Nominal Direction Estimation for Slightly Distributed Scatterers Using the SAGE Algorithm

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Abstract

In this paper, two SAGE (Subspace-based Alternating Generalized Expectation-maximization) algorithms [1] are derived using a deterministic (D-) version and stochastic (S-) version of the generalized array manifold (GAM) model proposed in [2] for estimation of nominal azimuths of arrival (NAoAs) of multiple slightly distributed scatterers (SDSs).

Monte-Carlo simulation studies in a single-SDS scenario show that the S-GAM SAGE algorithm returns lower root mean squared estimation error than the D-GAM SAGE algorithm, as well as the Spread-ESPRIT technique based on a two-ray model proposed in [3]. Furthermore in a two-SDS scenario with strong power unbalance between the SDSs, the S-GAM SAGE algorithm demonstrates best performance in estimating the NAoA of the SDS with weakest power compared to the other above-mentioned estimators. The simulation results further demonstrate that when multiple slightly distributed scattering occurs, the investigated estimators can estimate the NAoA with an effective accuracy provided the SDSs are separated by more than two times the intrinsic azimuth resolution of the array.

The proposed algorithms can be easily extended to include the nominal direction, i.e. azimuth and elevation, of arrival and direction of departure of the SDS when multiple-input and multiple-output (MIMO) applications are considered.

I. INTRODUCTION

Conventional direction-of-arrival (DoA) estimators are derived based on the specular-scatterer (SS) model which assumes point scattering in the propagation environment. In a scenario where a scatterer has a certain geometrical extent that is small in the view of the receiver (Rx) or local scattering around a transmitter (Tx) located far away from the Rx occurs, the contribution to the received signal can be conceived as the sum of the contributions of multiple sub-scatterers with slightly different DoAs [2] [3] [6]. We refer to such scatterers or clusters of local scatterers as slightly distributed scatterers (SDSs).

It has been shown in [7] that, in propagation environments with SDSs the DoA estimators derived based on the SS model generate estimation errors with a heavy-tailed probability distribution function. This indicates that large estimation errors might happen with high probability. As alternatives, estimators based on approximation models characterizing the signal contribution of SDSs have been proposed. One of the models is the generalized array manifold (GAM) model [2]. In the same reference, three estimators for the nominal DoA of SDSs have been derived based on subspace-based techniques. Application of these methods requires a common prerequisite, i.e. the propagation environment has to be time-invariant. Another approximation model is the two-ray model proposed in [3]. In this reference, the Spread-F technique is proposed, which can be used to estimate the nominal DoAs and angular spreads of SDSs using uniform linear arrays (ULAs) in time-variant environments.

In this paper we derive two SAGE algorithms based on respectively a deterministic (D-) version and a stochastic (S-) version of the GAM model for estimation of nominal azimuth of arrival (NAoA) of multiple SDSs. The terms "deterministic" and "stochastic" emphasize that the (unknown) parameters of the underlying signal model are assumed to be respectively deterministic and random. The algorithms are derived with the assumptions that the propagation environment is time-variant and the transmitted signal is known to the Rx. The used arrays can have arbitrary layouts and characteristics. The proposed algorithms can be applied with slight modifications in time-invariant environments with unknown transmitted signals. As a byproduct an estimate of the direction spread (DS) of a SDS [6] can be computed from the estimates of the parameters in the GAM model [4]. Extension of the algorithms to include other dispersive dimensions is straightforward. In particular when multiple-input multiple-output (MIMO) systems or techniques are considered, these algorithms can be extended to include estimation of the nominal direction, i.e. azimuth and elevation, of departure and direction of arrival of the SDSs. Joint characterization of these two parameters is of paramount importance for MIMO applications.

The organization of the paper is as follows. Section II and III describe respectively the signal model and the proposed algorithms. Section IV reports the simulation results. Finally concluding remarks are addressed in Section V.

