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MODELS OF THIN-WALLED BEAM CONNECTIONS

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INTRODUCTION

In thin-walled beam theory the kinematics of the beam is described by 6 degrees of freedom for a rigid body displacement of each cross-section plus an additional degree of freedom for cross-section warping. Torsion and warping are coupled, and this coupling requires special attention at joints. In general the transfer of warping through a joint will create deformation of the cross-section. Although distortion in the form of cross-section deformation is not accounted for in classical thin-walled beam theory this effect is often local, and for cross-sections containing two main flanges – such as I, U and C-profiles – simple joint models compatible with classical thin-walled beam theory can be developed.

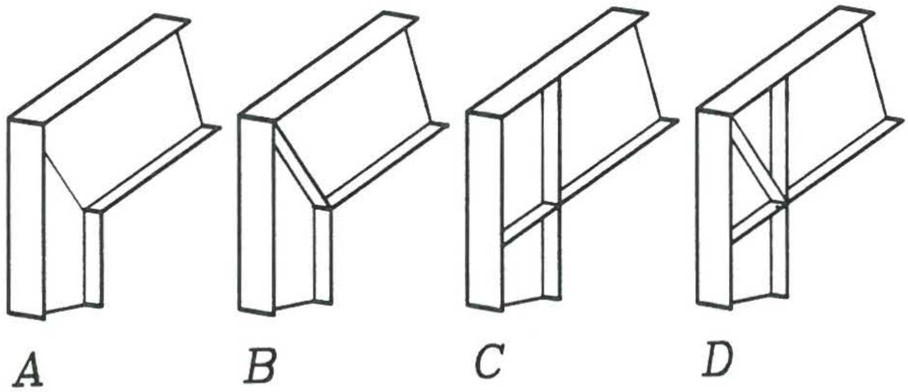

Fig. 1. Four different types of I-beam joints.

In the present paper models are presented for the four I-beam joints shown in Fig. 1. These joints have been considered by Vacharajittiphan & Trahair (1974), who represented the effect of the joint as a numerically calibrated warping restraint stiffness. In the present approach the key issue is to identify suitable kinematical continuity conditions, and then – if necessary – to include any additional stiffness associated with distortion of the beams and torsion of the plates added to the joints of type B, C and D.

The height of the beams is h_1 and h_2, respectively. At the joints the warping intensity of the beams is denoted by θ_1 and θ_2. The warping of a cross–section of beam 1 according to classical thin–walled beam theory consists of a relative inclination of the two flanges of magnitude $h_1\theta_1$ in the plane of the flanges as shown in Fig. 2. At the joint the continuity of the inside and outside flanges leads to mutual inclination of the flanges of magnitude ψ_1 in the plane of the cross–section as shown in Fig. 3. At any of the four types of joint the four kinematical parameters θ_1, ψ_1, θ_2 and ψ_2 are related by two continuity conditions. Thus the deformation of the beam cross–sections at the joint can be expressed in terms of the warping intensities θ_1 and θ_2. In joint types B and C an additional equation is obtained from the very large in–plane bending stiffness of the added plates, while in type D two additional equations are obtained from the plates in the joint. Thus type A has two free warping parameters, types B and C have one, while warping as well as cross–section deformation is fully restrained in type D.

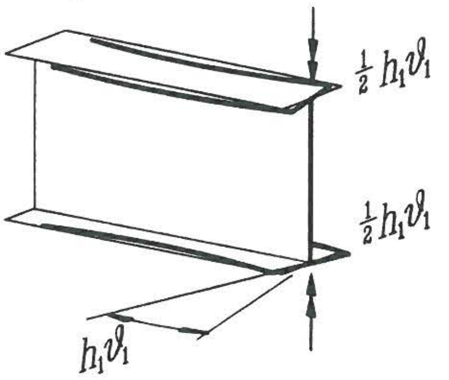

Fig. 2. Warping of beam 1.

