi-agent systems. The Lyapunov technique is adopted to derive fully distributed control protocols for each agent, such that each agent only requires its own information and the information of its agents on the communication graph.

Electric power microgrids are small-scale power systems containing distributed generators (DG). The dynamics of DGs are nonlinear and non-identical. Microgrids may get islanded form the main power grid. Once islanded, the DG voltage amplitudes start to deviate. To maintain these voltage amplitudes in stable ranges, the so-called primary control is applied. However, primary control may not return the DG voltage amplitudes to the nominal voltage. This function is provided by the secondary control, which compensates for the voltage and frequency deviations caused by the primary control [14]-[19]. The secondary voltage control of microgrids resembles the tracking synchronization of a multi-agent system with nonlinear and non-identical dynamics. The effectiveness of the proposed control scheme, hence, is verified by implementing the secondary voltage control of a typical microgrid.

## II. Preliminaries of Graph Theory

The communication network of a multi-agent cooperative system can be modeled by a directed graph (digraph). A digraph is usually expressed as $\mathcal{G}=(V_G,E_G,A_G)$ with a nonempty finite set of $N$ nodes $V_G=\{v_1,v_2,\ldots,v_N\}$, a set of edges or arcs $E_G\subset V_G\times V_G$, and the associated adjacency matrix $A_G=[a_{ij}]\in\mathbb{R}^{N\times N}$. In a microgrid, DGs are considered as the nodes of the communication digraph. The edges of the corresponding digraph of the communication network denote the communication links. In this paper, the digraph is assumed to be time invariant, i.e., $A_G$ is constant. An edge from node $j$ to node $i$ is denoted by $(v_j,v_i)$, which means that node $i$receives information from node $j$. $a_{ij}$ is the weight of edge $(v_j,v_i)$, and $a_{ij}>0$ if $(v_j,v_i)\in E_G$, otherwise $a_{ij}=0$. Node $i$ is called a neighbor of node $j$if $(v_i,v_j)\in E_G$. The set of neighbors of node $j$ is denoted as $N_j=\{i \mid (v_i,v_j)\in E_G\}$. For a digraph, if node $i$ is a neighbor of node $j$, then node $j$can get information from node $i$, but not necessarily vice versa. The in-degree matrix is defined as $D=diag\{d_i\}\in\mathbb{R}^{N\times N}$ with $d_i=\sum_{j\in N_i} a_{ij}$. The Laplacian matrix is defined as $L=D-A_G$. A direct path from node $i$ to node $j$is a sequence of edges, expressed as $\{(v_i,v_k),(v_k,v_l),\ldots,(v_m,v_j)\}$. A digraph is said to have a spanning tree, if there is a root node with a direct path from that node to every other node in the graph [3].

## III. Synchronization and Feedback Linearization

Consider $N$ nonlinear and heterogeneous systems or agents that are distributed on a communication graph $\mathcal{G}$ with the node dynamics

$$\begin{cases}\dot{x}_i = f_i(x_i)+g_i(x_i)u_i \\ y_i = h_i(x_i)\end{cases}, \tag{1}$$

where $x_i(t)\in\mathbb{R}^{n_i}$ is the state vector, $u_i(t)\in\mathbb{R}$ is the control input, and $y_i(t)\in\mathbb{R}$ is the output of$i^{\text{th}}$ node. It is assumed that $f_i(\bullet):\mathbb{R}^{n_i}\to\mathbb{R}^{n_i}$ is locally Lipschitz in $\mathbb{R}^{n_i}$ and $f_i(0)=0$. The agent state dynamics and state dimensions $n_i$ do not need to be the same. The usual assumptions are made to ensure existence of unique solutions.

In the tracking synchronization problem, it is desired to design distributed control inputs $u_i(t)$ to synchronize the output of all nodes to the output of a leader node $y_0(t)$, i.e. one requires $y_i(t)\to y_0(t), \forall i$. The leader node can be viewed as a command generator that generates the desired trajectory

$$\begin{cases}\dot{x}_0 = f_0(x_0) \\ y_0 = h_0(x_0)\end{cases}. \tag{2}$$

The functions $f_0$ and $h_0$ are assumed to be of class $C^\infty$.

