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Published in:
Proceedings of the 8th International Modal Analysis Conference

Publication date:
1990

Document Version
Publisher's PDF, also known as Version of record

IDENTIFICATION OF LIGHT DAMPING IN STRUCTURES

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Abstract

Different methods to identification of linear and nonlinear damping in lightly damped structures are discussed in this paper. The discussion is based on experiments with a 4 meter high monopile. Two alternative methods have been used for experimental cases of linear and nonlinear damping. Method 1 is identification by ARMA models assuming a white noise input. Method 2 is identification by simulation of a free decay response. Experimental data on the free decay response has been obtained directly by measurement as well as by the random decrement technique. Two experimental cases has been considered. The first case was a naturally damped monopile which was considered to be linear viscous damped. The second case was nonlinear viscous damping of the monopile due to a mounted damper on the monopile. The two cases illustrate identification of lightly damping in the linear and the nonlinear case.

Nomenclature:

$a(t)$	Time series of noise
$h_i(t)$	Impulse response function of DOF no i
c_{ij}	Element in the linear viscous damping matrix
c_{ij}^{cou}	Element in the coulomb damping matrix
c_{ij}^{nlv}	Element in the damping matrix due to nonlinear viscous damping
$D_{ji}^{X_0}[\tau]$	Random decrement signature of DOF no j due to trigger condition X_0 on DOF no i
f_n	Eigen frequency no n
F_i	Damping force
k_{ij}	Element in the stiffness matrix
m_i	Element in the mass matrix
n	Number of degrees of freedom
N	Number of data points in time series
$p(x_1)$	Probability density function
$R_{x_ix_j}(\tau)$	Cross correlation function
$t,\ t_i$	Continuous time and discrete time
$V(\overline{\Theta})$	Error function
x_t	Discrete time series of $x(t)$
$x_i(t)$	Time series of the response of DOF no i
μ_{12}	*Eigen value of damped eigen value problem*
λ_i	Root of a characteristic polynomial
σ_a^2	Variance of noise
δ	Variation coefficient
$\epsilon(t\|\overline{\Theta})$	Error
ζ_n	Damping ratio
$\overline{\Phi}_i$	Eigen mode vector
$\overline{\Theta}$	Parameter vector

Introduction

*The light damping in civil structures is very uncertain de*termined. This is due to several reasons. First of all there exists an insufficient knowledge about damping mechanisms in general. Secondly the damping estimates are often distorted by a bias error e.g. in the FFT analysis, see Bendat and Piersol [1]. Thirdly *when viscous damping is assumed*, estimates of small damping become per definition more uncertain as the damping decreases. In fact for a white noise driven SDOF system Kozin [2] has shown that the variation coefficients become:

$$\delta_{f_n^2} = \sqrt{\frac{\zeta_n}{2\pi f_n}} \quad \delta_{\zeta_n f_n} = \sqrt{\frac{1}{\zeta_n 2\pi f_n}} \tag{1}$$

Thus the very unreliable nature of the damping estimates of lightly damped structures makes it necessary to considered alternative methods which reduces the uncertainty and the error of the estimate to a minimum. In this paper identification by ARMA models has been considered to overcome the bias problems of the traditional FFT analysis. Furthermore identification by simulation of the impulse reponse has been considered to make it possible to identify nonlinear damping mechanisms. The random decrement technique has been considered as a link between the analysis of the response due to a random excitation and the analysis of a free decay response.

Beyond the methods mentioned above, the method of the logarithmic decrement, $\delta_{ni} = \ln \frac{A_n}{A_i}$ has also been applied:

$$\zeta = \frac{\ln \frac{A_n}{A_i}}{2\pi(n-i-1)} \tag{2}$$

to provide a quick estimate of the damping ratio from a free decay. A_n is here the nth amplitude and A_i is the ith amplitude.

Identification By ARMA Models

Identification by ARMA models provides an alternative to FFT analysis. The ARMA model gives a direct relation to the modal quantities while the FFT-analysis gives a non-parametric model which followed by a curvefitting algorithm gives the estimates of the modal quantities .

