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Recent developments in the area of probabilistic design of timber structures

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ABSTRACT: Timber is a complex building material with properties which are highly variable both spatially and in time. In structural engineering, the important material properties are the strength and stiffness related properties of standard test specimen under given (standard) loading and climate conditions and the timber density. JCSS (Joint Committee of Structural Safety) has recently developed a probabilistic model code for timber properties for structural design, (JCSS 2006). This paper describes the main content of this probabilistic model code and gives a short example for possible application.

1 BACKGROUND

Typical problems in structural engineering such as design, assessment, inspection and maintenance planning are decision problems subject to a combination of inherent, modeling and statistical uncertainties. Recent developments in the field of structural reliability together with the formulation of probabilistic models for the structural response and for loads enable the structural engineer to quantify these uncertainties and establish his decisions on a consistent basis. The results of these advances are summarized in the Probabilistic Model Code (PMC) published by the Joint Committee on Structural Safety (JCSS 2006).

The PMC contains general guidelines for uncertainty modeling, reliability assessment and probabilistic models for loads and material resistance for building materials as concrete and steel. Lately, also a probabilistic model code for structural timber has been developed by an international group of researchers organized under the umbrella of the European COST action E24 ‘Reliability of Timber Structures’ (COST 2005).

2 INTRODUCTION

Timber is a rather complex building material. Its properties are highly variable, spatially and in time. In structural engineering, material properties of timber are defined as the stress and stiffness related properties of standard test specimen under given (standard)

| Reference Material Properties | Other Material Properties |
|---------------------------------|-------------------------------------|
| r_m = bending moment capacity | $r_{t,0/90}$ = tension strength |
| moe_m = bending MOE | $moe_{t,0/90}$ = tension MOE |
| ρ_{den} = density | $r_{c,0/90}$ = compression strength |
| | $moe_{c,0/90}$ = compression MOE |
| | r_v = shear strength |
| | mog_v = shear modulus |
| | $r_{h,0/90}$ = embedding strength |

Figure 1. Reference material properties and other material properties.

loading and climate conditions and the timber density. Timber is a graded material. Due to the grading process, the material properties are associated with some control scheme, whereas only the so-called reference material properties are considered explicitly. The so-called other material properties are only assessed implicitly. The distinction between reference properties and other properties is made as illustrated in Figure 1. The bending strength r_m , the bending modulus of elasticity moe_m and the timber density ρ_{den} are referred to as the reference material properties.

When modeling timber material properties in a structure, i.e. at any generic point, in time and in space, several issues have to be taken into account. As illustrated in Figure 2 the cornerstone of the modeling of

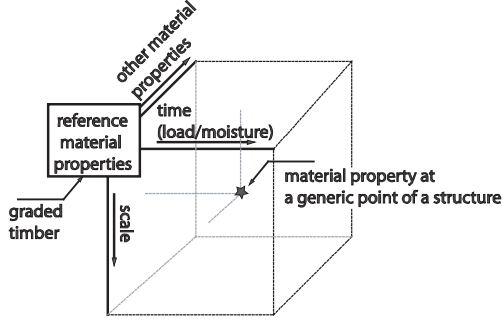


Figure 2. Outline of the modeling of timber material properties.

Table 1. Basic material properties in different context.

| | [MPa] | [kg/m ³] |
|---|---------------------------------------|----------------------|
| Properties of test specimen, size, climate and loading conditions are specified by test standards. ^{1,2} | $r_{m,s}$ $mo_{e_{m,s}}$ | $\rho_{den,s}$ |
| Properties of cross sections, climate and loading conditions are specified by test standards. | r_m mo_{e_m} | ρ_{den} |
| Properties of cross sections under any climate and loading condition. | $mo_{e_{m,\alpha}}$ $r_{m,\alpha}$ | $\rho_{den,\alpha}$ |

¹ ISO 8375: symmetrical 4-point bending test, span $18h(3 \cdot 6 \cdot h)$ with $h \approx 150 \text{ mm}$, ramp load test duration $300 \pm 120 \text{ s}$, specimen conditioned at nominal climate, $20 \pm 2^\circ \text{C}$, $65 \pm 5\%$ relative humidity.

² ISO 3131: from a disc of full cross section, free defects.

timber material properties is constituted by the reference material properties under test conditions. The material property of interest at any generic point may deviate in terms of type ('other material properties'), of dimensions and size ('scale') and of specific loading and climate conditions ('time (load/moisture)').

3 PROBABILISTIC MODEL FOR TIMBER MATERIAL PROPERTIES

3.1 Definitions

The three reference material properties are defined in three different contexts; as illustrated in Table 1.

