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Modal Identification from Ambient Responses using Frequency Domain Decomposition

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Abstract

In this paper a new frequency domain technique is introduced for the modal identification from ambient responses, i.e. in the case where the modal parameters must be estimated without knowing the input exciting the system. By its user friendliness the technique is closely related to the classical approach where the modal parameters are estimated by simple peak picking. However, by introducing a decomposition of the spectral density function matrix, the response can be separated into a set of single degree of freedom systems, each corresponding to an individual mode. By using this decomposition technique close modes can be identified with high accuracy even in the case of strong noise contamination of the signals.

Nomenclature

x, y	Input, output
G	Power spectral density matrix
H	Frequency Response function matrix
R, A	Residues
λ	Poles
ϕ	Mode shapes
γ	Modal participation vector
ω	Angular frequency
u	Singular vectors
s	Singular values

Introduction

Modal identification from ambient responses is normally associated with the identification of modal parameters from the natural responses of civil engineering structures, space structures and large mechanical structures. Normally, in these cases the loads are unknown, and thus, the modal identification has to be carried out based on the responses only. Real case examples on some civil engineering structures can be found in Ventura and Horyna [1] or Andersen et al. [2].

The present paper deals with a new way of identifying the modal parameters of a structure from the responses only when the structure is loaded by a broad-banded excitation.

The technique presented in this paper is an extension of the classical frequency domain approach often referred to as the Basic Frequency Domain (BFD) technique, or the Peak Picking technique. The classical approach is based on simple signal processing using the Discrete Fourier Transform, and is using the fact that well separated modes can be estimated directly from the power spectral density matrix at the peak, Bendat and Piersol [3]. Other implementations of the technique make use of the coherence between channels, Felber [4].

The classical technique gives reasonable estimates of natural frequencies and mode shapes if the modes are well separated. However, in the case of close modes, it can be difficult to detect the close modes, and even in the case where close modes are detected, estimates becomes heavily biased. Further, the frequency estimates are limited by the frequency resolution of the spectral density estimate, and in all cases, damping estimation is uncertain or impossible.

The main advantage of the classical approach compared to other approaches, such as two-stage time domain identification technique by Polyreference, Vold et al [5], Ibrahim Time Domain, Ibrahim and Milkulcik [6] ERA, Juang and Papa [7], or one-stage time domain identification tehcniques such as the Stochastic Subspace Identification algorithm, Van Overschee and De Moor [8], is its user-friendliness. It is fast, simple to use, and gives the user a "feeling" of the data he or she is dealing with.

The technique presented in this paper is a Frequency Domain Decomposition (FDD) technique. It removes all the disadvantages associated with the classical approach, but keeps the user-friendliness.

In this paper it is shown that taking the Singular Value Decomposition (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system. This result is exact in the case where the loading is white noise, the structure is lightly damped, and when the mode shapes of close modes are geometrically orthogonal. If these assumptions are not satisfied, the decomposition into SDOF systems is approximate, but still the results are significantly more accurate than the results of the classical approach.

Theoretical Background of Frequency Domain Decompostion

The relationship between the unknown inputs $x(t)$ and the measured responses $y(t)$ can be expressed as, Bendat & Piersol [9]:

$$G_{yy}(j\omega) = \overline{H}(j\omega)G_{xx}(j\omega)H(j\omega)^T \qquad (1)$$

Where $G_{xx}(j\omega)$ is the $r \times r$ Power Spectral Density (PSD) matrix of the input, r is the number of inputs, $G_{yy}(j\omega)$ is the $m \times m$ PSD matrix of the responses, m is the number of responses, $H(j\omega)$ is the $m \times r$ Frequency Response Function (FRF) matrix, and "—" and superscript T denote complex conjugate and transpose, respectively.

