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Operational Modal Analysis Tutorial

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Abstract

In this paper the basic principles in operational modal testing and analysis are presented and discussed. A brief review of the techniques for operational modal testing and identification is presented, and it is argued, that there is now a wide range of techniques for effective identification of modal parameters of practical interest – including the mode shape scaling factor – with a high degree of accuracy. It is also argued that the operational technology offers the user a number of advantages over traditional modal testing. The operational modal technology allows the user to perform a modal analysis in an easier way and in many cases more effectively than traditional modal analysis methods. It can be applied for modal testing and analysis on a wide range of structures and not only for problems generally investigated using traditional modal analysis, but also for those requiring load estimation, vibration level estimation and fatigue analysis.

Notation

Scalar	a
Vector	$\mathbf{a}$
Matrix	$\mathbf{A}$
Diagonal matrix	$\left[a_r\right]$
Free time response function vector	$\mathbf{h}$
Un-scaled mode shapes	$\boldsymbol{\varphi}$
Scaled mode shape	$\boldsymbol{\psi}$
Mode shape matrix of un-scaled mode shapes	$\boldsymbol{\Phi}$
Mode shape matrix of scaled mode shapes	$\boldsymbol{\Psi}$
Mode shape matrix of right hand side mode shapes	$\boldsymbol{\Xi}$
Natural frequency	ω
Damping	ς
Continuous time pole	λ
Discrete time pole	μ
Sampling time step	Δt
Modal participation vector	$\boldsymbol{\gamma}$
Modal participation matrix	$\boldsymbol{\Gamma}$
Unknown loading	$\mathbf{x}(t)$
Measured responses	$\mathbf{y}(t)$
Modal co-ordinates	$\mathbf{q}(t)$
Covariance matrix	$\mathbf{C}(\tau)$
Power spectral matrix	$\mathbf{S}(\omega)$
Frequency Response Function (FRF) matrix	$\mathbf{H}(\omega)$

1. Introduction

In recent years operational modal testing and analysis has become more popular due to some advantages of the technology compared to traditional modal testing and analysis. However, there still seem to remain some uncertainties, for instance including how the testing should be performed, which techniques should be used, and how reliable the results are. This paper is addresses some of these issues and presents a discussion of some of the advantages of the operational technology.

Adequate testing procedures are discussed. Attention is drawn to the fact that it is of great importance to make sure to be dealing with multiple input loading to improve the quality of the identification process. Typical examples of loading that can contribute to this include moving loads on the structure (the ideal case) or distributed loads with a limited spatial correlation length.

Some of the most well known identification techniques are presented and the basic concepts of these techniques are discussed. All of the well-known techniques of today can handle multiple input data, and the importance of this aspect is illustrated by a Frequency Domain Decomposition, as well as, a Stochastic Subspace Identification technique. These two techniques represent two very different classes of identification, but they clearly illustrate what is believed to be a common tendency for all techniques: they work much better with multiple-input data.

One of the links that from the beginning was missing in operational identification – the estimation of the mode shape scaling – is addressed. Attention is drawn to the fact that a simple and reliable way of scaling the mode shapes is now available.

Finally the advantages of output only modal testing and analysis are discussed. It is argued that it is a reliable technique that can be used on a broader range of structures, and that it can be used for solving a broader range of practical engineering vibration problems.

2. The easy way of modal testing

In operational modal testing, the testing is normally done by just measuring the responses under the natural (ambient or operational) conditions. This means, for instance, that if a bridge is going to be tested, the bridge traffic and normal operation need not be interrupted during the test. On the contrary, the traffic will be used as the excitation source, and the natural response of the bridge to that loading – and to other natural loads acting on the structure at the same time - will be measured and used to perform an operational modal identification.

Similarly, if an engine is going to be subjected to output only modal testing, it is more desirable to perform such test with the engine running under normal operating conditions. The engine responses will be measured, and the operational identification will be performed on this load condition.

In cases where the number of sensors selected for a test is less than the desired number of measured DOFs, it will be necessary to use some of the sensors as references (they remain in the same points), and the remaining sensors will be roved through the desired DOFs in order to obtain a series of datasets containing vibration information of all the DOFs of interest. In many cases the number of datasets can be rather high. It is not unusual to have 20 to 30 data sets from a single test.

The number of sensors, their orientation and the selection of reference sensors must be made during the test planning stage so that all modes of interest are clearly identifiable in all data sets. Special care has to be taken in cases where closely spaced modes are likely to exist. In such cases the user must make sure that the closely spaced modes are not only clearly visible in all data sets, but also clearly distinguishable in all data sets.

