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BEYOND MULTIPLEXING GAIN IN LARGE MIMO SYSTEMS

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Preliminary Notations

Notation 1 *We denote the binary entropy function (BEF) as*

$$H(p) = (p-1)\log_2(1-p) - p\log_2 p, \quad p \in [0,1]$$

with the convention $0\log_2 0 = 0$.

Notation 2 *Consider a* $\phi N \times N$ *random matrix* $\boldsymbol{X}$. *As* $N \to \infty$, *let* $\boldsymbol{X}\boldsymbol{X}^\dagger$ *have a limiting eigenvalue distribution (LED), denoted by* $\mu_{\boldsymbol{X}}$.

Introduction

Consider a $\phi T \times T$ MIMO channel $\boldsymbol{H}$. Assume that $\mu_{\boldsymbol{H}}$ exits as $T \to \infty$. Further define

$$\alpha \triangleq 1 - \mu_{\boldsymbol{H}}(0)$$

$$\tilde{\mu}_{\boldsymbol{H}}(x) \triangleq (1 - 1/\alpha)u(x) + \mu_{\boldsymbol{H}}(x)/\alpha$$

with $u(x)$ being the unit-step function.
For no CSI at Tx and perfect CSI at Rx, the mutual information (MU) is given by

$$\mathcal{I}(\gamma;\mu_{\boldsymbol{H}}) = \underbrace{\alpha\int_0^\infty \log_2(\gamma x)d\tilde{\mu}_{\boldsymbol{H}}(x)}_{\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}})} + \underbrace{\alpha\int_0^\infty \log_2\left(1+\frac{1}{x\gamma}\right)d\tilde{\mu}_{\boldsymbol{H}}(x)}_{\mathcal{I}_{\sim}(\gamma;\mu_{\boldsymbol{H}})}.$$

where γ denotes the SNR.

- $\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}})$ is a lower bound of the MU; it gives exact capacity in high SNR regime.
- $\mathcal{I}_{\sim}(\gamma;\mu_{\boldsymbol{H}})$ is the deviation of the lower bound from the exact MU.

Definitions

Definition 1 *A projector* $\boldsymbol{P}_\beta$ *is a diagonal matrix* $\boldsymbol{P}_\beta \in \{0,1\}^{N\times N}$ *with the ratio* $\beta = tr(\boldsymbol{P}_\beta)/N$ *fixed as* $N \to \infty$.

Definition 2 *Let* $\boldsymbol{H} \in \mathbb{C}^{R\times R}$ *have a LED and be free of an* $R \times R$ *projector* $\boldsymbol{P}_\beta$ *as* $R \to \infty$. *Then we define* ***the deviation of the linear capacity growth*** *in the LSL as*

$$\Delta\mathcal{L}_{\boldsymbol{H}\boldsymbol{P}_\beta} \triangleq \mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}\boldsymbol{P}_\beta}) - \beta\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}}).$$

- $\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}})$ is the growth rate of the normalized multiplexing gain β.

References

[1] Burak Çakmak, Ralf R. Müller, and Bernard H. Fleury. Beyond multiplexing gain in large MIMO Systems. *arXiv:1306.2595*, June 2013.

[2] C. Chuah, D. Tse, J. Kahn, and R. Valenzuela. Capacity scaling in MIMO wireless systems under correlated fading. *IEEE Transactions on Information Theory*, 48(3), 2002.

[3] Uffe Haagerup and Sören Möller. The law of large numbers for the free multiplicative convolution. *arXiv:1211.4457v1*, November 2012.

[4] Sergio Verdú and Shlomo Shamai. Spectral efficiency of CDMA with random spreading. *IEEE Transactions on Information Theory*, 45:622–640, March 1999.

[5] Ralf R. Müller. A random matrix model for communication via antenna arrays. *IEEE Transactions on Information Theory*, 48(9):2495–2506, September 2002.

Binary Entropy Increase

Consider a MIMO system with ϕT receive and T transmit antennas. Let the $M \times M$ matrix $\boldsymbol{P}_\beta$ be a projector with $M \triangleq \max(T, \phi T)$ and $\beta \triangleq \min(\phi, 1/\phi)$. Assume the following:

1. The channel matrix $\boldsymbol{H}$ has full rank with probability one.
2. T tends to infinity with $\boldsymbol{H}$ having a limiting singular value distribution and being free of $\boldsymbol{P}_\beta$.

