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Dynamic production monitoring in pig herds II. Modeling and monitoring farrowing rate at herd level

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ABSTRACT

Good management in animal production systems is becoming of paramount importance. The aim of this paper was to develop a dynamic monitoring system for farrowing rate. A farrowing rate model was implemented using a dynamic generalized linear model (DGLM). Variance components were pre-estimated using an expectation-maximization (EM) algorithm applied on a dataset containing data from 15 herds, each of them including insemination and farrowing observations over a period ranging from 150 to 800 weeks. The model included a set of parameters describing the parity-specific farrowing rate and the re-insemination effect. It also provided reliable forecasting on weekly basis. Statistical control tools were used to give warnings in case of impaired farrowing rate. For each herd, farrowing rate profile, analysis of model components over time and detection of alarms were computed. The model provided a good overview of the development of the parity specific farrowing rate over time and the control charts were able to detect impaired results. Suggestions for future improvements include addition of parity-specific control charts, calibration of the charts for use in practice and inclusion of a sow effect in the farrowing model.

1. Introduction

One aspect of paramount importance in the swine industry is reproduction. According to Hughes and Varley (1980), reproduction includes different aspects, of which one of them is conception. Conception is conditioned by several factors, such as boar and time of insemination, seasonal effects, feed intake, age and genotype, artificial insemination, lactation length, etc. The measurement of "conception rate" is not very precise, since it has to be measured indirectly as the percentage of sows that do not return to oestrus 21 days after service, or be based on pregnancy diagnosis at about 30 days post-service. The farrowing rate is a more reliable numeric indication of the success of the conception: it is defined as the ratio of the total number of farrowings divided by the total number of matings, expressed as a percentage (Hughes and Varley, 1980).

The influence of several aspects on reproductive performance has been largely reported in the literature. Several authors have reported an influence of parity on farrowing rate in sows (Hoving et al., 2010; Hughes, 1998; Jørgensen and Ali, 1993; Koketsu et al., 1997; Kristensen and Søllested, 2004a; Le Cozler et al., 1998; Tummaruk

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Table 1
Number of farrowings according to parities and inseminations.

Parity	Event	Insemination			
		1	2	3	4
1	Inseminated	46 266	4449	453	90
	No farrowing	7050	1298	224	51
2	Inseminated	34 139	2707	218	27
	No farrowing	4132	665	84	17
3	Inseminated	26 830	1656	118	13
	No farrowing	2787	443	38	7
4	Inseminated	21 206	1321	107	15
	No farrowing	2339	357	51	4
5	Inseminated	16 259	1000	85	6
	No farrowing	1860	304	34	5
6	Inseminated	11 460	593	53	3
	No farrowing	1332	199	26	2
7	Inseminated	5834	306	34	7
	No farrowing	771	122	23	5
8	Inseminated	2411	111	7	0
	No farrowing	318	31	4	0

et al., 2010). The general pattern seems to be that first parity sows have a relatively low farrowing rate which increases over the first few parities with a maximum around Parity 3, whereafter it again decreases. In addition, Jørgensen and Ali (1993) estimated a modest reduction in farrowing rate for gilts and sows returning to oestrus. The reduction increased with the number of times the sow returned to oestrus. Lower farrowing rates after insemination 2, 3, and 4 have been assumed in several published replacement models, as for instance Huirne et al. (1991) and Jalvingh et al. (1992) (the latter referring to Bisperink, 1979). Even though the referred studies agree on reduced farrowing rates for sows returning to oestrus, there are diverging opinions about the magnitude. According to Jørgensen and Ali (1993) it is only modest (2–3 percentage units per re-insemination), whereas the other authors assume it to be very significant, with 20 percentage units less for second insemination and 35 for third (compared to first insemination).

Literature provides detailed studies in many fields concerning the reproductive performance of sows. However, no information is available on dynamic monitoring of farrowing rate influenced by parity. A simple average of parities is not suitable for monitoring, because it depends heavily on the age structure and, to some extend, on the number of re-inseminations in the herd. An appropriate monitoring system for farrowing rate must adjust for these systematic effects, be able to capture correlations between categories (parity and insemination number), and to develop over time.

An attempt to improve the monitoring of pig production has recently been presented by Bono et al. (2012). In that study, the static litter size model proposed by Toft and Jørgensen (2002) was re-parameterized and implemented in a dynamic linear model. The dynamic setting allowed for sequential weekly updating of parameters at herd and sow level. Furthermore, statistical control tools were applied and implemented to give warnings in case of impaired litter size results. Also the possibility of making predictions was taken into consideration. Nevertheless, other factors, such as farrowing rate, need to be included in order to achieve a more complete monitoring system.

The aim of the work presented in this paper was to further improve methods for monitoring pig production by also including dynamic monitoring of the farrowing rate in sow herds. Farrowing being a binary trait required that a new multivariate binomial filtering technique had to be developed. The paper presents the results of this work in terms of a dynamic generalized linear model (DGLM). Parity and insemination number are the main effects included in the model. To detect systematic deviations, changes or other factors that may influence the farrowing rate, statistical control tools are implemented in order to give warnings in case of impaired farrowing rate results.

The paper is organized as follows: an explorative data analysis is carried out in order to support the formulation of a farrowing rate model. The filtering technique with sequential estimation of parameters is described in the next section with details of the method given in an appendix. After introduction of the detection techniques, the results from the model (including examples of its use) are presented and discussed.

2. Explorative data analysis

Data used in the current study have been provided by the Danish Advisory Center. The dataset consists of 15 herds (also used in Bono et al., 2012), which are only identified by numbers to ensure the anonymity of the farmers. The traits included in the study are: sow identity, parity number, inseminations (by date and insemination number) and resulting farrowings (by date).

An explorative data analysis was performed on the dataset. Farrowing events for up to eight consecutive parities and up to four inseminations are taken into account for each sow in the explorative analysis. Table 1 shows the number of inseminations per parity, according to the insemination number. For instance, for Parity 1, 46 266 sows were inseminated for the first time. Out of the 7050 that failed to conceive, 4449 were inseminated for the second time. Then out of the 4449, 1298 failed to conceive and 453 were inseminated for the third time. This process is repeated for the four inseminations, for each parity. The difference, in number of sows, between the empty sows and the re-inseminated ones is likely to be culled sows.

Table 2 presents an overview of the farrowing rate according to parity and insemination number. For any given parity, the first insemination has a noticeable higher rate than the following inseminations, even at Parity 8 (0.87). Already from the second insemination there is a reduction of the farrowing rate, as compared to the first insemination (0.87 vs 0.70 on average). Inseminations 3 and 4 have low values and high standard deviations due to a fewer number of observations as compared to the first and the second inseminations.

