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Performance Analysis of a Self-Locating Mobile Sensor *

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ABSTRACT

We consider the ability of a mobile sensor to locate its own geographical location, the so-called self-localization problem. The need to locate people and objects has inspired the development of many systems for automatic localization. Most systems are based on location information and measured radio propagation characteristics for received signals from sensors in the proximity of the mobile sensor. It is of fundamental importance that such systems also works in critical situations such as loss of observability or the presence of multipath. The present paper suggest a framework to assess the performance of localization algorithms in mobile and critical situations. This is done by exploring the performance of various filtering techniques for self-localization of a mobile sensor in a field of sensors. More specifically, we model the mobility of the sensor such that the velocity varies according to an autoregressive model. Measurement uncertainty is assumed to follow a Gaussian distribution and the probability for detecting a distance to a given sensor is assumed to fall off exponentially with squared distance. The combined model is formulated as a nonlinear state space model and Bayesian inference is performed with the extended Kalman filter (EKF) and a particle filtering method. Precision of the position estimate is evaluated by the root mean square error (RMSE). A lower bound on the RMSE of the estimate is derived, thus providing important information on the best achievable precision for any algorithm. We report a number of simulation experiments which validate our proposed algorithms and theoretical results. We conclude that the performance of the EKF and particle filtering methods are comparable and that the derived lower bound is a useful lower limit on the RMSE.

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1. INTRODUCTION

1.1 Related work

The ability of a mobile sensor to determine its own geographical location – the so called self-localization problem – is of fundamental importance in many applications. Examples of applications include tracking of goods in warehouses and road pricing systems. Such needs have inspired development of many systems to automatically locate people and objects. The most well-known system is the Global Positioning System (GPS), a satellite-based self-location service. Self-localization with GPS works well outdoors but is not suitable indoors and in densely populated areas. Hence various systems based on self-localization by aid of cellular systems [6], WLAN received signal strengths (RSS) [2] or ultrasonic time-of-flight (TOF) measurements [19] are being developed for commercial use. Currently, there are also a number of emerging research activities in methods for fusing such systems into one single localization system [8, 9].

There exists a huge literature on solving localization problems and a full characterization is beyond the scope of the present paper. A useful taxonomy categorizing localization algorithms into triangulation, proximity and fingerprint based methods is introduced in [14]. The present setting belongs to triangulation, where one generally assumes that the object has available noisy distance measurements to neighbouring sensors. These measurements can be e.g. TOF or RSS measurements suitably inverted to distance measurements. A straightforward approach to solving such a localization problem is to minimize the sum of the squares of the differences between inter-sensor distances and measured distances, leading to a nonlinear least-squares (NLS) optimization problem [20, 21]. A more formal and model-based approach is to use maximum likelihood (ML) estimators [18, 17] or to use the configuration of sensors that has overall maximum probability given the observations. In a Bayesian framework this corresponds to maximum a posteriori (MAP) estimation [13]. An alternative Bayesian approach is to use the expected mean of the posterior distribution, which minimizes the mean square error loss function. It turns out, however, that this approach is quite computer intensive, see [16]. Although, the combination of positioning and mobility has a profound history in the tracking literature, see [3], investigations of the combination between self-localization and mobility seems to be an emerging research topic, with applications for GPS and inertial navigation systems (INS) as the most prominent examples, see e.g. [3, Chapter 12] and [12].

1.2 Our contribution

In our opinion a formal framework for performance analysis of a localization methods for a mobile object is composed of 1) a performance measure, 2) a model for signal measurements, 3) a mobility model, and 4) an evaluation of the localization method with respect to the performance measure. Although, a number of authors have treated various aspects of issues 1)–4), a formal framework for performance analysis of localization methods in critical situations such as loss of observability or presence of multipath is still an open research topic.

In accordance with previous literature [5, 10] we choose the root mean square error of the position estimate as performance measure. It is worth noting that an optimal estimator for the mean square error is the posterior mean of the position given the previously obtained distance measurements.

For the distance measurements in the model we adopt the approach of [15] and [17], where the probability of detecting nearby sensors falls off exponentially with squared distances. This model is motivated by the fact that the probability of obtaining a distance observation decreases with distance due to a decreasing probability for line of sight, increased interference and simple power decay.

We are dealing with the estimation of the state of a discrete-time linear dynamic system with nonlinear measurements. Optimal filters exist in principle but are computationally demanding. Hence, the need for suboptimal filters is evident. This paper focuses on the comparison of two suboptimal filtering techniques based on the extended Kalman filter and a particle filter.

A standard mobility model in the tracking literature is a random walk model for the velocity [3, Section 6.3]. Although the resulting position process is not a stationary model, the Kalman filter yields a steady-state filter for linear measurement models. However, it also well-known that optimal filters for nonlinear measurement models often yields nonstationary filters. As the scope of the present paper is to study the performance under critical radio propagation regimes, we circumvent the problem with the nonstationarity by introducing an autocorrelation model for the velocity. This model turns out to provide asymptotically steady state filtering.