Then we have

$$\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}}) - \mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{P}_\beta\boldsymbol{H}}) = H(\beta); \quad \phi \geq 1$$

$$\mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}^\dagger}) - \mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{P}_\beta\boldsymbol{H}^\dagger}) = H(\beta); \quad \phi < 1$$

Additivity of the Deviation

Consider a random matrix $\boldsymbol{H} = \boldsymbol{X}\boldsymbol{Y}$ with $\boldsymbol{X} \in \mathbb{C}^{R\times R}$ and $\boldsymbol{Y} \in \mathbb{C}^{R\times R}$. Assume that

1. $\boldsymbol{H}$ has almost surely full rank.
2. $\boldsymbol{X}$, $\boldsymbol{Y}$ and the $R \times R$ projector $\boldsymbol{P}_\beta$ have a LED each and are free of each other as $R \to \infty$.

Then we have

$$\Delta\mathcal{L}_{\boldsymbol{H}\boldsymbol{P}_\beta} = \Delta\mathcal{L}_{\boldsymbol{X}\boldsymbol{P}_\beta} + \Delta\mathcal{L}_{\boldsymbol{Y}\boldsymbol{P}_\beta}.$$

Example

We consider the random matrix

$$\boldsymbol{H} = \prod_{n=1}^{N} \boldsymbol{A}_n$$

where the entries of $R \times R$ matrices $\boldsymbol{A}_n, n = 1,\ldots,N$, are iid zero mean and variance $1/R$. Then as $R \to \infty$ we have

$$\Delta\mathcal{L}_{\boldsymbol{H}\boldsymbol{P}_\beta} = N\left(H(\beta) + \beta\log_2\beta\right).$$

Non-High SNR Deviation

Introduce the following parameters

$$m \triangleq \int x d\tilde{\mu}_{\boldsymbol{H}}(x); \quad \hat{m} \triangleq \frac{1}{\int x^{-1}d\tilde{\mu}_{\boldsymbol{H}}(x)}.$$

Let $\mu_{\boldsymbol{H}}$ be not a Bernoulli distribution and have a fixed mean. Define

$$\lambda \triangleq \frac{m}{m-\hat{m}}; \quad \gamma' \triangleq (m-\hat{m})\gamma.$$

Let μ_{MP} be the Marčenko-Pastur distribution with rate parameter λ. Then we have

$$\mathcal{I}_{\sim}(\gamma;\mu_{\boldsymbol{H}}) \approx \alpha\mathcal{I}_{\sim}(\gamma';\mu_{\mathrm{MP}})$$