As we shall see in the next section, current techniques for operational identification can formulated for multiple-inputs. To clearly identify closely spaced modes, the loading must also be multiple-input. The question is, however: How can the user be sure that the loading is multiple-input?

In order to answer this question, one has to make sure that at least one of two different types of loads produces a clear multiple-input is present:

- A loading that is moving over a large part of the structure
- A distributed loading with a correlation length significantly smaller than the structure

The first type of loading may result, for instance, when a car is crossing a bridge. The car passes over the bridge and thus loads the bridge at infinitely many points. Not only does this kind of loading provide multiple loading, it also helps us ensure that all modes that are sensitive to vertical loading will be excited. This is the ideal kind of loading.

The second kind of loading results, for instance, when wind is acting along the height of a building or waves are loading an offshore structure. Such loading is random in time and space, but as there is a correlation time at a fixed point, there is also a correlation length at a fixed time. To make sure that the wind load on a building is multiple-input, the correlation length of the wind loading must be significantly smaller than the width and height of the building. The same can be said about traffic on a road nearby a building. If the road is close to the building, then the traffic is actually loading the structure in many points, however, if the road is at a fair distance from the building, then a car passing by would produce a single wave that will be propagated toward the structure. In this case the building could be considered to be excited by a single-input load.

This explains why one has to take special care in cases of testing of scaled structural models. If we scale a building down to 1/30 or 1/50 of its original size but keep the distance to the traffic source the same as for the prototype, or keep the correlation length for the wind loading, then we may get a loading that resembles a single input, and thus, we loose quality.

For such cases we either have to scale also the loading, or – if this is not feasible - it would then be desirable to provide some kind of artificial loading that resembles a multiple input. Requirements for artificial loading to ensure multiple input is much easier to satisfy when compared to the work required to setup a forced vibration test, in which shakers have to be installed, forces have to be controlled and measured, etc. For the output only modal test we just need to make sure that the loading is reasonable random in time and space. Let some cats chase some rats on the structure, let it rain on it, let somebody walk on it, or somebody drive a cart on it. All of these are examples of suitable random loads necessary for good testing practices.

One could say, then that in operational testing there are in fact only two main rules to be followed:

- make sure that you have multiple input loads, and
- make sure that you have good quality data.

The first rule was explained above, but a good question may be: how can the user check that? The answer is that one of the easiest ways to check it is to perform a singular value decomposition of the spectral matrix of the measured data and plot the singular values as a function of frequency. If a family of singular values is observed, this is a good indication that multiple loading is exciting the structure. If the excitation is single input, then only one singular value curve will be significant, and all the other singular values will be close to zero mainly describing noise in the data. Figure 1 helps clarify this concept.

The second rule is of general applicability for all experimental cases - get good data, because if you don't, then you are going to have a hard time performing a modal identification. You must ensure that your data has a good signal-to-noise ratio, and you must check and remove outliers, drop-outs and spikes.

Furthermore, it is not enough to have good-quality data. You also need to make sure that you have enough data. To make sure that you have enough data, one can use the following simple rule of thumb. The correlation time for a certain mode can be defined as $(\varsigma_k \omega_k)^{-1}$. This means that the data segments should at least have that length to reasonably suppress the influence of leakage. If one accepts that about 100 averages are needed to have a reliable spectral estimate, then the minimum duration of the continuous measurement should be:

$$T = \max\left\{\frac{100}{\varsigma_k \omega_k}\right\} \tag{1}$$

If you manage to collect good data, the rest of the process is the easy part. One of the reasons why operational modal had very little appeal to engineers in the past is that is that the principles of good testing were not well understood, and erroneous testing procedures were adopted in many projects, leading to severe problems in the identification phase.

3. The easy way of modal identification

For many people operational identification is related to Time Domain identification. There seem to be a strong expectation that if one is performing an operational ID, then one is only working in the time domain. The truth is that there is no “natural domain”, both time domain and frequency domain work well with good data, and perform poorly if one has bad data.

For quite some time people working with modal analysis have been performing operational modal ID using the commonly accepted fact that if only one mode contributes to the spectral matrix, then any column or row in that matrix can be used as a mode shape estimate. This makes the procedure seem relatively simple: pick a peak in one of the spectral density functions and infer the mode shape from one of the columns or rows in the spectral matrix. This approach is effective if one does not have to deal with noise and closely spaced modes. However, in reality we nearly always have to do that – so even though it is a practical way of getting the first clue of the modes – this "technique" is not the best choice for real modal identification. In fact, this estimation procedure is a way of getting Operational Deflection Shapes (ODS) that in some cases might be close to the mode shapes.

