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Published in:
Proceedings of the 15th International Modal Analysis Conference

Publication date:
1997

Document Version
Publisher's PDF, also known as Version of record

APPLICATION OF VECTOR TRIGGERING RANDOM DECREMENT

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Abstract *This* paper deals *with* applications *of the vector triggering Random Decrement technique. This technique is new and developed with the* aim *of* minimizing *estimation time and* identification *errors. The theory* behind *the technique* is *discussed* in an *acccompanying* paper. *The results presented* in *this paper should be regarded as a futher documentation of the technique. The key point in Random Decrement estimation is the formulation of* a *triggering* condition. *If the triggering* condition is *fulfilled a time segment from each measurement* is *picked out and averaged with previous time segments. The final result is a Random Decrement function from each measurement. In traditional Random Decrement estimation the triggering* condition is a *scalar* condition, *which should only be fulfilled in a single measurement. In vector trig*gering *Random Decrement the triggering condition* is a *vector* condition. *The advantage of this new approach should be a reduction* in estimation *time without a significant loss of accuracy, since the vector triggering conditions ensure cross information between the measurements in the Random Decrement functions. The* different *problems with this technique is highlighted in two examples. A simulation study of a 4 degree of freedom system* and *the* identification *of a laboratory bridge model, both loaded by white noise, is made.*

Nomenclature

a, a_1, a_2	Triggering level/bounds
$D_{X_iX_j}$	Random Decrement function.
$D^V_{X_i\mathbf{X}}$	Vector Random Decrement function.
N	Number of triggering points.
t, τ, **t**	Time variables, vector time variable.
$T_{X_j(t)}$	Triggering condition on $X_j(t)$ (scalar).
$T_{\mathbf{X}(\mathbf{t})}$	Vector triggering condition on X(t).
f	Eigenfrequency.
ζ	Damping ratio.
Φ	Mode shape vector

1 Introduction

Since the introduction of the Random Decrement (RDD) technique, see Cole [1], this technique has been used for identification of several different structures, such as off shore platforms, bridges, aeroplanes etc., see reference [2] [4]. The basic idea in RDD estimation is to average time segments of the measurements. The time segments are picked out if the reference measurement fulfils a triggering condition. Since only a single measurement should fulfil the triggering condition it is denoted a scalar triggering condition. The resulting averages from each measurement are called RDD functions. To each reference measurement a set of RDD functions belonging together is estimated. A set of RDD functions is usually interpreted as free decays or impulse response functions from a virtual test. This depends on the actual formulation of the scalar triggering condition. If the measurements are Gaussian a relationship between the RDD functions and the covariance functions of the time series is established, see e.g. Brincker et al. [5]. By changing the reference measurement a number of sets of RDD functions corresponding to the number of measurements can be obtained. This approach was introduced as the multiple reference RDD technique, see Ibrahim [6]. The advantage is higher identification accuracy by increasing the number of virtual tests. The modal parameters can be extracted from the RDD functions using methods like the Ibrahim Time Domain technique or the Polyreference Time Domain technique, etc., which are based on impulse response functions or free decays.

The main advantage of the RDD technique compared to other identification techniques is the speed. Since the estimation procedure only consists of a simple averaging process followed by the solution of an overdetermined set of linear equations and an eigenvalue solution the total estimation time is low. Only if the number of measurements is high, the time segments are chosen with too many points or extremely long time series are used, the estimation time of the RDD functions will be unacceptably high, especially if all measurements are used as reference measurements. Alternatively, only one or a few reference measurements can be used. This will reduce the estimation time, and also the amount of information leading to higher identification errors. Another problem is the signal-to-noise ratio in the cross RDD functions. Cross RDD functions are the functions which are obtained from any measurement besides the reference measurement. Cross RDD functions in most cases have a high signal-to-noise ratio, since they generally contain less energy (lower amplitudes) compared to auto RDD functions.