The explorative analysis gave an overview of the data and confirmed the influence of the parity number on the farrowing rate, as documented by several authors (Hoving et al., 2010; Hughes, 1998; Jørgensen and Ali, 1993; Koketsu et al., 1997; Kristensen and Søllested, 2004a; Le Cozler et al., 1998;

Table 2
Observed farrowing rate according to parities and inseminations.

Parity	Inseminations			
	1	2	3	4
1	0.85	0.71	0.51	0.43
2	0.88	0.75	0.61	0.37
3	0.90	0.73	0.68	0.46
4	0.89	0.73	0.52	0.73
5	0.89	0.70	0.60	0.17
6	0.88	0.66	0.51	0.33
7	0.87	0.60	0.32	0.29
8	0.87	0.72	0.43	–
Mean ± SD	0.87 ± 0.015	0.70 ± 0.084	0.50 ± 0.111	0.43 ± 0.177

Tummaruk et al., 2010). It also confirmed a lower farrowing rate for gilts and sow returning to oestrus, and that the farrowing rate is further reduced after each consecutive insemination (except here, for the fourth insemination at Parity 4; the number of sows was only 15). The farrowing model applied in this study will therefore reflect the literature findings and the patterns observed in this explorative analysis.

3. The farrowing rate model

A simple average is not suitable for monitoring farrowing rate, because it depends heavily on the age structure and, to some extend, on the number of re-inseminations in the herd. A monitoring system for farrowing rate must adjust for these systematic effects and be able to capture the correlations between categories (parity and insemination number) and their development over time.

Since conception is a binary trait it is natural to model the farrowing rate on the logistic scale. We shall denote the farrowing rate as p_{nj} for parity n insemination j (where $j>1$ corresponds to sows returning to oestrus) and the corresponding logistic transform as η_{nj} where

$$\eta_{nj} = \log \frac{p_{nj}}{1-p_{nj}}. \tag{1}$$

A simple model for the systematic effects of parity and insemination number could be as follows:

$$\eta_{nj} = \begin{cases} \theta_n - (j-1)\theta_7, & n \le 5 \\ \theta_5 - (n-5)\theta_6 - (j-1)\theta_7, & n > 5. \end{cases} \tag{2}$$

The model allows the adaptation of the farrowing rate based on the parity number. Thus, the mean farrowing rate of the first five parities (θ_1 to θ_5) is directly specified in the parameter vector θ, whereas the following parities need the consideration of the negative slope (θ_6). The effect of re-insemination (θ_7) is also included. It reflects the decrease of farrowing rate according to the insemination number for a given reproductive cycle of a sow.

4. Sequential estimation technique

A multivariate dynamic generalized linear model (DGLM) consisting of an observation equation and a system equation will be applied. We will use weekly observations of farrowing to update the herd profile as described by Eq. (2). The latent parameter vector for week t will be

$$\theta_t = (\theta_{1t}, \theta_{2t}, \theta_{3t}, \theta_{4t}, \theta_{5t}, \theta_{6t}, \theta_{7t})', \tag{3}$$

where the parameters θ_{1t}–θ_{5t} correspond to the farrowing rate for the first five parities at week t, θ_{6t} represents the negative slope and θ_{7t} is the effect of the re-insemination.

4.1. Observation equation

The observation vector Y_t consists of elements, y_{njt}, corresponding to all combinations of parity n and insemination number j. The individual observation y_{njt} is the number of inseminations resulting in a farrowing no later than week $t+17$ out of N_{njt} inseminated at week t, where 17 weeks correspond to the maximum gestation length of sows. Combinations of n and j where $N_{njt}=0$ are left out of the observation vector.

The observation equations linking the observations to the parameters have the general form

$$y_{njt}|\theta_t \sim \mathcal{B}(N_{njt}, p_{njt}), \tag{4}$$

where $\mathcal{B}$ denotes the binomial distribution. The farrowing rate p_{njt} is equal to $(\exp(-\eta_{njt})+1)^{-1}$ (cf. Eq. (1)), and it depends on the parameter vector θ_t as follows:

$$\eta_t = F'_t \theta_t, \tag{5}$$

where F_t is called the *design matrix*. The number of columns corresponds to the size of θ_t, and the number of rows corresponds to the number of non-zero values of N_{njt}.

Now assume that, in week t, N_{njt} sows are inseminated and y_{njt} of them will farrow no later than week $t+17$. Assume for week t that the sows for different combinations of parity n and insemination number j are characterized as shown in Table 3. The observation vector would then be $Y_t = (10, 1, 1, 9, \ldots, 4, 0, 3)'$, and the design matrix will then look as follows (cf. Eq. (2)):

$$F' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 & 0 \end{pmatrix} \tag{6}$$

Table 3
Example of insemination results for sows inseminated in week t grouped by parity and insemination number within parity in a herd.

Parity n	Insemination number j	Inseminations N_{njt}	Resulting farrowings y_{njt}
1	1	12	10
1	2	2	1
1	3	1	1
2	1	10	9
⋮	⋮	⋮	⋮
8	1	5	4
8	2	1	0
9	1	3	3

with the corresponding parameter vector $\theta_t = (\theta_{1t}, \theta_{2t}, \theta_{3t}, \theta_{4t}, \theta_{5t}, \theta_{6t}, \theta_{7t})'$.

4.2. System equation

The system equation expresses how the parameter values may change over time. The general form of the system equation is

$$\theta_t = G_t\theta_{t-1} + w_t, \tag{7}$$

where G_t is called the *system matrix*, and $w_t \sim \mathcal{N}(\underline{0}, W)$ where $\underline{0}$ is a vector of zeros and W_t is a variance–covariance matrix describing the evolution variance of each of the parameters (and the covariance). Since no particular systematic trend or pattern is expected, we assume that $G_t = I$, where I is the identity matrix. For the variance–covariance matrix W_t the following structure is assumed:

$$W_t = \begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} & W_{15} & 0 & 0 \\ W_{12} & W_{22} & W_{23} & W_{24} & W_{25} & 0 & 0 \\ W_{13} & W_{23} & W_{33} & W_{34} & W_{35} & 0 & 0 \\ W_{14} & W_{24} & W_{34} & W_{44} & W_{45} & 0 & 0 \\ W_{15} & W_{25} & W_{35} & W_{45} & W_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & W_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & W_{77} \end{pmatrix}. \tag{8}$$

As it is seen, the changes of the parameters $\theta_1, \ldots, \theta_5$ are assumed to be mutually correlated, but independent of changes in other model elements.

4.3. Weekly updating

The concept of *univariate* DGLM is theoretically well described in the literature (e.g. West and Harrison, 1997; West et al., 1985). For an example of application in pig production using binomially distributed data, reference is made to Cornou and Lundbye-Christensen (2012). When it comes to *multivariate* binomial models no applications known to the authors have been published even though Jørgensen et al. (1996) presented a generalized approach for multivariate time series of mixed types. However, the method does not allow for binary data to be analyzed. In this paper, the univariate binomial technique is extended also to cover multivariate models. In Appendix A an updating technique relying on Taylor expansion of the conditional probability function of y_{njt} given η_{njt} is briefly described. A key property of the technique is the fact that, for given η_t, the observations y_{njt} are independent. Using the described technique, it is possible to obtain weekly updated estimates for θ_t and thus η_t.