The first techniques that really became known for serious operational identification was the time domain techniques. The Ibrahim Time Domain (ITD) was the first, Ibrahim [1], and shortly after came the Polyreference, Vold [3] and the Eigen Realization Algorithm (ERA), Juan and Pappa [4]. The two last techniques were born multiple input, but also the ITD can be formulated as multiple input, Fukuzono [2]. Also a common formulation has been given for all these techniques, Zhang et al [5].

What they have in common is that they assume that a free response function can be obtained. If the structure is given an impulse in a certain point, if the initial conditions are specified in a certain point, if correlation functions are determined correlating one channel with the other channels, or if a Random decrement function is determined using one of the channels as the triggering channel, then we can obtain a single-input time response function $h_{1k} = h_1(k\Delta t)$. Now, if the system is linear, this function can always be written as a linear combination of the modal free decays

(2) $$\begin{aligned}\mathbf{h}_{1k} &= \gamma_1 \boldsymbol{\varphi}_1 e^{\lambda_1 k \Delta T} + \gamma_2 \boldsymbol{\varphi}_2 e^{\lambda_2 k \Delta T} + \ldots \\ &= \boldsymbol{\Phi}\left[\mu_r^k\right]\boldsymbol{\gamma}^T\end{aligned}$$

The "old" time domain techniques are all based on fitting this function by exponential decays. However, the present case is just single input because the time response function is a vector. If you in stead is giving the structure an impulse in several points, if the initial conditions are specified in several points, if correlation functions are determined correlating the other channels with several channels, or if a Random Decrement function is determined using several of the channels as the triggering channels, then we can obtain a multiple-input time response function

(3) $$\left[\mathbf{h}_{1k}, \mathbf{h}_{2k}, \ldots\right] = \boldsymbol{\Phi}\left[\mu_r^k\right]\boldsymbol{\Gamma}^T$$

In operational modal we are restricted to correlation function and Random Decrement function estimation. The channels that are used for correlation estimation, or as triggering channels in the Random Decrement technique, are often referred to as the reference channels (this concept of references in doing ID should not be confused with the reference points that is used when doing the test). If we use all channels as ID reference channels, then, estimating correlation functions the participation matrix is $\boldsymbol{\Gamma} = \left[\gamma_r\right]\boldsymbol{\Phi}$, and thus, the time response function is given by the correlation matrix

(4) $$\mathbf{C}_y(k\Delta t) = \boldsymbol{\Phi}\left[\gamma_r \mu_r^k\right]\boldsymbol{\Phi}^T$$

A similar equation can be stated for the Random Decrement matrix, in fact the Random Decrement matrix is (under the assumption of Gaussian processes) always a linear combination of the correlation matrix and its derivative, Brincker et al, [6].

In the classical exponential decay approach there is no separate noise modeling. Noise is modeled by adding extra modes, the co-called noise modes. A more modern approach is to use one of the Stochastic Subspace (SSI) time-domain techniques: Principal Components, Un-weighted Principal Components, and Canonical Variate Analysis all of which were originally introduced in the literature in different ways but all given a common formulation by Overschee and De Moor [7].

Basically the SSI techniques are doing the same thing as the normal time domain techniques, but a noise modeling is included that is believed to reduce the necessary number of noise modes. This might be important especially in cases with many data sets where the problem of pole pairing between data sets (a certain mode must be visible in all data sets) might become significant. In SSI the data is modeled like an ARMA model

(5) $$\mathbf{y}_t + \mathbf{A}_1\mathbf{y}_{t-1} + \ldots + \mathbf{A}_n\mathbf{y}_{t-n} = \mathbf{e}_t + \mathbf{B}_1\mathbf{e}_{t-1} + \ldots + \mathbf{B}_n\mathbf{e}_{t-n}$$

However, in the SSI world everything takes place in the state space where the left hand side of equation (5) (the autoregressive part that corresponds to the physics) is modeled by the state space equation

(6) $$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}\mathbf{x}_t + \mathbf{w}_t \\ \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \mathbf{v}_t\end{aligned}$$

The state variables $\mathbf{x}_t$ are found by a so-called projection, that is a conditional mean. In general, a conditional mean is the same as a Random Decrement estimator, and for zero mean Gaussian processes a conditional mean is known to be expressed by the covariance matrix. The projection channels corresponds to the previously mentioned ID references. Thus, we allow for multiple input loading to the same degree we have projection channels. When the state variables are known, the system matrices $\mathbf{A}$ and $\mathbf{C}$ are obtained by regression on equation (6). The noise modeling corresponding to the left hand side of equation (5) (the moving average part) is introduced by the Kalman Gain matrix $\mathbf{K}$ by rewriting equation (6) as