These problems motivate the formulation of the Vector RDD (VRDD) technique. By formulating a vector triggering condition the number of possible setups is reduced with the size of the vector condition. This reduces the estimation time. Furthermore, since the cross information is preserved using the vector triggering condition, no significant loss in accuracy is expected. The theoretical aspects are considered in an accompanying paper, see Ibrahim et al. [7]. The application of the VRDD technique is justified in this paper by comparing speed and accuracy with the traditional RDD technique. The comparison is based on simulated data of a linear 4DOF system loaded by white noise, see section 3 , and measurements of a laboratory bridge model also loaded by white noise through a shaker, see section 4

The simulation study shows that the VRDD technique is capable of reducing the estimation time without any significant loss of accuracy compared to the traditional RDD technique. The VRDD technique produced more accurate results than the traditional RDD technique, when only a single reference measurement is used. The analysis of the bridge data results in a high correlation between the modal parameters of the VRDD technique and the traditional RDD technique. But still the VRDD has lower estimation time.

2 Estimation Procedures

In traditional applications of the RDD technique a set of RDD functions is estimated by.

$$\hat{D}_{X_iX_j}(\tau) = \frac{1}{N}\sum_{i=1}^{N} X_i(t+\tau)|T_{X_j(t)} \qquad (1)$$

Where $i = 1, 2, ..., n$, n is the number of measurements and N is the number of triggering points in X_j detected by the triggering condition $T_{X_j(t)}$. Only a single set is required to estimate the modal parameters. To use all the information in the measurements, n sets can be estimated by applying the triggering condition to all measurements and repeat the averaging process in eq. (1). Two different triggering conditions are tested with the traditional RDD technique, the level triggering condition, $T^L_{X_j(t)}$, and the positive point triggering condition, $T^P_{X_j(t)}$.

$$T^L_{X_j(t)} = \{X_j(t) = a\} \qquad (2)$$

$$T^P_{X_j(t)} = \{a_1 \leq X_j(t) < a_2\} \qquad (3)$$

Where the triggering levels are restricted by $0 \leq a_1 < a_2$. The decisive difference between these two triggering conditions is the number of triggering points. The positive point triggering condition produces more triggering points resulting in a higher identification accuracy, but also higher estimation time. In the examples the following three estimation procedures are used for the traditional RDD technique.

- RDD_S^P RDD functions estimated using Positive point triggering. Only a single setup is used.

- RDD_A^L RDD functions estimated using Level crossing triggering. All possible setups are used.

- RDD_A^P RDD functions estimated using Positive point triggering. All possible setups are used.

- *VRDD* VRDD functions estimated using positive point triggering.

When RDD_S^P is applied the reference measurement is chosen as the measurement with the highest standard deviation. The signal-to-noise ratio is expected to be hightest in this measurement. Common for all methods is that the modal parameters are extracted using the ITD technique, see Ibrahim [8].

In application of the VRDD technique a vector formulation of the level crossing triggering condition, see eq. (2), will not produce sufficient triggering points to obtain a reasonable convergence in the averaging process, see eq. (1) and (5). Instead $T^P_{X_j(t)}$ is used in a vector form.

$$T^P_{\mathbf{X}(t)} = \{\mathbf{a}_1 \leq \mathbf{X}(t) < \mathbf{a}_2\} \quad (4)$$

Where the triggering bounds fulfil $0 \leq a_1^i < a_2^i$, $i = 1, 2, \ldots. m$ and $m \leq n$. a_1^i and a_2^i are the elements of $\mathbf{a}_1$ and $\mathbf{a}_2$.