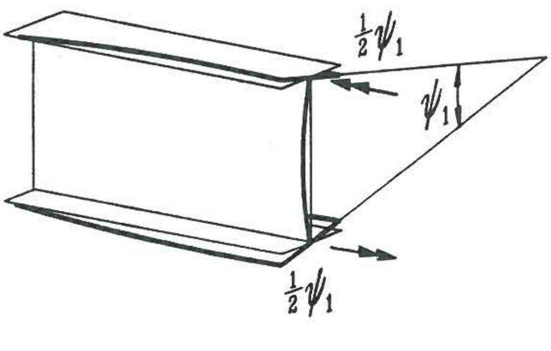

Fig. 3. Distortion of beam 1.

FLANGE CONTINUITY

The displacement of a joint may be considered as consisting of a rigid body motion and a deformation of the joint permitting warping and distortion of the associated beams. The present analysis is only concerned with the latter part. The rotation of the intersection line of the inner flanges must be the same whether described in terms of the parameters $h_1\theta_1$, ψ_1 or $h_2\theta_2$, ψ_2. Projection of the rotation vectors shown in Fig. 4 then gives the continuity conditions

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \frac{1}{\sin\alpha} \begin{bmatrix} \cos\alpha \, h_1 & -h_2 \\ h_1 & -\cos\alpha \, h_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \qquad (1)$$

This relation enables elimination of the distortion parameters ψ_1 and ψ_2 locally at the joint, provided the beam distortion does not couple to the neighbouring joints.

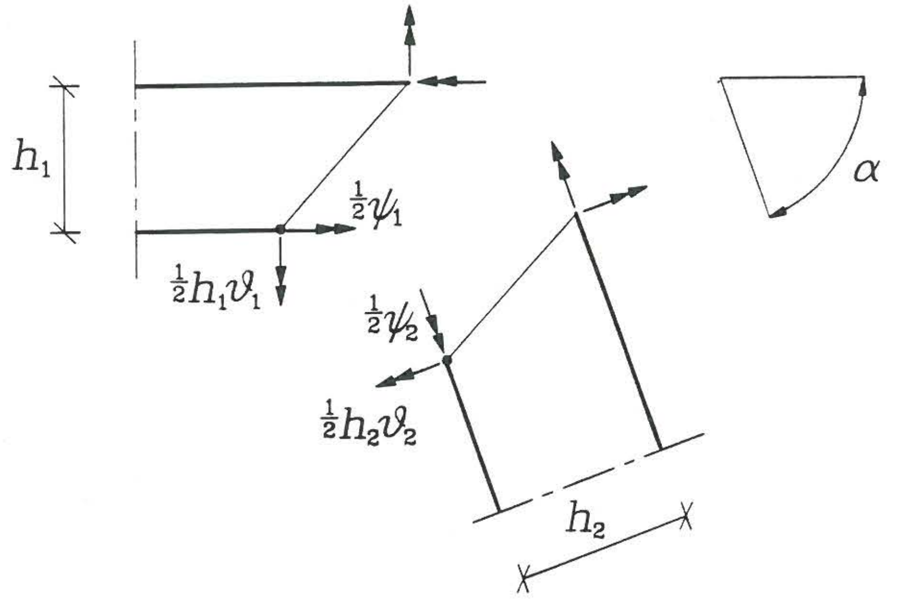

Fig. 4. Flange continuity at joint.

ENERGY AND LENGTH SCALES

The elastic energy per unit length of beam associated with warping is

$$W_\theta = \frac{1}{2} E\, C\, \theta'^2 + \frac{1}{2} G\, K\, \theta^2 \tag{2}$$

where E is the modulus of elasticity and G is the shear modulus. K is the St. Venant torsion constant of the full beam cross–section, and $C = h^2 I_f/2$ is the warping stiffness when I_f is the in–plane bending stiffness of one flange.

The elastic energy associated with distortion can be approximated by a similar expression.

$$W_\psi = \frac{1}{2} G\, 2K_f(\tfrac{1}{2}\psi')^2 + \frac{1}{2}\frac{1}{h} D_w\, \psi^2 \tag{3}$$

K_f is the St. Venant torsion stiffness of one flange, and $D_w = Et_w^3/12(1-\nu^2)$ is the bending stiffness of the web.

