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# A DYNAMIC MODEL OF A BUOYANCY SYSTEM IN A WAVE ENERGY POWER PLANT

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**Abstract:** A nonlinear dynamic model of the buoyancy system in a wave energy power plant is presented. The plant (“Wave Dragon”) is a floating device using the potential energy in overtopping waves to produce power. A water reservoir is placed on top of the WD, and hydro turbines lead the water to the sea producing electrical power. Through air chambers it is possible to control the level, the trim and the heel of the WD. It is important to control the level (and trim, heel) of the WD in order to maximize the power production in proportion to the wave height, here the amount of overtopping water and the amount of potential energy is conflicting. Five separate air chambers, all open to the sea, makes the device float. The pressures in the air chambers may be individually controlled by an air fan through an array of valves. In order to make a model-based control system, this paper presents a model describing the dynamics from the air inlet to the level, trim and heel. The model is derived from first principles and is characterized by physical parameters. Results from validation of the model against plant data are presented.

## 1 INTRODUCTION

Renewable energy is an important issue due to the global warming problem and utilisation of wave power is one of the energy resources to be exploited.

The wave power system “Wave Dragon”, on which this paper focuses, was invented by Erik Friis Madsen, Löwenmark and tested at Aalborg University and University of Cork. An EU based European consortium has been involved in the construction and implementation of a 1:4 scaled test site - 57x27 m wide and with a weight of 237 tonnes- which is placed in Nissum Bredning in Denmark. Large numbers of tests have been carried out during a two years operating period. One goal for energy production improvement is a better control of the Wave Dragon buoyancy.

Wave Dragon (WD) is an offshore wave energy converter of the overtopping type, a description is found in (Kofoed, 2006) and (W.D.Aps, 2006). The main structure consists of a ramp where the waves are overtopping and led to a reservoir (basin). Two reflectors are focusing the waves towards the ramp as seen on figure 1. WD is fastened to an anchor

making it possible to turn the ramp towards the dominant wave direction.

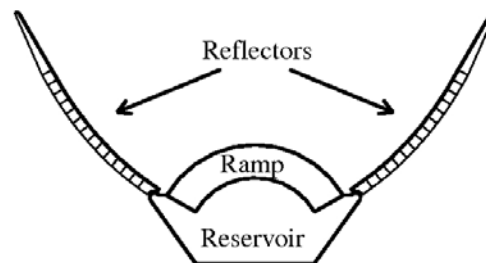


Figure 1: Main components of the Wave Dragon (Kofoed, 2006).

The WD use the potential energy of the waves, meaning that for a given wave type there exist an optimal level of the reservoir. As shown on figure 2 the reservoir water is led through a turbine.

The WD floats on open air chambers used to adjust the floating level. Control of the floating level is a part of optimizing the overtopping and a dynamic model for a model-based control system is the topic of this paper. It should be noted that the

wave conditions are measured online and may be used as reference to the level control system.

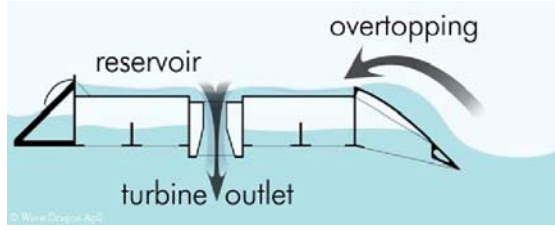


Figure 2: The basic principle of WD showing waves loading the reservoir via the ramp. (Kofoed, 2006).

First the wave dragon buoyancy system is presented. A dynamic model of the air supply system controlling the pressure in the air chambers is set up. The model parameters have been adjusted to the test site Wave Dragon. Finally the buoyancy model is verified by comparing simulation results and measurements from the test site Wave Dragon.

## 2 DYNAMIC MODEL OF THE BUOYANCY SYSTEM

The buoyancy system consists of five air chambers all open to the water surface. The five chambers are shown on figure 3 (21,22,23,24,25,26,27), (15,16,18,19,20), (3,4,5,10,11), (1,2,8,9,14) and (6,7,12,13,17). Furthermore 9 small chambers contain a constant amount of air.

The air pressures in the chambers are controlled by an air supply system using an on/off driven air fan and input/output valves to each chamber. The valves are operated as on/off valves and only one valve is allowed to be active at the time in order to prevent pressure equalizing in the chambers. A PWM scheme is in (Andersen, 2007) proposed to handle this problem. The air supply system model consist of two parts, one describing air inlet to the chamber and one describing air flow out of the chamber. In both models tube pressure drops are ignored. The inlet air mass flow,  $m_{ai}$ , is given by

$$m_{ai} = -K_{pa}(p_c - p_a) + K_{pb}$$

which is an approximation to the fan characteristic.  $p_c$  is the chamber pressure,  $p_a$  is the inlet pressure to the fan (atmospheric pressure) and the two constants  $K_{pa}$ ,  $K_{pb}$  are from the fan data sheet.