The FRF can be written in partial fraction, i.e. pole/residue form

$$H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \frac{\overline{R}_k}{j\omega - \overline{\lambda}_k} \qquad (2)$$

where n is the number of modes, λ_k is the pole and R_k is the residue

$$R_k = \phi_k \gamma_k^T \qquad (3)$$

where ϕ_k, γ_k is the mode shape vector and the modal participation vector, respectively. Suppose the input is white noise, i.e. its PSD is a constant matrix, i.e. $G_{xx}(j\omega) = C$, then Equation (1) becomes

$G_{yy}(j\omega) =$

$$= \sum_{k=1}^{n}\sum_{s=1}^{n} \left[\frac{R_k}{j\omega - \lambda_k} + \frac{\overline{R}_k}{j\omega - \overline{\lambda}_k}\right] C \left[\frac{R_s}{j\omega - \lambda_s} + \frac{\overline{R}_s}{j\omega - \overline{\lambda}_s}\right]^H \qquad (4)$$

where superscript H denotes complex conjugate and transpose. Multiplying the two partial fraction factors and making use of the Heaviside partial fraction theorem, after some mathematical manipulations, the output PSD can be reduced to a pole/residue form as follows

$$G_{yy}(j\omega) = \sum_{k=1}^{n} \frac{A_k}{j\omega - \lambda_k} + \frac{\overline{A}_k}{j\omega - \overline{\lambda}_k} + \frac{B_k}{-j\omega - \lambda_k} + \frac{\overline{B}_k}{-j\omega - \overline{\lambda}_k} \qquad (5)$$

where A_k is the k th residue matrix of the output PSD. As the output PSD itself the residue matrix is an $m \times m$ hermitian matrix and is given by

$$A_k = R_k C \left(\sum_{s=1}^{n} \frac{\overline{R}_s^T}{-\lambda_k - \overline{\lambda}_s} + \frac{R_s^T}{-\lambda_k - \lambda_s} \right) \qquad (6)$$

The contribution to the residue from the k th mode is given by

$$A_k = \frac{R_k C \overline{R}_k^T}{2\alpha_k} \qquad (7)$$

where α_k is minus the real part of the pole $\lambda_k = -\alpha_k + j\omega_k$. As it appears, this term becomes dominating when the damping is light, and thus, is case of light damping, the residue becomes proportional to the mode shape vector

$$A_k \propto R_k C \overline{R}_k = \phi_k \gamma_k^T C \gamma_k \phi_k^T$$
$$= d_k \phi_k \phi_k^T \qquad (8)$$

where d_k is a scalar constant. At a certain frequency ω only a limited number of modes will contribute significantly, typically one or two modes. Let this set of modes be denoted

by $Sub(\omega)$. Thus, in the case of a lightly damped structure, the response spectral density can always be written

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)} \frac{d_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{\bar{d}_k \bar{\phi}_k \bar{\phi}_k^T}{j\omega - \bar{\lambda}_k} \quad (9)$$

Identification Algorithm

In the Frequency Domain Decomposition (FDD) identification, the first step is to estimate the power spectral density matrix. The estimate of the output PSD $\hat{G}_{yy}(j\omega)$ known at discrete frequencies $\omega = \omega_i$ is then decomposed by taking the Singular Value Decomposition (SVD) of the matrix

$$\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^H \quad (10)$$

where the matrix $U_i = [u_{i1}, u_{i2}, \ldots, u_{im}]$ is a unitary matrix holding the singular vectors u_{ij}, and S_i is a diagonal matrix holding the scalar singular values s_{ij}. Near a peak corresponding to the k th mode in the spectrum this mode or may be a possible close mode will be dominating. If only the k th mode is dominating there will only be one term in Equation (9). Thus, in this case, the first singular vector u_{i1} is an estimate of the mode shape

$$\hat{\phi} = u_{i1} \quad (11)$$

and the corresponding singular value is the auto power spectral density function of the corresponding single degree of freedom system, refer to Equation (9). This power spectral density function is identified around the peak by comparing the mode shape estimate $\hat{\phi}$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found that has high MAC value with $\hat{\phi}$ the corresponding singular value belongs to the SDOF density function.

From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained. In this paper the piece of the SDOF PSD was taken back to time domain by inverse FFT, and the frequency and the damping was simply estimated from the crossing times and the logarithmic decrement of the corresponding SDOF auto correlation function.

In the case two modes are dominating, the first singular vector will always be a good estimate of the mode shape of the strongest mode. However, in case the two modes are orthogal, the first two singular vectors are unbiased estimates of the corresponding mode shape vectors.