The Euler equations corresponding to (2) and (3) are

$$E\, C\, \theta'' - G\, K\, \theta = 0 \tag{4}$$

$$G\, \tfrac{1}{2}K_f\, \psi'' - (D_w/h)\, \psi = 0 \tag{5}$$

These equations have exponential solutions with parameters given by

$$k_\theta^2 = \frac{GK}{EC} \;, \quad k_\psi^2 = \frac{2\,D_w}{GK_f h} \tag{6}$$

The parameters k_θ and k_ψ determine the attenuation of warping and distortion with distance from the joint, respectively. Usually k_ψ is greater than k_θ implying faster attenuation of the distortion mode.

In the following the distortion modes are assumed to be local. The elastic distortion energy in each of the beams can then be represented by the value corresponding to a semi–infinite beam, e.g.

$$E_{\psi_1} = \frac{1}{2}\sqrt{\frac{D_{w1}}{2\,h_1}\, GK_{f1}}\; \psi_1^2 \tag{7}$$

This expression is accurate to within 10 pct for $k_{\psi_1} l \geq 1.5$.

UNSTIFFENED JOINTS, TYPE A

In the unstiffened joint of type A the warping intensities θ_1 and θ_2 are used as independent parameters, while the distortion parameters ψ_1 and ψ_2 are expressed by the relation (1) as

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = [A] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{8}$$

The distortion energy associated with the joint then follows from (7) in the form

$$E_\psi = \tfrac{1}{2}(\psi_1, \psi_2) \begin{bmatrix} \sqrt{(D_{w1}/2h_1)GK_{f1}} & 0 \\ 0 & \sqrt{(D_{w2}/2h_2)GK_{f2}} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} \tag{9}$$

When the diagonal stiffness matrix is denoted [D], the distortion energy is expressed in terms of the warping parameters as

$$E_\psi = \tfrac{1}{2}(\theta_1, \theta_2)\ [A]^T [D]\ [A] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{10}$$

Thus the unstiffened joint of type A acts in warping as a hinge with two independent warping parameters (θ_1, θ_2) and an additional 2 by 2 elastic spring stiffness matrix given by the matrix product in (10).

STIFFENED JOINTS, TYPE B

In stiffened joints of type B the in–plane bending stiffenss of the cross plate in the joint is usually sufficiently stiff to effectively prevent in–plane deformation. This leaves only one parameter to describe the combined warping and distortion of this type of joint.

The geometry of the joint is shown in Fig. 5, where the parameters are determined by

$$a_1 = (h_2 - h_1 \cos\alpha)/\sin\alpha \tag{11}$$

$$a_2 = (h_1 - h_2 \cos\alpha)/\sin\alpha \tag{12}$$

and

$$\tan\alpha_1 = a_1/h_1 \tag{13}$$

$$\tan\alpha_2 = a_2/h_2 \tag{14}$$

The length of the cross plate is

$$h_c = h_1/\cos\alpha_1 = h_2/\cos\alpha_2 \tag{15}$$

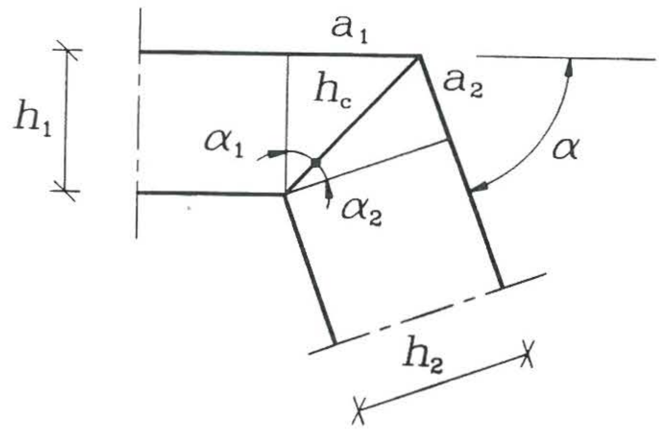

Fig. 5. Geometry of type B joint.