The reference material properties are in general communicated in form of the properties of standard test specimens; $r_{m,s}$, $mo_{e_{m,s}}$, $\rho_{den,s}$.

However, these properties are sensitive to the deviations from the standard test conditions. The reference material properties of a cross section in situ

(i.e. at any generic point in time and in space) can be estimated as:

Bending moment capacity in situ, $r_{m,\alpha}$:

$$r_{m,\alpha} = \alpha \left(Ex(s, \omega, \tau, T) \right) r_m \quad (1)$$

Bending MOE in bending in situ, $mo_{e_{m,\alpha}}$:

$$mo_{e_{m,\alpha}} = mo_{e_m} / \left(1 + \delta(Ex(s, \omega, \tau, T)) \right) \quad (2)$$

Density in situ, $\rho_{den,\alpha}$:

$$\rho_{den,\alpha} = \rho_{den} \quad (3)$$

where $Ex(s, \omega, \tau, T)$ is the exposure of the structure to loads s , humidity ω and temperature τ , in the time interval $[0, T]$; $\alpha(Ex(\cdot))$ is a strength modification function, in general defined for a particular set of exposures; $\delta(Ex(\cdot))$ is a stiffness modification function, in general defined for a particular set of exposures.

r_m , mo_{e_m} and ρ_{den} are the bending moment capacity, the modulus of elasticity and the density of a cross section under test (climate and loading) conditions.

Other material properties are estimated based on the reference material properties.

Expressions for the expected values $E[.]$ and the coefficient of variation $COV[.]$ are given in Table 2. (Note that the capital letters in Table 2 indicate that the material properties are now represented as random variables, i.e. $r_m \rightarrow R_m$, $mo_{e_m} \rightarrow MOE_m$, $\rho_{den} \rightarrow P_{den}$, etc.)

The relations are derived for standard test specimen properties tested under reference conditions. However, it is assumed that the relations can be used at any level, i.e. for components of any size and/or for other climate and load conditions.

3.2 Typical limit state functions

3.2.1 Ultimate limit states

The ultimate limit state equation for a cross section subjected to stress in one particular direction is given as:

$$g = z_d R X_M - \sum_i S_i \quad (4)$$

where z_d is a design variable, e.g. cross-sectional area, R is the resistance, e.g. tension strength, $\sum_i S_i$ is the sum of all possible load effects, e.g. axial stresses, X_M is the model uncertainty.

Ultimate limit state equations for cross sections subjected to combined stresses can be formulated similarly.

$$g = 1 - \left(\sum_i S_{i,i} / (z_{d,i} R_{i,0}) + \sum_i S_{m,i} / (z_{d,M} R_m) \right) X_M \quad (5)$$

Table 2. Relation between reference properties other properties.

| Property | Expected values $E[X]$ | Coef. of variation $COV[X]$ |
|---------------------------------------|----------------------------------|------------------------------------|
| $r_{t,0} \rightarrow R_{t,0}$: | $E[R_{t,0}] = 0.6 E[R_m]$ | $COV[R_{t,0}] = 1.2 COV[R_m]$ |
| $r_{t,90} \rightarrow R_{t,90}$: | $E[R_{t,90}] = 0.015 E[P_{den}]$ | $COV[R_{t,90}] = 2.5 COV[P_{den}]$ |
| $moe_{t,0} \rightarrow MOE_{t,0}$: | $E[MOE_{t,0}] = E[MOE_m]$ | $COV[MOE_{t,0}] = COV[MOE_m]$ |
| $moe_{t,90} \rightarrow MOE_{t,90}$: | $E[MOE_{t,90}] = E[MOE_m]/30$ | $COV[MOE_{t,90}] = COV[MOE_m]$ |
| $r_{c,0} \rightarrow R_{c,0}$: | $E[R_{c,0}] = 5 E[R_m]^{0.45}$ | $COV[R_{c,0}] = 0.8 COV[R_m]$ |
| $r_{c,90} \rightarrow R_{c,90}$: | $E[R_{c,90}] = 0.008 E[P_{den}]$ | $COV[R_{c,90}] = COV[P_{den}]$ |
| $mog_v \rightarrow MOG_v$: | $E[MOG_v] = E[MOE_m]/16$ | $COV[MOG_v] = COV[MOE_m]$ |
| $r_v \rightarrow R_v$: | $E[R_v] = 0.2 E[R_m]^{0.8}$ | $COV[R_v] = COV[R_m]$ |

where $z_{d,A}$, $z_{d,M}$ are design variables, e.g. the cross sectional area and the section modulus, $R_{t,0}$, R_m are the tension strength and the bending moment capacity, $\Sigma S_{t,i}$, $\Sigma S_{m,i}$ are the sum of all possible load effects, e.g. axial stresses and bending stresses, X_M is the model uncertainty.

