Random Decrement and Regression Analysis of Traffic Responses of Bridges

Asmussen, J. C.; Ibrahim, S. R.; Brincker, Rune

Published in:
Proceedings of the 14th International Modal Analysis Conference, Dearborn, Michigan, USA, February 12-15, 1996

Publication date:
1996

Document Version
Publisher's PDF, also known as Version of record

RANDOM DECREMENT AND REGRESSION ANALYSIS OF TRAFFIC RESPONSES OF BRIDGES

J.C. Asmussen[1], S.R. Ibrahim[2] & R. Brincker'

'Department of Building Technology and Structural Engineering
Aalborg University, Sohngaardsholmsvej 57, 9000 Aalborg, Denmark.

*Department of Mechanical Engineering, Old Dominion University,
238 Kaufman/Duckworth Hall, Norfolk, Virginia, USA.

Abstract The topic of this paper is the estimation of modal parameters from ambient data by applying the Random Decrement technique. The data from the Queensborough Bridge over the Fraser River in Vancouver, Canada have been applied. The loads producing the dynamic response are ambient, e.g. wind, traffic and small ground motion. The Random Decrement technique is used to estimate the correlation function or the free decays from the ambient data. From these functions, the modal parameters are extracted using the Ibrahim Time Domain method. The possible influence of the traffic mass load on the bridge is investigated by assuming that the response level of the bridge is dependent on the mass of the vehicle load. The eigenfrequencies of the bridge are estimated as a function of the response level. This indicates the degree of influence of the mass load on the estimated eigenfrequencies. The results of the analysis using the Random Decrement technique are compared with results from an analysis based on fast Fourier transformations.

Nomenclature

a	Response value at t_i
C_y	Trig condition on y.
D	Random Decrement signature.
f	Natural frequencies.
N	Number of trig points.
R_{xy}	Cross correlation between x and y.
R'_{xy}	Time-derivative of R_{xy}
t_i	Discrete time point (trig point).
v	Value of time-derivative of response.
x	Response time series.
y	Response time series.
$\dot{y}$	Time derivative of y.
σ_y	Standard deviation of y.
τ	Time segment.
ζ	Damping ratio.

1 Introduction

This paper deals with the analysis of acceleration responses of a civil engineering structure subjected to ambient forces. The actual structure is the Queensborough Bridge across the Fraser River in Vancouver. In principle the ambient loads include both wind, traffic and small ground motions, however, the main part of the excitation is caused by traffic loads, i.e. different kinds of vehicles in service crossing the bridge during the measuring period.

The data are analysed using the Random Decrement (RDD) technique. The main purpose is to investigate and illustrate the application of the technique on a real case with non-stationary and probably non-white load. The basic theory of the RDD technique is described in Vandiver et al. [1], Brincker et al. [2], [3] and Ibrahim et al. [4]. The background of using the technique on non-white loaded structures is further developed in Ibrahim et al. [5]. The main advantages of the RDD technique are the very simple and fast estimation procedure that produces short data sequences called RDD signatures, representing the loading process, and the physical properties of the system, and furthermore, that using RDD signatures, the well-known leakage problem of FFT analysis is avoided, Brincker et al. [2].

From the RDD signatures the modal parameters can be extracted using different regression techniques like the Polyreference method, Deblauwe et al. [6], or Auto Regressive models, Pandit [7]. However, in this investigation, the Ibrahim Time Domain method is used.

When applying a regression technique to RDD signatures in the case of non-white loading, and when noise is present in the response, it is necessary to add some physical, but non-structural degrees of freedom to model the loading process, and some non-physical degrees of freedom to model the noise. It is illustrated how to distinguish the physical and the non-physical modes in the modal estimation process. The modal parameters estimated in this way are compared with an independent FFT analysis, Ventura et al. [8], and a good agreement is observed.

Due to the traffic load the observed modal parameters can be different from the true modal parameters of the bridge. The mass of the vehicles crossing the bridge changes the mass of the system, so, the modal parameters are estimated for the total system: Bridge and traffic. To investigate the influence of the mass-load, it is assumed that a high response level indicates a high mass-load, and the influence is analysed estimating the RDD signatures for different trig levels. The results indicate that the influence of the mass-load is smaller than the random uncertainty of the estimation technique. Finally, the quality of the estimated mode shapes is illustrated by one translational and one rotational mode.

