



Aalborg Universitet

AALBORG UNIVERSITY  
DENMARK

## Automated Modal Parameter Estimation of Civil Engineering Structures

Andersen, Palle; Brincker, Rune; Goursat, Maurice; Mevel, Laurent

*Publication date:*  
2007

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Andersen, P., Brincker, R., Goursat, M., & Mevel, L. (2007). *Automated Modal Parameter Estimation of Civil Engineering Structures*. Paper presented at The International Conference of Experimental Vibration Analysis for Civil Engineering Structures, Porto, Portugal.

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Automated Modal Parameter Estimation of Civil Engineering Structures

Palle Andersen

*Structural Vibration Solutions A/S, Aalborg, Denmark*

Rune Brincker

*Department of Civil Engineering, University of Aalborg, Aalborg, Denmark*

Maurice Goursat

*SISTHEM/INRIA, Le Chesnay Cedex, France*

Laurent Mevel

*SISTHEM/IRISA, Rennes Cedex, France*

## ABSTRACT

In this paper the problems of doing automatic modal parameter extraction of ambient excited civil engineering structures is considered. Two different approaches for obtaining the modal parameters automatically are presented: The Frequency Domain Decomposition (FDD) technique and a correlation-driven Stochastic Subspace Identification (SSI) technique. Finally, the techniques are demonstrated on real data.

## 1 INTRODUCTION

For several reasons there has for many years existed a wish for a reliable automatic approach for estimation of modal parameters in civil engineering.

If Structural Health Monitoring (SHM) is going to use modal information effectively it is necessary to avoid constant user interaction in the modal extraction process. Today, most monitoring systems have to rely on estimation and automatically inspection of e.g. spectral densities, an approach widely used in monitoring of rotating machinery. However, this cannot be used as the sole tool in monitoring of civil engineering structures, where changes of excitation levels and distribution then will cause too many false alarms. It is necessary to be able to distinguish between the structure and the excitation, which can only happen by inspection of physical parameters such as the modal parameters.

For large ambient tests involving many measurement channels, many setups or many modes it can be a quite time consuming task to extract the modal parameters manually. In such case it is desirable to be able to estimate most of the modes automatically and only spend time on a few difficult modes.

Finally, there might be cases where manually extraction of modes might not be possible; cases where the modes are maybe weakly excited, or highly damped and broad-banded excitation cover the exact locations of the modal peaks.

This paper will focus on the problem of extracting modal parameters of a civil engineering structure automatically. Two different approaches for obtaining the modal parameters automatically using Operational Modal Analysis (OMA) will be presented.

The Frequency Domain Decomposition (FDD) technique is known as one of the most user friendly and powerful techniques for operational modal analysis of structures. However, the classical implementation of the technique requires some user interaction. In this paper an automatic algorithm for FDD is presented.

The Stochastic Subspace Identification (SSI) is a well known identification procedure well suited to handle data with non stationarity. Many variants exist. Here, we will focus on an automated version of the multipatch covariance driven subspace. This variant has the advantage over data driven versions of using less memory and merges data from different records measured at different periods of time. Special attention will be given to the problem of automated extraction of the modes. Finally, the techniques will be demonstrated on real data.

## 2 THE FREQUENCY DOMAIN DECOMPOSITION (FDD) TECHNIQUE

In order to do proper modal analysis of civil engineering structures it is often necessary to measure for very long time due to the low-frequency modes originating from the large structural dimensions. At the same time many sensors locations are often needed, especially when doing a modal survey of a new structure. The inevitable result is many measurement channels with many samples to process. When dealing with many channels and many samples, the frequency domain is a desirable domain to work with. The estimation of spectral densities can easily be performed because only small segments of data will stay in the computers memory at the same time. Also the estimation time becomes an insignificant problem, due to the extremely fast implementations of the Fast Fourier Transform (FFT) available for various PC processors.

### 2.1 *The basic algorithm*

Though it is still popular to work directly with spectral densities it is cumbersome to deal with all the auto- and cross spectral densities, and the accuracy of the modal parameter estimates extracted will depend very much on how well-separated the modes are. The Frequency Domain Decomposition (FDD) technique is a way to solve these two problems, Brincker et al [1], [2].