Ultimate limit state equations for cross sections subjected to other combined stresses can be formulated similarly.

3.2.2 Serviceability limit states

E.g. when a deflection exceeds an allowable limit:

$$g(t) = \delta_L - W_{\Delta}(\Sigma S_i, MOE_m, t) X_M \quad (6)$$

where δ_L is an allowable deflection limit and $W_{\Delta}(\Sigma S_i, MOE_m, t)$ is the deflection at time t , depending on load effects ΣS_i and modulus of elasticity and X_M is the model uncertainty.

3.3 Formulation of prior models

A proposal for the quantification of probabilistic models is presented in this section. In general this information can be utilized for the formulation of prior models when new information, e.g. information of test data becomes available. However, the given specifications may also be seen as a common reference for, e.g. code calibration procedures.

3.3.1 Reference material properties

The distribution type and the recommended coefficient of variation (COV) of the basic material properties for European softwood are given in Table 3. Recommended distribution functions of other material properties are given in Table 4. The distribution parameters can be determined with the information given in Table 2. and 3. If additional information about the material property of interest of the considered population becomes available, e.g. inform of test data, this information should be integrated using a Bayesian updating scheme.

Table 3. Probabilistic models for reference properties.

| | Distribution | COV |
|----------------------------------|--------------|-------|
| Bending strength $R_m = R_{m,s}$ | Lognormal | 0.25 |
| Bending MOE $MOE_m = MOE_{m,s}$ | Lognormal | 0.13 |
| Density $P_{den} = P_{den,s}$ | Normal | 0.1 |

Table 4. Distribution functions for other material properties.

| Property | Distribution |
|---|--------------|
| Tension strength par. to the grain, $R_{t,0}$: | Lognormal |
| Tension strength perp. to the grain, $R_{t,90}$: | 2-p Weibull |
| MOE – tension par. to the grain, $MOE_{t,0}$: | Lognormal |
| MOE – tension perp. to the grain, $MOE_{t,90}$: | Lognormal |
| Compression strength par. to the grain, $R_{c,0}$: | Lognormal |
| Compression strength perp. to the grain, $R_{c,90}$: | Normal |
| Shear modulus, MOG_v : | Lognormal |
| Shear strength, R_v : | Lognormal |

3.3.2 Correlation matrix

The relations to other material properties are given in Table 2. Indicative values of the correlation coefficient matrix are given in Table 5.

3.3.3 Strength and stiffness modification functions

Values for the strength modification function $\alpha(\cdot)$ are quantified for discrete exposures $Ex(s, \omega, \tau, t)$ as specified in Table 6.

The particular sets of exposures are defined as in Eurocode (EC) 5 (EN 1995-1 2004); different load duration classes and different service classes (sc) depending on the expected moisture content (mc) of the timber (sc 1, 2, 3 is associated with mc <12%, <20%, <20%). The values for $\alpha(\cdot)$ are taken from EC 5.

Values for the stiffness modification function $\delta(\cdot)$ are quantified for discrete exposures $Ex(s, \omega, \tau, T)$ as specified in Table 7. The particular sets of exposures

Table 5. Correlation coefficient matrix – indicative values.

| | MOE_m | P_{den} | $R_{t,0}$ | $R_{t,90}$ | $MOE_{t,0}$ | $MOE_{t,90}$ | $R_{c,0}$ | $R_{c,90}$ | MOG_v | R_v |
|--------------|---------|-----------|-----------|------------|-------------|--------------|-----------|------------|---------|-------|
| R_m | .8 | .6 | .8 | .4 | .6 | .6 | .8 | .6 | .4 | .4 |
| MOE_m | | .6 | .6 | .4 | .8 | .4 | .6 | .4 | .6 | .4 |
| P_{den} | | | .4 | .4 | .6 | .6 | .8 | .8 | .6 | .6 |
| $R_{t,0}$ | | | | .2 | .8 | .2 | .5 | .4 | .4 | .6 |
| $R_{t,90}$ | | | | | .4 | .4 | .2 | .4 | .4 | .6 |
| $MOE_{t,0}$ | | | | | | .4 | .4 | .4 | .6 | .4 |
| $MOE_{t,90}$ | | | | | | | .6 | .2 | .6 | .6 |
| $R_{c,0}$ | | | | | | | | .6 | .4 | .4 |
| $R_{c,90}$ | | | | | | | | | .4 | .4 |
| MOG_v | | | | | | | | | | .6 |

The values in Table 5 are quantified by judgment (COST 2005), such that 0.8 → high correlation, 0.6 → medium correlation, 0.4 → low correlation, 0.2 → very low correlation.