II. SIGNAL MODEL

In a propagation scenario with a single SDS the output signal of a M-element Rx array can be viewed as composed of the contributions of multiple sub-scatterers distributed with respect to the azimuth of arrival (AoA):

$$\boldsymbol{Y}(t)=\Big[\sum_{\ell=1}^{L}a_\ell(t)\boldsymbol{c}(\bar{\theta}+\tilde{\theta}_\ell)\Big]\cdot s(t)+\boldsymbol{W}(t),\ t=t_1,\ldots,t_N. \quad (1)$$

The components of the M-dimensional (M-D) complex vector $\boldsymbol{Y}(t)$ denote the M output signals of the Rx array at time t, $s(t)$ represents the complex envelope of the transmitted signal, and the noise vector $\boldsymbol{W}(t)$ is a spatially and temporally white M-D Gaussian process with component variance σ_w^2. We assume that totally N observation samples are collected at time instances t_n, $n = 1, \ldots, N$. Moreover, in (1) ℓ denotes the index of the sub-scatterers with total number of L, and $a_\ell(t)$ represents the complex gain of the propagation path via the ℓth sub-scatterer. The AoA of the ℓth sub-scatterer is decomposed as the sum of the NAoA $\bar{\theta}$ of the SDS and a small deviation $\tilde{\theta}_\ell$ from $\bar{\theta}$. Finally, $\boldsymbol{c}(\theta) = [c_1(\theta), \ldots, c_M(\theta)]^{\mathrm{T}}$ with $[\cdot]^{\mathrm{T}}$ denoting transposition, is the array response. We assume that $s(t)$ is known to the Rx. Without loss of generality $s(t) = 1$.

The function $\boldsymbol{c}(\bar{\theta}+\tilde{\theta}_\ell)$ in (1) can be approximated by its first-order Taylor series expansion at $\bar{\theta}$. Inserting the first-order Taylor approximation for each $\boldsymbol{c}(\bar{\theta}+\tilde{\theta}_\ell)$ in (1) yields the so-called GAM model [2]

$$\begin{aligned}\boldsymbol{Y}(t) &= \sum_{\ell=1}^{L} a_\ell(t)[\boldsymbol{c}(\bar{\theta})+\tilde{\theta}_\ell \boldsymbol{c}'(\bar{\theta})] + \boldsymbol{W}(t), \\ &= \alpha(t)\boldsymbol{c}(\bar{\theta}) + \beta(t)\boldsymbol{c}'(\bar{\theta}) + \boldsymbol{W}(t), \quad t = t_1, \ldots, t_N, \end{aligned} \tag{2}$$

where $\alpha(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)$, $\beta(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)\tilde{\theta}_\ell$, and $\boldsymbol{c}'(\theta) = \frac{\partial \boldsymbol{c}(\theta)}{\partial \theta}$. Using matrix notation, (2) can be written as $\boldsymbol{Y}(t) = \boldsymbol{F}(\bar{\theta})\boldsymbol{\xi}(t) + \boldsymbol{W}(t)$ with $\boldsymbol{F}(\theta) = [\boldsymbol{c}(\theta)\ \boldsymbol{c}'(\theta)]$ and $\boldsymbol{\xi}(t) = [\alpha(t), \beta(t)]^{\mathrm{T}}$.

We assume that the deviations $\tilde{\theta}_1, \ldots, \tilde{\theta}_L$ are zero-mean uncorrelated random variables with identical variance $\sigma_{\tilde{\theta}}^2$. Moreover the gain processes $a_1(t), \ldots, a_L(t)$ are uncorrelated complex zero-mean circularly-symmetric wide-sense stationary (WSS) sequences with autocorrelation function $R_{a_\ell}(\tau)$. Additionally, the azimuth deviations and the gain processes are uncorrelated. As a consequence of these assumptions, $\alpha(t)$ and $\beta(t)$ are uncorrelated complex circularly-symmetric zero-mean WSS sequences with correlation functions $R_\alpha(\tau) = \sum_{\ell=1}^{L} R_{a_\ell}(\tau)$, and $R_\beta(\tau) = \sigma_{\tilde{\theta}}^2 R_\alpha(\tau)$ respectively. In this paper we focus on time-variant environments and assume that $R_\alpha(|t_{n'} - t_n|) = 0$, $n \neq n'$, $n, n' = 1, \ldots, N$, or equivalently that, $\alpha(t)$ and $\beta(t)$ are uncorrelated white random sequences. We make the additional assumption that $\alpha(t)$ and $\beta(t)$ are Gaussian random sequences in the S-GAM model.