It is convenient to introduce the parameters θ_c and ψ_c representing warping and in–plane bending of the cross plate, see Fig. 6. Projection of the rotation vectors gives the following expression for the parameters of beam 1.

$$\begin{bmatrix} h_1\theta_1 \\ \psi_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix} \begin{bmatrix} h_c\theta_c \\ \psi_c \end{bmatrix} \tag{16}$$

The similar formula for $(h_2\theta_2, \psi_2)$ follows by exchange of α_1 with $-\alpha_2$.

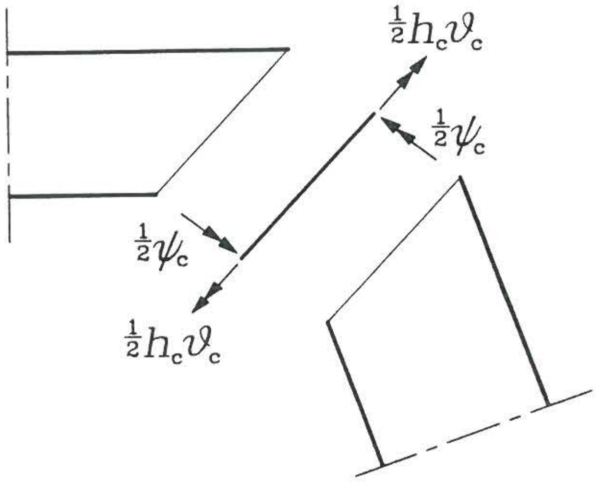

Fig. 6. Joint warping θ_c and in–plane bending ψ_c.

In practice the in–plane bending stiffness of the cross plate is much larger than the warping stiffness, and thus $\psi_c = 0$ is a good approximation. By (16) and the similar formula for beam 2 this implies

$$h_1\theta_1/\cos\alpha_1 = h_2\theta_2/\cos\alpha_2 = h_c\theta_c \tag{17}$$

Elimination of h_1 and h_2 by use of (15) then gives the continuity condition

$$\theta_1 = \theta_2 = \theta_c \tag{18}$$

Thus constraint of the in–plane bending of the cross plate – $\psi_c = 0$ – implies that the two beams have identical warping parameters at the joint – $\theta_1 = \theta_2$.

There are two stiffness contributions associated with the warping parameter θ_c – warping of the cross plate and distortion of the beam cross–sections. The elastic energy required for warping of the cross plate with dimensions $h_c \times b_c \times t_c$ is

$$E_\theta = \frac{1}{2}\left\{\frac{1}{3}G\ h_c b_c t_c^3\right\}\theta_c^2 \tag{19}$$

The distortion parameters ψ_1 and ψ_2 follow from (1) with $\theta_1 = \theta_2 = \theta_c$. When the geometric parameters a_1 and a_2 shown in Fig. 5 are introduced from (11) and (12)

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \theta_c \tag{20}$$

This relation is substituted into the energy expression (9) to give the energy of distortion

$$E_\psi = \frac{1}{2} \left\{ a_1^2 \sqrt{(D_{w1}/2h_1)GK_{f1}} + a_2^2 \sqrt{(D_{w2}/2h_2)GK_{f2}} \right\} \theta_c^2 \tag{21}$$

The only dependence on the joint angle α is through the lengths a_1 and a_2.

STIFFENED JOINTS, TYPE C

In stiffened joints of type C the warping is conveniently expressed in terms of the out–of–plane displacement $\pm\Delta$ of the four points of flange intersection. The analysis has been carried out by Krenk et al. (1990). The result is that there is a single warping parameter

$$\theta_c = 4 \frac{\Delta}{A} \tag{22}$$

where $A = h_1 h_2 / \sin\alpha$ is the web area enclosed by the flanges. The warping at the beam ends is determined by

$$\theta_1 = -\theta_2 = \theta_c \tag{23}$$

Thus warping is of the same magnitude, but of different sign in the two beams.