The VRDD functions, $D^V(\tau)$, are estimated by.

$$\hat{D}^V_{X_i\mathbf{X}}(\tau) = \frac{1}{N}\sum_{i=1}^{N} X_i(t+\tau)|T^P_{\mathbf{X}(t)} \quad (5)$$

Where $i = 1, 2, ..., n$. Notice that the triggering condition does not has to be applied to all measurements. If e.g. $T^P_{\mathbf{X}(t)}$ only covers half of the measurements then a new set of VRDD functions could be estimated by applying $T^P_{\mathbf{X}(t)}$ to the second half of the measurements. Vector time variable t in $T^P_{\mathbf{X}(t)}$ makes it possible to time shift the triggering condition at different measurements. This possibility is important in order to formulate $T^P_{\mathbf{X}(t)}$ to obtain the maximum number of triggering points.

3 Example 1 - Simulation

A viscous damped linear 4-DOF system is considered. The eigenfrequencies and damping ratios are shown in table 1 and the mode shapes, which are approximately in or out of phase, are plotted in figure 1.

Mode	1	2	3	4
f [Hz]	1.62	4.61	6.86	9.00
ζ [%]	3.70	2.07	1.16	1.52

Table 1: Modal parameters of *4 DOF system.*

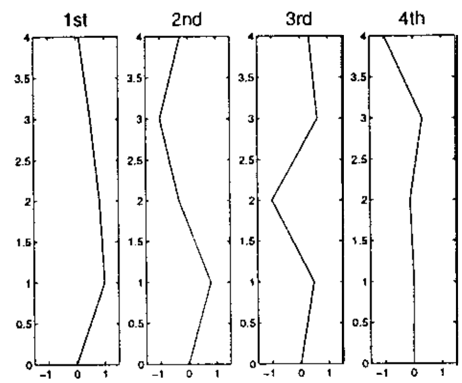

Figure 1: *Mode shapes of 4 DOF system.*

The response of this system is simulated by an ARMAV-model, see Andersen et al. [9]. The sampling frequency is 50 Hz and 8000 points are simulated in each time series. In order to describe the accuracy of the different methods statistically the simulation and estimation procedure are repeated 500 times. The following three quality measures are used.

$$Relative\ Error = \frac{1}{N_{Par}}\sum_{i=1}^{N_{Par}} \frac{|x_i - \hat{x}_i|}{x_i} \quad (6)$$

$$Bias = \frac{1}{N_{Par}}\sum_{i=1}^{N_{Par}} \frac{|x_i - \hat{x}_i|}{\sigma_{x_i}} \quad (7)$$

$$Variance = \frac{1}{N_{Par}}\sum_{i=1}^{N_{Par}} \frac{\sigma_{x_i}}{x_i} \quad (8)$$

In the above equations x_i describes an eigenfrequency, damping ratio or the absolute value of a mode shape component. The mode shapes are normalized with the largest absolute value.

The triggering levels, see eq. (2) and (3), are chosen to $a = 1.4\ \sigma_{X_j}$, $a_1 = 0$, $a_2 = \infty$. The triggering levels of the VRDD approach are $a_1 = 0$, $a_2 = \infty$. This choice ensures the maximum number of triggering points for a given time vector t. The time vector t is chosen from an initial RDD estimation with short time segments, see figure 2.

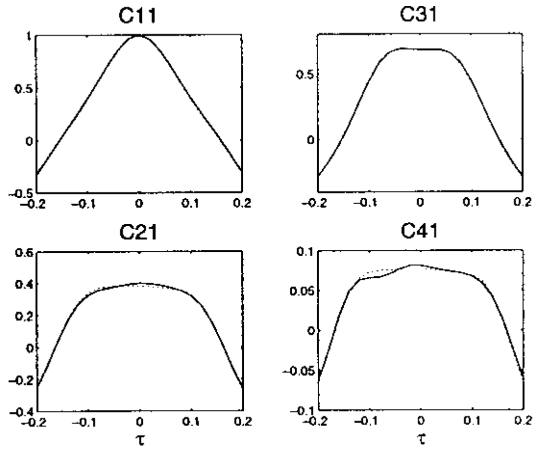

Figure 2: Initial *RDD* ***functions for selection of*** triggering *time delays. Estimated* ***(full)*** *and theoretical (dotted). C21 is an abbreviation* ***for*** *the correlation* ***function*** *between measurement 2 and 1.*

Since the simulated time series are zero mean Gaussian the estimated RDD functions are proportional to the correlation functions of the time series, see figure 2. The elements of the time vector t is chosen as the time lags where the correlation is maximum. In this case the time vector is t = 0, but this is a coincidence. The quality measures in eq. (6) - eq. (8) are shown in table 2 4.