Table 6. Strength modification function table.

| sc | Permanent ($t > 10$ yrs) | Long term ($.5 < t < 10$ yrs) | Medium term ($.25 < t < 6$ mths) | Short term ($t < 1$ wks) | Instant. |
|-----|------------------------------|-----------------------------------|--------------------------------------|------------------------------|----------------|
| 1/2 | $\alpha = 0.6$ | $\alpha = 0.70$ | $\alpha = 0.80$ | $\alpha = 0.9$ | $\alpha = 1.1$ |
| 3 | $\alpha = 0.5$ | $\alpha = 0.55$ | $\alpha = 0.65$ | $\alpha = 0.7$ | $\alpha = 0.9$ |

Table 7. Stiffness modification function table.

| sc | Permanent ($t > 10$ yrs) | Long term ($.5 < t < 10$ yrs) | Medium term ($.25 < t < 6$ mths) | Short term ($t < 1$ wks) | Instant. |
|----|------------------------------|-----------------------------------|--------------------------------------|------------------------------|----------------|
| 1 | $\delta = 0.6$ | $\delta = 0.5$ | $\delta = 0.25$ | $\delta = 0.0$ | $\delta = 0.0$ |
| 2 | $\delta = 0.8$ | $\delta = 0.5$ | $\delta = 0.25$ | $\delta = 0.0$ | $\delta = 0.0$ |
| 3 | $\delta = 2.0$ | $\delta = 1.5$ | $\delta = 0.75$ | $\delta = 0.3$ | $\delta = 0.0$ |

are defined as in EC 5. The values for $\delta(\cdot)$ are taken from EC 5.

3.3.4 Model uncertainties for different ultimate limit states

The model uncertainties cover deviations and simplifications related to the probabilistic modeling and the limit state equations. The reference properties are determined by standardized tests. Therefore, model uncertainties related to estimation of other material parameters (e.g. tension and compression strengths) have to be accounted for. Geometrical deviations from specified dimensions, durations of load and moisture effects (damage accumulation) also contribute to model uncertainties if not explicitly accounted for in the probabilistic modeling. Furthermore, the idealized and simplified limit state equations introduce model uncertainties. In Table 8 indicative values for model uncertainties are shown. The model uncertainty depends on the limit state (bending or e.g. combined stress effects) and how much the actual condition deviates from the standard test conditions.

Table 8. Model uncertainties X_M for different component limit states.

| | mean | st.dev. | | Distribution |
|------------|------|---------|------|--------------|
| Short term | 1 | 0.05 | 0.10 | Lognormal |
| Long term | 1 | 0.10 | | Lognormal |

4 REFINEMENTS OF THE PROBABILISTIC MODELS

4.1 Modeling the spatial variation of timber properties

4.1.1 Bending moment capacity

Following a model proposed by Isaksson (1999), the bending strength $r_{m,ij}$ at a particular point j in the component i of a structure can be modeled by:

$$r_{m,ij} = \exp(v + \varpi_i + \chi_{ij}) \quad (7)$$

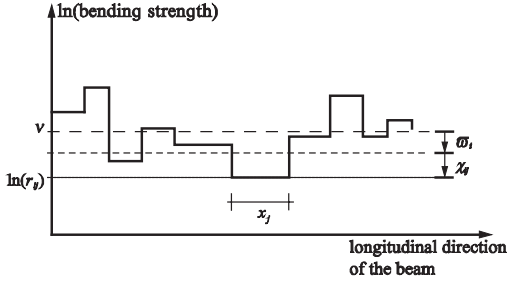


Figure 3. Section model for the longitudinal variation of bending strength.

where v is the unknown logarithm of the mean strength of all sections in all components (compare with Figure 3), w_i is the difference between the logarithm of the mean strength of the sections within a component i and v . w_i is normal distributed with mean value equal to zero and standard deviation σ_w , χ_{ij} is the difference between the strength weak section j in the beam i and the value $v + w_i$. χ_{ij} is normal distributed with mean value equal to zero and standard deviation σ_χ . w_i and χ_{ij} are statistically independent. The bending strength $r_{m,ij}$ can be seen as the bending strength of a particular cross section $r_{m,0}$; $r_{m,ij}$ is a realization of the random variable $R_{m,ij}$ which is the lognormal distributed bending strength of a cross section.

It is assumed that the bending strength of a cross section $R_{m,0}$ is related with the bending strength of a test specimen $R_{m,s}$ as:

$$R_{m,0} = R_{m,s}^\vartheta \quad (8)$$

where ϑ is a constant depending on the applied bending test standard and the type of timber.

The bending moment capacity $R_{m,0}$ is assumed to be constant within one section. The discrete section transition is assumed to be Poisson distributed, thus the section length follows X an exponential distribution with mean value $1/\lambda$. A realization of X , x_j is illustrated in Figure 3.