There are two stiffness contributions – warping of the additional flange lengths, and distortion of the beam cross–sections. The additional energy from warping of flanges with dimensions $b_1 \times t_1$ and $b_2 \times t_2$ is

$$E_\theta = \frac{1}{2} \left\{ \frac{1}{3} G \frac{h_2 b_1}{\sin\alpha} t_1^3 + \frac{1}{3} G \frac{h_1 b_2}{\sin\alpha} t_2^3 \right\} \theta_c^2 \tag{24}$$

The distortion energy is computed by introducing the common warping parameter θ_c from (23) into (1) and (9). Experience indicates that the main point is the enforcement of the continuity condition (23), while the additional stiffness contributions are of secondary importance.

STIFFENED JOINTS, TYPE D

The stiffened joints of type D must satisfy the continuity conditions of joints of type B as well as joints of type C. This implies that

$$\theta_1 = \theta_2 = 0 \tag{25}$$

for joints of type D. The distortion determined by (1) then also vanish, and the type D joint can be considered as a full warping and distortion restraint.

EXAMPLES AND FINITE ELEMENT RESULTS

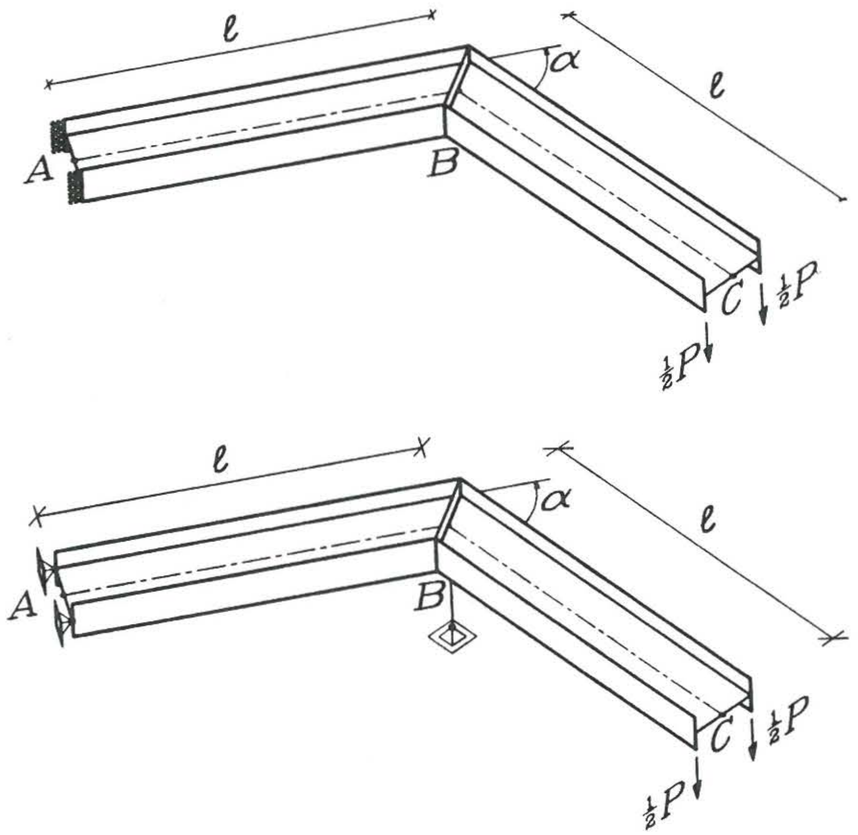

Fig. 7. Cantilever and simply supported angle beams.

The continuity and stiffness conditions for warping of thin–walled I–beam joints of types A and B have been investigated using the angle beam configurations shown in Fig. 7a and 7b for joint type B. Two identical I–beams of length ℓ are joined at an angle α, and a transverse load P is applied at the point C. Two support conditions are considered: in case a) the beam AB is rigidly fixed at A, and in case b) the flanges of the beam AB are simply supported at A, while a single simple support is introduced at B.

In the classical theory for thin–walled beams of open cross–section warping is determined by vanishing shear strain in the mid–surface. The warping parameter θ then equals minus the rate of twist φ'.

$$\theta = -\varphi' \tag{26}$$

Torsion and warping of the beams AB and BC in Fig. 7 are then governed by the differential equation

$$(EC\varphi'')'' - (GK\varphi)'' = 0 \tag{27}$$

and the appropriate boundary and continuity conditions.