We compare the precision of the suggested position estimates by the root mean square error. In particular we derive a lower bound on the RMSE for any estimator. Such a lower bound is of great practical importance in real-world applications. For instance in most civil and military applications, the user of a localization system requires the error of a position estimate to be below a certain value. Hence, this study may reveal if such precision is unachievable. The need for RMSE lower bounds of estimates in nonlinear filtering problems have generated a large literature in signal processing.

We will base our lower bound on an extension of the classical Cramér-Rao bound for biased estimators due to van Trees [24]. This bound is usually referred to as the van Trees inequality or the posterior Cramér-Rao bound (PCRB) [22]. In our setting a recursive formula allowing us to compute the PCRB for position estimates is easily derived from Proposition 2 in [23], although it involves numerical integration. The PCRB has been derived for a tracking problem in [4]; see also [25] for an example of a tracking problem with noisy observations. Finally, Giremus et al. [12] considered a PCRB for a GPS/INS nonlinear filtering approach.

By simulations we conclude that the EKF and particle filtering approaches show similar and good localization performance. They also compare well with the lower bound on the best achievable performance.

The paper is organized as follows. Section 2 presents the model. The model is split into three terms arising from mobility, signal propagation and measurement uncertainty. Section 3 presents an extended Kalman filter and a particle filter to self-localization, details implementation issues and provides a posterior Cramér-Rao lower bound on the mean square error of any estimator. In Section 4 we demonstrate performance of the proposed algorithms and compare them with posterior Cramér-Rao lower bounds. Section 5 provides some conclusive remarks and discusses various open questions for future research.

2. STATE SPACE MODEL

2.1 Notation

One mobile sensor with unknown position is assumed to be located in $\mathbb{R}^2$, and a (maybe infinite) grid of stationary sensors is assumed to be located on $K \subseteq \mathbb{Z}^2$. The position of the mobile sensor is considered at discrete times $t_0, t_1, t_2, \ldots$, where for notational convenience we assume the times to be equispaced with $t_k = k$. We denote the position of the mobile sensor at time t_k by $z_k = (x_k, y_k)^\top$ and the velocity by $\dot{z}_k = (\dot{x}_k, \dot{y}_k)^\top$. The state vector of the mobile sensor is denoted by $\tilde{z}_k = (x_k, \dot{x}_k, y_k, \dot{y}_k)^\top$. The positions of the stationary sensors are given by $z_{(i)} = (x_{(i)}, y_{(i)})$ for $i \in K$.

Measurements are made between the sensors at times $t = t_1, t_2, \ldots$. At time t_k the sensor with unknown position tries to obtain a measurement of the distance to sensor i, where $i \in K$. We define

$$o_{k,i} = \begin{cases} 1, & \text{if measurement } i \text{ is obtained,} \\ 0, & \text{otherwise,} \end{cases}$$

and let $o_k = (o_{k,i})_{i \in K}$. The collection of successfully measured distances $d_{k,i}$ is denoted by d_k. Furthermore, we use the short notations $o_{1:k} = (o_1, \ldots, o_k)$ and $d_{1:k} = (d_1, \ldots, d_k)$.

2.2 Mobility model

The sensor with unknown position is assumed to follow a mobility model, where the velocity $(\dot{x}_k, \dot{y}_k)$ varies according to an autoregressive model. More specifically, at time t_k we model $\tilde{z}_k$ by

$$\tilde{z}_k = G_1 \tilde{z}_{k-1} + G_2 \omega_k \tag{1}$$

where

$$G_1 = \begin{pmatrix} 1 & \alpha & 0 & 0 \\ 0 & \alpha & 0 & 0 \\ 0 & 0 & 1 & \alpha \\ 0 & 0 & 0 & \alpha \end{pmatrix}, \qquad G_2 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix},$$

$\omega_k \sim N_2(0, \sigma_m^2 I_2)$, and $\alpha \in [0, 1]$.

Since we have assumed that the sensors with known positions are stationary, we will not need a mobility model for these sensors, but the setup is easily adapted to the case with mobile sensors with known positions.

2.3 Signal propagation model

For the distance measurements in the model we adopt the approach of [15] and [17] in the following way. We model the variables $o_{k,i}$ as independent Bernoulli variables with parameter $P_o(z_k, z_{(i)}) \in [0, 1]$; that is,

$$p(o_k|\tilde{z}_k) \propto \prod_i P_o(z_k, z_{(i)})^{o_{k,i}} \left(1 - P_o(z_k, z_{(i)})\right)^{1-o_{k,i}}. \tag{2}$$

The probability of getting a successful measurement of the distance between z_k and $z_{(i)}$ is assumed to fall of exponentially with squared distance. More specifically,

$$P_o(z_k, z_{(i)}) = \exp\left(-\frac{\|z_k - z_{(i)}\|^2}{2R^2}\right), \tag{3}$$

where the parameter $R > 0$ is a constant that defines the decay rate on the detection probability.

