

On a new boundedness result for pseudodifferential operators with exotic symbols

The α -modulation spaces $M_{p,q}^{s,\alpha}(\mathbb{R}^d)$, $\alpha \in [0, 1]$, form a family of spaces that contain the Besov and modulation spaces as special cases. In this paper we prove that a pseudodifferential operator $\sigma(x, D)$ with symbol in the exotic Hörmander class $S_{\rho,0}^b$ extends to a bounded operator $\sigma(x, D) : M_{p,q}^{s,\alpha}(\mathbb{R}^d) \rightarrow M_{p,q}^{s-b,\alpha}(\mathbb{R}^d)$ provided $\alpha \leq \rho \leq 1$, $0 < \alpha \leq 1$, and $1 < p < \infty$. The result extends the well-known result that pseudodifferential operators with symbol in the class $S_{1,0}^b$ maps the Besov space $B_{p,q}^s(\mathbb{R}^d)$ into $B_{p,q}^{s-b}(\mathbb{R}^d)$.