(7) $$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}\mathbf{x}_t + \mathbf{K}\mathbf{e}_t \\ \mathbf{y}_t &= \mathbf{C}\mathbf{x}_t + \mathbf{e}_t\end{aligned}$$

The meaning of the matrices is better understood by performing a modal decomposition and substitution

(8) $$\begin{aligned}\mathbf{A} &= \mathbf{V}[\mu_r]\mathbf{V}^{-1} \\ \mathbf{z}_t &= \mathbf{V}^{-1}\mathbf{x}_t\end{aligned}$$

obtaining the alternative form of equation (7)

(9) $$\begin{aligned}\mathbf{z}_{t+1} &= [\mu_r]\mathbf{z}_t + \mathbf{\Xi}\mathbf{e}_t \\ \mathbf{y}_t &= \mathbf{\Phi}\mathbf{z}_t + \mathbf{e}_t\end{aligned}$$

Shortly after the millennium a new frequency domain approach was introduced that brings things back to the old way of performing frequency domain identification by just picking information form the spectral matrix, Brincker et al [8]. The principle in the Frequency Domain Decomposition (FDD) techniques is easiest illustrated by realizing that any response can by written in modal co-ordinates

(10) $$\mathbf{y}(t) = \boldsymbol{\varphi}_1 q_1(t) + \boldsymbol{\varphi}_1 q_2(t) + \ldots = \mathbf{\Phi}\mathbf{q}(t)$$

Now obtaining the covariance matrix of the responses

(11) $$\mathbf{C}_{yy}(\tau) = E\{\mathbf{y}(t+\tau)\mathbf{y}(t)^T\}$$

and using equation (10) leads to

(12) $$\begin{aligned}\mathbf{C}_{yy}(\tau) &= E\{\mathbf{\Phi}\mathbf{q}(t+\tau)\mathbf{q}(t)^T\mathbf{\Phi}^T\} \\ &= \mathbf{\Phi}\mathbf{C}_{qq}(\tau)\mathbf{\Phi}^T\end{aligned}$$

Note the similarity to equation (4). Then by taking the Fourier transform

(13) $$\mathbf{S}_{yy}(\omega) = \mathbf{\Phi}\mathbf{S}_{qq}(\omega)\mathbf{\Phi}^T$$

Since the modal co-ordinates are un-correlated, the power spectral density matrix $\mathbf{S}_{qq}(\omega)$ of the modal co-ordinates is diagonal, and thus, if the mode shapes are orthogonal, then equation (13) is a singular value decomposition of the response spectral matrix.

This explains the importance of independent loading as it was presented in the preceding section. If we have a single input case, then the response spectral matrix $\mathbf{S}_{yy}(\omega)$ is forced to rank one. Thus, only the first singular value will show anything that is related to the physics of the system. However, if we have a loading that allows the spectral matrix to have full rank (the necessary rank is the largest number of closely spaced modes plus the number of independent noise sources), then the singular values will give you a nice picture of the auto spectral density functions of the individual SDOF systems corresponding to each of the modes.

This is illustrated in Figure 1 where it can be seen, that when the loading is natural (top picture: multiple input) then it is easy to see all 16 modes immediately, however, when the loading is single input (bottom picture), it is hard to determine how many modes are present.

One might ask if the time domain techniques can do any better in such a case. In fact they do not do better - they also prefer full rank data. What happens if the same data is analyzed by an SSI technique is illustrated in figure 2.