2.4 Measurement model

The distance measurements $d_{k,i}$ are assumed to be independent normally distributed variables with the true distance $\|z_k - z_{(i)}\|$ as mean and a fixed known variance σ_o^2; that is,

$$p(d_k|o_k, \tilde{z}_k) \propto \prod_{i:o_{k,i}=1} \exp\left(-\frac{(d_{k,i} - \|z_k - z_{(i)}\|)^2}{2\sigma_o^2}\right). \tag{4}$$

3. INFERENCE

The aim of the filters is to provide an estimate of the positions and velocities $\tilde{z}_k$ given the observed data $d_{1:k}$ and $o_{1:k}$. Assuming that d_k and o_k are conditionally independent of $d_{1:k-1}$ and $o_{1:k-1}$ given $\tilde{z}_k$ and using Bayes formula, we get the following factorization

$$\begin{aligned} & p(\tilde{z}_k|d_{1:k}, o_{1:k}) \\ & \quad \propto \; p(\tilde{z}_k|d_{1:k-1}, o_{1:k-1}) p(o_k|\tilde{z}_k) p(d_k|o_k, \tilde{z}_k). \end{aligned} \tag{5}$$

The three terms on the right hand side in (5) correspond to the mobility model, the signal propagation model, and the measurement model.

If we assume that $\tilde{z}_{k-1}$ conditional on $(d_{1:k-1}, o_{1:k-1})$ is normally distributed with mean μ_{k-1} and precision P_{k-1}, then $\tilde{z}_k$ conditional on $(d_{1:k-1}, o_{1:k-1})$ is normally distributed with mean and precision

$$\mu_k = G_1 \mu_{k-1}, \; P_k = (G_1 (P_{k-1})^{-1} G_1^\top + \sigma_m^2 G_2 G_2^\top)^{-1}. \tag{6}$$

3.1 Extended Kalman filter

In the extended Kalman filter (EKF), we approximate the posterior $p(\tilde{z}_k|d_{1:k}, o_{1:k})$ by a normal distribution, and represent this distribution by its estimated mean $\hat{\tilde{z}}_k$ and precision $\hat{P}_k$. To find the appropriate expressions for $\hat{\tilde{z}}_k$ and $\hat{P}_k$, consider a normal distribution with mean μ and precision P. If we denote its density by f, the first and second order derivatives of the logarithm of f is given by

$$\begin{aligned} f^{(1)}(z) &= \frac{\partial}{\partial z} \log f(z) = -P(z - \mu), \\ f^{(2)}(z) &= \frac{\partial^2}{\partial z \partial z^\top} \log f(z) = -P, \end{aligned}$$

which yields

$$\begin{aligned} P &= -f^{(2)}(z), \\ \mu &= z + P^{-1} f^{(1)}(z). \end{aligned}$$

We then make a Taylor approximation by substituting $f^{(1)}(z)$ and $f^{(2)}(z)$ with $p^{(1)}(\tilde{z}_k) = \frac{\partial}{\partial \tilde{z}_k} \log p(\tilde{z}_k|d_k, o_k, \tilde{z}_{k-1})$ and $p^{(2)}(\tilde{z}_k) = \frac{\partial^2}{\partial \tilde{z}_k \partial \tilde{z}_k^\top} \log p(\tilde{z}_k|d_k, o_k, \tilde{z}_{k-1})$, and evaluate these functions at a point $\tilde{z}'$, which should be chosen close to the posterior mode. This leads to the following recursive step for $k = 1, 2, \ldots$ in the extended Kalman filter,

1. $\hat{P}_k = -p^{(2)}(\tilde{z}')$,
2. $\hat{\tilde{z}}_k = \tilde{z}' + \hat{P}_k^{-1} p^{(1)}(\tilde{z}')$,

where we use $\tilde{z}' = G_1 \hat{\tilde{z}}_{k-1}$. Usually an EKF is formulated using a prediction step and an update step, but here we have instead chosen to combine these steps into one step.

Now all that remains is to calculate $p^{(1)}(\tilde{z}_k)$ and $p^{(2)}(\tilde{z}_k)$. Using (5), we immediately obtain that $p^{(1)}(\tilde{z}_k)$ and $p^{(2)}(\tilde{z}_k)$ is given by the sum of the derivates of $\log p(\tilde{z}_k|d_{1:k-1}, o_{1:k-1})$, $\log p(o_k|\tilde{z}_k)$, and $\log p(o_k|\tilde{z}_k)$. Using (6), we obtain the first and second order derivatives of the first term,

$$\begin{aligned} & \frac{\partial}{\partial \tilde{z}_k} \log p(\tilde{z}_k|d_{1:k-1}, o_{1:k-1}) \\ & \qquad = -\left(G_1 (\hat{P}_{k-1})^{-1} G_1^\top + \sigma_m^2 G_2 G_2^\top\right)^{-1} \\ & \qquad\qquad (\tilde{z}_k - G_1 \mu_{k-1}), \end{aligned} \tag{7}$$

$$\begin{aligned} & \frac{\partial^2}{\partial \tilde{z}_k \partial \tilde{z}_k^\top} \log p(\tilde{z}_k|d_{1:k-1}, o_{1:k-1}) \\ & \qquad = -\left(G_1 (\hat{P}_{k-1})^{-1} G_1^\top + \sigma_m^2 G_2 G_2^\top\right). \end{aligned} \tag{8}$$