The technique simplifies the user interaction because the user has only to consider one frequency domain function - the singular value diagram of the spectral density matrix. This diagram concentrates information from all spectral density functions. Further, if some simple assumptions are fulfilled, the technique directly provides a modal decomposition of the vibration information, and the modal information for each mode can be extracted easily and accurately. The technique works even in the case of closely spaced modes and when a lot of noise is present.

The principle in the FDD technique is easiest illustrated by realizing that any response  $\mathbf{y}(t)$  at time  $t$  can be written in modal coordinates,  $q_i$ , as

$$\mathbf{y}(t) = \boldsymbol{\varphi}_1 q_1(t) + \boldsymbol{\varphi}_2 q_2(t) + \dots = \boldsymbol{\Phi} \mathbf{q}(t) \quad (1)$$

where  $\boldsymbol{\varphi}_i$  are the corresponding mode shape vectors,  $\boldsymbol{\Phi}$  is a matrix that contains the modes shape vectors as its columns, and  $\mathbf{q}(t)$  is the vector of corresponding modal coordinates. Now obtaining the covariance matrix of the responses

$$\mathbf{C}_{yy}(\tau) = E\{\mathbf{y}(t + \tau)\mathbf{y}(t)^T\} \quad (2)$$

and using equation (1) leads to

$$\mathbf{C}_{yy}(\tau) = E\{\boldsymbol{\Phi} \mathbf{q}(t + \tau) \mathbf{q}(t)^H \boldsymbol{\Phi}^H\} = \boldsymbol{\Phi} \mathbf{C}_{qq}(\tau) \boldsymbol{\Phi}^H \quad (3)$$

Expressing that the covariance of the measurements,  $\mathbf{C}_{yy}(\tau)$ , is related to the covariance of the modal coordinates,  $\mathbf{C}_{qq}(\tau)$ , through the mode shape matrix. The  $^H$  is the Hermitian transposed operator.

The equivalent relation in frequency domain, giving the spectral density matrix  $\mathbf{G}_{yy}(f)$  of the response, is obtained by taking the Fourier transform

$$\mathbf{G}_{yy}(f) = \boldsymbol{\Phi} \mathbf{G}_{qq}(f) \boldsymbol{\Phi}^H \quad (4)$$

Thus if the modal coordinates are uncorrelated, the spectral density matrix  $\mathbf{G}_{qq}(f)$  of the modal co-ordinates is diagonal, and thus, if the mode shapes are orthogonal, then equation (4) is a singular value decomposition (SVD) of the response spectral matrix  $\mathbf{G}_{yy}(f)$ . Therefore, FDD is based on taking the SVD of the spectral density matrix

$$\mathbf{G}_{yy}(f) = \mathbf{U}(f) [\mathbf{s}_i] \mathbf{U}(f)^H \quad (5)$$

The matrix  $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots]$  is a matrix of singular vectors and the matrix  $[\mathbf{s}_i]$  is a diagonal matrix of singular values. As it appears from this explanation, plotting the singular values of the spectral density matrix will provide an overlaid plot of the auto spectral densities of the modal coordinates. Note here that the singular matrix  $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots]$  is a function of frequency because of the sorting process that is taking place as a part of the SVD algorithm.

#### 2.4 Data Reduction – Projection Channels

Since the spectral density matrices typically consist of much more columns than there are modes participating at the difference frequencies, many of the columns of  $\mathbf{G}_{yy}(f)$  are linear dependent upon each other. For large channel counts there are no need to process all the columns and the following SVD of equation (5). Good and

reliable estimates of the modal parameters can be obtained from a limited number of columns. The columns we choose will be called the projection channels.

In the example used in this paper five reference channels were used, and these references were placed carefully in order to ensure that all analyzed modes were present in at least one of them. We choose these five reference channels as our projection channels in this case. The quality of the choice of projection channels should be verified by looking at the SVD of the spectral densities. If the last plotted singular value forms a horizontal line over the frequency band of interest, and if the other singular values display a good mode separation, then the choice is fine. If not other and / or more projection channels should be included. If more projection channels are needed, the channels to look for should contain as much new information as possible about the system compared to the channels already selected. This evaluation can be performed using a simple analysis of the correlation coefficients between the difference measurement channels.