For Nordic spruce the following information basis can be given (Isaksson 1999): The variation of the logarithm of the bending capacity $\ln(R_{m,0})$ is related by 40% to the variable w and by 60% to the variable χ . The expected length of a section is $1/\lambda = 480$ mm.

4.2 Duration of load effect

The mechanism leading to strength reduction of a timber member under sustained load is referred to as creep rupture and is modeled by so-called cumulative damage models with the general form:

$$\frac{d\alpha_D}{dt} = h(s(t), r_0, \alpha_D, \theta) \quad \text{for } 0 \leq \alpha_D \leq 1 \quad (9)$$

where t is time, α_D is the damage state variable which commonly ranges from 0 (no damage) to 1 (failure), the function $h(\cdot)$ contains parameters θ that must be determined from experiment observations, $s(t)$ is the applied stress and r_0 the failure stress under short term ramp loading.

The following long term limit state function is utilized:

$$g = 1 - X_M \alpha_D (S(t)) \quad (10)$$

where α_D is the damage state variable at time t and X_M is the model uncertainty. More information about damage accumulation modeling can be found in the JCSS Probabilistic Model Code, section 3.5 (JCSS 2006).

4.3 Updating scheme for the basic properties

When information has been collected about the basic material properties the new knowledge implicit in that information might be applied to improve any previous (prior) estimate of the material property. For the type of information e.g. it can be differentiated between direct and indirect information; i.e. direct measurements of the material property and the measurement of some indicator of the property respectively.

In the proposed framework new information can be introduced at any stage of modeling. In the following the principle of considering new direct information e.g. in form of test results is presented. The utilization of new indirect information, e.g. information from grading indications, is illustrated with two different methods.

In the following updating is illustrated with three different methods based on Bayesian updating as described in JCSS Probabilistic Model Code (JCSS 2006), section 3.0.

4.3.1 Updating – direct information

The bending strength R_m and the bending modulus of elasticity E_m are modelled by lognormal distributed random variables which can be represented through the normal distributed random variables $R_m^* = \ln(R_m)$ and $E_m^* = \ln(E_m)$. All basic properties may be represented with the uncertain mean value M and standard deviation Σ as illustrated by:

$$R_m = \exp(R_m^*) \rightarrow R_m^* : N(M_R, \Sigma_R) \quad (11)$$

$$E_m = \exp(E_m^*) \rightarrow E_m^* : N(M_E, \Sigma_E) \quad (12)$$

$$\rho_{den} : N(M_\rho, \Sigma_\rho) \quad (13)$$

The parameters M and Σ of R_m^* , E_m^* and ρ_{den} are quantified with a Normal-Inverse-Gamma-2 distribution with the parameters m, s, n, v which is equivalent to

the natural conjugate prior of a normal distribution with unknown mean and standard deviation. Given the parameters m, s, n, v the predictive distribution of R_m^* , E_m^* and ρ_{den} can be derived as:

$$F_{X|\hat{x}}(x|m, n, s, v) = T_v\left(\frac{x-m}{s} \sqrt{\frac{n}{n+1}}\right) \quad (14)$$

where $T_v(\cdot)$ is the student-t-distribution with v degrees of freedom.

The prior predictive distribution can be quantified with parameters $(m, s, n, v) = (m', s', n', v')$.

New measurements on the material properties can be used for updating the parameters given above. For a sample of n observations $(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$, the posterior predictive distribution function of X is obtained using the JCSS Probabilistic Model Code (JCSS 2006), section 3.0.

4.3.2 Indirect information I – machine grading

In this section a simple model for updating the statistical parameters of the Lognormal distribution for e.g. the bending strength of a given timber grade when new information becomes available in the form of machine grading results is described.

The Lognormal distributed strength parameter R is assumed to have a coefficient of variation COV_R . Then $X = \ln R$ is Normal distributed with expected value M_X and standard deviation $\sigma_X^2 = \ln(COV_R^2 + 1)$. σ_X is assumed to be known and M_X is assumed to be Normal distributed with expected value μ_0 and standard deviation σ_0 .

When machine grading is based on a measured indicator i , typically related to the stiffness of a timber test specimen, the following relation between the indicator i and the strength parameter can initially be fitted to tests results where both the indicator value and the strength R are measured:

$$i = b_0 \cdot R^{b_1} \cdot \varepsilon \quad (15)$$

where b_0 and b_1 are constants and ε is an error term which is assumed Lognormal distributed. $\ln(\varepsilon)$ is then Normal distributed and is assumed to have zero mean value and standard deviation $\sigma_{\ln(\varepsilon)}$. The parameters b_0 , b_1 and $\sigma_{\ln(\varepsilon)}$ can be estimated from the tests using the Maximum Likelihood method.