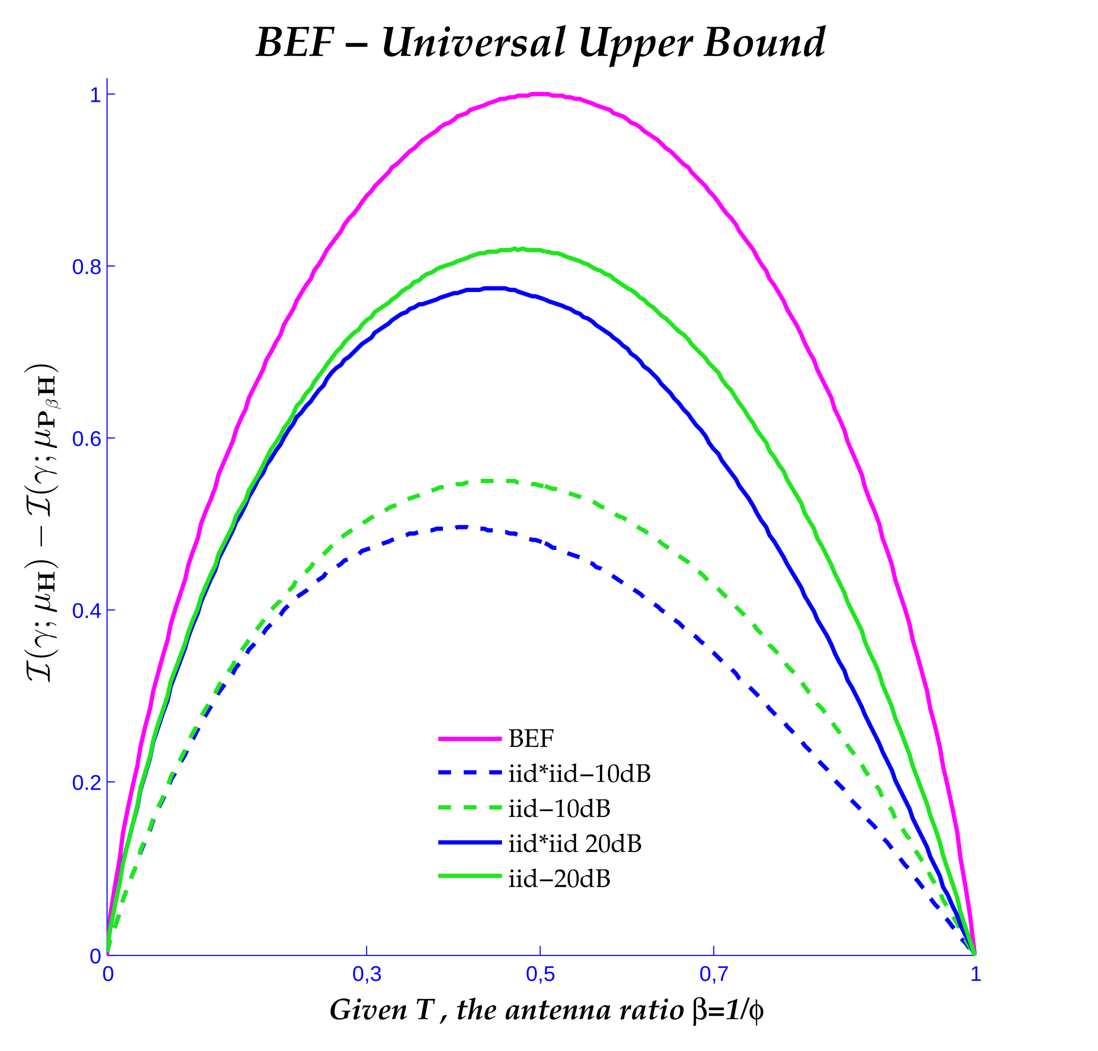
with

$$\mathcal{I}_{\sim}(\gamma;\mu_{\mathrm{MP}}) = \mathcal{I}(\gamma;\mu_{\mathrm{MP}}) - \log_2\gamma - \lambda H(\lambda^{-1}) + 1.$$