Method	RDD_S^P	RDD_A^L	RDD_A^P	VRDD
f	0.016	0.004	0.003	0.003
ζ	1.260	0.214	0.115	0.214
Φ	2.430	0.763	0.508	0.117

Table *2: Relative error measure* ***for*** *f, ζ and Φ*

Method	RDD_S^P	RDD_A^L	RDD_A^P	VRDD
f	0.774	0.843	0.804	0.803
ζ	0.675	0.876	0.809	0.858
Φ	2.322	2.170	2.228	0.937

Table 3: Bias measure ***for*** f, ζ and Φ

Method	RDD_S^P	RDD_A^L	RDD_A^P	VRDD
f	0.040	0.010	0.006	0.008
ζ	4.637	0.475	0.285	0.481
Φ	6.428	2.042	1.800	0.288

Table 4: *Variance measure* ***for*** *f, ζ and Φ*

Table 5 shows the estimation time (CPU in [sec]) for the different methods and the number of triggering points. The initial estimation process for the VRDD technique is also included.

	RDD_S^P	RDD_A^L	RDD_A^P	VRDD	VRDD ini
Time	1.39	1.40	5.56	1.05	0.31
N	3950	470	3950	1590	3950

Table 5: *Estimation* Time (CPU-time [sec]) *and triggering points, N.*

Table 6 illustrates the conclusion of the results in table 2 table 4 and table 5. The number of + indicates the value of the methods with respect to either estimation time or accuracy.

Method	1	2	3	4
Quality	+	+ +	+ + + +	+ + +
CPU Time	++	++	+	++

Table 6: *Value* ***of*** *the different methods.*

The simulation study indicates that the VRDD approach is efficient in the sense of having low estimation time and high accuracy. Method 3 is recommended only if estimation is not a problem and high accuracy modal parameters are needed.

4 Example 2 - Experiment

This example is based on the measured acceleration response of a laboratory bridge model loaded by white noise. The bridge model is a 3-span simply supported 0.01 m thick steel plate with a total length of 3 m and a width of 0.35 m. Figure 3 shows the bridge model. The bridge is excited by a shaker attached at the right-hand span. Only identification of the mid-span bridge is considered in this paper. The 16 different locations of the accelerometers at the mid span are indicated in the figure. The measurements are sampled at 150 Hz and the white noise load is exciting the frequency span O-60

Hz. The measurements are filtered analogously and digitally after sampling.

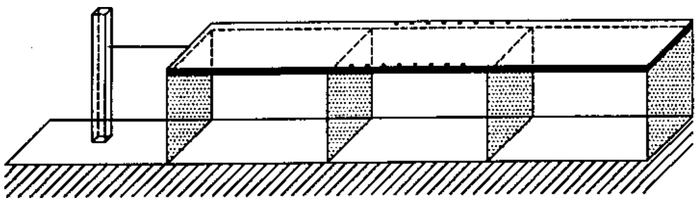

Figure *3:* ***Laboratory bridge model and location Of accelerometers.***

Each measurement consists of 32000 points. The measurements are collected using three setups with 7,7 and 6 measurements in each setup, since two reference points are used.

In order to apply the VRDD approach, the time shifts between the elements of the vector triggering condition should be chosen. As an example, the last setup with 6 measurements is considered. Figure 4 shows the initial estimate of the RDD function using level crossing triggering condition $(a = 1.4 \cdot \sigma_{X_1})$.