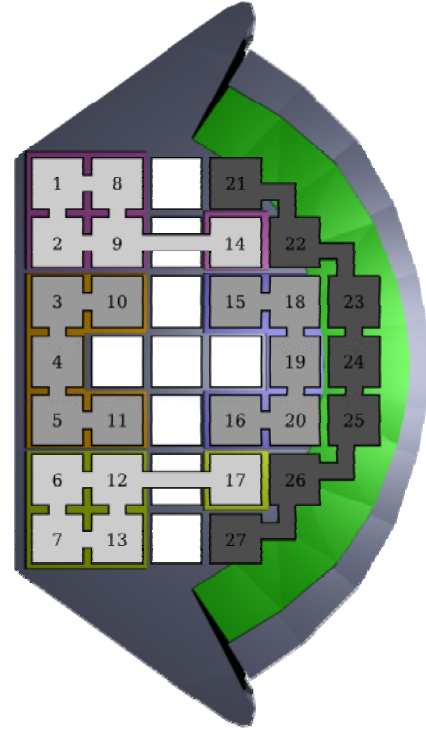


Figure 3: The air chambers in the Wave Dragon.

Air outlet mass flow,  $m_{ao}$ , is given by the Bernoulli equation:

$$m_{ao} = K_{vo} \sqrt{p_c - p_a}$$

where the constant  $K_{vo}$  depend on the outlet tube dimension and the air density.

Each input/output valve pair is controlled by a signal  $u$ , where  $u=1$  allows an airflow into the chamber,  $u=0$  closes both valves and  $u=-1$  open the outlet valve. This gives the air mass flow equation:

$$m_a = m_{ai} g_1(u) - m_{ao} g_2(u) \quad (1)$$

where

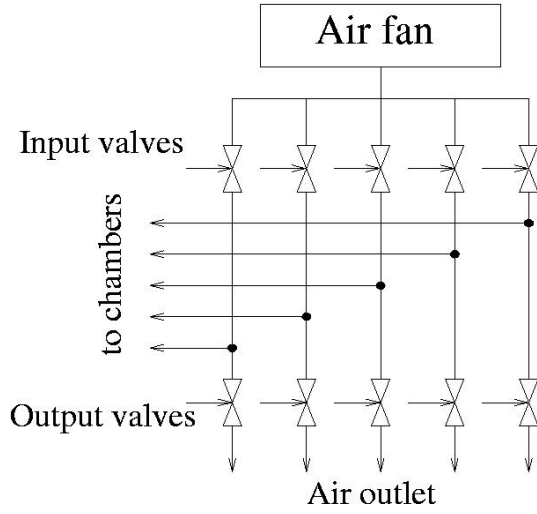


Figure 4: Fan/valve system.

$$g_1(u) = \begin{cases} 1 & \text{if } u \geq 1 \\ 0 & \text{if } u < 1 \end{cases}$$

$$g_2(u) = \begin{cases} 1 & \text{if } u \leq -1 \\ 0 & \text{if } u > -1 \end{cases}$$

Eq. (1) describes the air mass flow to the chamber and is valid when only one chamber is operated at a time.

### 3 ONE CHAMBER DYNAMIC MODEL

This section outlines the dynamic describing equations for a single chamber. The equations assumes that the air density is constant (variations could be included using the ideal gas equation), the chamber is only moving along a vertical axis perpendicular to the water surface, the cross section area is constant along this axis and there is only one rigid moving body. The model then consists of a mass balance equation describing the air in the chamber and a Newton equation describing the motion of the chamber.

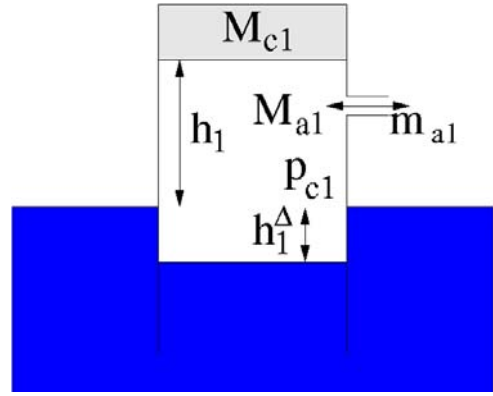


Figure 5: One chamber model variables.