2 Ambient data

A full description of the test setup, equipment arrangements and bridge geometry can be found in Ventura et al. [8]. The main structure of the bridge is a 3 span continuous structure with a length of 204 m. The main structure is supported at the ends and 56 m from both ends. During the measuring period the main load on the bridge was traffic, Ventura et al. [8]. Measurements were collected at 23 positions along the bridge at both sides of the bridge. The data were sampled at 40 Hz for about 800 sec. Before converting the data to a digital signal they were lowpass filtered. To remove the higher frequency energy more efficiently the time-series were low-pass filtered digitally using a Butterworth filter with a cut-off frequency equal to 12 Hz.

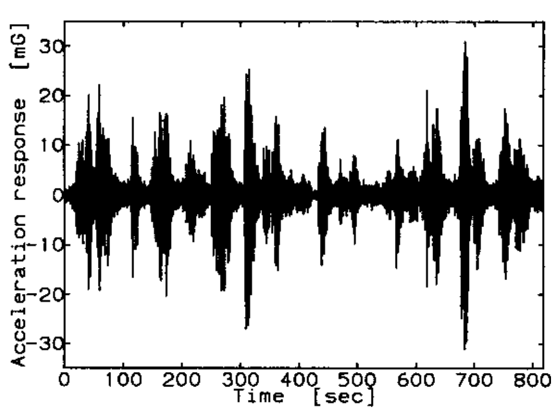

Figure 1: Typical acceleration *response of the bridge.*

Figure 1 shows a typical measured response of the bridge after digital filtering.

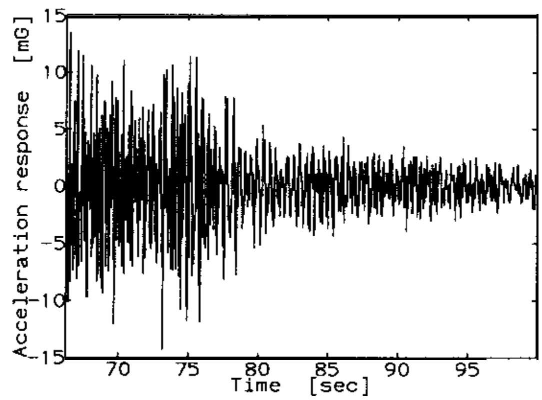

Figure 2: *Decaying part of figure 1.*

Figure 2 shows a decaying part of the response shown in figure 1. The time axes are equivalent. Figure 1 and figure 2 illustrate that in general the time series cannot be considered stationary. In Ventura et al. [8] the responses from two opposite positions of the bridge are subtracted or added to separate translational and rotational modes. This can be done in the analysis if the bridge can be considered symmetric. A similar assumption is not made in the present analysis. The loads produced by the vehicles are not measured. This means that no information is available to characterize the loads.

3 RDD-Technique

The Random Decrement (RDD) technique is a method for estimating auto- and cross-correlation functions of time series, Vandiver et al. [1], Brincker et al. [2], [3]. RDD-signatures are calculated as a sum of time segments of the time series given some initial conditions.

$$\hat{D}_{xy}(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(t_i+\tau)|C_y \tag{1}$$

In eq. (1), $\hat{D}_{xy}$ is the estimate of the cross RDD-signature between the time series x and y, N is the number of trig points, τ is the time lag and C_y a trig condition such as:

$$C_y = [y(t_i) = a \vee \dot{y}(t_i) = v] \tag{2}$$

If instead the trig condition is applied to the time series x, the auto RDD-signature is estimated. In general if x and y are stationary zero-mean Gaussian time series, the following relation between the RDD-signature and the cross correlation function exists, Brincker et al. [3].

$$\begin{aligned} D_{xy} &= E[x(\tau)|y(0) = a \wedge \dot{y}(0) = v] \\ &= \frac{R_{xy}(\tau)}{\sigma_y^2}\cdot a - \frac{R'_{xy}(\tau)}{\sigma_{\dot{y}}^2}\cdot v \end{aligned} \tag{3}$$