4.4. Initialization

In order to have a full specification of the DGLM, the initial information $\theta_0 \sim \mathcal{N}(m_0, C_0)$ before anything has been observed in the herd must be defined. Because a binomial model is considered for farrowing rate, the values are on logistic scale. The initial means based on the results of the explorative data analysis (Table 2) of the seven parameters are, in order from θ_1 to θ_7: 1.71, 1.98, 2.15, 2.08, 2.04, 0.05 and 1.03. For the variance–covariance matrix it is just assumed, that the seven standard deviations correspond to a coefficient of variation of 40% and that the parameters are mutually independent. This crude approach is justified by the fact that as soon as the DGLM is applied to data from a specific herd, the model will automatically adapt to the conditions of that herd. The initial settings are therefore of minor importance.

4.5. EM-algorithm

System variance (W) is estimated trough the use of the expectation-maximization (EM) algorithm. It is an iterative algorithm based on maximum likelihood (ML) estimation. The free software R (R Development Core Team, 2012) was used to compute the algorithms. The EM technique is described in more details by Bono et al. (2012).

5. Detection of impaired farrowing rate results

Monitoring tools are applied in order to detect any critical changes in the farrowing rate. The deviations between the observations and the predicted values are analyzed in a short and long time periods. For the short term control, charts inspired by Shewhart (Montgomery, 2005) are used in order to detect alarms on a weekly basis. For the long term, a V-mask applied to the cumulative sum (Cusum) control chart is used to detect level changes in the farrowing rate. For further details about these monitoring methods reference is made to Bono et al. (2012).

For both short and long term monitoring, the following components are extracted from the model: the forecast for the observation vector at time t which has the mean μ_t (A.14) and the variance Σ_t (A.16), shown in Appendix A.

Let $\underline{1} = (1, , 1)$ be a row vector consisting of only elements with value 1. The forecast for the total number of farrowings in week t is therefore $\underline{1}\mu_t$ with variance $\underline{1}\Sigma_t\underline{1}'$ and the observed total number is $\underline{1}Y_t$. Thus the weekly forecast error, e_t, is

$$e_t = \underline{1}Y_t - \underline{1}\mu_t. \tag{9}$$

For the control charts, the observation in week t is e_t and the standard deviation used for control limits, is

$$s_t = \sqrt{\underline{1}\Sigma_t\underline{1}'}. \tag{10}$$

Thus, the numerical value of s_t will depend heavily of the number of sows farrowing at week t.

Because $\underline{1}Y_t$ is a sum of several different binomial distributions with unknown value of the probability parameter, the distribution of the forecasted number of farrowings is not normal, so a standard Shewhart chart with symmetric control limits is not well suited. Due to the basically binomially distributed data with unknown probabilities, the control limits must be un-symmetric. In order to adapt the limits to this kind of data, a beta-binomial distribution was fitted in such a way that the mean and variance corresponded to the mean, $\underline{1}\mu_t$, and variance, $\underline{1}\Sigma_t\underline{1}'$, of the forecast distribution. The lower control limit was defined as the 0.025 quantile of the beta-binomial distribution, and the upper was defined as the 0.975 quantile. Both the limits were defined as integers: for the upper control limit, the integer was rounded up, and for the lower control limit, the integer was rounded down. This procedure broadens slightly the control limits, implying that for each limit (upper and lower) there is a significance level that corresponds to less than 2.5%.

For the V-mask, the cumulative sum (Cusum) is defined as the sum of the standardized forecast errors

$$C_t = \sum_{t=1}^{t} \frac{e_t}{s_t}. \tag{11}$$

The value of the lead distance of the V-mask was set to $d=10$ and the slope of the arms of the mask was set to 0.4 in the examples shown in this paper. Detailed description of the setting and its choice are available in Barnard (1959) and Montgomery (2005).

6. Results

Results of the system variance estimation, of the model application and of the monitoring methods are shown in this section. All 15 herds are included in the analysis.

6.1. System variance

The values of the variance–covariance matrix converged after 45 000 iterations of the EM-algorithm. Convergence of the variance components of farrowing rate for the first five parities and the correlation between them are shown in Fig. 1(a) and (b). Convergence of the slope and the re-insemination effect is shown in Fig. 1(c) and (d).

The values of the variance–covariance matrix are presented in Table 4. The variance of the parities parameters θ_1, θ_2, θ_3, θ_4, θ_5, the slope θ_6 and the effect of the re-insemination θ_7 are presented in the diagonal. Correlations between the five parities are shown below the diagonal. The highest correlation was found between Parities 3 and 4 (0.99) and the lowest between Parities 1 and 5 (0.62).

6.2. Farrowing rate profiles

Fig. 2(a) shows the farrowing rate profiles of the 15 herds, for first insemination. The shape of the profiles is not always as expected. A few herds show an increase of farrowing rate between Parities 4 and 5. This may be explained by a lower number of observations at Parity 5 (combined with the considerably higher variance for θ_5 seen in Table 4), as compared to Parity 4. Herds 6 and 7 have a lower profile, as compared to the other herds, and particularly the gilts seem to have problems with conception in these herds. The farrowing rate at Parity 1 in the most cases confirms that the farrowing rate is lower for gilts than for sows. On the other hand Herd 1 shows a high farrowing rate at Parity 1, which subsequently decreases at Parity 2. Herd 9 presents the highest farrowing rate's profile. Fig. 2(b) presents the profile of the first four inseminations for Herd 8. The distance between two consecutive inseminations reflects the effect of the re-insemination.