*Definition 1.* For the smooth function $h(x):\mathbb{R}^n\to\mathbb{R}$ and smooth vector field $f(x):\mathbb{R}^n\to\mathbb{R}^n$, the Lie derivative is defined as $L_f h=\frac{\partial h}{\partial x}f(x)$, with $\frac{\partial h}{\partial x}$ being the Jacobian matrix [20].

A direct relationship between the dynamics of the outputs $y_i(t)$ and the control inputs $u_i(t)$ is generated by differentiating the $y_i(t)$. Differentiating the output of the $i^{\text{th}}$ agent yields

$$\dot{y}_i = L_{f_i}h_i + L_{g_i}h_i u_i. \tag{3}$$

An auxiliary control $v_i$ is defined as

$$v_i = L_{f_i}h_i + L_{g_i}h_i u_i. \tag{4}$$

If $L_{g_i}h_i$ is invertible over $\Omega$, the control input $u_i$ can be expressed as

$$u_i = (L_{g_i}h_i)^{-1}(-L_{f_i}h_i + v_i). \tag{5}$$

If $L_{g_i}h_i$ is equal to zero, the differentiation process is continued until $u_i$ can be written as a function of $v_i$ as follows.

*Definition 2.* For the smooth vector field $h(x):\mathbb{R}^n\to\mathbb{R}$ and smooth matrix field $f(x):\mathbb{R}^n\to\mathbb{R}^n$, $L_f{}^k h = L_f(L_f{}^{k-1}h)=\frac{\partial(L_f{}^{k-1}h)}{\partial x}f$, $k>1$, where $L_f h$ has been defined in Definition 1 [20].

*Assumption 1.* There exists an $r>1$ such that

a. $L_{g_i}L_{f_i}{}^l h_i = 0, \quad for \quad l<r-1, \quad \forall i$.

b. $L_{g_i}L_{f_i}{}^{r-1}h_i \neq 0, \forall i$.

According to Definition 2 and Assumption 1a, the $r^{\text{th}}$ derivative of $y_i(t)$ can be written as

$$y_i^{(r)} = L_{f_i}{}^r h_i + L_{g_i}L_{f_i}{}^{r-1}h_i u_i. \tag{6}$$

Define the auxiliary control $v_i$ as

$$v_i = L_{f_i}{}^r h_i + L_{g_i}L_{f_i}{}^{r-1}h_i u_i. \tag{7}$$

If Assumption 1b holds, then the control input $u_i$ is implemented as

$$u_i = (L_{g_i}L_{f_i}{}^{r-1}h_i)^{-1}(-L_{f_i}{}^r h_i + v_i). \tag{8}$$

Equation(7) results in the $r^{\text{th}}$-order linear system

$$y_i^{(r)} = v_i, \forall i. \tag{9}$$

Equation (9) and the first $(r-1)$ derivatives of $y_i$ can be written as

$$\begin{cases}\dot{y}_i \equiv y_{i,1} \\ \dot{y}_{i,1} \equiv y_{i,2} \\ \vdots \\ \dot{y}_{i,r-1} = v_i\end{cases}, \forall i. \tag{10}$$

or equivalently

$$\dot{\mathcal{Y}}_i = \mathbf{A}\mathcal{Y}_i + \mathbf{B}v_i, \forall i, \tag{11}$$

where $\mathcal{Y}_i = [y_i \quad y_{i,1} \quad \dots \quad y_{i,r-1}]^T$, $\mathbf{B} = [0 \quad 0 \quad \dots \quad 1]^T_{r\times 1}$, and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{r\times r}. \tag{12}$$

Using this input-output feedback linearization, the dynamics of each agent is decomposed into the $r^{\text{th}}$-order dynamical system in (11), and a set of internal dynamics denoted as (Slotine & Li, 1991)

$$\dot{\mu}_i = W_i(\mathcal{Y}_i, \mu_i), \forall i. \tag{13}$$

The commensurate reformulated dynamics of the leader node in (2) can be expressed as

$$\dot{\mathbf{Y}}_0 = \mathbf{A}\mathbf{Y}_0 + \mathbf{B}y_0^{(r)}, \tag{14}$$

where $\mathbf{Y}_0 = [y_0 \quad \dot{y}_0 \quad \dots \quad y_0^{(r-1)}]^T$.