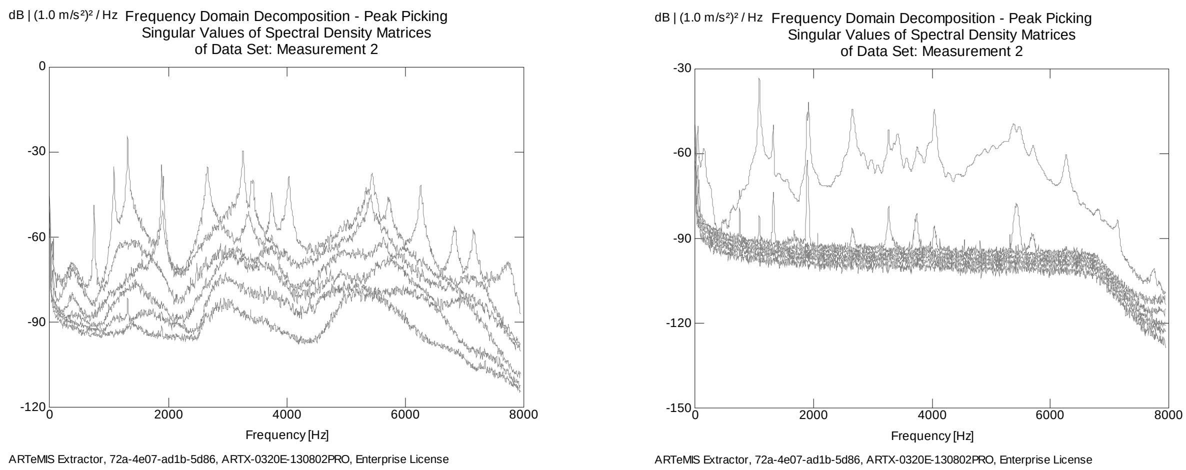

Figure 1. Analysis by FDD. Free-free plate illustrating the importance of multiple input loading. Top: multi-input loading by one loading source moving on the plate. Bottom: Same load but acting as a stationary load (acting in one point only)

As it appears, the same problems arise. If we have natural loading the SSI can see the closely spaced modes (there are two close modes at 2 kHz) and we can easily count the 16 modes like in the FDD analysis case. However, if we have single input loading, modes disappear because of low excitation, closely spaces modes cannot be so well identified and modes appear where they do not belong.

What is the conclusion then ? The conclusion is that we have available a nice basket of techniques that all works very well if you just make a good test where we make sure to have a natural (multiple input) loading and if we take advantage of using the ID techniques in their multiple-input (multiple reference) formulation.

This does not make life any more difficult for the user, the user just have to know that this is important, respect the way it works, and then he will without any further problems have all the modes nicely estimated.

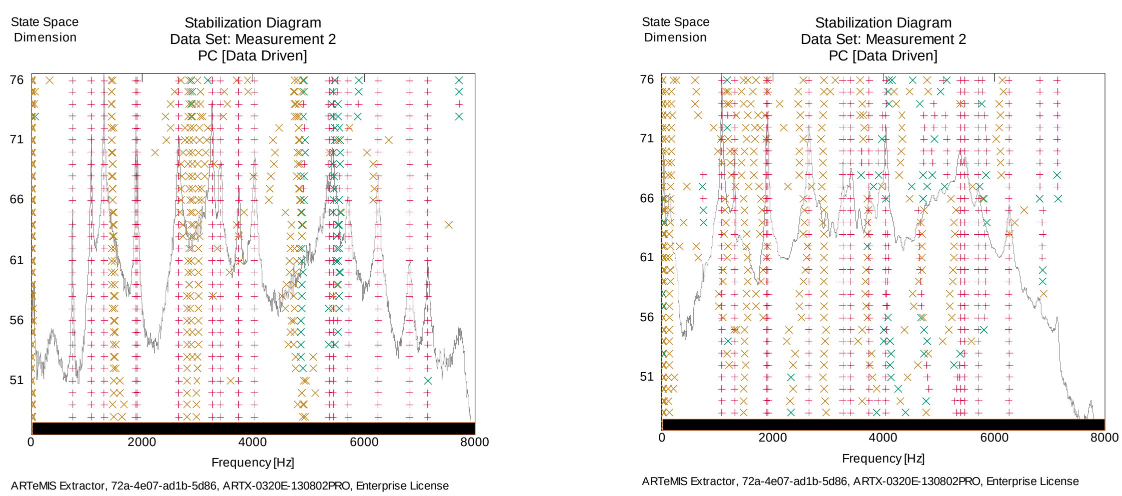

Figure 2. Analysis by SSI. Free-free plate illustrating the importance of natural loading. Top: multi-input loading by one loading source moving on the plate. Bottom: Same load but acting as a stationary load (acting in one point only)

4. The mode shape scaling factor

One of the hard argues against operational testing has been the lack of possibility of scaling the mode shapes. If the identified modal model is going to be used for structural response simulation or for structural modification, then the scaling factors of the mode shapes must be known. Also in health monitoring applications and in cases where damage is to be identified, the scaling factors might be useful.

The last years some suggestion has been given in the literature for solving this problem. One solution has been suggested by Bernal and Gunes [9] based on the assumption that partition of the inverse of the mass matrix associated with the measured coordinates is diagonal. However, the approach gives exact answers only when there is a full set of modes, and robustness for a truncated modal space has not been demonstrated. Parloo et al [10] and Brincker & Andersen [11] have suggested another approach based on a more extensive testing procedure that involves repeated testing where mass changes are introduced at the points where the response is measured. This approach seems more appealing, since to scale a certain mode, only that particular mode has to be known.