The estimated value of the seven model parameters for the 15 herds, obtained at the end of the observation period, is available in Table 5. In order to have an easier

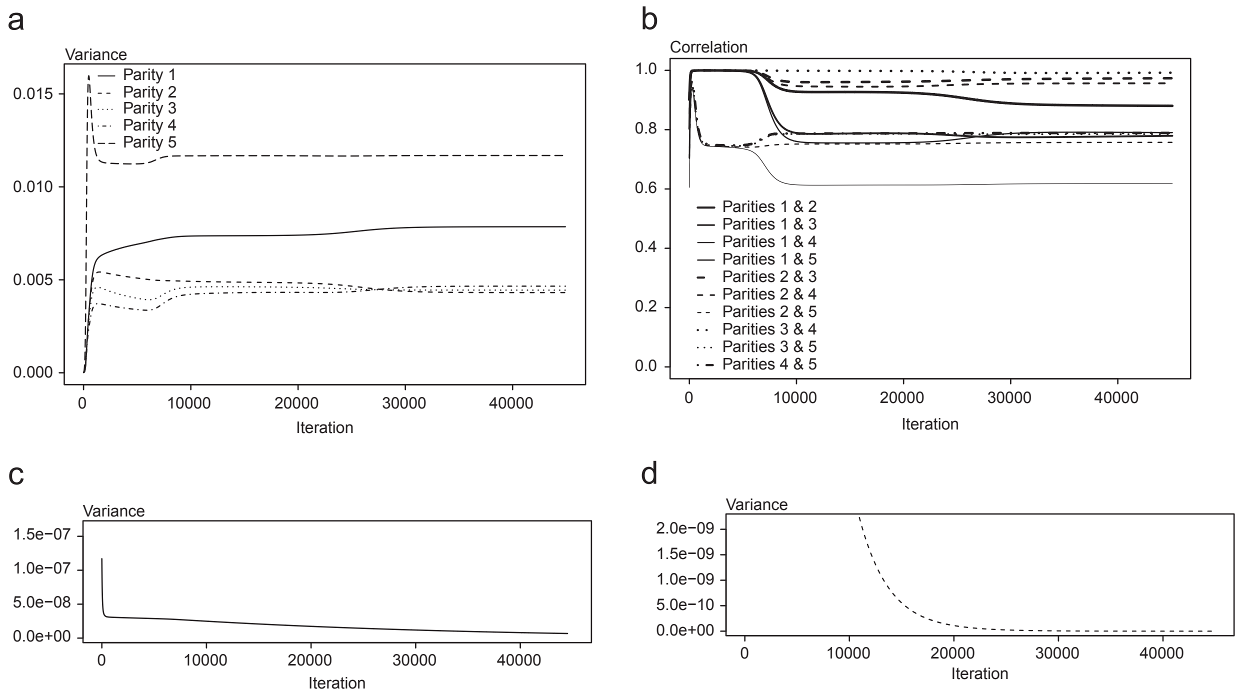

Fig. 1. Representation of convergence using the EM-algorithm, 45 000 iterations. (a) Convergence of the system variance in the first five parities. (b) Convergence of system correlations in the first five parities. (c) Convergence of the system variance of the slope. (d) Convergence of the system variance of the re-insemination.

Table 4
System variance–covariance $W(7\times7)$. Values of the correlations are shown below the diagonal.

W	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
θ_1	0.00785	0.00512	0.00462	0.00477	0.00592	0	0
θ_2	*0.88*	0.00431	0.00426	0.00428	0.00537	0	0
θ_3	*0.79*	*0.97*	0.00444	0.00451	0.00565	0	0
θ_4	*0.78*	*0.95*	*0.99*	0.00465	0.00582	0	0
θ_5	*0.62*	*0.76*	*0.78*	*0.79*	0.01168	0	0
θ_6	*0*	*0*	*0*	*0*	0	5.45e−09	0
θ_7	*0*	*0*	*0*	*0*	0	*0*	4.25e−13

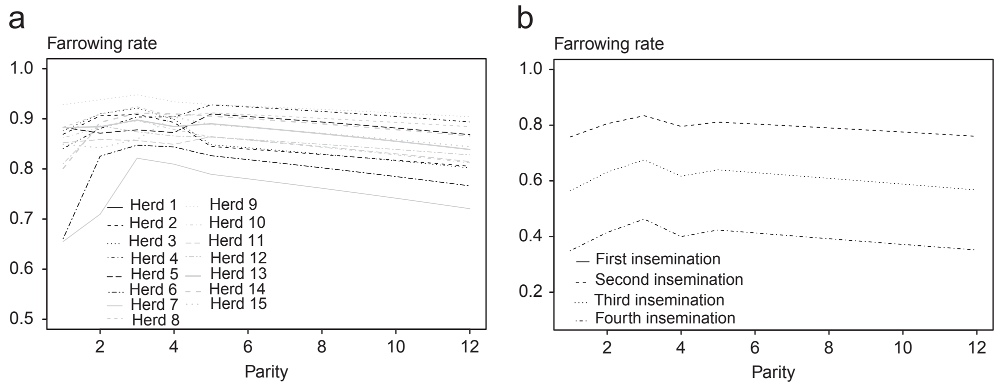

Fig. 2. Farrowing rate profiles from the DGLM. (a) For first insemination, 15 herds. (b) For four inseminations, Herd 8.

Table 5
Estimated values of the model parameters for the 15 herds at the end of the observation period. The estimated farrowing rates p_{nj} for first insemination at Parities 1–5 are calculated as $p_{n1}=(\exp(-\theta_n)+1)^{-1}$, $n=1,\ldots,5$. Bottom part: mean, minimum, maximum and standard deviation.

Herd	Farrowing rates (1st ins.), Par. 1–5					Slope	Re-insemination
	p_{11}	p_{21}	p_{31}	p_{41}	p_{51}	θ_6	θ_7
1	0.90	0.85	0.86	0.86	0.84	0.05	0.75
2	0.87	0.91	0.91	0.89	0.84	0.04	0.96
3	0.88	0.91	0.92	0.90	0.85	0.05	0.78
4	0.84	0.88	0.90	0.90	0.93	0.06	0.59
5	0.88	0.87	0.88	0.87	0.91	0.06	0.59
6	0.66	0.83	0.85	0.84	0.83	0.05	0.93
7	0.66	0.71	0.82	0.81	0.79	0.05	0.69
8	0.88	0.91	0.92	0.90	0.91	0.04	0.88
9	0.93	0.94	0.95	0.93	0.93	0.05	0.88
10	0.86	0.88	0.90	0.88	0.86	0.06	0.91
11	0.85	0.86	0.86	0.85	0.86	0.05	0.69
12	0.81	0.89	0.87	0.87	0.86	0.04	0.93
13	0.88	0.88	0.90	0.88	0.89	0.06	0.69
14	0.80	0.89	0.91	0.91	0.91	0.06	1.11
15	0.85	0.84	0.86	0.90	0.89	0.06	0.79
Mean	0.84	0.87	0.89	0.88	0.87	0.05	0.81
Min	0.66	0.71	0.82	0.81	0.79	0.04	0.59
Max	0.93	0.94	0.95	0.93	0.93	0.06	1.11
SD	0.08	0.05	0.03	0.03	0.04	0.01	0.15

interpretation of the values, the first five parameters are presented in a probabilistic scale (i.e. $p_{n1}=(\exp(-\theta_n)+1)^{-1}$), while the last two ($\theta_6$, θ_7) remain in the logistic scale. Mean, minimum, maximum and standard deviation are shown in the bottom part. The standard deviations indicate the variation between herds, and Parity 1 shows the most variation between herds, as depicted in Fig 2(a).