An ARMA model can be found from the stationary gaussian zero mean response of a linear system excited by white noise, $a(t)$. The ARMA model of the measured response of a given point, $x(t)$ at discrete time intervals is defined by:

$$x_t = \underbrace{\sum_{i=1}^{n} \Phi_i x_{t-i}}_{AR-part} + a_t - \underbrace{\sum_{i=1}^{m} \Theta_i a_{t-i}}_{MA-part} \tag{3}$$

This is called an ARMA(n,m) model (Auto Regressive Moving Average of order n,m). The parameters in the ARMA model are real numbers. The appropiate order should be ($2n$,$2n$-1) for a white noise excited system with n degrees of freedom. This choice will be a proper choice since it can be shown that for the assumed white noise excitation the covariance function of the response due to the ARMA-model and that of the white noise excited structure will be identically, see Kozin and Natke [3]. In other words an ARMA model will provide an unbiased estimate of the response spectrum provided that the assumptions hold.

The parameters of the ARMA model are estimated from the time series x_t. This is done by minimising the error function which in the present paper is identical with the computed variance of a_t :

$$\sigma_a^2(\Phi_i, \Theta_j, x_t) = \frac{1}{N}\sum_{t=1}^{N} a_t^2 \tag{4}$$

The error function will be nonlinear w.r.t. the parameters which means that methods of nonlinear least squares has to be applied.

When the ARMA parameters and the residual a_t have been estimated it must be checked whether or not a_t is a realisation of white noise. If not it indicates that the model order is too low. In that case it means that the residual a_t consists of a white noise contribution plus a model error. Hence the model order should be increased.

The dynamic parameters are found from the $2n$ roots, λ_i of the characteristic polynomial of the AR-parameters:

$$\lambda^{2n} - \Phi_1\lambda^{2n-1} - \cdots - \Phi_{2n-1}\lambda - \Phi_{2n} = 0 \tag{5}$$

In e.g. Jensen [4] it is shown that the roots are related to the modal parameters trough the $2n$ relations :

$$(\ \lambda_i\ ) = (\ \exp(\mu_i\Delta)\ ) \tag{6}$$

where Δ is the sampling interval and μ_i has the following relation to modal parameters:

$$\mu_{(i)12} = -\omega_i\zeta_i \pm \mathrm{i}\omega_i\sqrt{1-\zeta_i^2} \qquad \zeta_i \ll 1.0 \tag{7}$$

The index (12) refers here to the fact that the λ_is are found as complex conjugated pairs if the modes are underdamped. This set of equations gives the relation between the estimated ARMA parameters and modal parameters if the sampling interval, Δ is known.

Identification By Response Simulation (IRS)

This method is rather straight forward: The error between the measured response and the simulated response of a free decay is minimized w.r.t. the unknown parameters:

$$V(\overline{\Theta}) = \frac{1}{N}\sum_{t=1}^{N} \overline{\epsilon}^T(t|\overline{\Theta})\overline{\epsilon}(t|\overline{\Theta}) \tag{8}$$

where

$$\overline{\epsilon}(t|\overline{\Theta}) = \overline{x}(t) - \hat{\overline{x}}(t|\overline{\Theta}) \tag{9}$$

and $\hat{\overline{x}}(t|\overline{\Theta})$ is the predicted response due to $\overline{\Theta}$ which is a vector containing the unknown parameters. The minimum of this error function w.r.t. the unknown parameters can be found by an iterative optimization , in the present paper by the algorithm NLPQL, Schittkowski [5].

The measured response, $\overline{x}(t)$ can either be obtained directly from a free decay or from an application of the random decrement technique on a white noise realisation. The latter will be presented in the next section.

The response simulation of $\hat{\overline{x}}(t|\overline{\Theta})$ has been performed by the Runga Kutta method of a general lumped mass system containing nonlinear damping mechanisms such as coulomb and nonlinear viscous damping mechanisms given by :

$$m_i\ddot{x}_i + c_{ij}\dot{x}_j + c_{ij}^{cou}\frac{\dot{x}_j}{|\dot{x}_j|} + c_{ij}^{nlv}|\dot{x}_j|\dot{x}_j + k_{ij}x_j = 0 \quad \text{for } j = 1,2,\dots n \tag{10}$$

for the ith degree of freedom.