The synchronization problem is to find a distributed $v_i$ in (8)such that $\mathcal{Y}_i \to \mathbf{Y}_0, \forall i$. To solve this problem, the cooperative team objectives are expressed in terms of the local neighborhood tracking error

$$\mathbf{e}_i = \sum_{j\in N_i} a_{ij}(\mathcal{Y}_i - \mathcal{Y}_j) + b_i(\mathcal{Y}_i - \mathbf{Y}_0). \tag{15}$$

The pinning gain $b_i \ge 0$ is nonzero only for the nodes that are directly connected to the leader node. From (15), the global error vector for graph $\mathcal{G}$ is written as

$$\mathbf{e} = \big((L+G)\otimes I_r\big)(\mathcal{Y} - \mathcal{Y}_0) \equiv \big((L+G)\otimes I_r\big)\boldsymbol{\delta}, \tag{16}$$

where $\mathcal{Y} = \begin{bmatrix}\mathcal{Y}_1^T & \mathcal{Y}_2^T & \cdots & \mathcal{Y}_N^T\end{bmatrix}^T$, $\mathbf{e} = \begin{bmatrix}\mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_N^T\end{bmatrix}^T$, $\mathcal{Y}_0 = \mathbf{1}_N \otimes \mathbf{Y}_0$, $G=diag\{b_i\}$, and $\boldsymbol{\delta}$ is the global disagreement vector. $\dot{\mathcal{Y}}$ can be written as

$$\dot{\mathcal{Y}} = (I_N \otimes \mathbf{A})\mathcal{Y} + (I_N \otimes \mathbf{B})v, \tag{17}$$

where $v = \begin{bmatrix} v_1 & v_2 & \cdots & v_N\end{bmatrix}^T$ is the global auxiliary control vector. $\dot{\mathcal{Y}}_0$ can be written as

$$\dot{\mathcal{Y}}_0 = (I_N \otimes \mathbf{A})\mathcal{Y}_0 + (I_N \otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)}, \tag{18}$$

where $\underline{\mathbf{y}}_0^{(r)} = \mathbf{1}_N \otimes y_0^{(r)}$.

*Definition 3.* The $\mathcal{Y}_i$ are cooperative UUB with respect to $\mathbf{Y}_0$ in (14) if there exists a compact set $\Omega \subset \mathbb{R}^r$ so that $\forall(\mathcal{Y}_i(t_0) - \mathbf{Y}_0(t_0)) \in \Omega$ there exists a bound $B$ and a time $t_f(B, (\mathcal{Y}_i(t_0) - \mathbf{Y}_0(t_0)))$, both independent of $t_0$, such that $\|\mathcal{Y}_i(t_0) - \mathbf{Y}_0(t_0)\| \le B, \forall t > t_0 + t_f$ [10].

The following lemmas are required.

*Lemma 1.* Let ( $\mathbf{A}$ , $\mathbf{B}$ ) be stabilizable**.** Let the digraph $\mathcal{G}$ have a spanning tree and $b_i \ne 0$ for at least one root node. Let $\lambda_i$ be the eigenvalues of $L+G$. The matrix

$$\mathbf{H} = I_N \otimes \mathbf{A} - c(L+G)\otimes \mathbf{B}K, \tag{19}$$

with $c \in \mathbb{R}$ and $K \in \mathbb{R}^{1\times r}$, is Hurwitz if and only if all the matrices $\mathbf{A} - c\lambda_i\mathbf{B}K, \ \forall i$ are Hurwitz [4], [21].

*Remark 1.* Note that $\mathbf{A}$ in(12) has $r$ eigenvalues at $s$=0 and, hence, is unstable. Therefore, Lemma 1 requires that $\lambda_i \ne 0, \ \forall i$, that is $L+G$ is non-singular. This requires the digraph $\mathcal{G}$ to have a spanning tree and $b_i \ne 0$ for at least one root node.