Next, it is assumed that n new observations $\hat{i}_1, \hat{i}_2, \dots, \hat{i}_n$ of the indicator is obtained from n specimens from a given timber grade. The mean value of these can be estimated as

$$\ln \bar{i} = 1/n \sum_{i=1}^n \ln \hat{i}_i \quad (16)$$

and the updated (predictive) distribution function for $X = \ln R$ is then Normal with expected value μ'' and standard deviation σ'' :

$$\mu'' = \frac{\frac{n}{\ln b_1} (\ln \bar{i} - b_0) \sigma_0^2 + \mu_0 \sigma_X^2}{n \sigma_0^2 + \sigma_X^2} \quad \text{and} \quad (17)$$

$$\sigma'' = \sqrt{\sigma_X^2 + \frac{\sigma_0^2 \sigma_X^2}{n \sigma_0^2 + \sigma_X^2} + \left(\frac{n/\ln b_1}{n \sigma_0^2 + \sigma_X^2}\right)^2 \sigma_{\ln(\varepsilon)}^2}$$

The updated predictive distribution for the strength R is then Lognormal with expected value $\mu_R'' = \exp(\mu'' + 0.5\sigma''^2)$ and standard deviation $\sigma_R'' = \mu_R''(\exp(\sigma''^2) - 1)^{0.5}$.

4.3.3 Updating – indirect information II – calibration of grading rules

The probabilistic model for bending strength described in this section can be used for machine graded timber and is based on the model described in Faber et al. (2004). The probabilistic model can be described by the following steps:

For a given geographic region and a given type of specie (e.g. Nordic Spruce) an initial (prior) distribution function $F_{R_m}(x)$ can be established for the bending strength R_m for non-graded timber. The recommended distribution function is Lognormal. The statistical parameters in the distribution function can be obtained using e.g. the Maximum Likelihood method. For the identification of lower grades it is recommended to fit the initial (prior) distribution function $F_{R_m}(x)$ to the data in the lower end (e.g. 30% of the data with lowest strengths); in order to obtain good models in the lower tail of the distribution function for the graded timber strength. This can be done using the Maximum Likelihood method, see e.g. in Faber et al. (2004).

Machine grading is based on a measured indicator i , typically related to the stiffness of a timber test specimen. For each grading technique the following linear relation with the bending strength is assumed:

$$i(r) = a_0 + a_1 r + \varepsilon \quad (18)$$

where a_0 and a_1 are constants and ε is the lack-of-fit quantity which is assumed Normal distributed with zero mean value and standard deviation σ_ε . The parameters a_0 , a_1 and σ_ε can be estimated using the Maximum Likelihood method which also gives the statistical uncertainty in form of standard deviations and correlation coefficients of the parameters a_0 , a_1 and σ_ε . It is noted that in Eq. (15) the logarithms are fitted with a Lognormal distributed lack-of-fit error, whereas in Eq. (18) a linear model with a Normal distributed lack-of-fit error is used.

After grading the updated (predictive) distribution function for the bending strength in grade no. j is obtained from:

$$F_{R_{m,j}}^U(x) = P(R_m \leq x | b_{L,j} \leq i(x) \leq b_{U,j}) \quad (19)$$

where $b_{L,j}$ and $b_{U,j}$ are lower and upper limits of the grading indicator i for grading no. j . The updated distribution function $F_{R_{m,j}}^U(x)$ can then be used in reliability analyses. A detailed description of the method can be found in Faber et al. 2004 and Köhler 2003.

5 EXAMPLE – RELIABILITY BASED CODE CALIBRATION

In Faber & Sørensen (2003) the principles of reliability based code calibration is demonstrated. In the following the approach followed there is illustrated along with an example on code calibration for a timber design code.

Reliability analysis of structures for the purpose of code calibration in general or for the reliability verification of specific structures requires that the relevant failure modes be represented in terms of limit state functions. The limit state functions define the realizations of resistance parameters, i.e. the material properties and the load variables resulting in structural failure.

In code based design formats such as the Eurocodes (EN 1990 2002), design equations are prescribed for the verification of the capacity of different types of structural components in regard to different modes of failure. The typical format for the verification of a structural timber component in Eurocode 5 (1990-1 2004) is given as a design equation in the following form:

$$G = k_{\text{mod}} z_d \frac{r_k}{\gamma_M} - \sum_i \psi_i \gamma_{s,i} S_{i,k} = 0 \quad (20)$$

where r_k is the characteristic value for the resistance, z_d is the design variable (e.g. the cross section of a timber beam), $S_{i,k}$ are the characteristic values of load effects which are considered in the design, γ_M and $\gamma_{s,i}$ are partial safety factors for the resistance and the loads respectively. When more than one variable load is acting, load combination factors ψ_i are multiplied on one or more of the variable load components to take into account the fact that it is unlikely that all variable loads are acting with extreme values at the same time. k_{mod} is a modification factor taking into account the effect of the duration of load and moisture. In this example k_{mod} is assumed to be unity, i.e. no load duration and moisture effects are considered.