However in this investigation, a simpler trig condition was used:

$$C_y = [y(t_i) > a_1\,\sigma_y \vee y(t_i) < a_2\,\sigma_y] \tag{4}$$

Since no requirement is made for the time derivative of y the second term in eq. (3) will average out. This reduces *eq. (3) to:*

$$D_{xy} = \frac{R_{xy}}{\sigma_y^2} \cdot a \tag{5}$$

This means that the RDD-signatures estimated from the trig condition in eq. (4) are proportional to the cross-correlation functions. The initial value a will approximately be:

$$a \approx 0.5\,(a_1 + a_2)\,\sigma_y \tag{6}$$

Figure 3 shows auto RDD-signatures for the 4 different intervalsused: $\sigma_y < y < 2\cdot\sigma_y$, $2\cdot\sigma_y < y < 3\cdot\sigma_y$, $3\cdot\sigma_y < y < 4\cdot\sigma_y$, $4\cdot\sigma_y < y < 5\cdot\sigma_y$. The corresponding number of trig points at the different intervals is approximately 2000, 600, 300 and 50. The accuracy of the signatures should increase with increasing number of trig points. However, if noise is present in the data, false trig points might appear at low trig intervals. In the estimation process of the RDD-signatures, the length of the signatures or time segments is chosen as 300 points or 7.5 seconds. This length is believed to be optimal in the sense of having as many points as possible in the signature influenced by a minimum of noise. If not all 300 points are used in the modal parameter estimation procedure, the estimation of RDD- signatures is not repeated with correspondingly shorter time segments, to obtain more trig points.

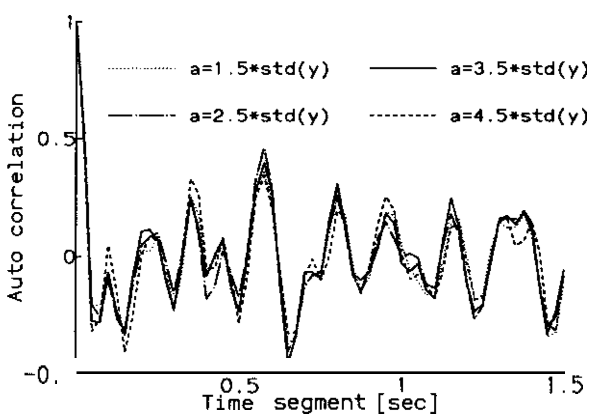

Figure 3: *4 Auto signatures obtained from the same time-series with different* trig levels.

Figure 3 illustrate the variation of the signatures on different trig level conditions. The variations could be due to noise or there could be a physical explanation. Figure 3 shows that it is common for all 4 trig levels, that the first part of the signatures is biased. Assume that the measured response is a sum of the true structural response and white noise, which is uncorrelated with the structural response. The estimated auto correlation function of the measured time series will be a sum of thr auto correlation function of the true structural response and of the noise process. If the noise process is white, it will only be non-zero close to the time $\tau = 0$. This indicates, that the first part of the signature should be omitted, as illustrated in figure 3.

Figure 4 shows 2 auto- and 2 cross-signatures estimated from two time series. The bias problem is apparent at the 2 auto signatures.

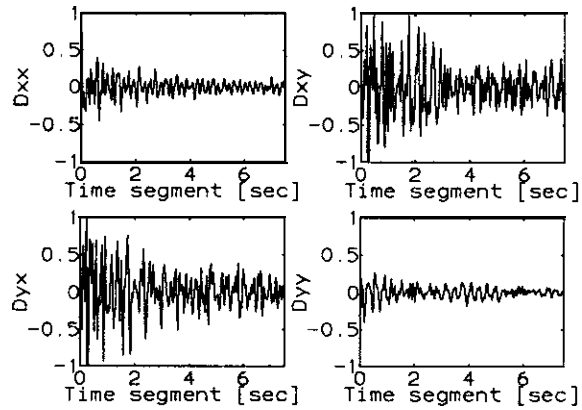

Figure 4: *Auto- and cross-signatures estimated* from 2 *different* acceleration responses of *the bridge.*