*Lemma 2.* Let the digraph $\mathcal{G}$ have a spanning tree and $b_i \ne 0$ for at least one root node. Let ( $\mathbf{A}$ , $\mathbf{B}$ ) be stabilizable and matrices $Q = Q^T$ and $R = R^T$ be positive definite. Let feedback gain $K$ be chosen as

$$K = R^{-1}\mathbf{B}^T P_1, \tag{20}$$

where $P_1$ is the unique positive definite solution of the control algebraic Riccati equation

$$\mathbf{A}^T P_1 + P_1\mathbf{A} + Q - P_1\mathbf{B}R^{-1}\mathbf{B}^T P_1 = 0. \tag{21}$$

Then, all the matrices $\mathbf{A} - c\lambda_i\mathbf{B}K, \ \forall i \in \mathcal{N}$ are Hurwitz if $c \ge \dfrac{1}{2\lambda_{\min}}$, where $\lambda_{\min} = \min_{i\in\mathcal{N}} Re(\lambda_i)$ [21].

*Assumption 2.* The vector $\underline{\mathbf{y}}_0^{(r)} = \mathbf{1}_N \mathbf{y}_0^{(r)}, \forall r$ is bounded so that $\left\|\underline{\mathbf{y}}_0^{(r)}\right\| \le Y_M^r$, with $Y_M^r$ a finite but generally unknown bound.

*Theorem 1.* Let the digraph $\mathcal{G}$ have a spanning tree and $b_i \ne 0$ for at least one root node. Assume that the zero dynamics of each node $\dot{\mu}_i = W_i(0, \mu_i), \forall i$ are asymptotically stable. Let the auxiliary control $v_i$ in (8)be chosen as

$$v_i = -cK\mathbf{e}_i, \tag{22}$$

where $c \in \mathbb{R}$ is the coupling gain, and $K \in \mathbb{R}^{1\times r}$ is the feedback control matrix. Then $\mathcal{Y}_i$ are cooperative UUB with respect to $\mathbf{Y}_0$ in (14)and all nodes synchronize to $\mathbf{Y}_0$ if $K$ is chosen as in (20) and

$$c \ge \frac{1}{2\lambda_{\min}}, \tag{23}$$

where $\lambda_{\min} = \min_{i\in\mathcal{N}} Re(\lambda_i)$.

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*Proof:* Consider the Lyapunov function candidate

$$V = \frac{1}{2}\mathbf{e}^T P_2 \mathbf{e}, \quad P_2 = P_2^{\,T}, P_2 > 0, \tag{24}$$

where $\mathbf{e}$ is the global error vector in (16). Then

$$\begin{aligned}\dot{V} &= \mathbf{e}^T P_2 \dot{\mathbf{e}} = \mathbf{e}^T P_2((L+G)\otimes I_r)\left(\dot{\mathcal{Y}} - \dot{\mathcal{Y}}_0\right) \\ &= \mathbf{e}^T P_2((L+G)\otimes I_r)\Big((I_N \otimes \mathbf{A})\boldsymbol{\delta} + (I_N \otimes \mathbf{B})(v - \underline{\mathbf{y}}_0^{(r)})\Big).\end{aligned} \tag{25}$$

Since the digraph $\mathcal{G}$ has a spanning tree, and $b_i \neq 0$ for at least one root node, $L+G$ is invertible. Therefore, (25) can be written as

$$\begin{aligned}\dot{V} &= \mathbf{e}^T P_2((L+G)\otimes I_r)((I_N \otimes \mathbf{A})((L+G)^{-1}\otimes I_r)\mathbf{e} \\ &+ (I_N \otimes \mathbf{B})(v - \underline{\mathbf{y}}_0^{(r)})).\end{aligned} \tag{26}$$

The global auxiliary control $\mathbf{v}$ can be written as

$$\mathbf{v} = -c(I_N \otimes K)\mathbf{e}. \tag{27}$$

Placing (27) into (26) and considering the fact that $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$ yields

$$\begin{aligned}\dot{V} &= \mathbf{e}^T P_2\left(I_N \otimes \mathbf{A} - c(L+G)\otimes \mathbf{B}K\right)\mathbf{e} - \\ &\mathbf{e}^T P_2((L+G)\otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)} \equiv \mathbf{e}^T P_2 \mathbf{H}\mathbf{e} - \\ &\mathbf{e}^T P_2((L+G)\otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)}.\end{aligned} \tag{28}$$