### 2.3 The Manual Approach

A mode is identified by looking at where the first singular value has a peak, let us say at the frequency  $f_0$ . This defines in the simplest form of the FDD technique - the peak picking version of FDD - the modal frequency. The corresponding mode shape is obtained as the corresponding first singular vector  $\mathbf{u}_1$  in  $\mathbf{U}$ .

$$\boldsymbol{\varphi} = \mathbf{u}_1(f_0) \quad (6)$$

## 3 AUTOMATING THE FDD TECHNIQUE

The process of finding peaks on a function is easily automated. However, to help distinguish between the different physical modes, harmonics and noise we introduce a set of indicators in the following.

### 3.1 Modal Coherence Indicator

Suppose a peak has been identified in the first singular value. The question is now if this is a liable modal peak or if it just a noise peak. Calculating the correlation between the first singular vector at the peak – the mode shape vector at that point - and the first singular vector at neighboring points defines the discriminator function called the modal coherence

$$d_1(f_0) = \mathbf{u}_1(f)^H \mathbf{u}_1(f_0) \quad (7)$$

If the modal coherence is close to unity, then the first singular value at the neighboring point correspond to the same modal coordinate, and therefore, the same mode is dominating. This function is helpful in discriminating between points dominated by modal information and points dominated by noise.

If the components of each of the vectors in equation (7) are random, then

$$E\{\mathbf{u}_1(f_0)^H \mathbf{u}_1(f)\} = 0 \quad (8)$$

and since the length is unity

$$\text{Var}\{\mathbf{u}_1(f_0)^H \mathbf{u}_1(f)\} = 1/N_m \quad (9)$$

where  $N_m$  is the number of measurement channels. Thus the more measurement channels we have the closer two points with random (non-physical) information will get to zero. A reasonable criterion for accepting the neighboring point as a point with similar physical information, and thus accepting the presence of physical information at that frequency, could be by introducing a threshold level  $\Omega_1$  and the requirement

$$d_1 \geq \Omega_1 \quad (10)$$

setting the limit  $\Omega_1$  equal to a number close to 1, say 0.8.

### 3.2 Modal Domain Indicator

Once a peak has been accepted as representing modal information, another discriminator function can be helpful in discriminating between different modes. In this case the discriminator function is defined as

$$d_2(f) = \mathbf{u}_1(f)^H \mathbf{u}_1(f_0) \quad (11)$$

Thus this discriminator function is not a function of the initial point given by the frequency  $f_0$ , but is a function of the frequency  $f$  of the considered neighboring point. If a high correlation is present over a certain frequency range around the considered peak it means that over that frequency range only that mode is dominating and introducing a similar criterion

$$d_2 \geq \Omega_2 \quad (12)$$

defines a frequency range  $[f_0 - \Delta f_1; f_0 - \Delta f_2]$  around each peak of modal dominance called the modal domain. The lower the value  $\Omega_2$ , the larger the size  $\Delta f = \Delta f_1 + \Delta f_2$  of the corresponding modal domain. Again a good initial value of  $\Omega_2$  would be 0.8. See Brincker et al. [3] for a more comprehensive discussion the choices of  $\Omega_1$  and  $\Omega_2$ .

### 3.3 The Automatic Approach

For a search set of interest, which usually will be maximum singular values at all discrete frequencies between DC and the Nyquist frequency, the following procedure is our proposal for an automatic FDD approach:

1. Identify a peak on the first singular value line representing a maximum
2. Check if the peak is likely to be physical
3. If so, establish the modal domain
4. If not define a noise domain around the peak
5. Exclude the modal domain or noise domain from the search set
6. Continue until the search set is empty, the peak is below the predefined excitation level, or a specified number of modes has been estimated

The key point of the algorithm is point 2). As described earlier, it is essential at this point to include a criterion concerning the correlation between neighboring points as described by the modal coherence function  $d_j$ . Also it is essential to be able to distinguish between a harmonic peak and a modal peak, see Andersen et al. [4] for a description of how to do this. Additional criteria can be based on for instance the size of the modal domain being larger than a certain value or the damping estimate being below a certain value. If we are looking for a certain number of modes, we can pick the modes that have the largest modal domain, or the modes that have the largest excitation level.