6.3. Model components

Fig. 3 shows a detailed analysis of the DGLM components for Herd 8, over the last three years. Filtered mean for the first five parities, θ_1–θ_5, are shown in Fig. 3(a), and the corresponding smoothed components in Fig. 3(b). The smoothed values of the slope (Fig 3(c)) and re-insemination effect (Fig. 3 (d)) appear almost constant. The evolution of the means over time (both filtered and smoothed) indicates that the first four parities follow a similar pattern, whereas Parity 5 shows a sudden level change at the beginning of the period (around week 300). The farrowing rate increases to a peak around week 360 and decreases to a local minimum around week 390.

Fig. 4 shows smoothed data of Herds 11 and 14, and illustrates specific patterns observed over time. Herd 11 shows a very clear seasonal pattern, and Herd 14 shows a parity-specific deviation (Parity 1). Whereas Herd 11 was the only herd showing a clear seasonal variation, parity-specific deviations were observed for six other herds.

6.4. Detections of alarms in farrowing rate

Monitoring methods on the short and the long term period were applied for all 15 herds individually. The use of a control chart for weekly monitoring is presented in Fig. 5(a). The central line (black plain line) corresponds to the differences between observed and predicted values. The dotted lines are the (asymmetric) control limits. No alarm was observed for the considered time-span

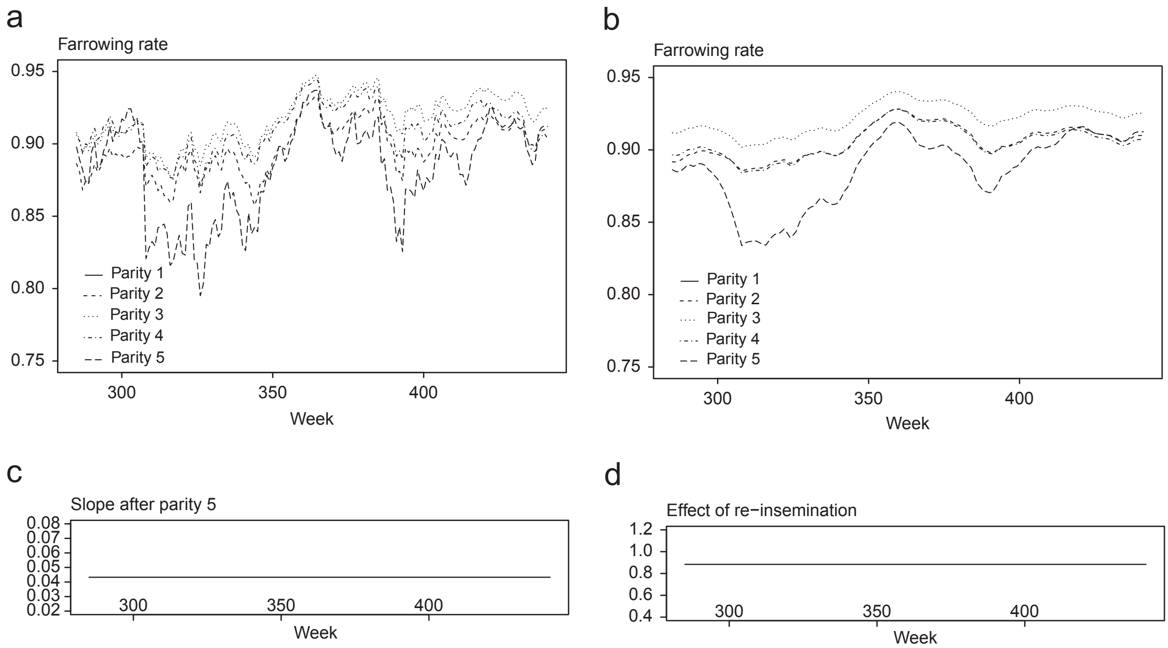

Fig. 3. Evolution of the model parameters for Herd 8 over the last three years. (a) Filtered parity components: θ_1, θ_2, θ_3, θ_4 and θ_5. (b) Smoothed parity components. (c) Smoothed slope (θ_6). (d) Smoothed re-insemination effect (θ_7).

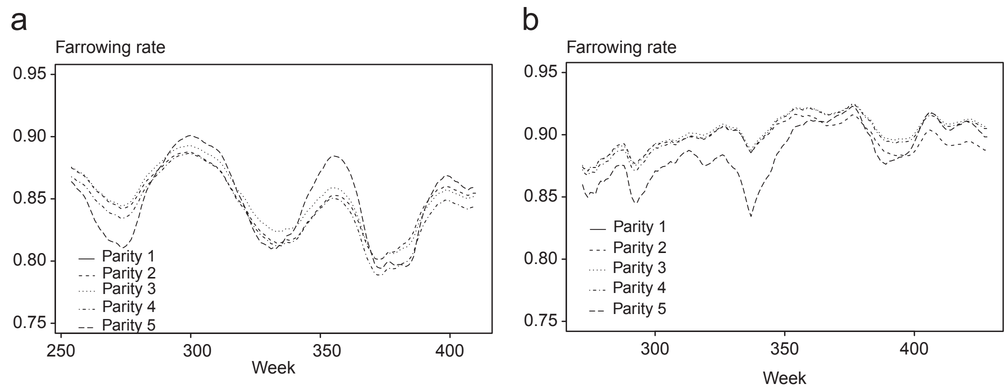

Fig. 4. Smoothed data of Herd 11 and Herd 14. (a) Seasonal pattern of Herd 11. (b) Parity-specific deviation (Parity 1) of Herd 14.

(26 weeks). The long term monitoring is performed by the use of Cusum combined with V-mask in Fig. 5(b). During the three years period, two level changes are detected: at weeks 360 and 390.

Table 6 shows the number of alarms detected applying the V-masks (left panel) and the control charts (right panel). A total of 42 alarms have been found in a 3 yr period using the V-masks, which indicates an average of 0.93 alarm per herd per year. If only the negative alarms are taken into account, the average is 0.64 alarm per herd per year. The larger number of negative alarms (29 vs. 13) may be explained by the fact that the V-mask is set equally sensitive to changes in both directions (despite the skew distribution). On the other hand, for the control charts, where asymmetric control limits are used, a total of 49 alarms is reported. This indicates an average of 1.08 alarms per herd per year (0.48 for negative alarms only). There the total of positive and negative alarms is more consistent (22 vs. 27).