From Lemma 1 and Lemma 2, $\mathbf{H}$ is Hurwitz. Given any positive real number $\beta$, choose the positive definite matrix $P_2$, such that the following Lyapunov equation holds,

$$P_2\mathbf{H} + \mathbf{H}^T P_2 = -\beta I_{Nr}. \tag{29}$$

Placing (29) in (28) yields

$$\begin{aligned}\dot{V} &= \frac{1}{2}\mathbf{e}^T(P_2\mathbf{H} + \mathbf{H}^T P_2)\mathbf{e} - \mathbf{e}^T P_2((L+G)\otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)} \\ &= -\frac{\beta}{2}\mathbf{e}^T I_{Nr}\mathbf{e} - \mathbf{e}^T P_2((L+G)\otimes \mathbf{B})\underline{\mathbf{y}}_0^{(r)}.\end{aligned} \tag{30}$$

According to Assumption 2

$$\dot{V} \leq -\frac{\beta}{2}\|\mathbf{e}\|^2 + \|\mathbf{e}\|\bar{\sigma}(P_2((L+G)\otimes \mathbf{B}))Y_M^r, \tag{31}$$

then, $\dot{V} \leq 0$ if

$$\|\mathbf{e}\| > \frac{2\bar{\sigma}(P_2((L+G)\otimes \mathbf{B}))Y_M^r}{\beta}. \tag{32}$$

Equation (32) shows that the global error vector $\mathbf{e}$ is UUB. Therefore, the global disagreement vector $\boldsymbol{\delta}$ is UUB and, hence, $\mathcal{Y}_i$ are cooperative UUB with respect to $\mathbf{Y}_0$ [9]. If zero dynamics are asymptotically stable, then (9) and (22) are asymptotically stable [20]. This completes the proof. □

*Remark 2.* If $y_0^{(r)} = 0$, the error bound in (32) is zero and the global error vector $\mathbf{e}$ is asymptotically stable.

## IV. Tracking Synchronization Problem in Microgrids

In this section, the feedback linearization-based tracking synchronization method presented in Section III is used to implement the secondary voltage control of microgrids.

Figure 1 shows the block diagram of an inverter-based DG. It contains the primary power source (e.g., photovoltaic panels), the voltage source converter (VSC), and the power, voltage, and current control loops. The control loops set and control the output voltage and frequency of the VSC. Outer volt-age and inner current controller block diagrams are elaborated in [22]. The power controller provides the voltage references $v_{odi}^*$ and $v_{oqi}^*$ for the voltage controller, and the

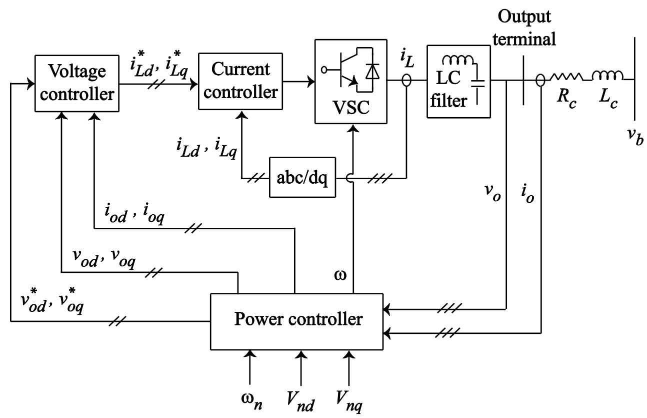


Fig 1. The block diagram of a DG.

operating frequency $\omega_i$ for the VSC. Note that nonlinear dynamics of each DG in a microgrid are formulated on its own *d-q* (direct-quadratic) reference frame. The reference frame of microgird is considered as the common reference frame, and the dynamics of other DGs are transformed to the common reference frame. The angular frequency of this common reference frame is denoted by $\omega_{com}$.

The nonlinear dynamics of the $i^{\text{th}}$ DG, shown in Fig. 1, can be written as

$$\begin{cases}\dot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + \mathbf{k}_i(\mathbf{x}_i)\mathbf{D}_i + \mathbf{g}_i(\mathbf{x}_i)u_i \\ y_i = h_i(\mathbf{x}_i)\end{cases}, \tag{33}$$

The term $\mathbf{D}_i$ is considered as a known disturbance. Detailed expressions for $\mathbf{f}_i(\mathbf{x}_i)$, $\mathbf{g}_i(\mathbf{x}_i)$, $h_i(\mathbf{x}_i)$, $d_i(\mathbf{x}_i)$, and $\mathbf{k}_i(\mathbf{x}_i)$ are adopted from the nonlinear model presented in [22].