In the following the accuracy of the continuity conditions and stiffness contributions for joints of type A and type B is evaluated by comparing the rotation φ_B of beam AB at B predicted by the present theory with three–dimensional finite element calculations. The results are normalized with respect to the rotation that would result if the beam AB were free to warp at both ends and loaded by the torsional moment

$$M_0 = P\,\ell \sin\alpha \tag{28}$$

Thus the results are expressed in terms of the non–dimensional parameter $\varphi_B GK/\ell M_0$.

The cross–section dimensions in all cases are h = 200 mm, b = 100 mm and $t_f = 8.5$ mm, $t_w = 5.6$ mm. With $\nu = 0.3$ the k–parameters are

$$k_\theta = 1.36\ 10^{-3}\ \text{mm}^{-1} \qquad k_\psi = 4.88\ 10^{-3}\ \text{mm}^{-1}$$

Thus the approximation of distortion as a local effect can be used for $k_\theta \ell \geq 0.4$ for this cross–section.

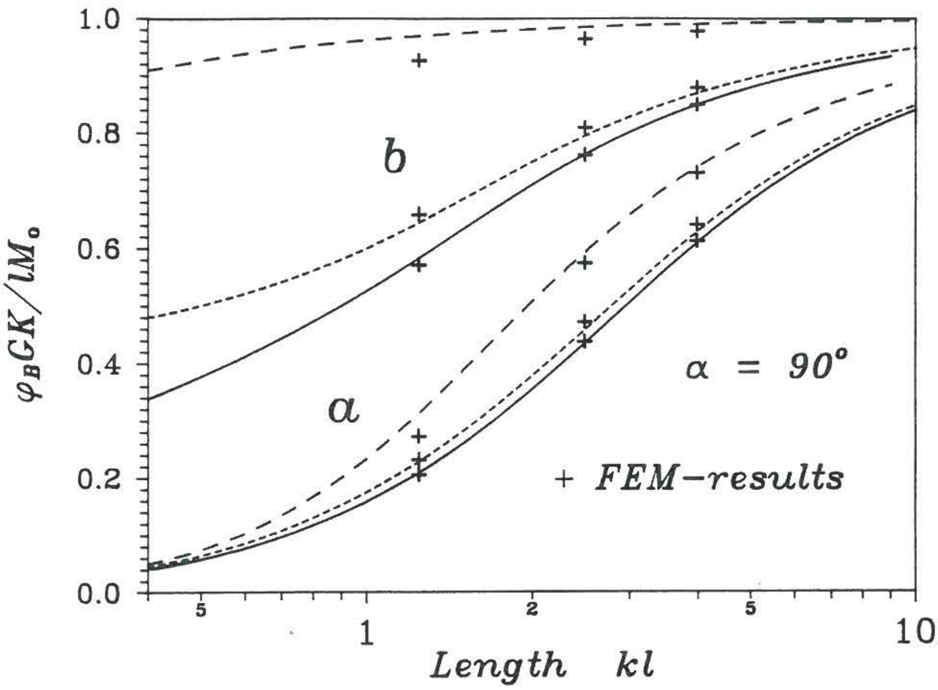

Fig. 8. Normalized angle φ_B from beam theory and FEM, $\alpha = 90^0$. (– –) type A, (· · ·) type B without joint stiffness, (——) type B with full stiffness.