In a scenario with D SDSs, (2) can be extended to

$$\boldsymbol{Y}(t) = \sum_{d=1}^{D} \alpha_d(t)\boldsymbol{c}(\bar{\theta}_d) + \beta_d(t)\boldsymbol{c}'(\bar{\theta}_d) + \boldsymbol{W}(t), \quad t = t_1, \ldots, t_N, \tag{3}$$

where d denotes the indexing variable for the SDSs.

III. The SAGE algorithms

A. SAGE algorithm based on the deterministic GAM model (D-GAM SAGE)

In a multi-SDS scenario as depicted by (3), the unknown parameter vector is $\boldsymbol{\Omega} \doteq [\sigma_w^2, \bar{\theta}_d, \alpha_d(t), \beta_d(t);\ d = 1, \ldots, D,\ t = t_1, \ldots, t_N]$. We choose the subsets of parameters updated in the iterations of the SAGE algorithm to be the sets including the parameters characterizing the individual signals and the unknown noise variance. Hence, at Iteration $i = 1, 2, \ldots$, $\boldsymbol{\Omega}_d \doteq [\sigma_w^2, \bar{\theta}_d, \alpha_d(t), \beta_d(t), t = t_1, \ldots, t_N]$ with $d = (i-1) \mod D + 1$ is updated. The admissible hidden-data [1] associated with $\boldsymbol{\Omega}_d$ reads

$$\boldsymbol{X}_d(t) = \alpha_d(t)\boldsymbol{c}(\bar{\theta}_d) + \beta_d(t)\boldsymbol{c}'(\bar{\theta}_d) + \boldsymbol{W}(t), \quad t = t_1, \ldots, t_N. \tag{4}$$

At Iteration i of the SAGE algorithm the objective function

$$Q_{\mathrm{D-SAGE}}(\boldsymbol{\Omega}_d|\hat{\boldsymbol{\Omega}}^{[i-1]}) \doteq \mathrm{E}\big[\Lambda(\boldsymbol{\Omega}_d; \boldsymbol{X}_d)|\boldsymbol{Y}(t) = \boldsymbol{y}(t), \hat{\boldsymbol{\Omega}}^{[i-1]})\big]$$

is computed in the expectation (E-) step of the SAGE algorithm, In the above expression $\hat{\boldsymbol{\Omega}}^{[i-1]}$ denotes the estimate of $\boldsymbol{\Omega}$ at the $i-1$th iteration. It can be shown that

$$Q_{\mathrm{D-SAGE}}(\boldsymbol{\Omega}_d|\hat{\boldsymbol{\Omega}}^{[i-1]}) = -MN \ln \sigma_w^2 - \frac{1}{\sigma_w^2}\sum_{t=t_1}^{t_N} \|\hat{\boldsymbol{x}}_d^{[i-1]}(t) - \boldsymbol{F}(\bar{\theta}_d)\boldsymbol{\xi}_d(t)\|^2, \tag{5}$$

where $\hat{\boldsymbol{x}}_d^{[i-1]}(t) = \boldsymbol{y}(t) - \sum_{d'=1, d'\neq d}^{D} \boldsymbol{F}(\hat{\bar{\theta}}_{d'}^{[i-1]})\hat{\boldsymbol{\xi}}_{d'}^{[i-1]}(t)$, $t = t_1, \ldots, t_N$ is an estimate of $\boldsymbol{X}_d(t)$ given $\boldsymbol{y}(t)$ and assuming $\boldsymbol{\Omega} = \hat{\boldsymbol{\Omega}}^{[i-1]}$. In the maximization (M-) step of the ith iteration $\hat{\boldsymbol{\Omega}}_d^{[i]} = \arg\max_{\boldsymbol{\Omega}_d}\{Q_{\mathrm{D-SAGE}}(\boldsymbol{\Omega}_d|\hat{\boldsymbol{\Omega}}^{[i-1]})\}$ is computed. Using a separable solution proposed in [8] the multiple-dimensional maximization reduces to a one-dimensional maximization problem with respect to $\bar{\theta}_d$.

In the initialization step of the SAGE algorithm, the initial estimates $\hat{\boldsymbol{\Omega}}_d^{[0]}$, $d = 1, \ldots, D$, are computed with a successive interference cancellation method similar to that used in [1].

