Load Estimation from Modal Parameters

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Abstract

In Natural Input Modal Analysis the modal parameters are estimated just from the responses while the loading is not recorded. However, engineers are sometimes interested in knowing some features of the loading acting on a structure. In this paper, a procedure to determine the loading from a FRF matrix assembled from modal parameters and the experimental responses recorded using standard sensors, is presented. The method implies the inversion of the FRF which, in general, is not full rank matrix due to the truncation of the modal space. Furthermore, some recommendations are included to improve the accuracy in the load estimation. Finally, the results of an experimental program carried out on a simple structure are presented.

Nomenclature

Stiffness matrix	$[k]$	Mass matrix	$[m]$
Damping matrix	$[c]$	Force vector	$\{f(t)\}$
FRF matrix	$[H(\omega)]$	Displacement vector	$\{u(t)\}$
Natural frequency	ω_r	Spectral matrix	$[S(\omega)]$
Un-scaled mode shape	$\{\psi\}_r$	Scaled mode shape	$\{\phi\}_r$
Damping factor	ζ_r	Scaling factor	α_r

1 Introduction

Operational modal analysis has become a powerful technology to estimate the modal parameters in a wider range of applications, mainly for big structures [1]. Testing is normally performed by just measuring the responses under the natural or operational conditions, i.e., the structure is excited by natural or operational loads such as wind loads, wave loads, traffic loads, etc. When the structure is tested in the laboratory, artificial loads are to be used applying some random tapping on the structure.

In operational modal analysis the forces are not recorded. Nevertheless, the force acting on a structure can still be estimated using the responses at several points of the structure together with the frequency response function, FRF [2]. The responses can be recorded using appropriate sensors and the FRF matrix can be constructed from the experimental modal parameters.

In operational modal analysis we can obtain the mode shapes, the natural frequencies and the damping ratios. However, the mode shape scaling is arbitrary causing incorrect modal participation factors. Recently, different techniques to obtain the right scaling have been developed and are now being tested on full scale structures [3, 4, 5, 6]. The results of these tests indicate that for even larger structures it is possible to obtain accurate scaling factors.

In this paper, a procedure to determine loading acting on a structure from a truncated modal model and the experimental responses is presented, together with some recommendations aiming to improve the accuracy of the results. Furthermore, an experimental program has been carried out on a steel cantilever beam to estimate the loading from the measurements and the experimental modal space. The modal parameters are obtained using natural input modal analysis and the scaling factors of the mode shapes by means of the mass change method [3, 4, 5, 6].

2 The method

As well know, the equation of motion of a structure subjected to a force $\{f(t)\}$ is given by:

$$[m]\cdot\{\ddot{u}\}+[c]\cdot\{\dot{u}\}+[k]\cdot\{u\}=\{f(t)\} \tag{1}$$

Transforming equation (1) in frequency domain by Fourier yields:

$$\left(-\omega^2\cdot[m]+j\omega\cdot[c]+[k]\right)\cdot\{U(\omega)\}=\{F(\omega)\} \tag{2}$$

After defining the frequency response function matrix (FRF) or transfer function matrix as:

$$[H(\omega)]=\left(-\omega^2\cdot[m]+j\omega\cdot[c]+[k]\right)^{-1} \tag{3}$$

and substituting equation (3) in equation (2), it results:

$$\{U(\omega)\}=[H(\omega)]\cdot\{F(\omega)\} \tag{4}$$

Thus, the loading in frequency domain can be calculated using:

$$\{F(\omega)\}=[H(\omega)]^{-1}\cdot\{U(\omega)\} \tag{5}$$

To solve equation (5), the FRF matrix and the responses have to be known. On one hand, the experimental responses are recorded using standard sensors. On the other hand, the FRF can be constructed from the experimental modal parameters obtained by operational modal analysis using a stationary broad band excitation. Alternatively, a finite element model can be updated using the modal parameters.

The force in time domain can be obtained applying the inverse Fourier transform to $\{F(\omega)\}$. The process is schematically shown in Figure 1.

The spectral density function load matrix can then be obtained from the spectral density function response matrix by means of the expression:

$$[S_{FF}(\omega)] = [H(\omega)]^{-1} \cdot [S_{UU}(\omega)] \cdot [H(\omega)]^{-H} \tag{6}$$

where the superscript ^{H} denotes complex conjugate transpose.