The approach is very simple. Make the mass changes described by the mass change matrix $\mathbf{\Delta M}$, identify the mode shape of a certain mode, identify the natural frequencies ω_1 and ω_2 before and after mass change, and then simply calculate the scaling factor as

$$\alpha = \sqrt{\frac{\omega_1^2 - \omega_2^2}{\omega_2^2 \boldsymbol{\varphi}^{\mathbf{T}} \mathbf{\Delta M} \boldsymbol{\varphi}}} \tag{14}$$

and the scaled mode shape is

$$\boldsymbol{\psi} = \alpha \boldsymbol{\varphi} \tag{15}$$

Now we have dealt with more or less the basic difficulties of operational testing and analysis and we have shown that “difficulties” are no big difficulties any more. Now let us turn to the advantages of this kind of modal testing.

5. It works easier

This is the well known argument for output only modal testing, you do not need an exciter, you do not need a fixture or a test rig, you do not need a lab. Go to the structure where it is, let it be loaded like it is naturally, let it remain under the circumstances where you found it. Just record the responses, go home and make your ID. All this is true, just keeping in mind that in some cases we have to “help” nature a little to ensure the well-behaving of the ID problem (multiple input loading).

What people tend to forget is that there are even more things to be said about making things easier – or more possible - if you prefer to put it this way.

Think about the necessary a’priori knowledge when performing a traditional modal test. You need to know where to put your exciters, and to put them in reasonably good positions, you need to have some ideas about the modes. If you don’t, then you have to shift the exciters around until you are sure that all modes are well excited. However, this takes time, and time is money. The weakness of traditional testing is that it cannot handle moving loads, and there is nothing as efficient in exciting ALL modes as this kind of loading. We can take any structure, make an output testing – only one – where we use a loading that is moving all over the structure, and be sure that we have ALL the modes captured. This is actually a very important quality in operational testing, and one of the reasons that this kind of testing might become very popular for prototype testing in the future.

Also another feature might have importance. Imagine that you want to estimate some modal parameters VERY accurately. It means a very long testing time, let us say for days, weeks or even month’s. This would not be feasible performing traditional modal testing because the lab, test-rig and equipment and also the structure itself would be tied up in testing. This is not the case using output only modal. Here it ties up the equipment it takes to measure the responses, but there is no lab and rig expenses, and at the same time the structure can be used as usual. All the techniques mentioned in the preceding section can in principle handle infinite data, so there is nearly no limitations to how well you can do things.

6. It works better

This is when the good guys in traditional modal get a smile on their face. How can things get any better by omitting information about the loading ? It cannot of course - but as soon we leave the ties of traditional modal, then we can use that freedom and the gained savings to make up for the loss of information about the inputs. We use moving loads to make sure that all modes are excited, we use cheap multiple input loading that makes sure that we can identify all closely spaced modes, we make longer tests (if needed) to obtain modal parameters with higher accuracy. It works better because we can afford to do things right.

It also works better in cases where boundary conditions plays a central role and where the loading level plays a central role. Think about cases of non-proportional damping or cases of non-linear systems. Here it only makes sense doing modal identification if the loading corresponds to reality. For instance, how will you get the information of damping of a car under different driving conditions ? Do we agree that the best way is simply to drive the car under that specific conditions and then make an operational ID ? Do we agree that finding the damping of an offshore structure under different wave loading conditions the best way is to make the response measurement under that specific loading conditions and then perform an operational ID ? Can it be done using forced vibration testing ? Yes it can, but the testing will be extremely difficult, or the results will be ambiguous.

7. It can be used on more structures

This argument is an old one. We all know that traditional modal analysis has it strong points and has been a successful tool for testing of middle-size structures. Structures that are small enough to go into the lab, small enough that we can excite them, but on the other hand not so small that it becomes a problem. Luckily enough must of our structures fall into this category, but problems start arising when we start thinking about trucks, trains, large engines, rockets, airplanes, buildings, dams, bridges and then the tiny little things like hard-discs, optical parts etc.

Also, since output only testing is the poor mans modal tool, modal technology becomes available for the vast majority of cases where it was to expensive in the past. For instance, it might be used for solving vibration problems on all kinds of consumers goods.

The range of structures where modal analysis can be applied extends dramatically when we move to the operational way of doing things. You can even say the bigger structure the better, because – as we have learned in the first section – the bigger the structure, the more independent inputs (the ratio between correlation length and structural size gets smaller), and as we saw in the preceding section, the better identification.