7. Discussion

Variance components were pre-estimated using the EM-algorithm technique. More than 40 000 iterations were necessary before convergence was obtained. This is far more than for the normally distributed litter size data in Bono et al. (2012), where only 400 iterations were needed. On the other hand, it is clear from Fig. 1(a) that the changes in values seen after iteration 800 are only small and insignificant. Even though the system variance of the slope and re-insemination effect did not converge as expected, the size of these values became so small that it was considered irrelevant to carry on with more iterations. High correlations between Parities 2 and 3 (0.97), Parities

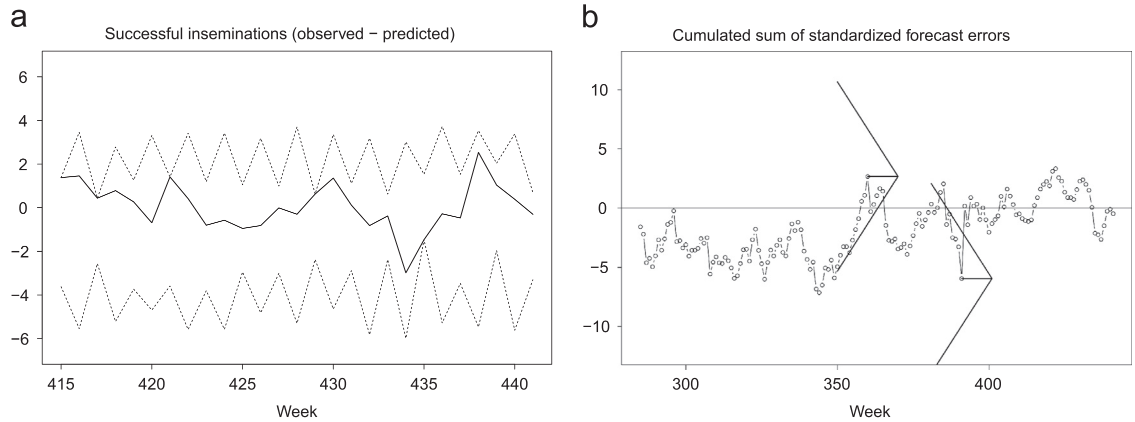

Fig. 5. Monitoring methods applied for Herd 8. (a) Short monitoring period (26 weeks). The central line represents the differences between observed and predicted values. The dotted lines are the upper and lower control limits. (b) Long monitoring period (156 weeks) using a V-mask applied on a Cusum.

Table 6
Number of alarms in a 156 weeks period according to the detection methods: V-mask (VM) and control chart (CC).

Herd number	VM decrease	VM increase	CC decrease	CC increase
1	1	0	3	2
2	1	1	1	0
3	0	0	2	0
4	2	1	0	1
5	4	0	1	1
6	3	2	4	3
7	5	1	2	3
8	1	1	2	1
9	1	1	1	4
10	2	0	1	0
11	3	3	0	1
12	0	0	1	0
13	3	2	2	4
14	2	0	2	3
15	1	1	0	4
Sum	29	13	22	27

3 and 4 (0.99) and Parities 2 and 4 (0.95) were found. The lowest value of correlation was found between Parities 1 and 5 (0.62). High correlations imply that the farrowing rate profiles will be maintained, i.e. that the farrowing rate of the first five parities will not drift independently of each other.

Coefficients of variation (CV) were calculated to improve the comprehension of the size of the system variance. For instance, the weekly variance of Parity 3 (0.00444) indicates a yearly variance of 0.23088. Compared to the average value for Parity 3, which is 2.09 (logistic transform of the probability 0.89 of Table 5) this implies an annual coefficient of variation of 0.23. This is a far higher value than seen for litter size profiles in Bono et al. (2012) where a similar coefficient of variation was estimated as low as 0.013. Thus, the litter size profile seems to be a far more stable property of a sow herd than the farrowing rate profile.

Herd 8 was used to illustrate the main results (Fig. 3(a) and (b)). The smoothed mean at a given time includes the knowledge from all observations (previous and future), and is computed backwards. It allows therefore to reduce the temporary random fluctuation observed in the filtered data. Filtered and smoothed data are not directly comparable. With the filtered data, the farmer is able to see what happens in real time. On the other hand smoothed data enables the farmer to identify and follow up on problems that may have occurred during the production process. See Bono et al. (2012) for further details. For practical application, if the smoothed data drift too much, hindering as such the interpretation of results, it can be suggested to reduce the size of the system variance (only for smoothing purpose).

The farrowing rate profiles for first insemination for the 15 herds do not look as homogeneous as the litter size profiles from Bono et al. (2012). This is a natural consequence of the much higher system variance for the farrowing rate. The peak of the farrowing rate is usually around Parity 3. For Parity 1, Herd 1 showed a high farrowing rate, likely due to a good quality of gilts. However, already at the second parity the percentage decreased sharply. Herds 4, 5 and 11 showed an increase of the farrowing rate between Parities 4 and 5. This may be due to the number of observations related to these parities and the high system variance for Parity 5 (cf. Table 4).

As it has been described by several authors, the parity number influences the farrowing rate (Koketsu et al., 1997; Le Cozler et al., 1998; Tummaruk et al., 2010). The negative slope used to describe the decrease in farrowing rate for high parities is in this study remarkably small (cf. Table 5). If there is a positive repeatability (i.e. a sow effect) in farrowing results as suggested by Jørgensen and Ali (1993), it means that data are censored because farmers tend to cull sows returning to oestrus. Sows that survive until high parity numbers will therefore also tend to have a better fertility resulting in overestimation of farrowing rates for high parities compared to an unrealistic situation with no culling. This is a possible explanation for the small value of the negative slope. A complementary explanation for the unexpected profile of herds 4, 5 and 11, could also be the absence of a sow effect in the model.

The value of the slope is an important component of the shape of the profile. It reflects the trend of the farrowing rate after Parity 5. A value of the slope close to 0

(Fig. 3(c)) indicates a persistent farrowing rate after Parity 5. This low reduction of farrowing rate was observed for all herds, and may indicate that farmers adopt, in general, a good culling strategy.

There are divergent opinions among authors about the magnitude of the reduction of farrowing rate for sows returning to oestrus. Jørgensen and Ali (1993) reported a reduction in percentage units of 2–3 per re-insemination. Other authors (Bisperink, 1979) found a much larger reduction. In this study, the decrease of farrowing rate per re-insemination is around 10 percentage units (Table 5). Again, an explanation for the larger reduction found in this study compared to Jørgensen and Ali (1993) could be the lack of a sow effect accounting for the repeatability in our model. As described by Jørgensen and Ali (1993), the size of the repeatability heavily influences the reduction of farrowing rate for sows returning to oestrus. If a sow effect had been included, the percentage of decrease of farrowing rate per re-insemination might approach the one reported by Jørgensen and Ali (1993).

The lowest farrowing rate was observed for Herd 7 and the highest (more than 90%) for Herd 9. A seasonal pattern was observed for Herd 11 only. A parity-specific deviation was observed for Parity 1 for Herd 14, and points out a problem specific to gilts from week 360 until the end of the period analyzed. Deviation of a single parity during the three years period was seen in almost half of the herds of the dataset.