Example, Simulation of a 2-Storey Building

In this example the response of a two-storey building is simulated used a lumped parameter system with 6 degrees of freedom. The measurements are assumed to be taken so that the rigid body motions of the floor slaps can be estimated. The geometry and the measurement points are shown in Figure 2.

This structure has two sets of close modes. The first two modes are bending modes, and the model was calibrated in such a way, that these two bending modes were close, but not very close. The third mode is a torsion mode, and the fourth and fifth modes are again close bending modes. The model was calibrated in such a way that the fourth and fifth modes were very close, nearly repeated poles.

The response was simulated using a vector ARMA model to ensure that the simulated responses were covariance equivalent, Andersen et al. [10]. The model was loaded by white noise, and the response was analysed using the identification technique introduced above. The simulated time series had a length of 10000 data points and three cases were considered: no noise, 10 % noise and 20 % noise added.

The singular values of the spectral density function matrix is shown in Figure 1. As it appears, the close modes are clearly indicated in this plot. Using the FDD identification procedure decribed above, the natural frequencies and damping ratios were identified with high accuracy, se Table 1 for the natural frequencies and Table 2 for the damping values. As it appears, the technique is not sensitive to the noise. Also the mode shape estimates were very close to the exact results. Note especially the mode shapes for the two nearly repeated modes (the fourth and the fifth) in Figure 5.

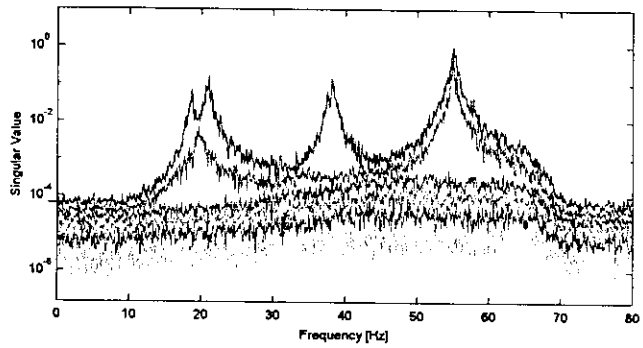


Figure 1. Singular values of the power spectral density matrix of the response.

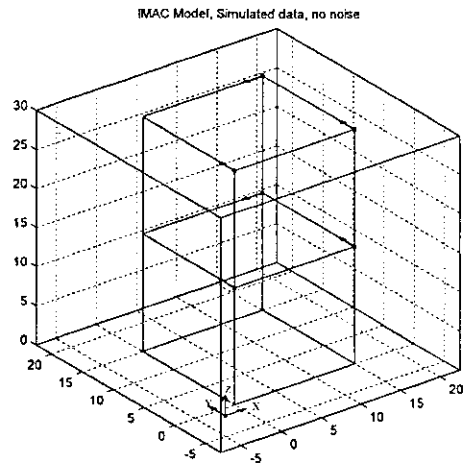


Figure 2. *Geometry of 2 storey-building model. Measurement points are indicated by arrows.*

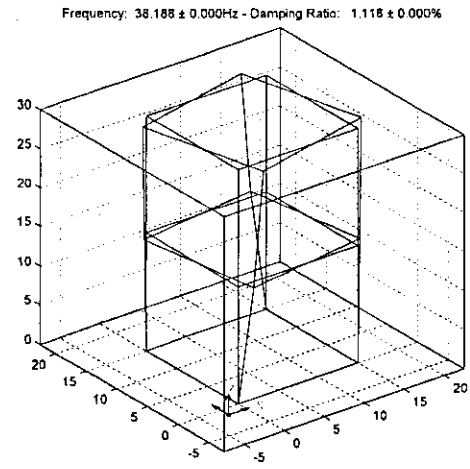
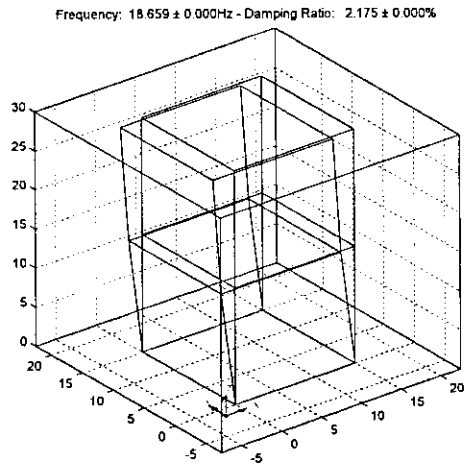


Figure 4. Estimated mode shape for the third mode (building torsion).