Differentiating (2), and simplifying the expressions using the particular form of P_o given by (3), we obtain

$$\frac{\partial}{\partial \tilde{z}_k} \log p(o_k|\tilde{z}_k) = -\frac{1}{R^2} \sum_i \left(\frac{o_{k,i} - P_o(z_k, z_{(i)})}{1 - P_o(z_k, z_{(i)})} \tilde{z}_{k,i}^0\right),$$

$$\frac{\partial^2}{\partial \tilde{z}_k \partial \tilde{z}_k^\top} \log p(o_k|\tilde{z}_k)$$
$$= -\frac{1}{R^2} \sum_i \left(\frac{1}{R^2} P_o(z_k, z_{(i)}) \frac{1 - o_{k,i}}{(1 - P_o(z_k, z_{(i)}))^2} \tilde{z}^0_{k,i} \tilde{z}^{0\top}_{k,i} + \frac{o_{k,i} - P_o(z_k, z_{(i)})}{1 - P_o(z_k, z_{(i)})} I^0 \right), \quad (9)$$

where

$$\tilde{z}^0_{k,i} = \begin{pmatrix} x_k - x_{(i)} \\ 0 \\ y_k - y_{(i)} \\ 0 \end{pmatrix}, \qquad I^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Finally, differentiating (4), we obtain

$$\frac{\partial}{\partial \tilde{z}_k} \log p(d_k|o_k, \tilde{z}_k) = \frac{1}{\sigma_o^2} \sum_{i:o_{k,i}=1} \left(\frac{d_{k,i}}{\|z_k - z_{(i)}\|} - 1 \right) \tilde{z}^0_{k,i},$$

$$\frac{\partial^2}{\partial \tilde{z}_k \partial \tilde{z}_k^\top} \log p(d_k|o_k, \tilde{z}_k)$$
$$= \frac{1}{\sigma_o^2} \sum_{i:o_{k,i}=1} \left(-\frac{d_{k,i}}{\|z_k - z_{(i)}\|^3} \tilde{z}^0_{k,i} \tilde{z}^{0\top}_{k,i} + \left(\frac{d_{k,i}}{\|z_k - z_{(i)}\|} - 1 \right) I^0 \right). \quad (10)$$

Since the EKF approximates $p(\tilde{z}_k|d_{1:k}, o_{i:k})$ using linearization the choice $\tilde{z}' = G_1 \hat{\tilde{z}}_{k-1}$ used in the linearization is of course important. For particular choices of the parameters in the model, $\tilde{z}'$ may be far away from the posterior mode of $\tilde{z}_k$ and then the normal approximation may not fit very well. In such cases we can use the iterated extended Kalman filter (IEKF). In the IEKF we repeat the step in the EKF a number of times for each k, where we update $\tilde{z}'$ by $\hat{\tilde{z}}_k$ between each of these times. This gradually improves the estimate $\hat{\tilde{z}}_k$. In cases of convergence, the approximated Gaussian density has the same mode as $p(\tilde{z}_k|d_{1:k}, o_{1:k})$. For the particular parameter values used in Section 4, it has not been necessary to iterate the EKF, but e.g. for higher values of σ_m, we have observed significant improvements using IEKF.

3.2 Particle filter

In the particle filter the posterior (5) is approximated by a weighted sample. Denote the sample size by n, the sample by $\tilde{z}_k^{(1)}, \ldots, \tilde{z}_k^{(n)}$, and the weights by $w_k^{(1)}, \ldots, w_k^{(n)}$. To obtain the sample, we use the sampling importance resampling (SIR) particle filter [1]; that is, we use the following algorithm recursively for $k = 1, 2, \ldots$:

1. Sampling:
   $\tilde{z}_k^{(j)} \sim p(\tilde{z}_k|\tilde{z}_{k-1}^{(j)})$, for $j = 1, \ldots, n$.
2. Calculation of weights:
   $\tilde{w}_k^{(j)} = p(d_k|o_k, \tilde{z}_k^{(j)}) p(o_k|\tilde{z}_k^{(j)})$, for $j = 1, \ldots, n$.
3. Normalization:
   $w_k^{(j)} = \tilde{w}_k^{(j)} / \sum_{j'} \tilde{w}_k^{(j')}$.
4. Resampling.