The algorithm applies a set of initial conditions which may be known or included as unknown parameters. Thus the parameter vector $\overline{\Theta}$ will in the general case be given as:

$$\overline{\Theta}^T = (M_i, C_{ij}, C_{ij}^{cou}, C_{ij}^{nlv}, K_{ij}, x_i(0), \dot{x}_i(0)) \quad i,j = 1,2\dots n \tag{11}$$

The method has been tested by a series of simulated examples with nonlinear viscous damping and coulomb damping present at one time. The results showed that even when noise was added it was possible to identify the nonlinear damping mechanisms. However the computer time increased seriously when noise was present.

Random Decrement Technique

The idea of the random decrement technique is to relate the white noise response of a linear system with the impulse response of the same system, see e.g. Vandiver et al [6]. The random decrement signature can be defined as:

$$D_{ii}^{X_0}[t_1, t_2] = E[x_i(t_2)|x_i(t_1) = X_0] \tag{12a}$$
$$D_{ji}^{X_0}[t_1, t_2] = E[x_j(t_2)|x_i(t_1) = X_0] \tag{12b}$$
$$i,j = 1,2\dots n$$

$D_{ii}^{X_0}$ is here the random decrement signature for the time series $x_i(t)$ for which a trigger condition X_0 has been given, while $D_{ji}^{X_0}$ is the random decrement signature of the DOF no j given the trigger condition on $x_i(t)$, see figure 1. The principle is that after having choosen a trigger level for a given time series no i, segments of each measured time series are identified and averaged for which the trigger condition of time series no i is fulfilled. This leads to information about auto and cross correlation and thus to the deterministic characteristics of the measured time series. However one of the essential problems of the random dec. technique is to avoid bias on the random dec. signature. Experience shows that both inter dependence of the segments, the way in which the theoretical trigger condition $x = X_0$ is realized on the discretized time series and the algorithm for choosing one trig point in each segment are of importance. To ensure independence of the averaged segments they must be long enough to let the trig condition fade out (the random dec. signature should be approximately zero at the end). Thus for lightly damped systems as a law of nature the random dec. technique demands long time series to avoid biasing.

It can be shown, see Vandiver et al [6] that this signature in general is related to the autocorrelation function of the reponse by:

$$R_{x_j x_i}(t_1, t_2) = \int_{x_j} x_j p(x_j) D_{ji}^{X_0}[t_1, t_2] dx_j \tag{13}$$

If the excitation is assumed to be a stationary gaussian (but not necessary white) random zero mean process it follows that eq(13) is simplified to:

$$D_{ji}^{X_0}[\tau] = \frac{R_{x_j x_i}(\tau)}{R_{x_i x_i}(0)} X_0 \tag{14}$$

If the system is nonlinear this equation will be an approximation for which the error will depend upon how well the response process can be approximated to a gaussian process.

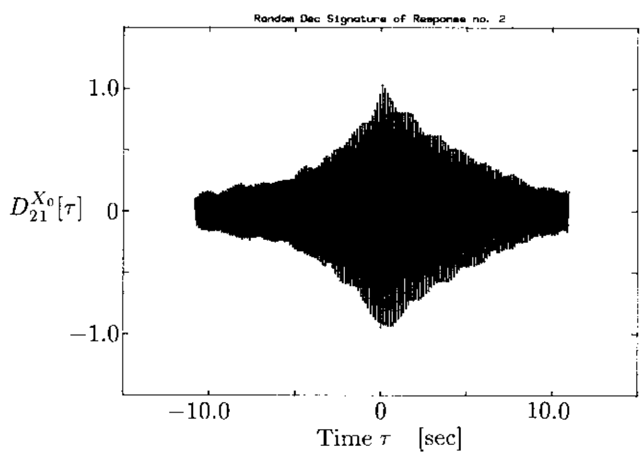

Figure 1. Random decrement signature of the response no. 2 with a choosen trigger condition on response no. 1. 100 means have been applied with $\Delta = 0.0213$ Sec. Total length of signature, 22 Sec.