For basic FDD we are satisfied when the peak is identified as a modal peak, and the corresponding mode shape vector is estimated. For the enhanced version of FDD the algorithm also works, but some additional steps are required, see e.g. Brincker et al [1],[2].

## 4 SSI MULTIPATCH MERGING

### 4.1 Algorithm

The multipatch subspace approach has been presented in Mevel et al. [5]. It is based on the fact that merging Hankel matrices from different measurement setups is only possible if setups share common reference sensors and the excitation is the same for all setups. If the first requirement is impossible to be avoided and is common for many identification methods, it has been shown that the second requirement can be dropped if proper normalization is applied to the Hankel matrix of each setup.

From the state space model,

$$\begin{cases} X_{k+1}^j &= F X_k^j + V_k^j, & \text{cov}(V_k^j) = Q_j \\ Y_{k,0}^{0,j} &= H^0 X_k^j & \text{(the reference)} \\ Y_k^j &= H^j X_k^j & \text{(sensor pool } n^o j) \end{cases} \quad (14)$$

one can define the following correlations between the reference sensors itself and between the moving and reference sensors for record  $j$  at time lag  $i$ , we got the matrix relation involving  $G_j$

$$\begin{aligned} R_i^{0,j} &= \mathbf{E}Y_k^{0,j}(Y_{k-i}^{0,j})^T = H^0 F^i G_j \\ R_i^j &= \mathbf{E}Y_k^j(Y_{k-i}^{0,j})^T = H^j F^i G_j \end{aligned} \quad (15)$$

The Hankel matrix corresponding to the reference sensors of record  $j$  is

$$\begin{aligned} \mathcal{H}^{0,j} &= \begin{pmatrix} R_0^{0,j} & R_1^{0,j} & R_2^{0,j} & \dots \\ R_1^{0,j} & R_2^{0,j} & R_3^{0,j} & \dots \\ R_2^{0,j} & R_3^{0,j} & R_4^{0,j} & \dots \\ \vdots & \vdots & \ddots & \vdots \end{pmatrix} \\ &= \mathcal{O}(H^0, F) \mathcal{C}(F, G_j) \end{aligned} \quad (16)$$

By juxtaposing the different reference Hankel matrices from all records, we got the merged reference Hankel matrix, built on the concatenation of the different autocorrelations, and thus we end up with the following decomposition

$$\begin{aligned} \mathcal{H}^0 &= \mathcal{O}(H^0, F) \mathcal{C}(F, G) \\ R_i^0 &= ( R_i^{0,1} \quad R_i^{0,2} \quad \dots \quad R_i^{0,J} ) \\ G &= ( G_1 \quad G_2 \quad \dots \quad G_J ) \end{aligned} \quad (17)$$

Finally, we obtain the different controllability matrices in the same modal basis by splitting the  $\mathcal{C}(F, G)$  matrix in the same manner  $G$  is split. Those matrices are also involved in the decomposition of  $\mathcal{H}^j = \mathcal{O}(H^j, F) \mathcal{C}(F, G_j)$ , where both left and right terms are different, and thus the matrices can not be stacked without proper normalization. By renormalizing each Hankel matrix like

$$\bar{\mathcal{H}}^j = \mathcal{H}^j (\mathcal{C}^T(F, G_j) [\mathcal{C}(F, G_j) \mathcal{C}^T(F, G_j)]^{-1} \mathcal{C}(F, G_1)) . \quad (18)$$

We can now stack them and get the matrix  $\bar{\mathcal{H}} = \mathcal{O}(H, F) \mathcal{C}(F, G_1)$ , where  $\mathcal{C}(F, G_1)$  is assumed to be the best conditioned controllability matrix. Finally, we do the SVD as usual for the subspace method.

This approach has been coupled with the usual subspace approach where identification is performed on each setup, and modes are extracted automatically. Usually subspace diagrams are noisy, but the multipatch diagram is much clearer, allowing determining the structural modes easily. By coupling both diagrams, using some automated extraction approach described below, one can extract modes and reconstruct modes and mode shapes. Future works will focus on improving the numerical efficiency of the method.

#### 4.2 The Automatic Approach

The identification provides a stabilization diagram. This diagram must be analyzed in order to distinguish physical from computational modes. The algorithm must also provide the frequency, the damping and the mode shape.