4 Analysing signatures

To extract eigenfrequencies, damping ratios and mode shapes, the Ibrahim Time Domain (ITD) method is used, see Ibrahim [10], Ibrahim et al. [11]. Input to the ITD-method is a matrix with RDD auto- and cross-signatures. To model noise the estimated model is oversized, i.e. it includes a number of non-physical modes. This means that a method for detecting modes should be applied in every identification performed. The chosen criterion for detecting a mode is: The modal confidence factor for the amplitude of a mode is greater than 90 (O-100) and the modal confidence factor for the phase of a mode is lower than 10 (0-100), see Ibrahim et al. [12]. Since the model fitted to the signatures is oversized, even modes modelling noise might fulfil this criterion. To detect only structural and load modes, a second method is applied. Every frequency detected by the first criterion is plotted along the x-axis of figure 5. Doing this for all estimations performed, a trend should be visual at a structural or load mode. This constitutes the basis for detecting modes. The results of the mode detection analysis are shown in figure 5. The actual frequencies and damping ratios of the bridge are estimated as an average of the detected modes. The averaging is an iterative procedure where the criterion for using a mode was that it was within 0.05 Hz of the actual average value.

Together with the estimated modes, (x) in figure 5, the solid lines show where it is decided to detect a mode. The averages of the estimated frequencies and damping ratios of the detected modes are shown in table 1 together with the estimated frequencies of Ventura et al. [8]. The second translational and the first rotational mode shape are shown in figure 6.

ζ_{RDD}	7.36	1.45	1.86	2.15	1.51	1.08	1.67
f_{RDD}	1.10	1.88	2.28	2.42	3.19	3.44	3.73
f_{FFT}	1.11	1.87	2.28	2.42	3.20	3.43	3.73
ζ_{RDD}	0.64	-	1.15	0.77	0.70	-	
f_{RDD}	5.15	-	5.74	7.01	7.54	-	
f_{FFT}	5.15	5.30	5.74	7.01	7.55	8.53	

Table 1: *Estimated modal parameters from RDD-signaures, f_{RDD}, ζ_{RDD}, from FFT, see Ventura et al. [8], f_{FFT}, frequency* in *[Hz], damping ratios in [%].*

From table 1, it is seen, that modes with about the same frequencies as in Ventura et al. [8] are detected, except the modes at about 5.30 (translational) and 8.43 (rotational). Since no information is available about the loads every detected mode is considered as structural, eventhough some of the modes could be load modes.

The results shown in table 1 and figure 5 are estimated from RDD signatures obtained from a trig level condition with $3\ \sigma_y < y(t_i) < 4\ \sigma_y$. As explained earlier, different trig conditions have been used corresponding to different energy levels at the response process. As it appears from figure 3, the difference between the resulting RDD signatures is small. However, to investigate if the eigenfrequencies are dependent on the trig level, the estimated eigenfrequencies corresponding to the second translational mode (1.88 Hz) are plotted as a function of the trig level in figure 7. To have a stable measure for the response level, the standard deviation of the reference measurements is used to charactererize the response level. The figure shows that the variations of the estimated eigenfrequencies are random. There is no visible trend in the value of the eigenfrequencies This means that the influence of the mass loads on the eigenfrequency is negligible compared to the estimation errors.

5 Conclusion

The applicability of the Random Decrement technique applied to ambient data is investigated. The influence of the trig level on the estimated eigenfrequencies is investigated. Under the given test-conditions it was not possible to detect any correlation between the initial response level and the eigenfrequencies. The variations of signatures estimated in the intervals $\sigma < a < 2 \cdot \sigma$, $2 \cdot \sigma < a < 3 \cdot \sigma$, $3\ \sigma < a < 4 \cdot \sigma$, are very small. In the investigation the trig level $3\sigma < a < 4\sigma$ was chosen, because good signatures were estimated based on a low number of trig points. Thus, for this case, this condition gave a good trade-off between accuracy and speed. Modes are detected in a time series if MCF-phase<10 and MCF-amplitude>90. Eigenfrequencies are detected if there is several close values, when the detected modes from all timeseries are compared. The numbers are about the same as FFT based analysis except for two eigenfrequencies, which could not be detected. The rotational frequency at 5.30 Hz is very close with a translational mode at 5.15 Hz. Since no procedure was applied to separate translational and rotational modes and since the 5.15 Hz generally has more energy than the mode at 5.30 Hz, see Ventura et al. [8], this might be an explanation. A corresponding explanation for the non-detected mode at 8.53 Hz does not exist. The mode shapes for the second translational mode and the first rotational mode are plotted. The quality of the mode shapes are comparabel with the results of the FFT-based analysis.