The application of operational modal is only limited by the possibility to measure the response. For very large structures that might still be a significant problem simply because with the measurement technology of today, it is costly to manage miles of cables. However, since we are moving in the direction of wireless digital sensors, cabling problems are disappearing and that means that in the future we will be able to apply operational modal on virtually any kind of large structure. The very small structures still remain a challenge, but with further development of the laser technology it is expected that soon affordable systems for response measurements of tiny components will be available.

8. It can deal with harmonics

One of the major problems working with real loads is that very often harmonics might be present. Somewhere in the building or in the neighborhood something is rotating. Even the smallest harmonic might create problems due the many over harmonics that are always presents, and to the corresponding false modes that might be estimated.

In Jakobsen [12] it is shown how harmonic components can be separated from structural modes and it is explained how an easy-to-use technique can be implemented based on Kurtosis. The idea is that a stochastic response signal and a harmonic singal has a very different density function, see Figure 3.

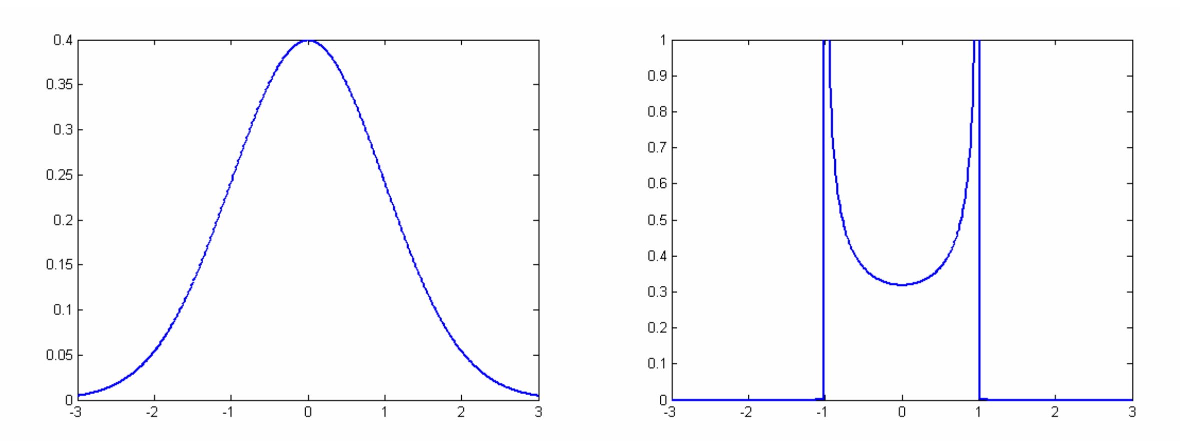

Figure 3. Normalized PDF of pure structural mode (left) and pure harmonic component (right)

9. It can be done automated

Recently an automated version of the FDD techniques has been introduced, Brincker [13], The approach does not imply any a' priory knowledge of the modes, the user gives a few inputs to the analysis among which the number of frequency lines in the spectral density estimates is the most important. The rest is automatic – and the modes arrive on the screen.

The idea is to introduce two new concepts; the first one is modal coherence which describes the correlation between singular vectors of neighboring spectral matrices. This measure gives a direct indication of modal influence on the signal at the considered frequency. Where there is a high modal coherence the signal is dominated by a mode and that mode is to be identified.

The second concept is the modal domain. Whenever a mode is identified, a frequency band around the mode is estimated where that particular mode is dominating the response, and the modal domain is excluded from the search set. This continues until all modes are identified.

10. It can be used to solve more problems

This is probably a new angle for the must of the readers. People have been looking at operational modal and saying, well it is a possibility for the large structures, it might even be better because it corresponds to real loading, but the traditional modal is the one that gives us the large range of practical applications.

Here comes the shocking facts: The output only modal has MORE applications than traditional modal. Let us try to explain the range of possibilities. Some of them are questionable since they have not been tried much in reality, but anyway the potential is there, which cannot be said about traditional modal.

First of all, output modal can be used for all the applications that is home ground for traditional modal. Now when we can also get the scaling factors on the mode shapes there are no limitations. Can be used for validation and updating of fem models, can be used response simulation, can be used for structural modification, can be used for damage detection, etc. Accuracy is also not an issue, operational can do as well. May be traditional modal is more competitive because one get the scaling factor for free, controlling the input is easy if the structure is middle size etc., so in many cases traditional will keep its lead. However, there is a range of applications where traditional modal can hardly compete. Here they come.