The mass balance equation for the chamber is

$$\frac{dM_{a1}}{dt} = m_{a1} = A_1 \rho_a \frac{d(h_1^\Delta + h_1)}{dt} \quad (2)$$

where  $M_{a1}$  is the air mass in the chamber,  $m_{a1}$  is the mass flow to the chamber,  $A_1$  is the cross section area,  $\rho_a$  is the air density,  $h_1^\Delta$  is the distance from the chamber water surface to the ambient water surface and  $h_1$  is the distance from the ambient water surface to the top of the chamber.

The pressure force from the chamber is assumed to equal the buoyancy force. This implies that the acceleration of the water volume in the chamber is small.

$$h_1^\Delta A_1 \rho_w g = p_{c1} A_1 \Rightarrow h_1^\Delta = \frac{1}{\rho_w g} p_{c1} \quad (3)$$

where  $\rho_w$  is the water density. Insertion of Eq. (2) in Eq. (3) gives

$$\frac{dp_{c1}}{dt} = \frac{g \rho_w}{A_1 \rho_a} m_{a1} - \rho_w g \frac{dh_1}{dt} \quad (4)$$

The other main equation for describing the dynamics of the chamber is Newton's 2<sup>nd</sup> law used on the free body with the mass  $M_{c1}$ . Pressure forces, gravity and a friction force proportional with the chamber velocity is assumed to act on the body.

$$M_{c1} \frac{d^2 h_1}{dt^2} = (p_{c1} - p_a) A_1 - M_{c1} g - K_{f1} \frac{dh_1}{dt} \quad (5)$$

In order to use an ODE-solver to simulate the one chamber model, the three states  $x_1 = p_{c1}$ ,  $x_2 = h_1$  and  $x_3 = \frac{dh_1}{dt}$  may be selected resulting in the differential equation system

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} -\rho_w g x_3 + \frac{g \rho_w}{A_1 \rho_a} ((-K_{pa}(x_1 - p_a) + K_{pb}) g_1(u) - K_{vo} \sqrt{x_1 - p_a} g_2(u)) \\ x_3 \\ (x_1 - p_a) A_1 - M_{c1} g - K_{f1} x_3 \end{bmatrix} \quad (6)$$

## 4 FIVE-CHAMBER DYNAMIC MODEL

In this section a 5 chamber dynamic model is described. Although the WD has a complex geometry the model assumes that each of the five chambers may be regarded as a control volume with position independent internal variables and that each chambers pressure force actuate the WD bode in a single point. The model consist of 8 differential equations, 5 equations describing the pressures in the five chambers derived from mass balance equations, and 3 equations describing height, trim and heel derived from Newton's law.

The five chambers are placed in a "wave dragon" coordinate system {WD} with the coordinates  $(\bar{x}_n, \bar{y}_n, \bar{z}_n)$ . An inertial coordinate system {I} is placed in the water level as shown on the figure. The states in the model are the five chamber pressures ( $p_{c1}$ ,  $p_{c2}$ ,  $p_{c3}$ ,  $p_{c4}$ ,  $p_{c5}$ ), the trim angle ( $\theta$ ), the heel angle ( $\gamma$ ) and the level ( $h$ ) of the wave dragon.

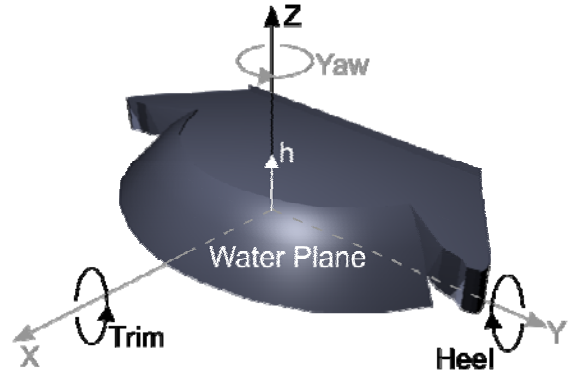


Figure 6: Wave dragon coordinate systems.