The primary voltage control is usually implemented as a local controller at each DG using the droop technique. Droop technique prescribes a desired relation between the voltage amplitude and the reactive power. The primary volt-age control is

$$v_{o,magi}^* = V_{ni} - n_{Q_i}Q_i, \tag{34}$$

where $v_{o,magi}^*$ is the voltage set point provided for the internal voltage control. $Q_i$ is the filtered reactive power at the DG's terminal. $n_{Qi}$ is the droop coefficient that is chosen based on the reactive power ratings of DGs. $V_{ni}$ is the primary control reference [19].

The secondary frequency control chooses $V_{ni}$ such that the output voltage amplitude of each DG synchronizes to its nominal value, i.e. $v_{o,magi} \rightarrow v_{ref}$ . For secondary voltage control, the input and output in (33) are $u_i = V_{ni}$ and $y_i = v_{o,magi}$ , respectively. Considering the nonlinear dynamics of each DG in (33), the input $V_{ni}$ appears in the second derivate of $v_{o,magi}$ , i.e., $r = 2$ in (9). According to the results of Theorem 1, adopting appropriate values for $c$ in (22) , and $P_1$ and $Q$ in (21), the control protocol in (8) and (22) synchronizes the output voltage amplitude of each DG to its nominal value.

## V. Simulation Results

The effectiveness of the proposed secondary voltage control is verified by simulation on an islanded microgrid. Figure 2 illustrates the single line diagram of the microgrid test system. This microgrid consists of four DGs. The lines between buses are modeled as series RL branches. The specifications of the DGs, lines, and loads are summarized in Table I. In this table, $K_{PV}$ , $K_{IV}$ , $K_{PC}$ , and $K_{IC}$ are the parameters of the voltage and current controllers in Fig. 1. The voltage and current controllers used in the following simulation are adopted from [21]. It is assumed that the DGs communicate with each other through the communication digraph depicted in Fig. 3. The associated adjacency matrix of this digraph is

$$A_G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{35}$$

TABLE I

Specifications of the Microgrid Test System

| | | | | | | |
|---|---|---|---|---|---|---|
| DGs | DG 1 & 2 (45 kVA rating) | | DG 3 & 4 (34 kVA rating) | | | |
| | $m_P$ | $9.4\times10^{-5}$ | $m_P$ | $12.5\times10^{-5}$ | | |
| | $n_Q$ | $1.3\times10^{-3}$ | $n_Q$ | $1.5\times10^{-3}$ | | |
| | $R_c$ | 0.03 Ω | $R_c$ | 0.03 Ω | | |
| | $L_c$ | 0.35 mH | $L_c$ | 0.35 mH | | |
| | $R_f$ | 0.1 Ω | $R_f$ | 0.1 Ω | | |
| | $L_f$ | 1.35 mH | $L_f$ | 1.35 mH | | |
| | $C_f$ | 50 µF | $C_f$ | 50 µF | | |
| | $K_{PV}$ | 0.1 | $K_{PV}$ | 0.05 | | |
| | $K_{IV}$ | 420 | $K_{IV}$ | 390 | | |
| | $K_{PC}$ | 15 | $K_{PC}$ | 10.5 | | |
| | $K_{IC}$ | 20000 | $K_{IC}$ | 16000 | | |
| Lines | Line 1 | | Line 2 | | Line 3 | |
| | $R_{l1}$ | 0.23 Ω | $R_{l2}$ | 0.35 Ω | $R_{l3}$ | 0.23 Ω |
| | $L_{l1}$ | 318 µH | $L_{l2}$ | 1847 µH | $L_{l3}$ | 318 µH |
| Loads | Load 1 | | Load 2 | | | |
| | $P_{L1}$ (per phase) | 12 kW | $P_{L2}$ (per phase) | 15.3 kW | | |
| | $Q_{L1}$ (per phase) | 12 kVAr | $Q_{L2}$ (per phase) | 7.6 kVAr | | |