One of the well known applications is *monitoring.* In this field traditional modal is out of question because we cannot go there and put an exciter or drop-load or hammer or whatever every time we are going to take a measurement to see if the structure is still ok. The obvious solution is to put the response measurement system there, take measurements whenever it is needed, and then – do your operational ID. This is expected to be a big business in the near future, measurements systems are getting cheaper and more accurate, to acquire data from long distance (for instance over the internet) is becoming within reach, and many structures needs an eye on them to get right maintenance at the right time.

This application is more or less well known. Here comes three that might not be so well known:

- Load estimation
- Vibration level estimation
- Fatigue estimation

These three very important applications are directly related to the fact that in operational ID we store the measured raw time series. We keep the full information about the measured responses due to real loading.

The first is obvious when we know that we can get the scaling factors of the mode shapes. When we have the scaling factors, then we also have the FRF matrix relating inputs and responses between all DOF's, thus we have the relations

(16) $$\mathbf{y}(\omega) = \mathbf{H}(\omega)\mathbf{x}(\omega)$$

(17) $$\mathbf{S}_{yy}(\omega) = \mathbf{H}(\omega)\mathbf{S}_{xy}(\omega)$$

and these relations can – in principle – be used to estimate the loading

(18) $$\mathbf{x}(\omega) = \mathbf{H}^{-1}(\omega)\mathbf{y}(\omega)$$

(19) $$\mathbf{S}_{xx}(\omega) = \mathbf{H}^{-1}(\omega)\mathbf{S}_{yx}(\omega)$$

Problems remains to be solved concerning numerical problems inverting the FRF matrix, problems concerning truncation of the modal space (if not all modes are known), and problems concerning the influence of errors on the modal estimates. However, these problems are about to be solved, and then we can find the loading in time domain and in frequency domain. We can use this to obtain valuable information about wind loads, wave loads, traffic loads etc. – all those loads that are so difficult to measure directly. We can also learn about the correlation length in the loading and thus learn how to perform better operational testing in the future.

Going to the next applications we isolate the modal coordinates in equation (10)

(20) $$\mathbf{q}(t) = \mathbf{\Phi}^{-1}\mathbf{y}_N(t)$$

which is possible if the number of observation points is as large (or larger) than the number of modes. If the number of observation points is larger than the number of modes then the problem is solved by linear regression. Now let us assume that a FEM solution has given us the corresponding detailed mode shapes $\mathbf{\Phi}_{fem}$ such that with reasonable approximation

(21) $$\mathbf{\Phi} = \mathbf{A}\mathbf{\Phi}_{fem}$$

where $\mathbf{A}$ is a an observation matrix containing only ones and zeros. Then the response can be calculated in any point of the structure by replacing the experimental mode shapes with the FEM mode shapes in equation (10)

(22) $$\mathbf{y}_{fem}(t) = \mathbf{\Phi}_{fem}\mathbf{q}(t)$$

Thus, by measuring the response in only a relatively small number of points, then after an operational modal ID, the corresponding response in any point of the structure can be calculated. This is of vital importance for vibration level estimation. For instance, the vibration levels in an office building do not have to be measured in all the offices. You can tell the vibration level in hundreds of offices just by taking measurements in a few points.

A similar approach can be used for fatigue analyses. The same procedure is followed, only now we use the FEM solution to relate the modal co-ordinate to the stress at a certain point

(23) $$\sigma(t) = C_1 q_1(t) + C_2 q_2(t) + \ldots$$

This solves one of the greatest problems in fatigue analysis: to find the exact stress history at any point of the structure. The constants C_k are the so-called stress concentration factors. In this formulation they relate the amplitude of the mode shape to the hot spot stress contribution from each of the modes. Doing an analysis like this, the engineer can perform an improved fatigue analysis by-passing all the uncertainty from loading modeling and from relating the loading with the stress history. Since a large uncertainty is removed, many structures that are already "dead" due to standardized fatigue analysis might be able to have their lives substantially extended. By making this kind of analysis under different loading conditions, and by building a statistical data base on the frequency of these loading conditions, a much better prognoses for the future damage accumulations might be made.

Closing remarks

This paper deals with some overview thoughts, some of which might be important, some not so important. Some of the mentioned ideas might never be applied in real life and some might dramatically change the ways engineers are doing their job. Some of the most important points might also be missing. This is the good thing about this issue, like in all other science – there is always more to learn.

The end of the story is that: we are at a point – or at least very close to a point – where operational modal analysis might become a widespread tool for solving a broad range of engineering problems. We have shared with you our thoughts on how and why. We hope this frank approach may generate some interesting discussions on the merits of modal methods of analysis among the engineering community.

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