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BEYOND MULTIPLEXING GAIN IN LARGE MIMO SYSTEMS

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Preliminary Notations

Notation 1 We denote the binary entropy function (BEF) as

$$H(p) = (p-1)\log_2(1-p) - p\log_2 p, \quad p \in [0, 1]$$

with the convention $0\log_2 0 = 0$.

Notation 2 Consider a $\phi N \times N$ random matrix \mathbf{X} . As $N \rightarrow \infty$, let $\mathbf{X}\mathbf{X}^\dagger$ have a limiting eigenvalue distribution (LED), denoted by $\mu_{\mathbf{X}}$.

Introduction

Consider a $\phi T \times T$ MIMO channel \mathbf{H} . Assume that $\mu_{\mathbf{H}}$ exits as $T \rightarrow \infty$. Further define

$$\alpha \triangleq 1 - \mu_{\mathbf{H}}(0)$$

$$\tilde{\mu}_{\mathbf{H}}(x) \triangleq (1 - 1/\alpha)u(x) + \mu_{\mathbf{H}}(x)/\alpha$$

with $u(x)$ being the unit-step function.

For no CSI at Tx and perfect CSI at Rx, the mutual information (MU) is given by

$$\mathcal{I}(\gamma; \mu_{\mathbf{H}}) = \underbrace{\alpha \int_0^\infty \log_2(\gamma x) d\tilde{\mu}_{\mathbf{H}}(x)}_{\mathcal{I}_-(\gamma; \mu_{\mathbf{H}})} + \underbrace{\alpha \int_0^\infty \log_2\left(1 + \frac{1}{x\gamma}\right) d\tilde{\mu}_{\mathbf{H}}(x)}_{\mathcal{I}_\sim(\gamma; \mu_{\mathbf{H}})}$$

where γ denotes the SNR.

- $\mathcal{I}_-(\gamma; \mu_{\mathbf{H}})$ is a lower bound of the MU; it gives exact capacity in high SNR regime.
- $\mathcal{I}_\sim(\gamma; \mu_{\mathbf{H}})$ is the deviation of the lower bound from the exact MU.

Definitions

Definition 1 A projector \mathbf{P}_β is a diagonal matrix $\mathbf{P}_\beta \in \{0, 1\}^{N \times N}$ with the ratio $\beta = \text{tr}(\mathbf{P}_\beta)/N$ fixed as $N \rightarrow \infty$.

Definition 2 Let $\mathbf{H} \in \mathbb{C}^{R \times R}$ have a LED and be free of an $R \times R$ projector \mathbf{P}_β as $R \rightarrow \infty$. Then we define the deviation of the linear capacity growth in the LSL as

$$\Delta\mathcal{L}_{\mathbf{H}\mathbf{P}_\beta} \triangleq \mathcal{I}_-(\gamma; \mu_{\mathbf{H}\mathbf{P}_\beta}) - \beta\mathcal{I}_-(\gamma; \mu_{\mathbf{H}})$$

- $\mathcal{I}_-(\gamma; \mu_{\mathbf{H}})$ is the growth rate of the normalized multiplexing gain β .

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Binary Entropy Increase

Consider a MIMO system with ϕT receive and T transmit antennas. Let the $M \times M$ matrix \mathbf{P}_β be a projector with $M \triangleq \max(T, \phi T)$ and $\beta \triangleq \min(\phi, 1/\phi)$. Assume the following:

1. The channel matrix \mathbf{H} has full rank with probability one.
2. T tends to infinity with \mathbf{H} having a limiting singular value distribution and being free of \mathbf{P}_β .

Then we have

$$\mathcal{I}_-(\gamma; \mu_{\mathbf{H}}) - \mathcal{I}_-(\gamma; \mu_{\mathbf{P}_\beta\mathbf{H}}) = H(\beta); \quad \phi \geq 1$$

$$\mathcal{I}_-(\gamma; \mu_{\mathbf{H}^\dagger}) - \mathcal{I}_-(\gamma; \mu_{\mathbf{P}_\beta\mathbf{H}^\dagger}) = H(\beta); \quad \phi < 1$$

Additivity of the Deviation

Consider a random matrix $\mathbf{H} = \mathbf{X}\mathbf{Y}$ with $\mathbf{X} \in \mathbb{C}^{R \times R}$ and $\mathbf{Y} \in \mathbb{C}^{R \times R}$. Assume that

1. \mathbf{H} has almost surely full rank.
2. \mathbf{X} , \mathbf{Y} and the $R \times R$ projector \mathbf{P}_β have a LED each and are free of each other as $R \rightarrow \infty$.

Then we have

$$\Delta\mathcal{L}_{\mathbf{H}\mathbf{P}_\beta} = \Delta\mathcal{L}_{\mathbf{X}\mathbf{P}_\beta} + \Delta\mathcal{L}_{\mathbf{Y}\mathbf{P}_\beta}$$

Example

We consider the random matrix

$$\mathbf{H} = \prod_{n=1}^N \mathbf{A}_n$$

where the entries of $R \times R$ matrices $\mathbf{A}_n, n = 1, \dots, N$, are iid zero mean and variance $1/R$. Then as $R \rightarrow \infty$ we have

$$\Delta\mathcal{L}_{\mathbf{H}\mathbf{P}_\beta} = N(H(\beta) + \beta\log_2\beta)$$

Non-High SNR Deviation

Introduce the following parameters

$$m \triangleq \int x d\tilde{\mu}_{\mathbf{H}}(x); \quad \hat{m} \triangleq \frac{1}{\int x^{-1} d\tilde{\mu}_{\mathbf{H}}(x)}$$