The partial safety factors together with the characteristic values are introduced in order to ensure a

certain minimum reliability level for the structural components designed according to design equations as e.g. given in Eq. (20). As different materials have different uncertainties associated with their material parameters the partial safety factors are in general different for the different materials.

In accordance with a given design equation, such as e.g. Eq. (20) a reliability analysis may be performed based on a limit state function of similar form as:

$$g = z_d X R - \sum_i S_i = 0 \quad (21)$$

where R and S_i are the resistance and the load effects as random variables and X a the model uncertainty. According to Eq. (21) failure F corresponds to an event defined by $F = \{g \leq 0\}$. With given probabilistic models for X , R and S_i the reliability of a structural timber component designed according to Eq. (20) with a given set of partial safety factors and characteristic values can be checked by using standard procedures as e.g. FORM/SORM (see e.g. Melchers 2002).

The aim of reliability based code calibration is the calibration of partial safety factors such that the reliability corresponding to different typical design situations are as close as possible to a specified value for the target reliability. Recommendations for suitable target reliabilities are provided by e.g. the JCSS (2006) or ISO 2394. Different design situation might be considered e.g. through different contributions of different load effects, in the case of the limit state functions in Eqs. (22) and (23), and considering two load effects due to permanent and variable load respectively, this might be introduced through a factor $\alpha_i = i/L$ $i = 1, 2, \dots, L$ modeling the relative importance of permanent and variable load as:

$$G_i = z_d \frac{r_k}{\gamma_M} - \alpha_i \gamma_G S_{G,k} - (1 - \alpha_i) \gamma_Q S_{Q,k} = 0 \quad (22)$$

$$g_i = z_d X R - \alpha_i S_G - (1 - \alpha_i) S_Q = 0 \quad (23)$$

$S_{G,k}$ and $S_{Q,k}$ are the characteristic values of the load effects due to permanent and variable load respectively, S_G and S_Q are the two load effects as random variables.

Accordingly, the following optimization problem can be formulated, (Faber & Sørensen 2003):

$$\min_{\gamma} W(\gamma) = \sum_{i=1}^L w_i (\beta_i(\gamma) - \beta_i)^2 \quad (24)$$

where β_i is the reliability index corresponding to the target reliability, $\beta_i(\gamma)$ is the reliability index corresponding to a design performed with a set of partial safety factors γ and w_i , $i = 1, 2, \dots, L$ are factors indicating relative frequency / importance of the different design situations.

Table 9. Parameters used in the example and results.

| | From the JCSS 2006 | | | Part.safety factors | |
|----------------------|--------------------|------------|----------|---------------------|-------------------------------------|
| | COV | Dist. Type | Quantile | EC 5 | Optimized, $\beta = 4.2$ (yrlly) |
| Resistance | 0.25 | Lognor. | 5% | 1.3 | 1.38 |
| Model uncertainty | 0.05 | Lognor. | – | – | – |
| Effect of perm. load | 0.1 | Normal | 50% | 1.35 | 1.39 |
| Effect of var. load | 0.4 | Gumbel | 98% | 1.5 | 1.41 |

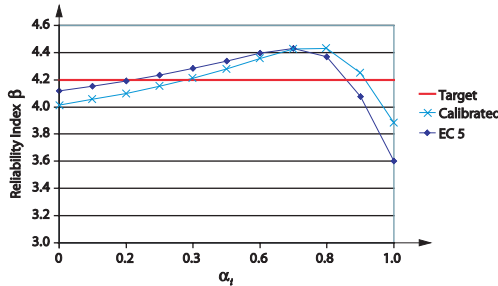


Figure 4. Reliability index corresponding to different design situations.

The code calibration procedure as described above is already included in the software package CodeCal, a MS EXCEL based software, which is provided as free-ware by the JCSS (2006). In the following, the partial safety factors $\gamma^T = (\gamma_M, \gamma_G, \gamma_Q)$ are calibrated using CodeCal. The calibration takes basis in Eqs. (22)–(23). The probabilistic model of the random variables of the problem are chosen as proposed in the JCSS PMC (JCSS 2006) and shown in Table 9.

$L = 10$ design situations are considered (see Eqs. (22), (eq23)).