Detection methods were applied to monitor changes in a short (weekly) and longer time-span. In Herd 8, no alarms were triggered during the period used to illustrate the short term monitoring (26 weeks). Nevertheless, the values observed around weeks 415, 417, 421, 429 and 435 were at the control limits (Fig. 5(a)). As mentioned in Section 5, the control limits were defined by integer values, for which the rounding procedure resulted in broader limits. It can therefore be discussed whether some of these weeks should have been considered as problematic. A potential tool to reduce the uncertainty during the decision process is the addition of “warning limits”, which would narrow the range of allowed deviations before a “warning” alarm is triggered.

As for the long term monitoring, V-masks were applied on the Cusums. As compared to the previous paper (Bono et al., 2012), the method does not appear entirely satisfactorily. In order to balance the number of negative and positive alarms, a different set up for the two arms of the V-mask could be implemented (one narrower than the other). Furthermore, it has been noticed that the method was unable to detect changes in the model specific parameters. For instance, in Fig. 3(b), the “drop” of Parity 5 around week 310 was not detected. Similar situations were noticed in Fig. 4(b) with a “drop” for Parity 1 after week 370, and for Herds 2–6 and 15. A suggestion for improving the monitoring method may therefore be to implement, concomitantly with the current method, a parity-specific alarm system. Finally, the seasonal pattern observed for Herd 11 triggered both positive and negative alarms at each fluctuation. Some monitoring systems may gain from including seasonal components in the model (Madsen and Kristensen, 2005), if this feature is inherent to the monitored variable. However, since it is desirable to keep a stable farrowing rate throughout the year, any cyclic variation should be detected (so that the farmer becomes aware of the problem), and hence not modeled.

Results of the total number of alarms were presented in Table 6. The percentage of alarms (positive and negative) for the control charts was, in average, 2%. This should be put in perspective with the 95% confidence intervals used in this study, which then should have resulted in about 5% of false alarms. It was arbitrarily decided to round the integers to the higher (upper limit) and lower (lower limit) values, which resulted in broader control limits. The opposite way to find the integer values would therefore result in more alarms, which may be closer to the expected 5%.

Further developments for the suggested monitoring system may include (i) the addition of a parity-specific monitoring system, (ii) a modified V-mask, and (iii) the inclusion of a sow effect in order to improve its accuracy. This may add value for the replacement strategy at the farm level (see Kristensen and Søllested, 2004a, 2004b).

The third and final step of this project is the development of a dynamic monitoring system for the mortality rate of sows and pre-weaned piglets which will be described in a subsequent paper.

8. Conclusion

A system to monitor farrowing rate was developed. It is based on a dynamic generalized linear model, with weekly updates, combined with monitoring methods for short (weekly) and long term periods. The farrowing rate profile does not appear as homogeneous as expected, and may be influenced by censoring. For practical implementation, a calibration of the settings of the control chart and V-mask under known production circumstances needs to be performed. The combination of this model with the previous work (Bono et al., 2012) and the inclusion of information about mortality rate, will help developing a management tool to help the farmers to monitor production, make decision, prevent problems, and reduce economical losses.

Conflict of interest statement

The authors report that there is no conflict of interest relevant to this publication.

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Appendix A. A multi variate binomial DGML

A.1. Sequential updating

The observation model is as specified in Eqs. (4) and (5) and the system equation is given by Eq. (7). Let $D_t = \{m_0, C_0\} \cup \{Y_1, \ldots, Y_t\}$ be the full information set at time t. Denote as

$$(\theta_{t-1}|D_{t-1}) \sim \mathcal{N}(m_{t-1}, C_{t-1}) \tag{A.1}$$

the posterior for the parameter vector at time $t-1$. It follows from standard arguments that the prior for the parameter vector at time t is

$$(\theta_t|D_{t-1}) \sim \mathcal{N}(a_t, R_t) \tag{A.2}$$

where

$$a_t = G_t m_{t-1} \quad \text{and} \quad R_t = G_t C_{t-1} G'_t + W_t. \tag{A.3}$$

Using standard rules in combination with Eq. (5) yields

$$(\eta_t|D_{t-1}) \sim \mathcal{N}(f_t, Q_t) \tag{A.4}$$

where

$$f_t = F'_t a_t \quad \text{and} \quad Q_t = F'_t R_t F_t. \tag{A.5}$$

We shall denote the farrowing rate corresponding to an individual element f_{njt} of the vector f_t as p^f_{ijt}. Thus

$$p^f_{ijt} = (\exp(-f_{njt}) + 1)^{-1}. \tag{A.6}$$

Since the individual observations y_{njt} are independent given η_t it follows from Bayes' theorem that the posterior distribution of η_t after observation of $Y_t = (y_{11t}, \ldots, y_{njt}, \ldots, y_{\bar{n}\bar{j}t})'$ is given as

$$p(\eta_t|D_t) \propto p(\eta_t|D_{t-1}) \prod_{n,j} p(y_{njt}|\eta_{njt}), \tag{A.7}$$

where p is the probability density function. It can be shown by Taylor expansion of $(\partial/\partial\eta_{njt}) \log p(y_{njt}|\eta_{njt})$ around $\eta_{njt} = f_{njt}$ that Eq. (A.7) can be approximated by

$$(\eta_t|D_t) \sim \mathcal{N}(f^*_t, Q^*_t), \tag{A.8}$$

where

$$Q^*_t = (Q_t^{-1} + \hat{V}_t^{-1})^{-1} \quad \text{and} \quad f^*_t = Q^*_t(Q_t^{-1} f_t + \hat{V}_t^{-1} \hat{\eta}_t), \tag{A.9}$$

with

$$\hat{V}_{njt} = \frac{1}{N_{njt} p^f_{njt}(1 - p^f_{njt})}, \quad \hat{V}_t = \mathrm{diag}(\hat{V}_{11t}, \ldots, \hat{V}_{\bar{n}\bar{j}t}) \tag{A.10}$$

and

$$\hat{\eta}_{njt} = f_t + \frac{y_{njt} - N_{njt} p^f_{njt}}{N_{njt} p^f_{njt}(1 - p^f_{njt})}, \quad \hat{\eta}_t = (\hat{\eta}_{11t}, \ldots, \hat{\eta}_{\bar{n}\bar{j}t})'. \tag{A.11}$$

Finally, the posterior for $(\theta_t|D_t) \sim \mathcal{N}(m_t, C_t)$ is identified by

$$m_t = a_t + R_t F_t Q_t^{-1}(f^*_t - f_t) \quad \text{and} \quad C_t = R_t - R_t F_t Q_t^{-1}(Q_t - Q^*_t) Q_t^{-1} F'_t R_t. \tag{A.12}$$

A.2. Dealing with singular variance–covariance matrix

In cases where the rank of F'_t is less than the number of rows, the variance–covariance matrix Q_t of Eq. (A.5) becomes singular and cannot be inverted as it must in Eq. (A.9). In those cases the following stepwise updating technique is applied:

- Denote as F'_t the full design matrix built as described in Section 4.1.
- Mark all rows of F'_t as "Not processed".
- Set $k = 0$.
- Continue until all rows of F'_t have been marked as "Processed":
  - Increment k by one.
  - Build the design matrix F'_{kt} for step k row by row by conditionally adding not processed rows from F'_t. A row is added if, and only if, the rank of F'_{kt} remains equal to the number of rows. If a row is added, it is marked as "Processed" in F'_t and the corresponding observations of N_{njt} and y_{njt} are added to the vectors N_{kt} and Y_{kt}.
  - The matrix F'_{kt} and the vectors N_{kt} and Y_{kt} are used for updating to m_{kt} and C_{kt} as described in A.1.
  - Set $a_t = m_{kt}$ and $R_t = C_{kt}$.
- Set $m_t = m_{kt}$ and $C_t = C_{kt}$.