In the sampling step, $\tilde{z}_k^{(j)}$ is drawn from $p(\tilde{z}_k|d_k, o_k, \tilde{z}_k^{(j)})$ using importance sampling, where $p(\tilde{z}_k|\tilde{z}_{k-1}^{(j)})$ is used as importance distribution. In the next step each $\tilde{z}_k^{(j)}$ is weighted by

$$\frac{p(\tilde{z}_k|d_k, o_k, \tilde{z}_k^{(j)})}{p(\tilde{z}_k|\tilde{z}_{k-1}^{(j)})} = p(d_k|o_k, \tilde{z}_k^{(j)}) p(o_k|\tilde{z}_k^{(j)}),$$

and then the weights are normalized. Finally, the sample is resampled by randomly drawing a new value for each $\tilde{z}_k^{(j)}$ from the set of current values $\tilde{z}_k^{(1)}, \ldots, \tilde{z}_k^{(n)}$ with probabilities given by the weights $w_k^{(1)}, \ldots, w_k^{(n)}$, and thereafter reset the weights to $w_k^{(j)} = 1/n$. The resampling step is included to avoid degeneracy, where one weight contains almost all of the probability mass [1].

As in the case of the EKF, the mean is used as a point estimate for the position and velocity. Here the mean is estimated by the weighted average of the sample $\hat{\tilde{z}}_k = \sum_j w_k^{(j)} \tilde{z}_k^{(j)}$ before the resampling step.

3.3 Posterior Cramer-Rao lower bounds for state space models

In the following we implicitly assume derivatives and expectations exists whenever necessary. Let $(d_{1:k}, o_{1:k})$ represent the vector of measured data and $\tilde{z}_k$ the vector to be estimated. If we let $p(d_{1:k}, o_{1:k}, \tilde{z}_k)$ be the joint probability density for $(d_{1:k}, o_{1:k}, \tilde{z}_k)$ and $\hat{\tilde{z}}_k(d_{1:k}, o_{1:k})$ be an estimator of $\tilde{z}_k$, then under mild regularity conditions the PCRB on the estimation error is given by [24]

$$E\left\{ [\hat{\tilde{z}}_k(d_{1:k}, o_{1:k}) - \tilde{z}_k][\hat{\tilde{z}}_k(d_{1:k}, o_{1:k}) - \tilde{z}_k]^\top \right\} \geq J^{-1} \quad (11)$$

where E denotes expectation over $(d_{1:k}, o_{1:k}, \tilde{z}_k)$ and

$$J = -E\left\{ \frac{\partial^2}{\partial \tilde{z}_k \partial \tilde{z}_k^\top} \log p(d_{1:k}, o_{1:k}, \tilde{z}_k) \right\}.$$

Applying (11) directly to state space models is not practical. Fortunately, several papers have recently given efficient recursively computationable lower bounds (e.g. [23, 4]). As the conditional distribution of $\tilde{z}_{k+1}$ given $\tilde{z}_k$ is not well-defined with respect to Lebesgue measure, we use the recursive algorithm provided in [23, Proposition 2].

From Sections 2.2 - 2.4 it follows that

$$\log p(\dot{z}_{k+1}|\tilde{z}_k) = -\frac{1}{2}\log 2\pi\sigma_m^2 - \frac{1}{2\sigma_m^2}(\dot{z}_{k+1} - \alpha \dot{z}_k)^2$$
$$\log p(d_{k+1}, o_{k+1}|\tilde{z}_k, \dot{z}_{k+1}) = \log p(o_{k+1}|z_k + \dot{z}_{k+1}) + \log p(d_{k+1}|o_{k+1}, z_k + \dot{z}_{k+1}).$$

Letting $\tilde{z}_0 \sim N(0, \epsilon^2 I_4)$ it follows that

$$z_k \sim N_2(0, \sigma_k^2 I_2)$$

where

$$\sigma_k^2 = \left[1 + \left(\frac{\alpha^{k+1}-1}{\alpha-1} - 1\right)^2\right]\epsilon^2 + \left[\alpha^2\frac{\alpha^{2k}-1}{\alpha^2-1} - 2\alpha\frac{\alpha^k-1}{\alpha-1} + k\right]\frac{\sigma_m^2}{(1-\alpha)^2}. \quad (12)$$

We are now able to formulate a lower bound for an estimator $\hat{\tilde{z}}_k(d_{1:k}, o_{1:k})$ of $\tilde{z}_k$ as follows

$$E\left[(\hat{\tilde{z}}_k(d_{1:k}, o_{1:k}) - \tilde{z}_k)(\hat{\tilde{z}}_k(d_{1:k}, o_{1:k}) - \tilde{z}_k)^\top\right] \geq J_k^{-1}$$

for $k = 0, 1, 2, \ldots$. A recursive algorithm for calculating J_k is given in the following way (see Proposition 2 in [23]),