For a chosen target reliability index $\beta_i = 4.2$ (yearly, as recommended in the JCSS PMC (JCSS 2006)) the partial safety factors are calibrated by solving Eq.(24). The calibrated set of partial safety factors together with the partial safety factors prescribed in EC5 (2004) is given in Table 9.

In Figure 4 the reliability index β is plotted over the different design situations represented by different values of α_i . It is shown that the set of partial safety factors prescribed in EC5 is not corresponding to the optimal set if all partial safety factors are subject to calibration. In load and resistance factor design (LRFD) formats as EC5, however, partial safety factors for load effects are in general similar and independent from the structural material which is utilized. Therefore, code calibration procedures should involve all possible building materials, i.e. calibrate partial safety factors considering all relevant design situations and materials. The software package CodeCal facilitates this option.

For the validity of the results of a code calibration procedure, it is of utmost importance that the basic random variables utilized in the analysis are quantified based on the best knowledge available. The JCSS PMC provides a set of probabilistic models, which can be used if no other information about the variables is available. E.g. for the timber bending strength, a lognormal distribution with $COV = 0.25$ is suggested. However, if new information (direct or indirect) is available, the suggested distribution should be considered as a prior distribution in a Bayesian updating scheme (as described in more detail in the JCSS PMC).

6 CONCLUSIONS, DISCUSSION AND OUTLOOK

A framework for probabilistic modeling of timber material properties is presented. The basic reference properties for timber strength parameters are described and some limit state equations are formulated. The recommended probabilistic model for the basic properties is presented and indicative numerical values for the parameters are given. Refinements related to updating of the probabilistic model given new information, spatial variation of strength and duration of load effects are described.

The proposal can be seen as a guideline and common reference for probability based code calibration of timber design codes as it is illustrated with an example. The parameters of the proposed models, however, need to be quantified on a broad and representative data base. Comprehensive experimental data concerning the basic timber phenomena already exist, especially resulting from research projects in North America, Europe and Australia. One major task for developing further the presented model code is to collect and assess existing experimental data. The timber research community is asked to contribute by making available experimental data for the quantification of model parameters for timber predominantly used for timber design.

The present version of the ‘JCSS – Timber Probabilistic Model Code’ does not cover all aspects of

the design of timber structures. An attempt to model the spatial variability of the timber bending strength is presented in this document. More experimental work should be performed or reviewed to quantify the parameters of the presented model. The description of size effects in regard to the modeling of other material properties has earned considerable recognition in the past research; no general consensus has been reached and future research should be directed in the development of a consistent framework for the description of size effects in structural timber elements. For timber structures, the structural performance depends to a considerable part on the connections between different timber structural members; connections can govern the overall strength, serviceability and fire resistance. Beside solid timber other timber materials are utilized in timber engineering. Glued laminated timber is an example of an interesting timber material, frequently used in high performance load carrying structures. It is of utmost importance to develop consistent probabilistic models for these timber materials and for timber connections, especially in the perspective of their potential competitiveness to other building materials such as steel and reinforced concrete.

The further development of the probabilistic model code for timber should constitute an important future task for the timber research community.

REFERENCES

- JCSS 2006. Joint Committee on Structural Safety – Probabilistic Model Code. <http://www.jcss.ethz.ch>.
- COST Action E 24 2005. Reliability of timber structures. Several meetings and Publications, Internet Publication: <http://www.km.fgg.uni-lj.si/coste24/coste24.htm>.
- ISO 8375: Solid timber in structural sizes – determination of some physical and mechanical properties. International Organisation for Standardisation, 1985.
- ISO 3131: Wood – Determination of density for physical and mechanical tests. International Organisation for Standardisation, 1975.
- ISO_2394: General Principles on Reliability for Structures 1998, International Organisation for Standardization.
- EN 1990–2002, EN 1990:2002 ‘Basis of Structural Design’. European Committee for Standardization (CEN).
- EN 1995-1 2004. Eurocode 5 – ENV 1995-1-1:2004 ‘Design of Timber Structures – Part 1-1: General’. 2004: European Committee for Standardization (CEN).
- Isaksson T. 1999. Modelling the Variability of Bending Strength in Structural Timber. PhD-thesis, Lund Institute of Technology, Report TVBK-1015.
- Faber M. H., Köhler J. & Sørensen, J. D. 2004. Probabilistic modelling of graded timber material properties. *Journal of Structural Safety*, 26(3), pp. 295–309.
- Faber M.H. & Sørensen J.D. 2003. Reliability Based Code Calibration – The JCSS Approach. Proceedings to the 9th International Mechanisms for Concrete Structures in Civil Engineering ICASP, San Francisco, USA.
- Melchers, R.E. 2002. *Structural Reliability Analysis and Prediction*. Second Edition ed. 2002: John Wiley & Sons. 437.