B. SAGE algorithm based on the stochastic GAM model (S-GAM SAGE)

In this case the parameter vector reads $\mathbf{\Omega}' \doteq [\sigma_w^2, \bar{\theta}_d, \sigma_{\alpha_d}^2, \sigma_{\beta_d}^2; d = 1, \ldots, D]$, where $\sigma_{\alpha_d}^2 = R_{\alpha_d}(0)$ and $\sigma_{\beta_d}^2 = R_{\beta_d}(0)$. The subsets of parameters re-estimated in the SAGE iterations are selected to be $\mathbf{\Omega}'_d \doteq [\sigma_w^2, \bar{\theta}_d, \sigma_{\alpha_d}^2, \sigma_{\beta_d}^2]$, $d = 1, \ldots, D$. The signal $\boldsymbol{X}_d(t)$ defined in (4) is an admissible hidden data associated with $\mathbf{\Omega}'_d$.

At Iteration i the objective function

$$Q_{\text{S-SAGE}}(\mathbf{\Omega}'_d|\widehat{\mathbf{\Omega}'}^{[i-1]}) = -\ln|\mathbf{\Sigma}_{\boldsymbol{X}_d\boldsymbol{X}_d}| - \operatorname{tr}\left[(\mathbf{\Sigma}_{\boldsymbol{X}_d\boldsymbol{X}_d})^{-1}\hat{\mathbf{\Sigma}}_{\boldsymbol{x}_d^{[i-1]}\boldsymbol{x}_d^{[i-1]}}\right]$$

is computed in the E-step of the SAGE algorithm. Here $\mathbf{\Sigma}_{\boldsymbol{X}_d\boldsymbol{X}_d} = \boldsymbol{F}(\bar{\theta}_d)\boldsymbol{P}_{\boldsymbol{\xi}_d}\boldsymbol{F}(\bar{\theta}_d)^{\mathrm{H}} + \sigma_w^2\boldsymbol{I}$ is the covariance matrix of $\boldsymbol{X}_d(t)$, $|\cdot|$ denotes the determinant of the matrix given as an argument, and

$$\hat{\mathbf{\Sigma}}_{\boldsymbol{x}_d^{[i-1]}\boldsymbol{x}_d^{[i-1]}} = \hat{\mathbf{\Sigma}}_{\boldsymbol{y}\boldsymbol{y}} - \sum_{d'=1, d'\neq d}^{D} \boldsymbol{F}(\hat{\bar{\theta}}_{d'}^{[i-1]})\hat{\boldsymbol{P}}_{\boldsymbol{\xi}_{d'}}^{[i-1]}\boldsymbol{F}(\hat{\bar{\theta}}_{d'}^{[i-1]})^{\mathrm{H}}$$

with $\hat{\boldsymbol{P}}_{\boldsymbol{\xi}_d}^{[i-1]} = \operatorname{diag}(\widehat{\sigma_{\alpha_d}^2}^{[i-1]}, \widehat{\sigma_{\beta_d}^2}^{[i-1]})$ is an estimate of the covariance matrix of $\boldsymbol{X}_d(t)$ given $\boldsymbol{y}(t)$ and assuming $\mathbf{\Omega}' = \widehat{\mathbf{\Omega}'}^{[i-1]}$. We use $\operatorname{diag}(\cdot)$ to represent a diagonal matrix with diagonal elements given as the argument. In the case where the matrix $\hat{\mathbf{\Sigma}}_{\boldsymbol{x}_d^{[i-1]}\boldsymbol{x}_d^{[i-1]}}$ is not positive semidefinite due to estimation errors in previous iterations, this matrix is modified by setting its negative eigenvalues equal to zero in its eigenvalue decomposition.

In the M-step, $\widehat{\mathbf{\Omega}'_d}^{[i]} = \arg\max_{\mathbf{\Omega}'_d}\{Q_{\text{S-SAGE}}(\mathbf{\Omega}'_d|\widehat{\mathbf{\Omega}'}^{[i-1]})\}$ is calculated. By applying a coordinate-wise updating procedure similar to that used in [1], the required multiple-dimensional maximization can be reduced to multiple one-dimensional maximization problems. It can be shown that this coordinate-wise updating still remains within the SAGE framework with the admissible data given in (4).