A.3. Forecast distribution

The forecast distribution $p(Y_t|D_{t-1})$ is deduced from the simultaneous distribution $p(Y_t, \eta_t|D_{t-1})$ (where p denotes the probability/density function). We first notice that (according to standard rules)

$$p(Y_t, \eta_t|D_{t-1}) = p(Y_t|\eta_t, D_{t-1}) p(\eta|D_{t-1}) = p(Y_t|\eta_t) p(\eta_t|D_{t-1}),$$

where the last expression follows from the fact that η_t summarizes all previous information. Similarly

$$p(Y_t, \eta_t|D_{t-1}) = p(\eta_t|Y_t, D_{t-1}) p(Y_t|D_{t-1}) = p(\eta_t|D_t) p(Y_t|D_{t-1}),$$

where the last term is the requested distribution. Combining the two expressions for $p(Y_t, \eta_t|D_{t-1})$ yields

$$p(Y_t|\eta_t) p(\eta_t|D_{t-1}) = p(\eta_t|D_t) p(Y_t|D_{t-1})$$

or

$$p\left(Y_t \middle| D_{t-1}\right) = p\left(Y_t \middle| \eta_t\right) \frac{p(\eta_t|D_{t-1})}{p(\eta_t|D_t)}. \tag{A.13}$$

In Eq. (A.13) the probability $p(Y_t|\eta_t)$ is just the product of probabilities from independent binomial distributions with natural parameters η_{njt}. We have

$$p\left(Y_t \middle| \eta_t\right) = \prod_{nj} \binom{N_{njt}}{y_{njt}} \frac{(1 - (e^{-\eta_{njt}} + 1)^{-1})^{N_{njt} - y_{njt}}}{(e^{-\eta_{njt}} + 1)^{y_{njt}}}.$$

The two conditional expressions $p(\eta_t|D_{t-1})$ and $p(\eta_t|D_t)$ are the multivariate normal density functions for the distributions known from Eqs. (A.4) and (A.8). Thus, for instance

$$p\left(\eta_t \middle| D_{t-1}\right) = \frac{1}{(2\pi)^{|Y_t|/2} \sqrt{\det Q_t}} \exp\left(-\frac{1}{2}(\eta_t - f_t)' Q_t^{-1} (\eta_t - f_t)\right).$$

For the forecast mean we get

$$\mathrm{E}(Y_t|D_{t-1}) = \mathrm{E}(\mathrm{E}(Y_t|\eta_t)|D_{t-1}) \approx \mathrm{diag}(N_{11t}, \ldots, N_{\bar{n}\bar{j}t}) p^f_t. \tag{A.14}$$

For the forecast variance–covariance matrix we use the general rule for random variables X and Y that

$\mathrm{Var}(X) = \mathrm{E}(\mathrm{Var}(X|Y)) + \mathrm{Var}(\mathrm{E}(X|Y))$ and obtain

$$\mathrm{Var}(Y_t|D_{t-1}) = \mathrm{E}(\mathrm{Var}(Y_t|\eta_t)|D_{t-1}) + \mathrm{Var}(\mathrm{E}(Y_t|\eta_t)|D_{t-1}). \tag{A.15}$$

The first term is approximated by the (independent) binomial variances, i.e.

$$\mathrm{E}(\mathrm{Var}(Y_t|\eta_t)|D_{t-1}) \approx \Delta_t$$

where

$$\Delta_t = \mathrm{diag}(N_{11t}p^f_{11t}(1-p^f_{11t}), \ldots, N_{\bar{n}\bar{j}t}p^f_{\bar{n}\bar{j}t}(1-p^f_{\bar{n}\bar{j}t})).$$

For the second term a Taylor expansion around f_{fjt} is used. Denoting the inverse of the logistic transform as the expit function (cf. Eq. (A.6)) we get, element by element

$$\begin{aligned}
\mathrm{E}(y_{njt}|\eta_{njt}) &= N_{njt}p_{njt} = N_{njt}\mathrm{expit}(\eta_{njt}) \\
&\approx N_{njt}\mathrm{expit}(f_{njt}) + N_{njt}\mathrm{expit}'(f_{njt})(\eta_{njt}-f_{njt}) \\
&= N_{njt}\mathrm{expit}(f_{njt}) + N_{njt}\mathrm{expit}(f_{njt}) \\
&\quad \times(1-\mathrm{expit}(f_{njt}))(\eta_{njt}-f_{njt}) \\
&= N_{njt}p^f_{njt} + N_{njt}p^f_{njt}(1-p^f_{njt})(\eta_{njt}-f_{njt})
\end{aligned}$$

where the third line follows from the fact that $\mathrm{expit}'(x) = \mathrm{expit}(x)(1-\mathrm{expit}(x))$. In matrix notation we get

$$\mathrm{E}(Y_t|\eta_t) \approx \mathrm{diag}(N_{11t}, \ldots, N_{\bar{n}\bar{j}t})p^f_t + \Delta_t(\eta_t - f_t),$$

and it follows, since $\mathrm{Var}((\eta_t - f_t)|D_{t-1}) = Q_t$, that

$$\mathrm{Var}(\mathrm{E}(Y_t|\eta_t)|D_{t-1}) \approx \Delta_t Q_t \Delta'_t.$$

With reference to Eq. (A.15) we therefore conclude that

$$\mathrm{Var}(Y_t|D_{t-1}) \approx \Delta_t + \Delta_t Q_t \Delta'_t. \tag{A.16}$$

Thus, the partially specified forecast distribution is approximately given by

$$(Y_t|D_{t-1}) \sim [\mu_t, \Sigma_t],$$

where

$$\mu_t = \mathrm{diag}(N_{11t}, \ldots, N_{\bar{n}\bar{j}t})p^f_t \quad \text{and} \quad \Sigma_t = \Delta_t + \Delta_t Q_t \Delta'_t.$$

In cases where a stepwise updating is done, the forecasting must be performed stepwise as well.

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