1. Let J_0 be the information matrix for $\tilde{z}_0$, i.e.
$$J_0 \;:=\; \epsilon^{-2} I_4.$$
2. For $k = 0, 1, \ldots$, let
$$J_{k+1} = \begin{pmatrix} H_k^{33} - J_k^{22} - H_k^{22} & H_k^{23\top} - J_k^{22} - H_k^{22} \\ H_k^{23\top} - J_k^{22} - H_k^{22} & J_k^{22} + H_k^{22} \end{pmatrix} - \left(H_k^{13} - J_k^{12} - H_k^{12} \quad J_k^{12} + H_k^{12}\right)^\top \left(J_k^{11} + H_k^{11}\right)^{-1} \left(H_k^{13} - J_k^{12} - H_k^{12} \quad J_k^{12} + H_k^{12}\right)$$

where

$$z_{k,i}^0 = \begin{pmatrix} x_k + \dot{x}_{k+1} - x_{(i)} \\ y_k + \dot{y}_{k+1} - y_{(i)} \end{pmatrix},$$

$$\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1}) = p(\dot{z}_{k+1}|\tilde{z}_k)p(d_{k+1}, o_{k+1}|\tilde{z}_k, \dot{z}_{k+1}),$$

$$\xi_k^{(1)} = \frac{1}{\sigma_o^2}E\sum_{i:o_{k,i}=1}\left[-\frac{d_{k,i}}{\|z_k - z_{(i)}\|^3}z_{k,i}^0 z_{k,i}^{0\top} + \left(\frac{d_{k,i}}{\|z_k - z_{(i)}\|} - 1\right)I^0\right] \quad (13)$$

$$\xi_k^{(2)} = \frac{1}{R^2}E\sum_i\left[\frac{1}{R^2}P_o(z_k, z_{(i)})\frac{1 - o_{k,i}}{(1 - P_o(z_k, z_{(i)}))^2}z_{k,i}^0 z_{k,i}^{0\top} + \frac{o_{k,i} - P_o(z_k, z_{(i)})}{1 - P_o(z_k, z_{(i)})}I^0\right] \quad (14)$$

$$\begin{aligned} H_k^{11} &= E\left(-\frac{\partial^2}{\partial \dot{z}_k \partial \dot{z}_k^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= \alpha^2\sigma_m^{-2}I_4 \\ H_k^{12} &= E\left(-\frac{\partial^2}{\partial z_k \partial \dot{z}_k^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= 0 \\ H_k^{13} &= E\left(-\frac{\partial^2}{\partial \dot{z}_{k+1} \partial \dot{z}_k^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= -\alpha\sigma_m^{-2}I_4 \end{aligned}$$

and

$$\begin{aligned} H_k^{22} &= E\left(-\frac{\partial^2}{\partial z_k \partial z_k^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= \xi_{k+1}^{(1)} + \xi_{k+1}^{(2)} \\ H_k^{23} &= E\left(-\frac{\partial^2}{\partial \dot{z}_k \partial z_k^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= \xi_{k+1}^{(1)} + \xi_{k+1}^{(2)} \\ H_k^{33} &= E\left(-\frac{\partial^2}{\partial \dot{z}_{k+1} \partial \dot{z}_{k+1}^\top}\bar{p}(\tilde{z}_k, \dot{z}_{k+1}, d_{k+1}, o_{k+1})\right) \\ &= \xi_{k+1}^{(1)} + \xi_{k+1}^{(2)} + \sigma_m^{-2}I_4. \end{aligned}$$

If we use the rule $E[\cdot] = E[E[E[\cdot|o,z]|z]]$, the expectations (13) and (14) can be simplified in the following way

$$\xi_k^{(1)} = \sigma_o^{-2}\sum_i E\left[P_o(z_k, z_{(i)})\frac{z_{k,i}^0 z_{k,i}^{0\top}}{\|z_k - z_{(i)}\|^2}\right] \quad (15)$$

$$\xi_k^{(2)} = \frac{1}{R^4}\sum_i E\left[\frac{P_o(z_k, z_{(i)})}{1 - P_o(z_k, z_{(i)})}z_{k,i}^0 z_{k,i}^{0\top}\right]. \quad (16)$$

The remaining mean values in (15) and (16) can be calculated by numerical integration.

4. SIMULATIONS

In this section we test the algorithms and the lower bound using simulation. Unless noted otherwise, we place $M = 16$ sensors with known position on a square 4×4 grid with distance one between neighbouring sensors. Furthermore, we let the initial position and velocity of the mobile sensor be normally distributed with the middle of the grid and zero velocity as mean and some small $\epsilon^2 I_4$ as variance at time $t_0 = 0$. The measurements of o_k and d_k are performed at times $t_k = 1, 2, \ldots, 50$.

In the first simulation, we consider the case where a specific (deterministic) path is followed by the sensor with unknown position, rather than simulating $\tilde{z}_k$ from the mobility model. This is done to test whether this movement can be reconstructed even though it does not follow the model assumed in the EKF and the SIR. The observed data o_k and d_k are simulated using (3) and (4), where the parameters are given by $\sigma_m = 0.02$, $\sigma_o = 0.01$, $R = 0.8$, and $\alpha = 0.9$. Figure 1 shows the path of z_k. Furthermore, the figure also shows the estimated paths using the EKF and the SIR with a sample size of 50. In this particular simulation, both of the filters manage to estimate z_k rather well. To see whether the velocities in $\tilde{z}_k$ are also estimated well, Figure 2 shows the speed of the sensor $\sqrt{\dot{x}_k^2 + \dot{y}_k^2}$ as a function of k. The almost linearly increasing speed seems to be well estimated by both filters.