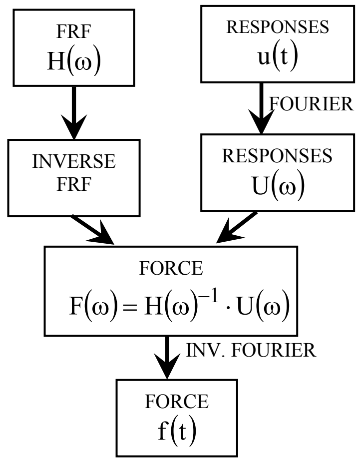

Figure 1. Process to estimate the loading.

3 The FRF matrix from modal parameters

When the modal space is used, the expression of the FRF matrix for complex modes is given by (see [7]):

$$[H(\omega)] = \sum_{r=1}^{N} \left(\frac{Q_r \{\psi_r\} \cdot \{\psi_r\}^T}{j\omega - \lambda_r} + \frac{Q_r^* \{\psi_r\}^* \cdot \{\psi_r\}^{*T}}{j\omega - \lambda_r^*} \right) \tag{7}$$

where:

- $\{\psi_r\}$ is the r-th un-scaled mode shape,
- $\lambda_r = -\zeta_r \omega_r + j\omega_r \sqrt{1-\zeta_r^2}$ is the pole of the r-th mode
- Q_r is a factor which takes into account the scale of the mode [7], and is related to the scaling factor α_r through $Q_r = \dfrac{\alpha_r^2}{2j\omega_r}$, and
- the superscript * denotes complex conjugate

From the natural input modal analysis all modal parameters are known except the scaling factors because the mode shapes are arbitrary scaled. In the last years, several methods have been proposed to estimate the scaling factors by the mass change method. This method consists of attaching several masses to the structure and performing operational modal analysis on both the modified and the unmodified structures [3, 4, 5, 6].The modification is carried out by attaching masses to the points of the structure where the mode shapes of the unmodified structure are known. In order to facilitate the mass modification and the calculation of the scaling factors, lumped masses are often used, so that the mass change matrix becomes, in general, diagonal.

Simple formulas can be used to estimate the scaling factors, such as [4, 6]:

$$\alpha_{01} = \sqrt{\frac{\left(\omega_0^2 - \omega_1^2\right)}{\omega_1^2 \cdot \{\psi_0\}^T \cdot [\Delta m] \cdot \{\psi_1\}}}, \tag{8}$$

or [6]:

$$\alpha^* = \frac{\sqrt{\frac{\left(\omega_0^2 - \omega_1^2\right)}{\omega_1^2 \cdot \{\psi_0\}^T \cdot [\Delta m] \cdot \{\psi_0\}}} + \sqrt{\frac{\left(\omega_0^2 - \omega_1^2\right)}{\omega_1^2 \cdot \{\psi_0\}^T \cdot [\Delta m] \cdot \{\psi_1\}}}}{2}. \tag{9}$$

Where the subscript $_0$ indicates original structure, subscript $_1$ indicates modified structure and the matrix $[\Delta m]$ is the mass change matrix.

The accuracy on the scaling factor estimation depends on both the accuracy obtained in the modal analysis and the mass change strategy (magnitude, number and location of the masses) used to modify the dynamic behaviour of the structure [6].

3.1 Inversion of the FRF matrix:

Equations (5) and (6) imply the inversion of the FRF matrix frequency by frequency, but this inversion can only be performed using standard methods when the FRF matrix is full rank, i.e., when the number of modes is equal to the number of observation points. Otherwise, when a truncated modal space is used, the FRF matrix is singular and normal inverse does not apply anymore.

However, the inversion of the FRF matrix may still be done using singular value decomposition (SVD), which for a complex matrix $[H]$ is given by:

$$[H] = [U] \cdot [\Sigma] \cdot [V]^H \tag{10}$$

where:

- $[U]$ and $[V]$ are unitary matrices (orthogonal in case of real matrices), and
- $[\Sigma]$ is a diagonal matrix containing the singular values. The number of non-zero singular values is equal to the number of modes active at the frequency considered, and
- the superscript ^{H} denotes complex conjugate transpose.