If stationary white noise is assumed and the system is assumed to be linear then it can be shown that the impulse response (/a free decay) will be proportional to the random decrement signature:

$$h_j(\tau|x_i = X_0 \text{ and } \overline{x}(t)) \propto D_{ji}^{x_0}[\tau] \tag{15}$$

This expression will also hold approximately for filtered white noise as input if the system is lightly damped. Thus the random decrement technique can be applied for lightly damped linear systems with a broad banded excitation to find an impulse response function of the excited modes. The relation will not hold in the case of nonlinearities. However as will be shown later it is believed that indicative information about nonlinearties can be obtained.

Experimental Model

The monopile structure is shown in figure 2. The structural response was either due to a free vibration or a displacement controlled base excitation, see Jensen [7]. The base excitation was lowpass filtered white noise. This meant that only two eigen modes were excited, primarily the second since the excitation force due to the white noise displacement input was a force autospectrum of the form:

$$S_{qq}(f) = (2\pi f)^4 S_0 \tag{16}$$

Hence when the response was due to forced excitation the second eigen mode was the dominating. In case of the performed free vibrations the most active eigen mode was the first. Thus the two kinds of response contained opposite weighting of the eigen modes and consequently also of the reliability of the respectively modal estimates.

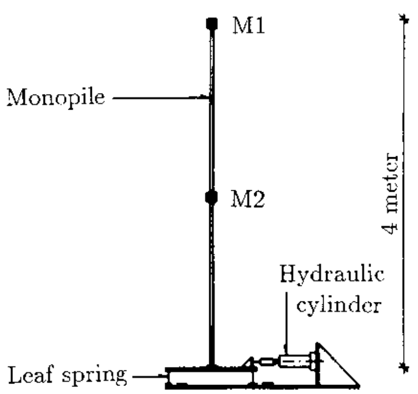

Figure 2. Monopile structure.

The two experimental cases which were considered were the monopile structure with two different damping configurations:

- The naturally damped monopile which was assumed to be proper modelled by a linear viscous damping model.
- The extra damped monopile due to a mounted nonlinear viscous damper on the concentrated mass in the middle of the monopile.

The first configuration is called the linear viscous damped case while the second is called nonlinear viscous damped case. The mathematical model of the mounted damper was confirmed by an calibration which showed that the damping force could be described by $F_2^{nlv} = (73.8\dot{x}_2 + 0.4)\dot{x}_2$ [N].

The response was measured at two locations. The response of the mass at the top was labelled response no. 1 while the reponse of the mass at the middle of the monopile structure was labelled reponse no. 2. The response was acceleration measured by accelerations.

The Linear Viscous Damped Case

An ARMA(8,7) model was found to be a proper ARMA model for the measured response of the two concentrated masses due to the random excitation. The arguments for this model order was:

- The variance of the residual obtained a minimum for this model order which indicated that the model was the best obtainable, (eq(4)).
- The autocorrelation of the residual seemed to be quite close to white noise, see figure 3. It is seen that the auto correlation function decreases fast to zero compared with the lowest eigen periode in the measured free decay, 0.14 Sec. This indicates that the resdiual is white noise and that no model error has been present.

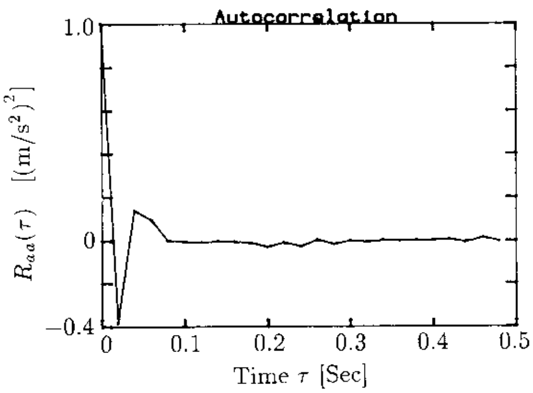

Figure 3. Autocorrelation of the residual. Time series: The measured response of mass at the middle, $\Delta = 0.02$ Sec. and time length 120 Sec.

Theoretical the model order was expected to be (4,3) since only two modes were excited. However due to the random but non-white force process the model order was in practice higher. Relative high model orders are quite common since the white noise assumption always will be an approximation.