To test and compare the two filters more thoroughly, we make 50 simulations of $\tilde{z}_k$ (this time using the model in Section 2.2), o_k and d_k, and estimate $\tilde{z}_k$ using both the EKF and the SIR. Figure 3 shows the RMSE estimated from the simulations for the SIR using a sample size of 50 and 500 and for the EKF. The simulations have been made using the parameters $\sigma_m = 0.002$, $\sigma_o = 0.01$, $R = 0.8$, and $\alpha = 0.9$.

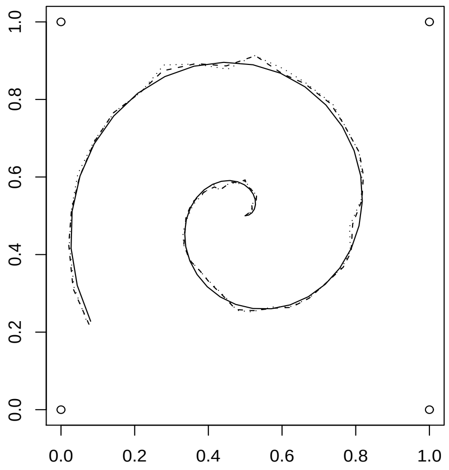

Figure 1: A spiral shaped path (solid line), and estimated paths based on the extended Kalman filter (dashed line) and the particle filter (dotted) line.

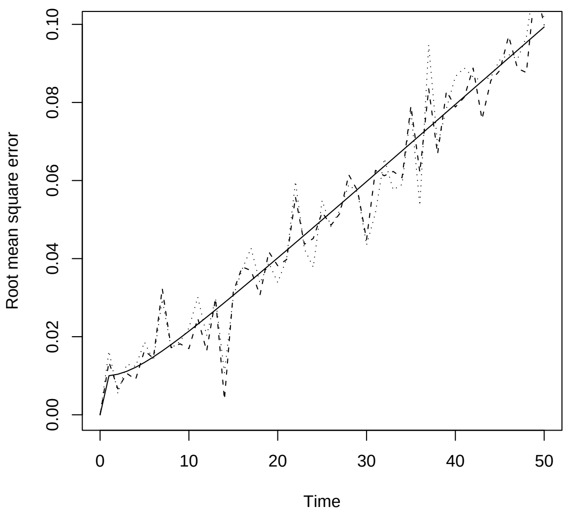

Figure 2: The speed of the sensor (solid line), and the estimated speeds based on the extended Kalman filter (dashed line) and the particle filter (dotted) line.

Furthermore, the root of the PCRB is also shown. From the plot it is evident that the SIR with a sample size of only 50 performs significantly worse than the SIR with a sample size of 500 or the EKF. Furthermore, it seems like the performance of the SIR with a sample size of 500 and the EKF is similar, perhaps with the EKF performing slightly better. The estimated RMSE of both filters are close to the lower bound.

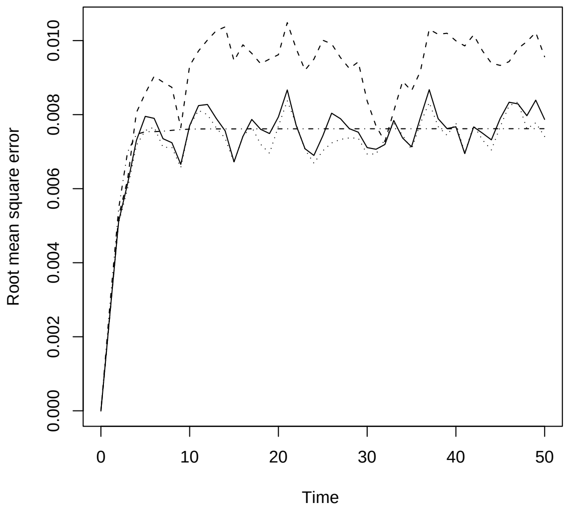

Figure 3: Estimated root mean square error for the particle filter with a sample size of 500 (solid line), the particle filter with a sample size of 50 (dashed line), the extended Kalman filter (dotted line), and the lower bound (dashed-dotted line).

In Figure 4 we show the root of the PCRB using parameters $\sigma_m = 0.002$, $\sigma_o = 0.01$, $\alpha = 0.9$, and different values of $R = 0.4, 0.6, 0.8, 1.0$. For the present choice of parameters, $R = 0.4$ means that the sensor with unknown position rarely observes more than one sensor with known position and often no sensors are observed at all at an arbitrary time t_k. On the other hand, $R = 1.0$ means that it usually observes all the sensors at each time t_k, so these values of R cover most of the interesting values. Not surprisingly, this plot shows that a lower value of R results in a higher PCRB.