Using equation (10) the inverse of the matrix $[H]$ can be obtained as:

$$[H]^{-1} = [V]^{-H} \cdot [\Sigma]^{-1} \cdot [U]^{-1} \tag{11}$$

and taking in account the properties of unitary matrices, i.e.:

$$[U]^H = [U]^{-1} \quad \text{and} \quad [V]^H = [V]^{-1} \tag{12}$$

the equation (11) becomes:

$$[H]^{-1} = [V] \cdot [\Sigma]^{-1} \cdot [U]^H \tag{13}$$

where only the non-zero singular values should be used in the calculation.

Equation (13) provides the exact solution when all modes are considered, but due to the truncation effect, the calculated FRF matrix and its inverse is only representing an approximation. As soon as more modes are considered, better accuracy is achieved. However, it must be taken into account that the singular matrix has to be inverted so that small errors in the singular values can be highly amplified by the inversion process. Therefore, only the biggest singular values in the inversion process should be taken into consideration.

3.2 The FRF matrix from modal updating.

An alternative to the method proposed in the last paragraph is to update a finite element model. The mass and the stiffness matrices can be estimated using standard modal updating procedures.

On the other hand, a proportional damping matrix can be constructed from the experimental modal damping parameters. In case of normal modes, the modal damping can be obtained from the damping matrix by:

$$[C_{ii}] = [\Phi]^T \cdot [c] \cdot [\Phi] \tag{14}$$

If the damping matrix is full rank, the inverse of the damping matrix can be obtained by:

$$[c]^{-1} = [\Phi] \cdot [C_{ii}]^{-1} \cdot [\Phi]^T \tag{15}$$

where $[C_{ii}]$ is a diagonal matrix with $C_{ii} = 2 \cdot \zeta_i \cdot \omega_i$.

The matrix $[c]^{-1}$ can be estimated from equation (13) using the experimental mode shapes and modal damping parameters, but it can not be inverted by standard procedures because of the truncation effect. However, the damping matrix can still be obtained when inverting the matrix $[c]^{-1}$ by singular value decomposition (the same method as was proposed to invert the FRF matrix).

It has to be noticed that an updated finite element modal is always useful. Both the FRF matrix from modal parameters and the FRF from the updated finite element model can be used to evaluate the importance of the truncated modes. A similar and easier interpretation can be done when comparing the singular values of both models (Figure 2).

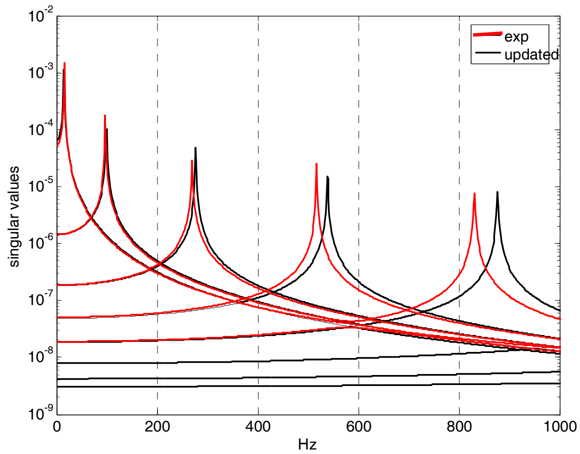

Figure 2. Singular values of a FRF matrix corresponding to a cantilever beam

The main advantage of the updated FRF matrix is that it is full rank and can be easily inverted. However, the natural frequencies of the updated model will not coincide exactly with the natural frequencies of the modal model, i.e., the peaks in the responses and in the FRF matrix will not be exactly the same at the same frequency, which could influences a lot in the final results.

4 Responses

If the responses are recorded with accelerometers, the displacement vector $u(t)$ can be obtained by double integration in time domain using standard numerical procedures.