The identification by response simulation (IRS) of a measured free decay was performed with a linear damping model. The following parameters were estimated:

Free Decay:

$$\begin{pmatrix} 29.8 & 0 \\ 0 & 34.5 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 1.38 & -1.49 \\ -1.49 & 1.75 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix}$$
$$+ \begin{pmatrix} 9819.8 & -22406.3 \\ -22406.3 & 61665.6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The parameters agree fairly well with the physical a priori knowledge. In figure 4a is shown an example of a fit of the free decay response. A good agreement is seen even though some deviation is seen. This reflects probably the difficulties of identifying the second eigen mode which was only weakly excited in the free decay response.

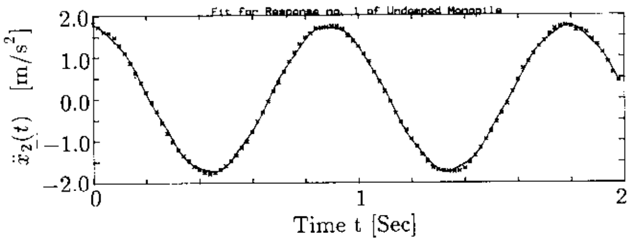

Figure 4. Identification by response simulation (IRS). Fit of the response no. 1 for linear viscous damped case. The figure shows a segment of 120 Sec time series sampled with $\Delta = 0.02$ Sec.

The estimated parameters were transformed to modal parameters for a comparison with the results of the ARMA method. In table 1 is shown the estimated modal parameters for the two methods together with the conventional logarithmic decrement. It is seen that the eigen frequencies estimated by the IRS and the ARMA method agrees very well. The damping ratio of the first mode estimated by the IRS and the logarithmic decay agrees fairly well but the ARMA method fails. This is due to the weak excitation of the first eigen mode in the response due to the white noise displacement input. The damping ratio of the second eigen mode has been determined with good agreement between the ARMA method and the IRS method even though the second eigen mode only was weakly excited in the measured free decay. Thus it is seen that the three applied methods seem to work well provided the eigen modes are excited sufficiently.

	ARMA (8,7)	Free Decay IRS	Free Decay Log.Dec.
f_1	1.1127	1.1104	-
ζ_1	0.0001	0.0009	0.0007
f_2	7.2386	7.2417	-
ζ_2	0.0011	0.0010	-

Table 1. Estimated modal parameters.

The Nonlinear Viscous Damping Case

The linear assumptions included in the ARMA method and the random dec. signature lead to models which are least squares approximations of a linear model to a nonlinear system. On the other hand, the application of the IRS method upon a measured free decay takes nonlinearities into account provided that all physical important mechanisms are included in the numerical IRS model.

As for the case of the linear structure an ARMA(8,7) model was found to be the best choice since the variance of the residual was minimized. However in this case the autocorrelation of the residual, $a(t)$ seemed not to be white noise as shown in figure 3, but showed periodicity, see figure 5. This can be taken as a possible sign of nonlinearity and a warning of the interpretation of the results.

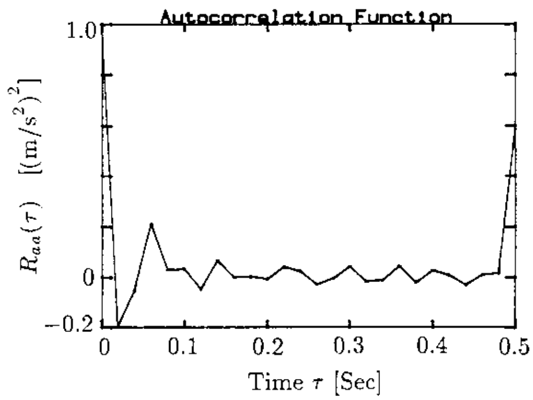

Figure 5. Residual of ARMA(8,7) for response no. 2, $\Delta =$ 0.02 Sec. and time length 120 Sec.

The IRS was performed with data obtained from the random decrement technique applied on the response due to random excitation as well as with data obtained directly from a free decay. An example of a random dec. signature is shown in figure 1.