Let $\mu_{\mathbf{H}}$ be not a Bernoulli distribution and have a fixed mean. Define

$$\lambda \triangleq \frac{m}{m - \hat{m}}; \quad \gamma' \triangleq (m - \hat{m})\gamma$$

Let μ_{MP} be the Marčenko-Pastur distribution with rate parameter λ . Then we have

$$\mathcal{I}_\sim(\gamma; \mu_{\mathbf{H}}) \approx \alpha\mathcal{I}_\sim(\gamma'; \mu_{\text{MP}})$$

with

$$\mathcal{I}_\sim(\gamma; \mu_{\text{MP}}) = \mathcal{I}(\gamma; \mu_{\text{MP}}) - \log_2 \gamma - \lambda H(\lambda^{-1}) + 1$$

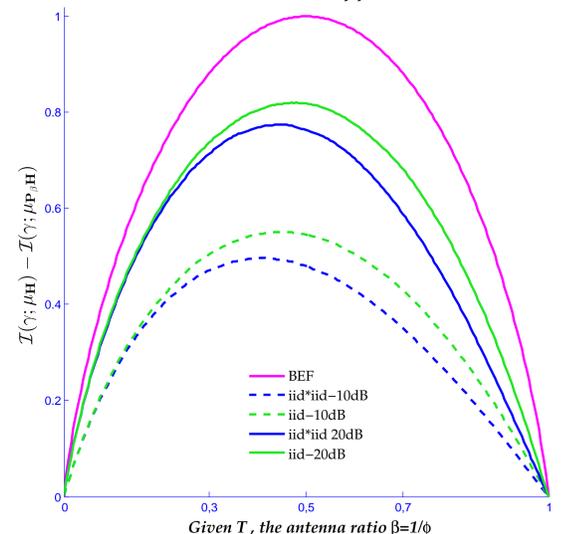
Here $\mathcal{I}(\gamma; \mu_{\text{MP}})$ is the asymptotic capacity of the MIMO channel with iid zero mean fading coefficients [4, Eq.(9)].

Finally we obtain the capacity approximation

$$\mathcal{I}_\approx(\gamma; \mu_{\mathbf{H}}) \triangleq \mathcal{I}_-(\gamma; \mu_{\mathbf{H}}) + \alpha\mathcal{I}_\sim(\gamma'; \mu_{\text{MP}})$$

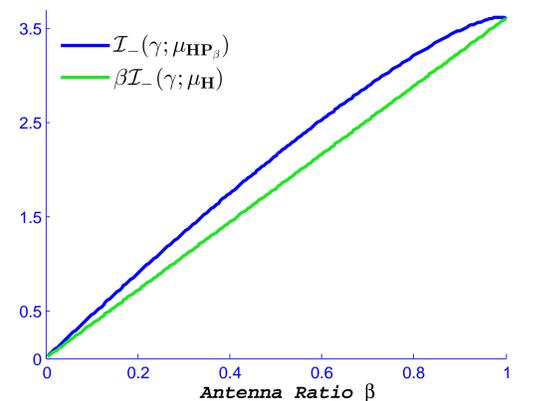
Numerics

BEF – Universal Upper Bound

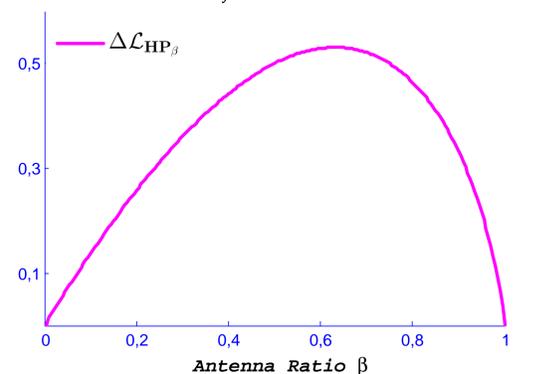


Numerics

iid-zero mean, with gamma=20dB

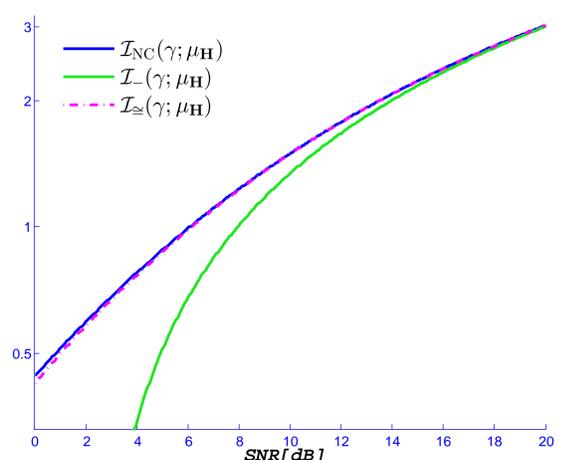


Deviation from the Linear Growth



Numerics

rho_0 = 1/2 and rho_1 = 2, so that m = 1 and m-hat = 0.338



Comparison of the asymptotic capacity of the concatenated scattering channel [5] with the numerical computation \mathcal{I}_{NC} [5, Eq.(54)] and our analytical approximation of it \mathcal{I}_\approx . The parameters $\rho_0 = T/R$ and $\rho_1 = S/R$ are the antenna ratio and the ratio between the number of scatterers around the receiver S and R respectively. For the sake of visual quality the y -axis is in logarithm scale.