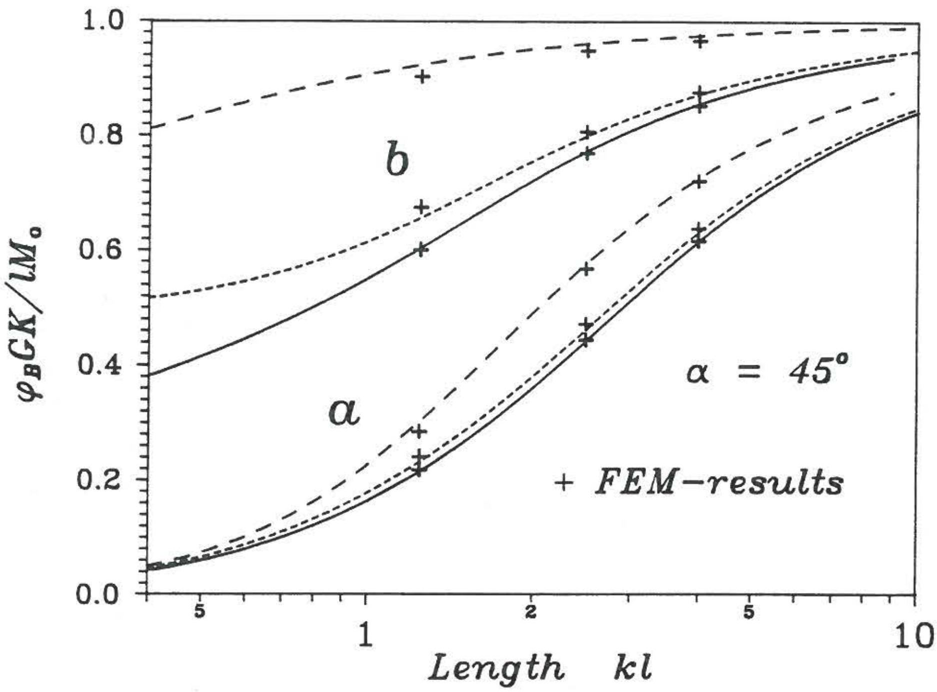

Fig. 9. Normalized angle φ_B from beam theory and FEM, $\alpha = 45^0$. (– –) type A, (· · ·) type B without joint stiffness, (——) type B with full stiffness.

The results according to the present theory have been evaluated in closed form for joints of type B by Krenk et al. (1990), and all four joint models have been incorporated in a finite element program for thin–walled frames by Damkilde et al. (1990). These results are given by curves in Figs. 8 and 9 for $\alpha = 90^0$ and 45^0, respectively. Note the limited influence of the angle α.

The results of the beam theory have been compared with three–dimensional finite element analyses using he PAFEC program. Three different lengths were considered with $k_\theta \ell = 1.25, 2.5, 4.0$. The element model used four approximately square 8–node shell elements for the web and four similar elements across each flange. A typical mesh is shown in Fig. 10.

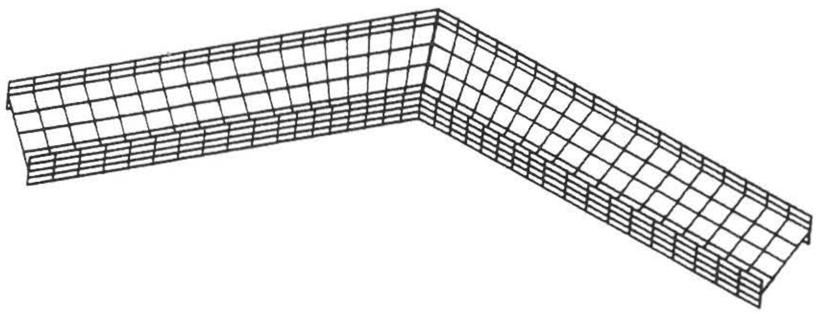

Fig. 10. Finite element mesh.

CONCLUSIONS

Continuity conditions and stiffness properties have been derived for warping and distortion of the four types of thin–walled I–beam joints shown in Fig. 1. The unstiffened joint, type A, has two independent warping parameters, and distortion of the joint appears as a 2 by 2 spring stiffness matrix. The stiffeners in joints of type B and C enforce warping of equal magnitude in both beams. In addition there are stiffness contributions from the added plates and disortion. Joints of type D act as full warping constraints.

The theory is expressed in tems of simple, explicit formulas suitable for use in connection with classical thin–walled beam theory. Three dimensional finite element calculations indicate high accuracy of the beam–type models of the joints.

ACKNOWLEDGMENT

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