The two model estimates were found to be:

Free Decay:

$$\begin{pmatrix} 30.6 & 0 \\ 0 & 31.9 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 1.86 & -3.67 \\ -3.67 & 12.38 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 49.3 \end{pmatrix} \begin{pmatrix} |\dot{x}_1|\dot{x}_1 \\ |\dot{x}_2|\dot{x}_2 \end{pmatrix} + \begin{pmatrix} 9812.1 & -22182.7 \\ -22182.7 & 60508.8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Random Decrement:

$$\begin{pmatrix} 20.5 & 0 \\ 0 & 44.1 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 0.38 & -0.05 \\ -0.05 & 19.03 \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} 9400.0 & -23750.4 \\ -23750.4 & 60000.0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The corresponding fits between the simulated and measured response are shown in figure 6. There are a large deviation between the estimated parameters of the two models. However this deviation was expected since the applied excitation was different and because the random decrement approach was an linear approximation.

It is seen that the IRS method applied on the free decay data gave in fact a fairly good estimate of the mounted damper characteristic. The damper calibration gave $c_{22}^{nlv} = 73.8$ kg/m while the estimate was $c_{22}^{nlv} = 49.3$ kg/m. It must be noticed that the calibration was performed by unified velocity while the mounted damper was excited randomly backwards and forwards. Thus this result shows that it is possible to identify a concentrated damping source.

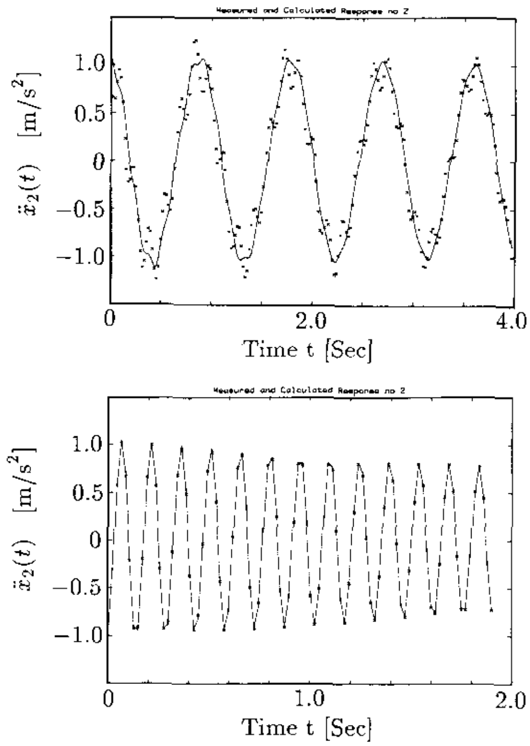

Figure 6. Top: IRS applied on the measured free decay, $\Delta = 0.02$ Sec. and time length 120 Sec. Bottom: IRS applied on the random dec. signature, $\Delta = 0.0213$ Sec. and time length 11 Sec. with 100 means.

The estimates of the two models were transformed into modal parameters for a comparison. For the nonlinear model due to the ISR method the equivalent damping ratios were determined be a least squares approach. The equivalent modal estimates are shown in table 2.

	ARMA (8,7)	Ran. Dec. IRS	Free Decay IRS
f_1	1.1024	0.0464/0.0271	1.1022
ζ_1	0.0027	1.0/-1.0	0.0029 (0.0035)
f_2	6.8037	6.7882	7.4094
ζ_2	0.0017	0.0039 (0.0041)	0.0058

Table 2. Estimated equivalent modal parameters of nonlinear viscous damped structure. The values in brackets are damping estimates obtained by the logarithmic decrement.

The first eigen frequency was well determined independent of the applied method and the damping ratio was also determined rather well by the ARMA method and IRS applied on a free decay. The IRS applied on the random decrement signature did fail to give information about the first eigen mode. This method and the ARMA method were based on the same time series thus the ARMA method seems to be more robust wrt. to identifying weakly excited eigen modes.

The modal parameters of the second eigen mode were estimated with some deviations. The eigen frequency estimate was almost the same for the ARMA method and the IRS applied on the ran.dec. signature while the IRS applied on a free decay gave some deviation. This was without doubt because the second eigen mode was only weakly represented in the free decay response.