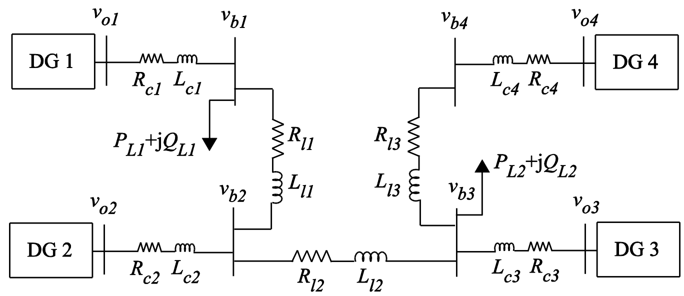


Fig 2. Single-line diagram of the microgrid test system.

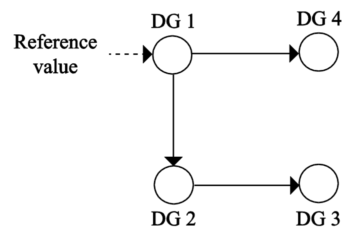


Fig 3. Topology of the communication digraph.

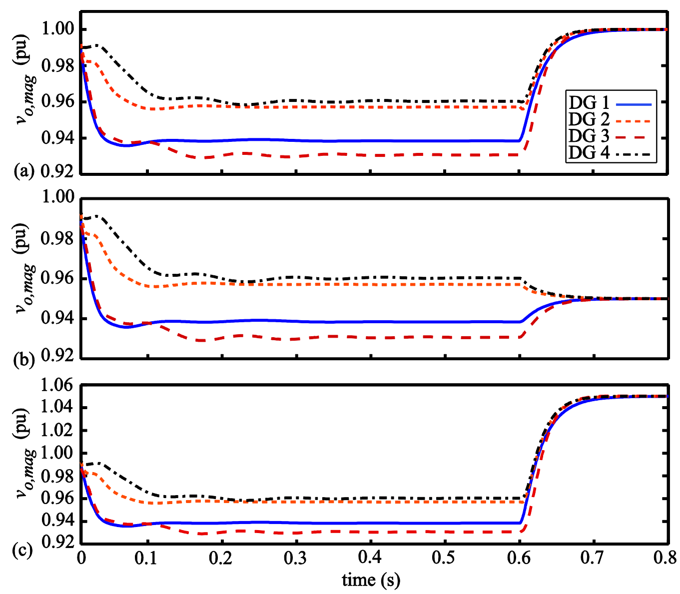


Fig. 4. DG output voltage magnitudes: (a) when $v_{ref}$=1 pu, (b) when $v_{ref}$=0.95 pu, and (c) when $v_{ref}$=1.05 pu.

DG 1 is the only digraph that is connected to the leader node with $g_1 = 1$. Since agent outputs need to track constant references, the leader node has no dynamics, i.e. $\dot{y}_0 = 0$ .The coupling gain in (22) is $c$=4 which satisfies(23). The solution of the algebraic Riccati equation in (21) is used to calculate the feedback control vector $K$in(22). In(21), the algebraic Riccati equation parameters are chosen as $Q=\begin{bmatrix}50000 & 0\\ 0 & 1\end{bmatrix}$ and $R$=0.01. The resulting feedback control vector is $K$=[2236 67.6]. It is assumed that the microgrid is islanded from the main grid at $t$=0, and the secondary control is applied at $t$=0.6 s. Figures 4a, 4b, and 4c show the simulation results when the reference voltage value is set to 1 pu, 0.95 pu, and 1.05 pu, respectively. As seen in Fig. 4, while the primary control keeps the voltage amplitudes stable, the secondary control establishes all the terminal voltage amplitudes to the pre-specified reference values after 0.1 seconds.

## VI. Conclusion

This paper proposes a control method for the tracker synchronization problem of multi-agent systems with nonlinear and heterogeneous dynamics. Input-output feedback linearization is used to transform the nonlinear dynamics of agents to linear dynamics. The distributed control inputs are designed such that the synchronization errors are bounded. The proposed control method is used to implement the secondary voltage control for microgrids. The proposed secondary control is fast and uses a sparse and cheap communication network. The effectiveness of the proposed secondary control is verified by simulating a microgrid test system.

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