The first step of the automated extraction procedure is searching for alignments using graph theory. Those alignments are defined neighborhood by neighborhood looking at the order of the points. We consider every point of the stabilization diagram as the vertex of a directed graph. For each point of a given order we define an edge from this point to the nearest point at an inferior order, and another edge from this point to the nearest point at a superior order.

The key of the algorithm is to define the nearest point. Each point of the stabilization diagram is associated to an order, a frequency, a damping and a mode shape. For any two points we can define 3 distances (frequency, damping, MAC). The nearest point will be the point which is the nearest for at least 2 distances. We look at the closest point in frequency (euclidian distance), then in modeshapes (MAC distance). Usually damping is not considered because of its high variance. As the diagram is associated with a directed graph, it is trivial to extract connected sub graphs using graph theory. Each sub graph defines an alignment. For each alignment, all sub alignments of a given length are considered, in order to find the most linear part. From this linear part, we will try to extend it to a longer alignment. The automated extraction algorithm need only the minimum length of an alignment as input parameter. This algorithm is very fast and very robust, so that it can be used for a monitoring routine. Finding and extending the linear part can be complex with the usual subspace algorithm because of the presence of many spurious modes. For the multi patch merging technique, the decrease in the number of spurious modes makes the automated extraction pretty trivial.

## 5 EXAMPLE – Z24 HIGHWAY BRIDGE

The two automatic approaches have been tested on a fairly large set of real data. The data is from the Z24 Bridge of the SIMCES project, and the test case used is the one called Progressive Damage Test no. 10. This case consists of 9 setups each with 33 channels, except setup 5 having only 27 channels. Five common reference channels were used and these have been selected as the Projection Channels in the analysis. The data has been sampled at 100 Hz with a measurement time of 655 seconds, resulting in 65516 samples per channel.

### 4.1 Results of the Automatic FDD Technique

The results in this section are obtained using the Automatic Modal Analysis component of the ARTeMIS *Extractor* 4.1. This component implements the automatic FDD approach described in section 2.5 using the indicators presented in section 2.4. The thresholds  $\Omega_1$  and  $\Omega_2$  have both been set to 0.8. In figure 4.1 the modal coherence is shown as the dark gray area on top of the diagram.

The top of the diagram corresponds to a modal coherence of 1 and the bottom to 0. Especially around the first mode it is seen that the modal coherence is almost 1 for a fairly broad frequency range. The modal domains of the different modes are displayed in light gray, and again this domain is quite large for the first mode as well.

It can also be observed how the automatic algorithm is capable of detecting modes at places where the peaks are not very distinct, as with the last of the modes

in figure 4.1. The total processing time from starting uploading data and to the final identification of the presented modes is around 2-5 minutes depending on the PC. So even for such a large set of data, it is possible very quickly to have results of the most well-excited modes.

#### 4.2 Results of the Automatic SSI Multipatch Merging Technique

Figure 4.2 compares the stabilization diagram of the multipatch technique with the traditional correlation driven SSI method of one of the nine setups. The multi patch stabilization diagram is much clearer, and therefore of course much easier to work with for the automatic mode identification algorithm, that in this case identifies the six modes below 14 Hz.

### 5 CONCLUSIONS

The problem of doing automatic modal parameter extraction has been addressed in this paper. Two new and completely different methods have been presented.

The first is the automated Frequency Domain Decomposition technique, where the peak picking has been automated in a robust way by the introduction of modal coherence, modal domain and harmonic indicators. This has made it possible to extract modes not even having distinct peaks as show in the example.

The second is an automated version of the multipatch covariance driven Stochastic Subspace Identification technique. The multipatch technology results in just a single stabilization diagram even when multiple setups of data are analyzed. In addition this stabilization diagram is much clearer as the only content that are the same between the different setups are the physical information of the system being measured. The noise differs from setup to setup, and is therefore suppressed in the multipatch stabilization diagrams. It has been described how an automatic mode selection can be implemented.