In Figure 5 we again show the root of the PCRB, but this time we vary the number of sensors with known position. The parameters are $\sigma_o = 0.01$, $\sigma_m = 0.002$, $R = 0.8$, and $\alpha = 0.9$. The number of sensors are $4, 16$, and 36, and the sensors are placed on a square grid with distance one between neighbouring sensors, and with the sensor with unknown distance in the middle. The plot shows a significant improvement to the lower bounds when the number of sensors with known positions are increased from 4 to 16; however, increasing the number of sensors from 16 to 36 changes the CPRB very little in this setup. The explanation for the latter observation is that the extra sensors added are rarely observed by the sensor.

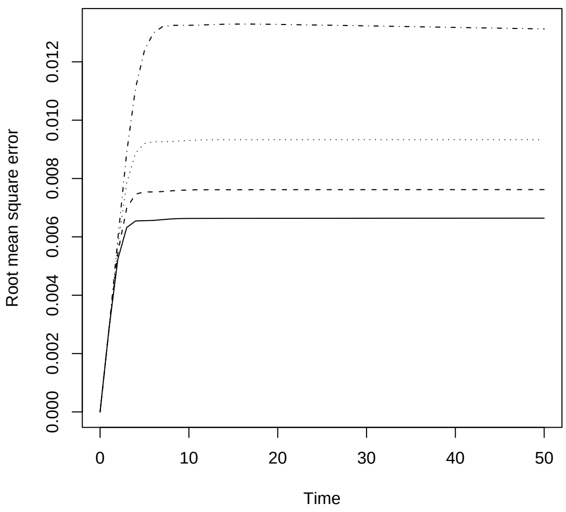

Figure 4: Lower bounds for the root mean square error for varying values of the reception decay rate: $R = 1.0$ (solid line), $R = 0.8$ (dashed line), $R = 0.6$ (dotted line), and $R = 0.4$ (dashed-dotted line).

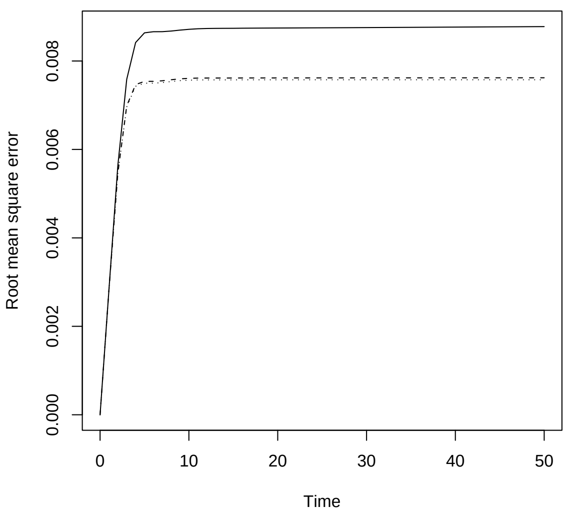

Figure 5: Lower bounds for the root mean square error for different numbers of stationary sensors: 4 (solid line), 16 (dashed line), and 36 (dotted line).

5. FUTURE RESEARCH DIRECTIONS

The evaluation of a localization method should be performed under realistic conditions including, sensible signal propagation models and realistic movements of the mobile sensors. The techniques developed and analyzed in the present paper is in principle directly applicable as long as the localization problem can be formulated as a filtering problem (i.e. estimation of the position z_n based on the observations $d_{1:n}$) for the following generalized state space model

$$\tilde{z}_{n+1} = f_n(\tilde{z}_n, w_n) \tag{17}$$
$$d_{n+1} = h_{n+1}(\tilde{z}_{n+1}, v_{n+1}) \tag{18}$$

where $\tilde{z}_{0:1}$ is defined in Section 2, $d_{1:n}$ is the obtained data, $w_{1:n}$ and $v_{1:n}$ are independent random variables or vectors. General methods for the EKF, particle filtering as well as the PCRB can be found in [3], [11] and [23] respectively. These references all provide methods applicable on the model formulated in (17) and (18).

The accuracy of a localization techniques depends on the propagation conditions of the wireless channel. Realistic models for the propagation characteristics are of particular importance in mobile scenarios where calibration in a fixed environment is impossible. These propagation characteristics should be modelled by the time varying function h_n. Models for signal propagation is an active research area and a full review is out of scope of the present paper. In passing we just notice that that a number of situations including TOF, power loss, and various mixtures of line-of-sight (LOS) and non LOS situations are expressible by a time varying function h_n.

Obviously, no generic mobility model can be formulated including all possible mobility patterns and must be designed to the given situation. For a comprehensive overview of mobility models used in wireless communication systems, see [7]. Quite many of the popular mobility are expressible by the time varying function f_n, including the popular random way point model and jump linear systems.

The generality and flexibility of the state space approach for performance analysis of self-locating mobile sensors clearly indicates that quite general situations can be analyzed and potentially applied in future hybrid and GPS free navigation systems. These generalizations are under current investigations.

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