Alternatively, the integration can be performed in the frequency domain by means of the expression:

$$\{U(\omega)\} = \frac{\{\ddot{U}(\omega)\}}{\omega^2} \tag{16}$$

and the response spectral density matrix by means of :

$$[S_{UU}(\omega)] = \frac{[S_{\ddot{U}\ddot{U}}(\omega)]}{\omega^4} \tag{17}$$

4.1 Numerical integration.

When a signal is integrated numerically, only the frequencies in the range $0.02f_s < f < 0.1f_s$, where f_s is the sampling frequency, will be treated accurately [8]. Below this range we will have gain and above this range we will have attenuation. Furthermore, when the integration approaches to zero frequency, the gain tends to infinity. The aforementioned problems are amplified because the acceleration has to be integrated twice.

The only solution consists in eliminating the low frequencies using a high pass filter. If it is necessary to determine the integration at very low frequencies, as it is the case of civil structures, then the best solution is to decimate the signal [8].

4.2 Noise in the responses.

When performing experimental measurements, some noise is always present in the signals. This type of error can be reduced using better sensors but can not be removed.

The noise present in the responses affects all the previous stages in the load estimation (modal identification, scaling factor calculation, inversion of the FRF matrix, signal processing, etc.).

When using accelerometers as sensors, the acceleration has to be integrated twice to calculate the force. If the integration is performed in the frequency domain, we have to use equation (14). This equation show that the noise presents in the responses is highly amplified at low frequencies which, on the other hand, represent the most important frequency range for civil structures. The singular value decomposition of the responses provides us useful information to determine qualitatively the noise present in the signals. In general, the first singular values correspond to the real responses meanwhile the rest of singular values represent the noise. Thus, a way to reduce the noise effect in the load calculation is to use only the first singular values. The method consists on the singular value decomposition of the responses, i.e.:

$$[S_{UU}(\omega)] = \sum_i U_i \cdot S_i \cdot V_i^H \tag{18}$$

and then to reassemble a response spectral density matrix using only the first singular values. This new matrix will be used in the load calculation. If we only consider two singular values, the new matrix becomes:

$$[S_{UU}(\omega)]_{new} = U_1 \cdot S_1 \cdot V_1^H + U_2 \cdot S_2 \cdot V_2^H \tag{19}$$

where $S_1 > S_2 > S_3 > \ldots S_N$.

Figure 3 shows the singular value decomposition of the responses recorded on a cantilever beam when an impact is applied. It can be observed that the responses are mainly represented by the first singular value whereas the rest of the singular values depict the noise.

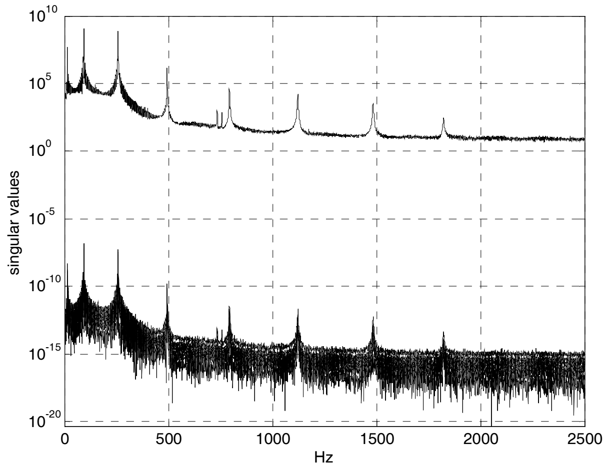

Figure 3. Singular values of the responses.

5 Load calculation

Once the inverse of the FRF matrix and the responses in displacement format have been estimated, the load calculation can be performed. The calculation is carried out in the frequency domain taking segments with an appropriate length (figure 4). The length of the segments must be large enough to allow for a low leakage effect.

The force and the spectral density can be calculated using equations (5) and (6), respectively. If the modal parameters of the structure are known and the responses are measured in real time, then the procedure proposed in this paper can be applied in real time.

Due to the errors in the modal analysis, mainly the damping and the scaling factors, some peaks can appear in the estimated force spectral density. If this type of irregularity is known not be present in load spectral density, a smooth technique can be used to improve the spectral density estimation.

5.1 Leakage reduction.

Equations (5) and (6) involve the Fourier transform of the responses so that the analysis of a finite time record can cause leakage.