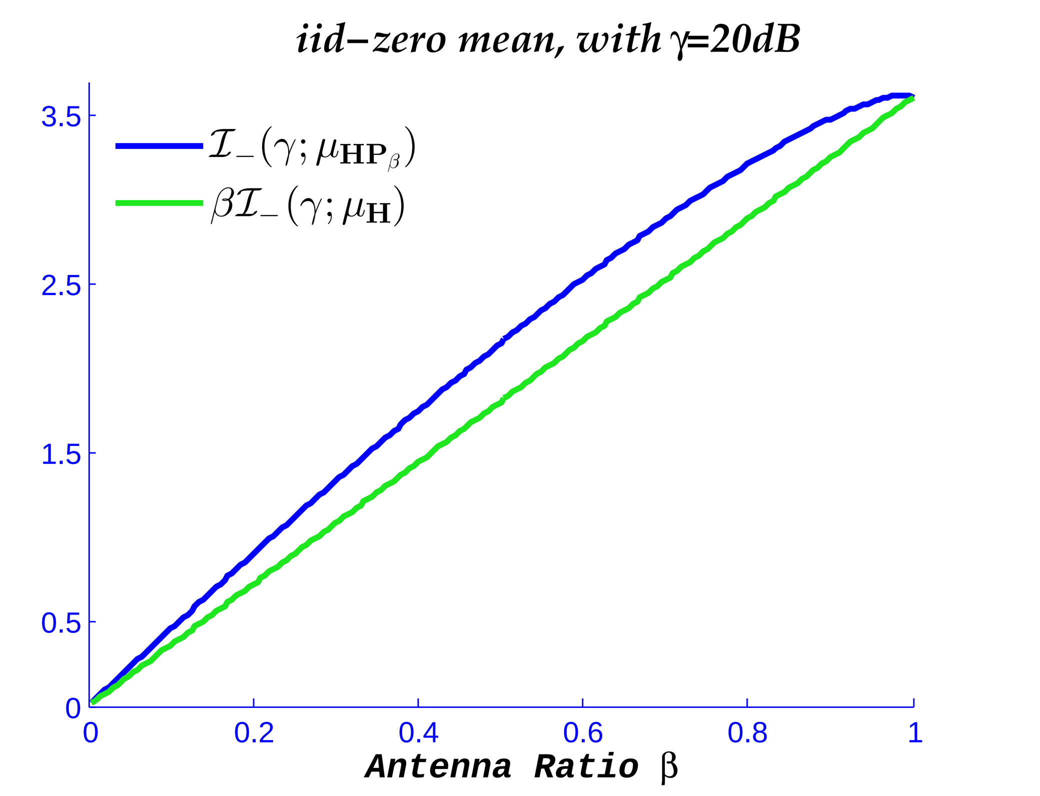
Here $\mathcal{I}(\gamma;\mu_{\mathrm{MP}})$ is the asymptotic capacity of the MIMO channel with iid zero mean fading coefficients [4, Eq.(9)].
Finally we obtain the capacity approximation

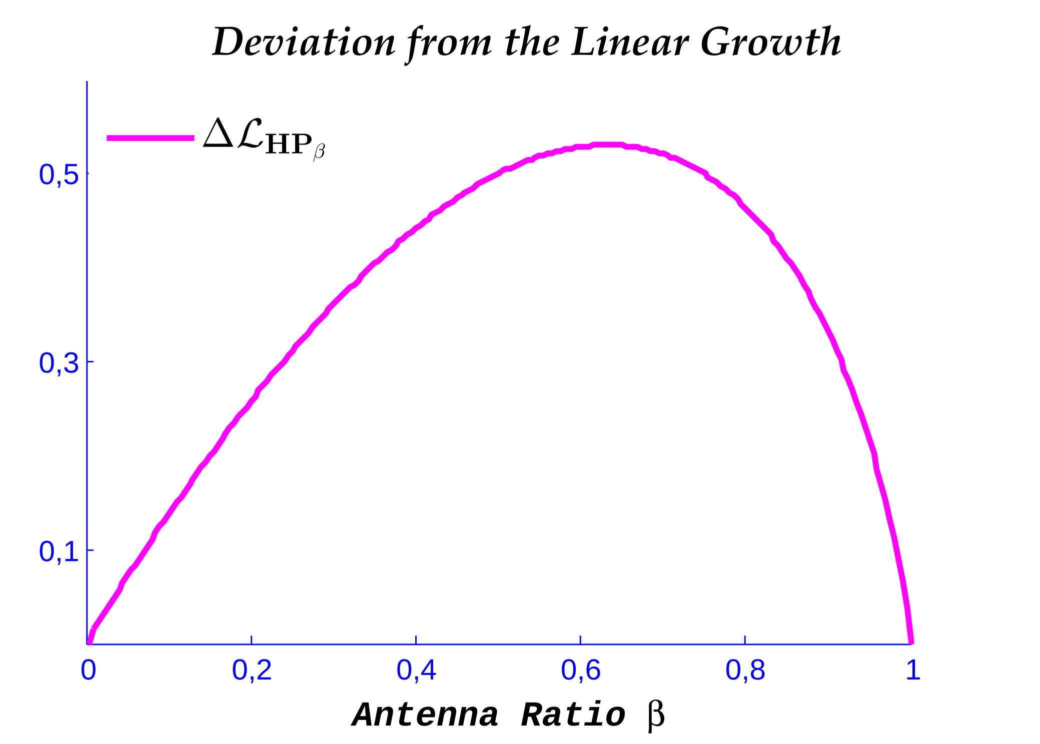
$$\mathcal{I}_{\cong}(\gamma;\mu_{\boldsymbol{H}}) \triangleq \mathcal{I}_{-}(\gamma;\mu_{\boldsymbol{H}}) + \alpha\mathcal{I}_{\sim}(\gamma';\mu_{\mathrm{MP}}).$$