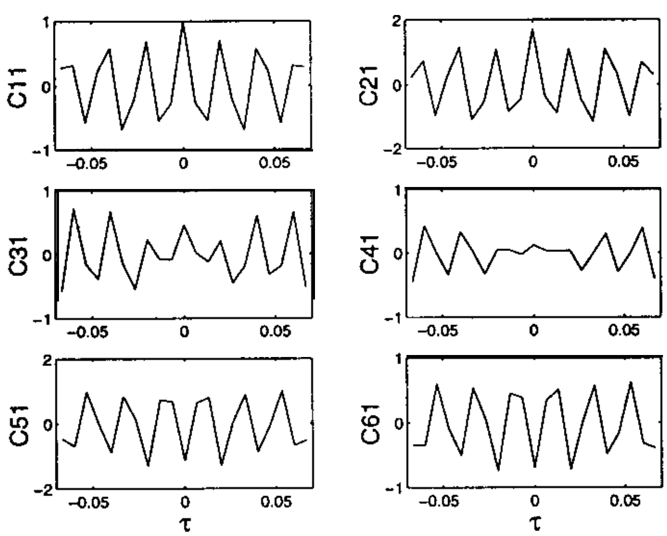

Figure ***4: Initial RDD*** functions ***for selection Of vector triggering time delays.*** *C31* ***is an abbreviation of the correlation function between measurements 3 and 1.***

The correlation are maximized by choosing the time vector as t = [0 0 0.06 0.06 0.0533 0.05331 or if the number of time lags is considered t = [0 0 9 9 8 8]. A time vector chosen as t = [0 0 9 9 − 3 − 3] would also have been a good choice. The size of the vector triggering condition does not have to be equal to the number of measurements. Table 7 shows the actual number of triggering points as a function of the size of the vector triggering condition. The elements of the triggering levels, see eq. (4), are all chosen as a; $= 0.5\sigma_X$ and a; = co.

Size	1	2	3	4	5	6
N	9900	8200	4600	4200	2600	2400

Table *7:* Size ***Of vector triggering condition and the corresponding number Of*** triggering ***points.***

The number of triggering points decreases with the size of the vector triggering condition. About 2000 triggering points are sufficient for a reasonable convergence in the averaging process. So the vector triggering condition is of size 6 (7).

For the traditional RDD technique the maximum number of sets of RDD functions is estimated. The triggering levels are chosen as $a_1 = 0.5\sigma_X$, $a_2 = \infty$. Any point between 0 and $0.5\sigma_X$ is omitted to avoid false triggering points. This level is expected to be dominated by noise. An average of the actual number of triggering points and the estimation time is shown in table 8 for the RDD technique and the VRDD technique. Level crossing triggering with $a = 1.4 \cdot \sigma_X$ is used for the initial estimate for the VRDD functions.

Method	RDD_A^P	$VRDD$	ini
Time	565	120	5
N	8700	1700	5000

Table 8: Estimation Time (CPU-time [sec]*)* ***and number Of triggering points (average), N.***

The modal parameters are extracted from the VRDD and the RDD functions using ITD. A stabilization diagram used with restrictions on the damping ratios (ζ < 10%) and the modal confidence factors is used to select the physical modes from the computational modes. Due to the loading every physical mode is taken as a structural mode. Table 9 shows the estimated modal parameters for the two appoaches.

F [Hz]	1	2	3
RDD_A^P	12.35	21.84	45.13
$VRDD$	12.35	21.84	45.15
F [Hz]	4	5	5
RDD_A^P	48.14	51.70	61.57
$VRDD$	48.11	51.64	61.59
ζ [%]	1	2	3
RDD_A^P	0.023	0.028	0.002
$VRDD$	0.026	0.019	0.003
ζ [%]	4	5	6
RDD_A^P	0.004	0.022	0.005
$VRDD$	0.004	0.010	0.002

Table *9: Estimated eigenfrequencies F in* Hz *and estimated damping ratios ζ in %*

From table 9 it is seen that there is a high correlation between the estimated modal parameters even for the damping ratios. The first pure bending and the first pure translational modes are shown in figure **5** and figure 6.