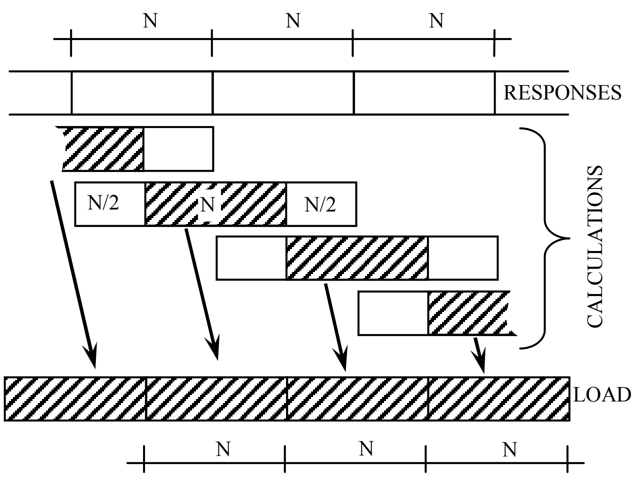

Figure 4. Leakage reduction with $\beta = 2$.

Leakage can be minimized by increasing the size of each data segment. If the force corresponding to N points is calculated, a way to reduce the leakage consists in calculating the force for a larger segment, i.e., a segment of βN points, where $\beta \geq 1$. The so defined segment of data is at the ends of each calculation appended with $N(\beta-1)$ points. Finally, we select the central N points. Thus, the $\frac{(\beta-1)N}{2}$ points of the estimated load on both the right and left sides are discarded. In Figure 2, the load calculation for $\beta = 2$ is shown.

The only disadvantage of this method is that more numerical operations have to be performed to obtain the load.

The aforementioned procedure can be improved combining a larger segment together with an appropriate window.

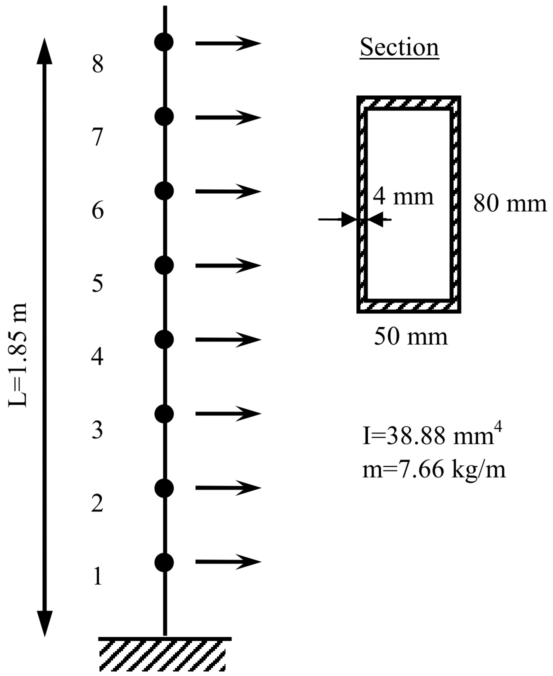

Figure 5. Cantilever beam used in the tests reduction

6 Experimental tests

A steel cantilever beam was used to perform the tests. The beam was 1.85 m long, showing an 80x50x4 tube rectangular section. The responses were measured in 8 degree of freedoms, regularly distributed along the beam (Figure 5). Two types of excitation were used: Stationary broad banded and impact.

6.1 Stationary broad band tests

In order to determine the modal parameters, a stationary broad banded excitation was applied to the structure. The loading was not measured so that a natural input modal analysis software was used to estimate the natural frequencies, the mode shapes and the damping. Only the first 5 modes were considered in the analysis. The modal identification, performed by the enhanced frequency domain decomposition (EFDD) [9], is shown in figure 6.

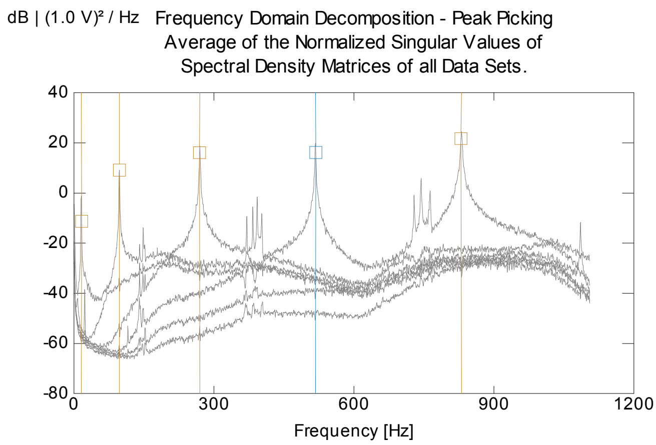

Figure 6. Frequency domain decomposition of measurements.