Numerics

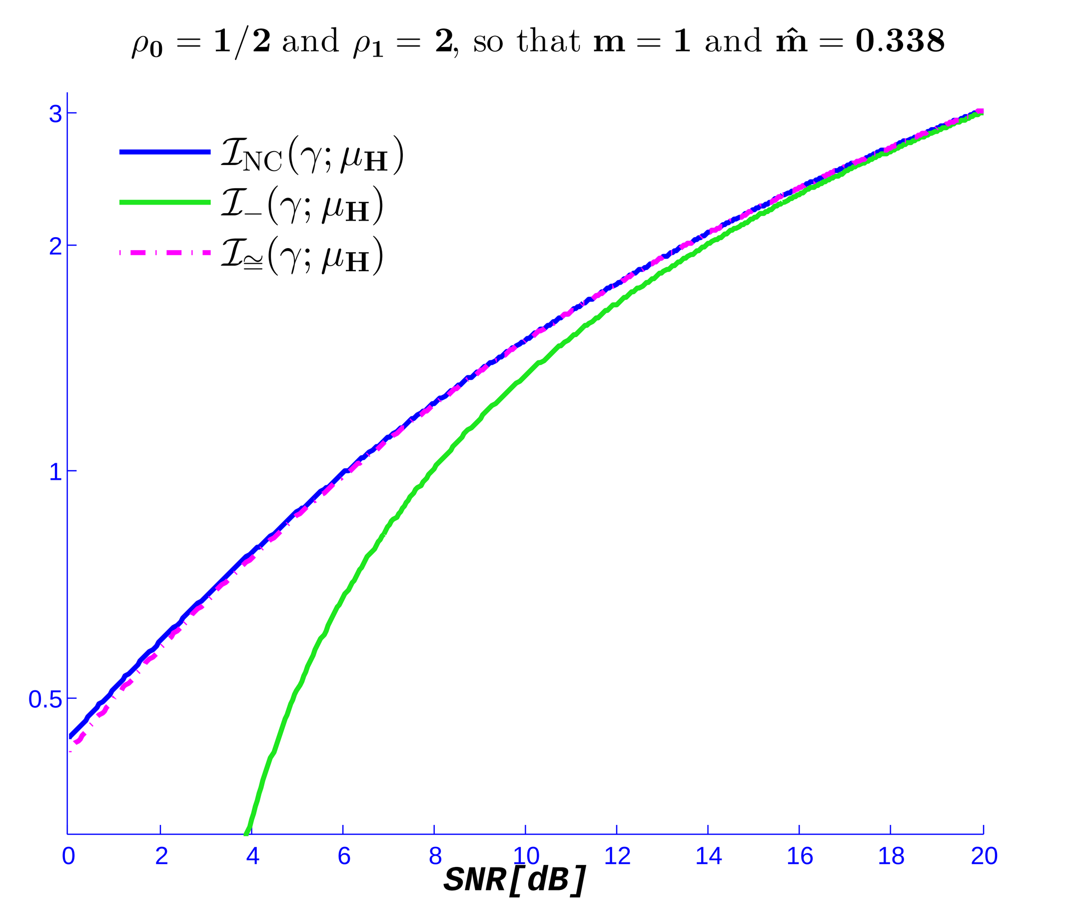


Numerics





Numerics



Comparison of the asymptotic capacity of the concatenated scattering channel [5] with the numerical computation $\mathcal{I}_{\mathrm{NC}}$ [5, Eq.(54)] and our analytical approximation of it $\mathcal{I}_{\cong}$. The parameters $\rho_0 = T/R$ and $\rho_1 = S/R$ are the antenna ratio and the ratio between the number of scatterers around the receiver S and R respectively. For the sake of visual quality the y-axis is in logarithm scale .