In the initialization step, the estimates $\widehat{\mathbf{\Omega}'_d}^{[0]}$, $d = 1, \ldots, D$ are computed by using the separable solution proposed in [8]. Notice that $\hat{\boldsymbol{P}}_{\boldsymbol{\xi}_d}^{[0]}$ is not necessarily diagonal in this case. The estimates $\widehat{\sigma_{\alpha_d}^2}^{[0]}$ and $\widehat{\sigma_{\beta_d}^2}^{[0]}$ are equated to respectively the largest eigenvalue and the smallest eigenvalue of $\hat{\boldsymbol{P}}_{\boldsymbol{\xi}_d}^{[0]}$.

IV. Simulation studies

The performance of the NAoA estimators of the D-GAM and S-GAM algorithms is assessed by means of Monte-Carlo simulations in both a single-SDS scenario and a two-SDS scenario. For comparison purpose the performance of the SAGE algorithm derived with the SS model (SS-SAGE) and the Spread-ESPRIT technique using the two-ray model [3] is reported as well. The environment is time-variant. Totally $N = 100$ observation samples are considered for each snapshot and 100 Monte-Carlo runs are collected for calculating the root mean square estimation error (RMSEE) of the SDS NAoAs. The Rx is equipped with a ULA consisting of 8 isotropic antennas spaced by half a wavelength. Each SDS consists of $L = 50$ sub-scatterers. The AoAs of the sub-scatterers are independent, identically von-Mises distributed random variables centred around the SDS NAoA. The complex gains of the propagation paths via the sub-scatterers have equal amplitude and independent $[0, 2\pi)$-uniformly-distributed random phases. In addition, the path gain phases and the AoAs are uncorrelated. Under these assumptions, the sequences $\alpha(t)$ and $\beta(t)$ in (2) are uncorrelated zero-mean complex circularly-symmetric nearly Gaussian random sequences. The gains of the propagation paths originating from each sub-scatterer have equal power, i.e. $R_{a_1}(0) = \ldots = R_{a_L}(0)$. As a consequence, $\sigma_{\tilde{\theta}}$ expressed in radian provides a close approximation of the DS of the SDS [6].

In the single-SDS scenario the NAoA of the SDS is set equal to $110°$ and $\sigma_{\tilde{\theta}} = 5°$. The SNR at the output of the estimators, which we denote with γ_o, ranges from 0 dB to 30 dB. Fig. 1 depicts the RMSEE of the NAoA versus the output SNR. It can be observed that the S-GAM SAGE algorithm outperforms the D-GAM SAGE algorithm, which outperforms the Spread-ESPRIT technique. The D-GAM SAGE algorithm and the Spread-ESPRIT technique exhibit better performance than the SS-SAGE algorithm beyond a certain SNR threshold which depends on the DS and the estimators, e.g. 7dB and 14dB for the D-GAM SAGE algorithm and the Spread-ESPRIT technique respectively when $\sigma_{\tilde{\theta}} = 5°$. The above observation remains valid for other choice of $\sigma_{\tilde{\theta}}$. In addition we also found that the Spread-ESPRIT technique fails in estimating the NAoA when $\sigma_{\tilde{\theta}}$ is small. This observation is in accordance with the discussion of the robustness of the Spread-F technique in [3].

In the two-SDS scenario, the SDS of interest (SDS_1) has a fixed NAoA equal to $110°$. The NAoA spacing $\Delta\bar{\theta}$ between SDS_1 and the second SDS (SDS_2) ranges from $5°$ to $40°$. For the two SDSs $\sigma_{\tilde{\theta}} = 3°$ is selected. Furthermore we consider a situation with strong SDS power unbalance, i.e. 9 dB. The output SNRs for SDS_1 and SDS_2 are 20 dB and 29 dB respectively. Fig. 2 (a) and Fig. 2 (b) depict respectively the average estimation error (AEE) and the RMSEE of the NAoA versus $\Delta\bar{\theta}$ for the weaker SDS (SDS_1). It can be observed that the AEE of the SS-SAGE and the D-GAM SAGE algorithms fluctuate significantly in the range $\Delta\bar{\theta} < 25°$. On the other hand the S-GAM SAGE algorithm shows a pretty stable AEE approximately equal to $-2°$. This observation shows that the performance of the S-GAM SAGE