The scaling factors were determined using the mass change method [3, 4, 5, 6]. A proportional mass change was applied to the structure attaching masses of 180 grams to each degree of freedom (except at the free border of the cantilever beam to which a 90 gram mass was attached). Finally, a new stationary broad banded excitation was applied to the modified structure.

The modal parameters are shown in table 1. As it can be seen, the damping remains very low. The scaling factors shown in the table correspond to mode shapes normalized to unity.

Table 1. Modal parameters of the cantilever beam.

Mode		1	2	3	4	5
Natural frequencies (Hz)	**Original structure**	15.65	97.46	269.71	517.41	830.43
	Modified structure	15.078	93.46	258.3	494.95	793.93
	Frequency shift (%)	3.654	4.10	4.23	4.34	4.40
Damping (%)		0.24	0.13	0.18	0.10	0.12
Scaling factors		0.450	0.425	0.391	0.354	0.354

6.2 Impact tests

In order to check the accuracy of the proposed method, several impacts were applied to each degree of freedom of the unmodified structure. Only one hit was applied every time, using an impact hammer with a rubber tip. The experimental responses, together with the FRF matrix estimated from the modal parameters, were used to estimate the load with the method proposed in this paper.

The force auto spectral density of both the estimated with the proposed method and the recorded from the hammer, corresponding to the 8th degree of freedom, are shown in Figure 7. 4096 frequency lines were used in the calculations. As can be seen, the estimation is reasonable good excepting at the peaks in the resonances which appear due to the errors in the modal parameter estimation.

The experimental responses were recorded at a sampling frequency $f_s = 5000\ \mathrm{Hz}$, so that only the results in the frequency range $100\ \mathrm{Hz} < f < 500\ \mathrm{Hz}$ are treated accurately due to the numerical integration and noise present in the responses. It can be seen that the error increases at low frequencies.

Figure 8 shows the force spectral density, corresponding to the 8^{th} degree of freedom, estimated applying decimation of order 10 ($f_s = 500\ \mathrm{Hz}$). It can be seen that the spectral density estimation is improved when decimation is used but removing the errors at frequencies below 40 Hz are not possible.

The singular values of the responses corresponding to noise (figure 3) are very low compared with the first singular value so that there is no significant differences between the spectral density obtained using one singular value or using eight singular values (figure 8), except in the uncertainty.

The force spectral density estimation can be improved when removing or reducing the peaks in the spectrum by means of a smoothing technique, but this has not been applied.

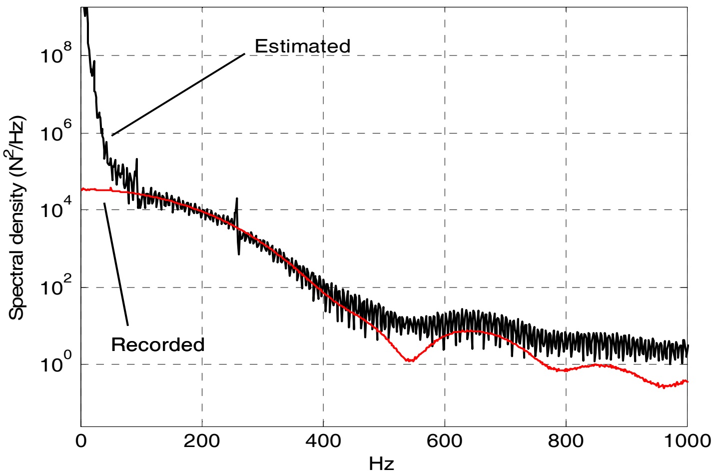

Figure 7. Force spectral density at the 8th degree of freedom. 4096 frequency lines.