This analysis shows that the RDD-technique can be used to identify modal parameters, even though the assumption of a stationary Gaussian response is not fulfilled. Problems arise when the trig condition, trig level and size of time segments are selected. More information about the influence of these options is needed to get signatures with a minimum of noise. All 3 options introduces uncertainty. Furthermore, uncertainty is introduced when the modal parameters are extracted.

6 Acknowledgement

This work was supported by the Danish Technical Research Council. Their support is gratefully acknowledged. The data were used with the permission of ED1 Ltd. The authors wish to thank Dr. Felber for helpful assistance and valuable discussions.

References

[1] Vandiver, J.K., Dunwoody, A.B., Campbell, R.B. & Cook, M.F. *A Mathematical Basis for the Random Decrement Signature Analysis Technique. Journal* of Mechanical Design, Vol. 104, April 1982.

[2] Brincker, R., Krenk, S., Kirkegaard, P.H. & Rytter, A. *Identification of Dynamical Properties from Correlation Function Estimates.* Bygningsstatiske Meddelelser, Vol. 63, No. 1, 1992, pp. 1-38.

[3] Brincker, R. Note about *the Random Decrement Technique*, Aalborg University 1995.

[4] Ibrahim, S.R., Wentz, K.R. & Lee, J. *Damping Identification from Nonlinear Random Responses using a Multi-Triggering Random Decrement Technique Journal of Mechanical System and Signal Processing*, Vol. 1(4, Oct. 1987, pp. 389-397.

[5] Ibrahim, S.R., Brincker. R. & Asmussen, J.C. *Modal Parameter Identification from Responses of General Unknown* Inputs. To be presented at the 14th International Modal Analysis Conference.

[6] Deblauwe, F., Brown, D.L. & Allemang, R.J. *The Polyreference Time Domain Technique.* Proc. 5th International Modal Analysis Conference, London, 1987, pp. 832.845.

[7] Pandit, S.M. *Modal and Spectrum Analysis: Data Dependent Systems* in *State* Space. JOHN WILEY & SONS, INC. 1991.

[8] Ventura, C.E., Felber, J.A. & Prim, H.G.L. *Seismic Evaluation of a Long Span Bridge by Modal Testing.* Proc. 12th International Modal Analysis Conference, Honolulu, Hawaii 1994, p. 1309-1315.

[9] Brincker, R., Kirkegaard, P.H. & Rytter. A. *Identification of Systems Parameters by the Random Decrement Technique.* Proc. 16th International Seminar on Modal Analysis, Florence, Italy Sept. 9-12.

[10] Ibrahim, S.R. *Incipient Failure Detection from Random Decrement Time Functions.* The International Journal of Analytical and Experimental Modal Analysis, April 1986, Vol. 1, No. 2, pp. l-8.

[11] Ibrahim, S.R. & Mikulcik, E.C. *A Method for the Direct Identification of Vibration Parameters from the Free Response.* The Shock and Vibration Bulletin, Bul. 47, pp 183.198, Sept. 1977.

[12] Ibrahim, S.R. & Pappa R.S. *Large Modal Survey Testing using the Ibrahim Time Domain Identification Technique.* AIAA Journal of Spacekraft and Rockets, Vol. 19, No. 5, Sept.-Oct. 1982, pp. 459-465.