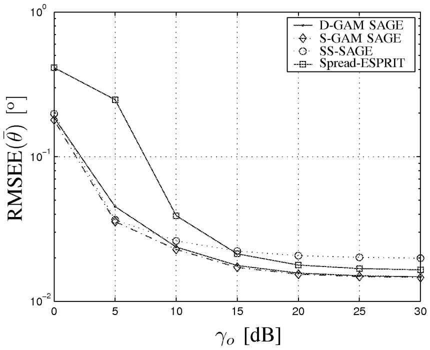
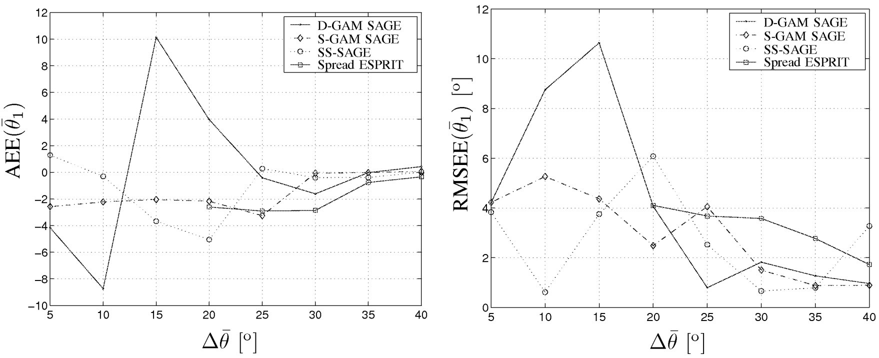


Fig. 1. RMSEE($\bar{\theta}$) vs. output SNR γ_o with $\sigma_{\tilde{\theta}} = 5°$.



(a) (b)

Fig. 2. AEE($\bar{\theta}_1$) (a) and RMSEE($\bar{\theta}_1$) (b) vs. the NAoA spacing $\Delta\bar{\theta}$ of the two SDSs.

algorithm in estimating weaker SDSs is more robust towards the impact of the stronger SDS than that of the D-GAM and SS-SAGE algorithms. The RMSEEs of all three SAGE estimators fluctuate at various intensity levels with a decreasing trend when $\Delta\bar{\theta} > 20°$. The fluctuations are strongly correlated with the absolute value of the corresponding AEE. The RMSEE graph of the S-GAM SAGE algorithm exhibits the less variations in accordance with the stable behaviour of the AEE. The figure shows clearly that the estimates are "interference" limited and the NAoA separation between the SDSs need to be larger than roughly 28° for the NAoA estimates returned by the S-GAM SAGE and D-GAM SAGE algorithms to be reasonably accurate. In this case the former algorithm slightly outperforms the latter. Notice that 28° equals twice the intrinsic azimuth resolution of the ULA [1]. Following [3], the Spread-ESPRIT technique is applied in a two-SDS scenario when the number of point scatterers estimated with the minimum description length (MDL) method is larger than 3. This condition is satisfied when $\Delta\bar{\theta} \geq 20°$ in our investigated scenarios. When the NAoA separation is larger than twice the intrinsic azimuth resolution of the ULA the D-GAM and S-GAM SAGE algorithms both perform better than the Spread-ESPRIT technique in terms of smaller AEE and lower RMSEE.

V. Conclusions

In this paper, we proposed two SAGE algorithms for estimation of nominal azimuths of arrival (NAoAs) of multiple slightly distributed scatterers (SDSs) in time-variant environments. The algorithms are derived based on the generalized array manifold (GAM) model proposed in [2]. The estimators yield low computational complexity and can be used with arbitrary arrays.

Monte-Carlo simulations show that in a single-SDS scenario, the SAGE algorithm derived based on the stochastic GAM model (S-GAM SAGE) returns lower root mean square estimation errors than the algorithm derived from the deterministic GAM model (D-GAM SAGE), as well as the Spread-ESPRIT technique using the two-ray approximation model proposed in [3]. Further simulations in a two-SDS scenario show that in case of significant power unbalance between the SDS the azimuth separation between these SDSs need to be larger than twice the intrinsic azimuth resolution of the used array for the NAoA estimators to perform reasonably accurately. However, the S-GAM SAGE algorithm shows more robust performance in such a scenario than the other above-mentioned estimators when the NAoA separation between the SDSs is small and outperform them when the separation is large.

The proposed algorithms can be applied in time-invariant environments with minor modifications. Extension of these algorithms to include estimation of the direction, i.e. azimuth and elevation, of arrival and the direction of departure is straightforward when multiple-input multiple-output (MIMO) applications are considered.

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