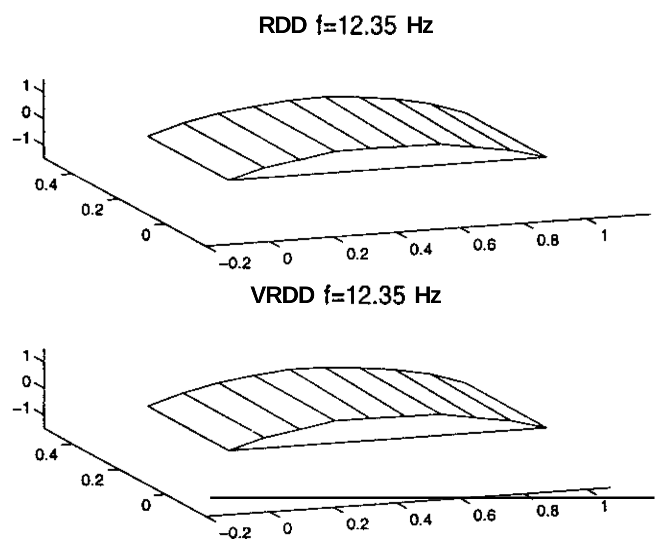

Figure *5: First bending mode of mid span. Estimated by RDD and VRDD.*

Visually there is practically no difference between the two different estimates of the modes in figure 5 and figure 6. The MAC value between the estimates are 0.9984 for the bending mode and 0.9999 for the rotational mode.

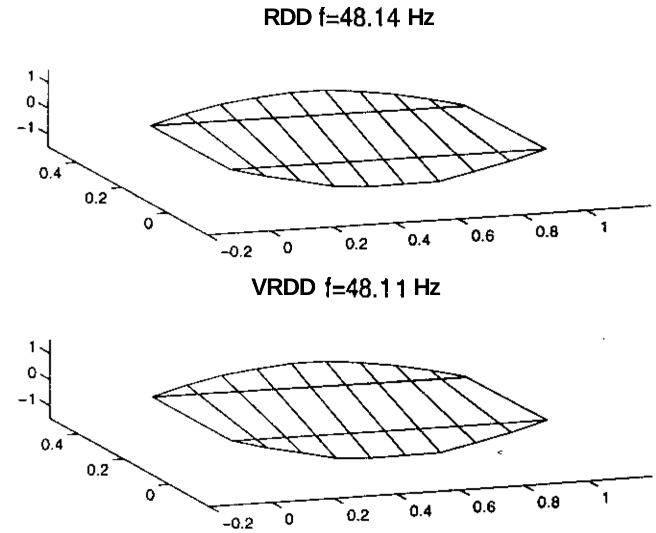

Figure *6: First rotational node of mid span. Estimated by RDD and VRDD.*

5 Conclusion

The application of the Vector triggering Random Decrement technique is justified through a simulation study and through analysis of the acceleration response of a laboratory bridge model. The theoretical arguments are discussed in an accompanying paper.

The simulation study show that the VRDD technique gives a good trade off between speed and accuracy. Only if all information is extracted by applying the positive point triggering condition and estimate the full number of setups, the RDD technique produces more accurate results. But the procedure is relatively slow.

The analysis of the bridge data resulted in a high correlation between the modal parameters estimated from RDD and VRDD functions. An approach to estimate the optimal time shifts for the formulation of the vector triggering condition is illustrated. The advantage of the VRDD technique is illustrated through a 5 time reduction of the estimation time.

6 Acknowledgement

Financial support from the Danish Technical Research Council is greatefully acknowledged.

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