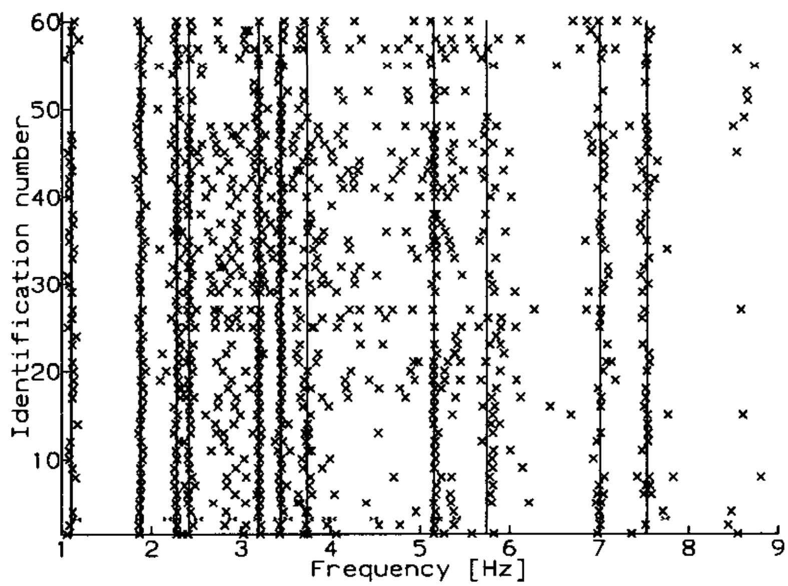

Figure *5. Identified modes with MCF-amplitude>90 and **MCF-phase<10 for** all identifications. The solid lines show where modes have been identified.*

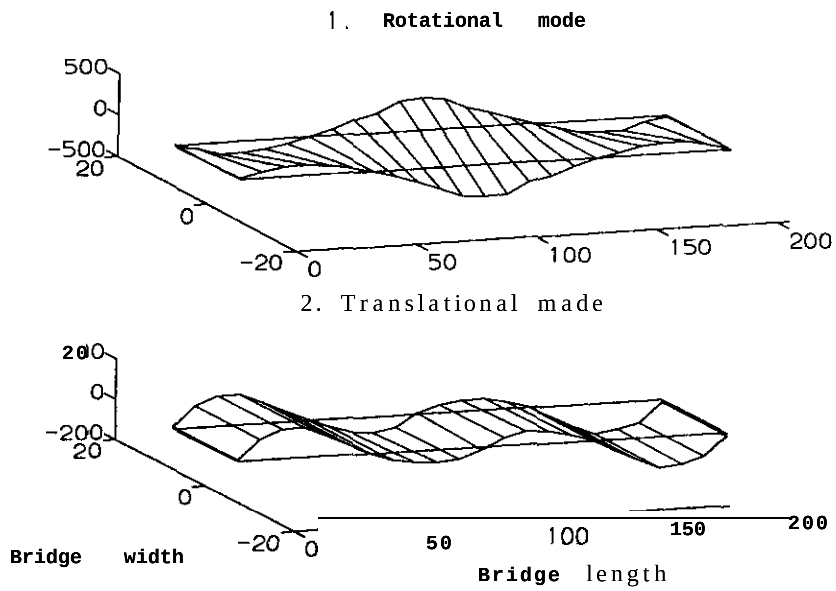

Figure 6. *The first rotational mode (2.28 Hz) and the second translational mode (1.88 Hz) together with the undeformed bridge.*

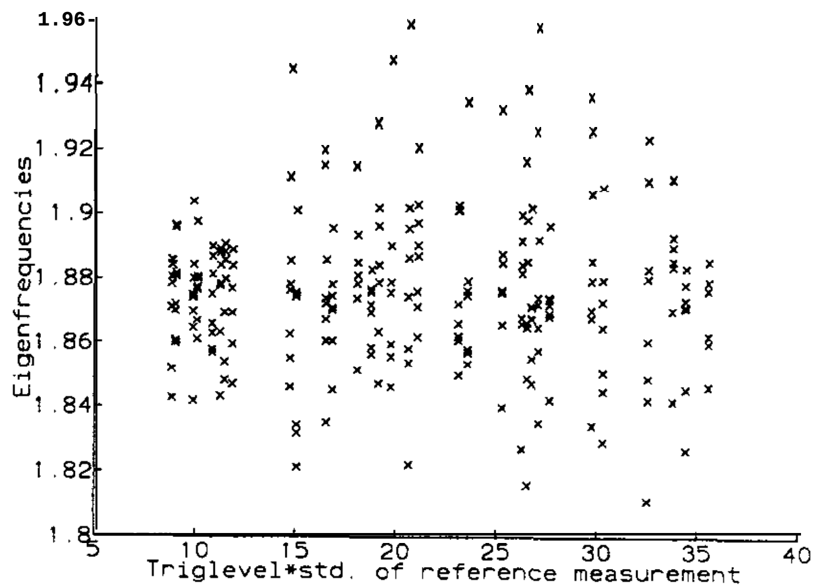

Figure 7. *Estimated frequencies plotted as a function of the trig level and the standard deviation of the reference measurements.*