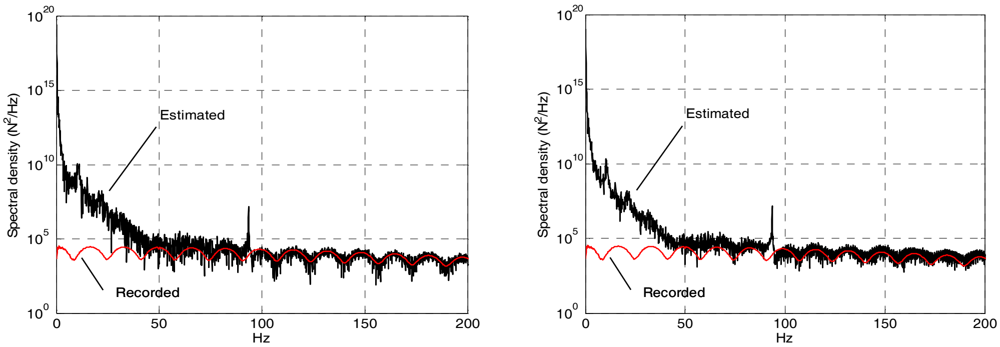

Figure 8. Force spectral density obtained using decimation of order 10. 4096 frequency lines. Left: using 8 singular values. Right: using only 1 singular value

The real force in time domain applied on the 8th degree of freedom is shown in Figure 9a whereas the estimated force is shown in Figure 9b. A high pass filter was applied to reduce the effect of numerical integration at low frequencies.

Due to the fact that the damping is very low, a large number of points (32768 points) were used to reduce the leakage effect. The calculations were carried out with a leakage factor $\beta = 2$. Windows to reduce the leakage effect were not applied

As can be observed, the impacts can be detected and the error is reasonable low. Due to the noise present in the responses, the numerical integration effect at low frequencies and the errors in the modal parameters (amplified in the inversion process) a low level force is estimated in all channels (Figure 9b). When the impact force is applied on a certain degree of freedom, a small peak is estimated in the adjacent degree of freedoms, due to the same reasons. Figure 9b shows a small peak estimated in the 8th degree of freedom when the impact is applied on the 7th degree of freedom.

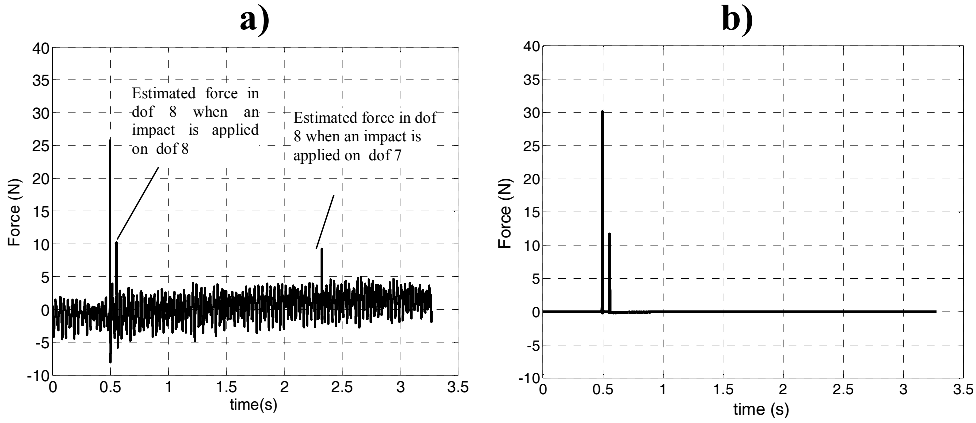

Figure 9 a). Force estimated in the 8^{th} degree of freedom. High pass filter at 25 Hz. **b).** Force recorded from the hammer in the 8^{th} degree of freedom. High pass filter at 25 Hz.

7 Conclusions

- A procedure is proposed to estimate the loading acting on a structure, using the experimental responses and the modal parameters estimated by Natural Input Modal Analysis.
- The main sources of errors have been identified and some recommendations are proposed to remove or reduce the errors.
- The proposed methodology has been applied to a steel cantilever beam. The modal parameters were estimated by natural input modal analysis using the enhanced frequency domain decomposition (EFDD). The FRF matrix estimated from the modal parameters together with the experimental responses were used to estimate the impact forces applied to the structure with an impact hammer.

8 Acknowledgements

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