The equivalent damping ratios showed also some deviation. Here the ARMA method seemed to have failed seriously. The damping estimate is seen to be small compared with the other estimates. This may have been due to the white noise approximation in the ARMA method which also was the cause to the increased order of the ARMA model.

Considering the frequency domain it can be explained by the fact that the force spectrum, according to eq(16) was very steep compared with steepness of the peak of frequency response function. This can very well have led to an under estimation of the damping. This was not a problem in the application of the ARMA method in the case of the linear viscous damped structure since the resonance peak of the the structure was steeper due to a lower damping.
It must be noted that where the ARMA method seemed to have failed, the IRS applied on the ran.dec. signature was also based on a white noise assumption but did give a reasonable estimate of the second damping ratio compared with estimate obtained by IRS applied on a free response. A plausible explanation is that while the ARMA method applies the hole time series for one model estimate, the IRS based on random dec. is based on averaging segments of the time series. Thus the ARMA model may lead to unbiased but uncertain estimates while the application of the random dec. signature may lead to biased but relative certain results.

In table 2 it can be noticed that the damping estimates due to the logarithmic decrement corresponded well to the estimates obtained by the more sophisticated methods.

In figure 7 is shown the auto correlation function of residuals for the fit of response no 2 for the ISR method applied on the measured free decay and the random dec. signature. It is seen that the IRS applied on the random dec. signature causes an auto correlation function which remains relative large for large time lags while the IRS applied on the measured free decay gives a smoothly decreasing auto correlation function of the residual. Thus the residual may indicate that linear model assumption have been applied on data obtained from a nonlinear structure. In the comparison of the two auto correlation function it should be noticed that the time scale was different because different modes was dominating the two time series.

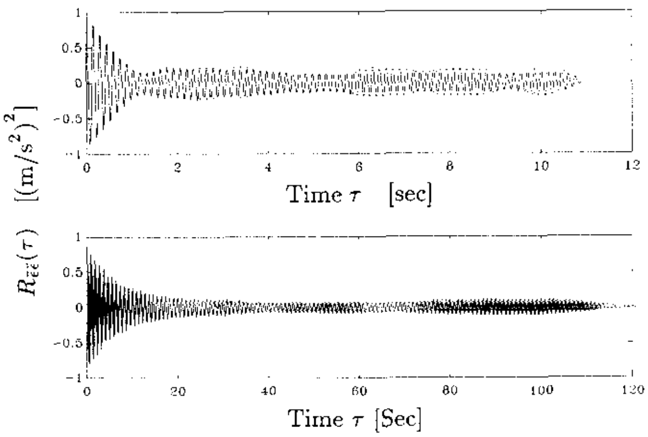

Figure 7. Top: Auto correlation of residual of the IRS method applied on the random dec. signature. Bottom: Auto correlation function of the residual of the IRS method applied on the measured free decay.

Conclusion

Linear and nonlinear damping models have been identified for a simple lightly damped structure. It has been shown that is in fact possible to identify light damping in the time domain avoiding the traditional bias due to FFT-analysis in the frequency domain.

The identification by simulation of response has shown that it is a possible way of identifying nonlinear damping mechanisms. However some difficulties exist w.r.t. choosing the initial estimates of the parameters Especially it has been found that the results are very sensitive to the start estimates of the initial conditions of the impulse response. In practice the method may be applied on simple reduced models of structures giving information about the principle performance of the structure.

The ARMA-model is able to give unbiased but uncertain damping estimates of linear lightly damped structures even when a mode is only weakly represented. The identification by the response simulation method (IRS) applied on random dec. signatures has shown that this is a possible way to extract information about the equivalent physical parameters of randomly excited systems.
It may be possible to obtain qualitative information about nonlinearities from the residuals of the IRS applied on a random dec. signature and the ARMA method. However this should be investigated further.

A fundamental future research topic should be an investigation of random and bias error on damping estimates obtained by the ARMA method and IRS based on the random dec. signature. Especially should the influence of nonlinearities on modal estimates be investigated further.

Acknowledgements

Financial support from the Danish Counsil for Scientific and Industrial Research is gratefully acknowledge.

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