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# On the Interpretation of the APP Algorithm as an LLR Filter

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**Abstract** — A channel decoder employing the a posteriori probability (APP) algorithm can be formulated so that its inputs and its outputs are log-likelihood-ratios (LLR): channel LLRs of the code bits are accepted, and a posteriori LLRs of the info bits and/or the code bits are delivered. Since decoding improves the reliability, the APP algorithm can be interpreted as a non-linear filter for LLRs. The “LLR amplification” depends on the distance properties of the channel code; for high signal-to-noise ratios it is dominated by the minimum distance.

## SUMMARY

The APP algorithm [1] accepts a priori probabilities and channel probabilities as inputs and delivers a posteriori probabilities as outputs. With additional computation of soft outputs for the code bits [2][3] and with usage of LLRs instead of probabilities [4], it can be extended to the logarithmic APP (LogAPP).

Consider a binary linear convolutional encoder of rate  $R = k/n$ . Let  $e$  denote the path through the trellis associated with the info word  $u(e)$  and the code word  $x(e)$ ,  $u, x \in \{+1, -1\}$ . The code bits are transmitted over a memoryless channel; the received value of a single bit is denoted by  $y$ , and the received word is denoted by  $\mathbf{y}$ .

The LogAPP algorithm takes the a priori LLRs of the info bits  $U$  and the channel LLRs of the code bits  $X$ ,

$$L^-(U) \triangleq \ln \frac{P(U=+1)}{P(U=-1)}, \quad L^-(X) \triangleq \ln \frac{P(X=+1|y)}{P(X=-1|y)}, \quad (1)$$

and computes the a posteriori LLRs of the info bits and of the code bits

$$L^+(U) \triangleq \ln \frac{P(U=+1|\mathbf{y})}{P(U=-1|\mathbf{y})}, \quad L^+(X) \triangleq \ln \frac{P(X=+1|\mathbf{y})}{P(X=-1|\mathbf{y})}. \quad (2)$$

These inputs and outputs of the LogAPP algorithm are depicted in Fig. 1. In the following, the info bits are assumed to be equally distributed, i.e.  $L^-(U) = 0$ .



Fig. 1: The input and the output LLRs of the LogAPP algorithm.

The purpose of decoding is to improve the reliability of the bits. This motivates to interpret decoding as non-linear filtering, as mentioned in [2]. In this paper, the LogAPP is treated as a *non-linear LLR filter*. This point-of-view suggests to define an *info bit LLR amplification* (ILA) and a *code bit LLR amplification* (CLA):

$$\text{ILA} \triangleq \left. \frac{E_{\mathbf{y}} L^+(U)}{E_{\mathbf{y}} L^-(X)} \right|_{L^-(U)}, \quad \text{CLA} \triangleq \left. \frac{E_{\mathbf{y}} L^+(X)}{E_{\mathbf{y}} L^-(X)} \right|_{L^-(U)}, \quad (3)$$

where  $E_{\mathbf{y}}$  denotes the expected value with respect to  $\mathbf{y}$ . The ILA can be regarded as the transfer function of a *soft-decoder*; since there are less output values than input values, the soft-decoder is similar to a decimator. The CLA can be regarded as the transfer function of a *soft-repeater*, i.e. a device which performs decoding and re-encoding using soft values.

For rate 1/2 convolutional codes with memories 2 to 8, binary transmission over an AWGN channel was simulated. In Fig. 2, the ILA and the CLA are depicted as a function of the mean channel LLR  $E_{\mathbf{y}} L^-(X)$  of the code bits. The following characteristics can be justified analytically:

1. For low input LLRs, the ILA approaches 0 and the CLA approaches 1.
2. For high input LLRs, both the ILA and the CLA approach a constant value which can be identified with the free distance of the code.

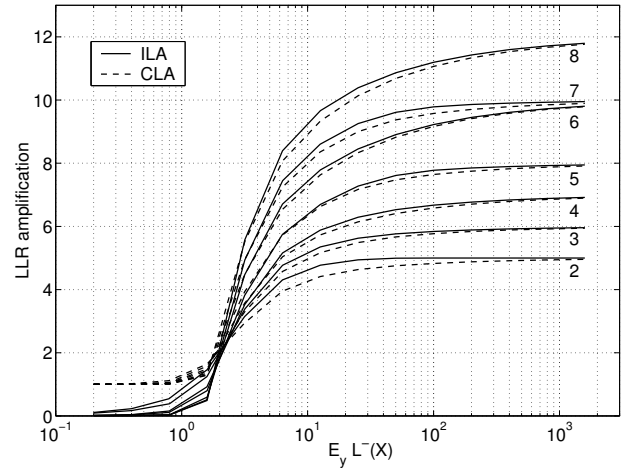


Fig. 2: The LLR amplifications of the convolutional codes with memories 2 to 8.

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