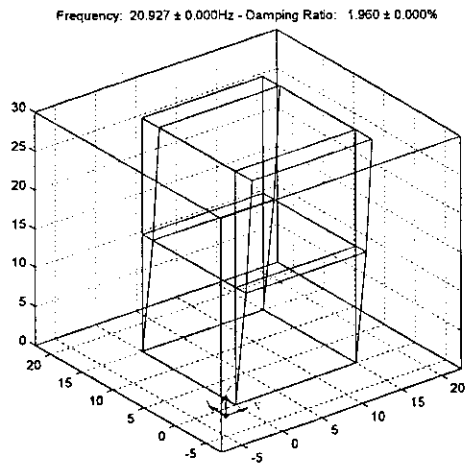
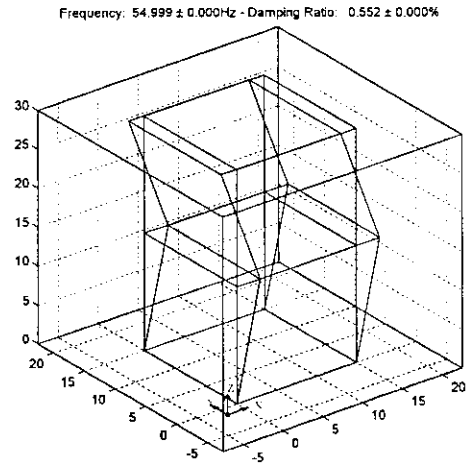


Figure 3. Estimated mode shapes for close modes, the first and the second mode (building bending).



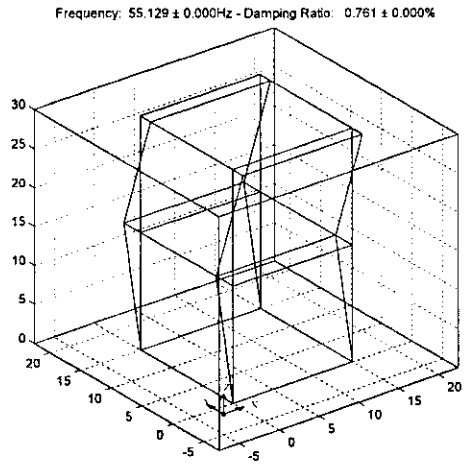


Figure 5. Estimated mode shapes for nearly repeated modes, the fourth and the fifth mode (building bending).

Table 1. Estimated Natural Frequencies

Exact (Hz)	Without noise (Hz)	10 % noise (Hz)	20 % noise (Hz)
18.686	18.676	18.661	18.665
21.054	20.930	20.927	20.938
38.166	38.188	38.188	38.206
55.055	55.036	55.011	54.999
55.121	55.129	55.133	55.125

Table 2. Estimated Damping Ratios

Exact (%)	Without noise (%)	10 % noise (%)	20 % noise (%)
2.13	2.22	2.19	2.33
1.89	1.97	1.98	1.97
1.04	1.12	1.11	1.13
0.72	0.61	0.61	0.55
0.72	0.76	0.76	0.77

Conclusions

In this paper a new frequency domain identification technique denoted Frequency Domain Decomposition (FDD) has been introduced.

The technique is based on decomposing the power spectral density function matrix using the Singular Value Decomposition. It has been shown that this decomposes the spectral response into a set of single degree of freedom systems, each corresponding to one individual mode.

The technique has been illustrated on a simulation example with noise and close modes. The results clearly indicate that the present technique is able to estimate close modes with high accuracy and that the technique is not sensitive to noise.

In the case of close modes that are not orthogonal, the mode shape of the dominant mode is still well estimated. However, if the other mode is not dominating a any frequency, other ways of estimating the mode shape for such a mode must be introduced.

The technique has been applied successfully to several civil engineering cases, Brincker et al. [11], [12] and to several cases of identification in mechanical engineering where the the structure was loaded by rotating machinery, Brincker et al. [13], [14] and Møller et al. [15].

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