### 6. FIGURES

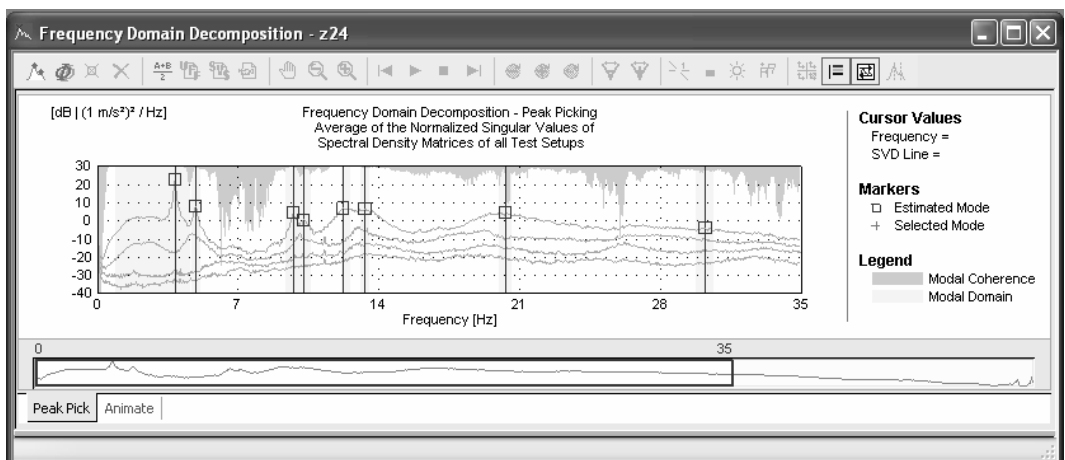


Figure 4.1: The Automatic Frequency Domain Decomposition. The red indicator shows the identified modes. The modal coherence is shown in light blue, and the modal domains in light green.

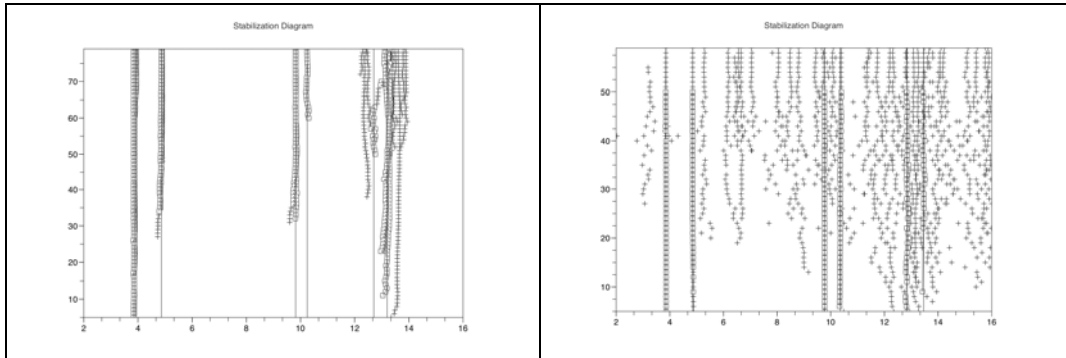


Figure 4.2: Multipatch stabilization diagram versus a traditional subspace stabilization diagram.

## 6 REFERENCE LIST

- [1] R. Brincker, P. Andersen, L. Zhang: *Modal Identification From Ambient Responses Using Frequency Domain Decomposition*. In Proceedings of The 18th International Modal Analysis Conference (IMAC), San Antonio, Texas, pp.625-630, 2000.
- [2] R. Brincker, L. Zhang and P. Andersen: *Modal identification of output-only systems using frequency domain decomposition*, 2001 Smart Mater. Struct. 10, 441-445
- [3] R. Brincker, P. Andersen, N.J. Jacobsen: *Automated Frequency Domain Decomposition for Operational Modal Analysis*. In Proceedings of The 25<sup>th</sup> International Modal Analysis Conference (IMAC), Orlando, Florida, 2007.
- [4] P. Andersen, R. Brincker, M. Goursat, L. Mevel: *Automated Modal Parameter Estimation For Operational Modal Analysis of Large Systems*. In Proceedings of The 2<sup>nd</sup> International Operational Modal Analysis Conference, Copenhagen, Denmark, 2007.
- [5] L. Mevel, M. Basseville, A. Benveniste, M. Goursat, *Merging sensor data from multiple measurement setups for nonstationary subspace-based modal analysis*, Journal of Sound and Vibration, 249(4):719-741, January 2002.