The mass balance for the n'th chamber is

$$\frac{dp_{cn}}{dt} = \frac{g \rho_w}{A_n \rho_a} m_{an} - \rho_w g \frac{dh_n}{dt}$$

$h_n$  is the height in the chamber and not a model state. Using a rotation matrix  ${}_{WD}^I R(\theta, \gamma, \alpha)$  (see (Craig, 1989)),  $h_n$  and the states are related through

$$h_n = -\bar{x}_n \sin(\gamma) + \bar{y}_n \cos(\gamma) \sin(\theta) + \bar{z}_n \cos(\gamma) \cos(\theta) + h$$

The time derivative of  $h_n$  is a tedious equation but using the approximations  $\cos(\theta) = 1$ ,  $\cos(\gamma) = 1$ ,  $\sin(\theta) = \theta$ ,  $\sin(\gamma) = \gamma$  assuming small heel and trim angles (all  $\bar{z}_n$ 's are 0) gives the simple relation

$$\frac{dh_n}{dt} = \bar{x}_n \frac{d\gamma}{dt} + \bar{y}_n \frac{d\theta}{dt} + \frac{dh}{dt}$$

which inserted gives

$$\frac{dp_{cn}}{dt} = \frac{g \rho_w}{A_n \rho_a} m_{an} - \rho_w g \left( \bar{x}_n \frac{d\gamma}{dt} + \bar{y}_n \frac{d\theta}{dt} + \frac{dh}{dt} \right) \quad (7)$$

Newton's 2'th law for the translational system is

$$\sum_{n=1}^5 A_n p_{cn} + K_{flh} \frac{1}{h} - M_{wd} g - p_w A_w - K_f \frac{dh}{dt} = (M_{wd} + \frac{p_w A_w}{g}) \frac{d^2 h}{dt^2} \quad (8)$$

where the sum is chamber pressure forces acting on the wave dragon body. The second term is the forces from the air chambers containing a constant air mass (see figure 3). In the model these are approximated with two chambers. They may be modelled using the mass balance Eq. (4) but in order to keep the model order low they are modelled using a static balance.  $K_{flh}$  may be found using the ideal gas law and the geometrics of the chambers.  $M_{wd}$  is the total mass of the wave dragon. The term  $p_w A_w$  represents the force from the water reservoir on top the Wave Dragon. It may be noted that the pressure  $p_w$  is an input to the model.

The rotational trim equation is

$$\sum_{n=1}^5 A_n p_{cn} \bar{y}_n - \frac{K_{fl\theta}}{h^2} \theta - K_\theta \frac{d\theta}{dt} = J_\theta \frac{d^2 \theta}{dt^2} \quad (9)$$

where the sum is the torque from the chambers. The second term is the torque from the constant air mass chambers modelled as two symmetrical chambers. It should be noted that  $K_{flh}$  as well as  $K_{fl\theta}$  are very dependent on the total air mass in the chambers. The moment of inertia is calculated as a constant although it depends on the water level in a very complex manner. In the simulation a situation with low water level has been used.

The heel equation is

$$\sum_{n=1}^5 A_n p_{cn} \bar{y}_n - K_{fl\gamma} \gamma - K_\gamma \frac{d\gamma}{dt} = J_\gamma \frac{d^2 \gamma}{dt^2} \quad (10)$$

This linear second order equation captures the gross behaviour of the heel dynamics.

The total model now consist of eleven equations, equation (7) used 5 times for the five chambers giving the five states  $x_1 = p_{c1}$ ,  $x_2 = p_{c2}$ ,  $x_3 = p_{c3}$ ,  $x_4 = p_{c4}$ ,  $x_5 = p_{c5}$ , equation (8)

with the states  $x_6 = h$ ,  $x_7 = \frac{dh}{dt}$  and the equation

$$\frac{dx_6}{dt} = x_7 \text{ gives 2 equations, equation (9) with the}$$

states  $x_8 = \theta$ ,  $x_9 = \frac{d\theta}{dt}$  and the equation

$$\frac{dx_8}{dt} = x_9 \text{ gives 2 equations, and finally equation}$$

(10) with the states  $x_{10} = \gamma$ ,  $x_{11} = \frac{d\gamma}{dt}$  and the

equation  $\frac{dx_{10}}{dt} = x_{11}$  gives 2 equations.

Inserting the states  $(x_1, x_2, \dots, x_{11})$  gives eleven nonlinear first order equations. The equations are solved using an ODE-solver.

## 5 SIMULATION

The model is tested using measured data from the wave dragon. The inputs to the model are the chamber pressures as well as the water level in the dragon represented by the pressure measurement  $p_w$ . The reason for not using the valve signals is that these were not recorded in the data acquisition equipment. As seen there is a good agreement between level in the model and the experimental data. (The agreements are found on the measurements from the same day. Measurements on different days are based on different initial pressures causing identification of slightly different model parameters) (This is also found on data measured on the same day.) Because the pressures in the constant air chambers are not measured these have been estimated from steady state observations.

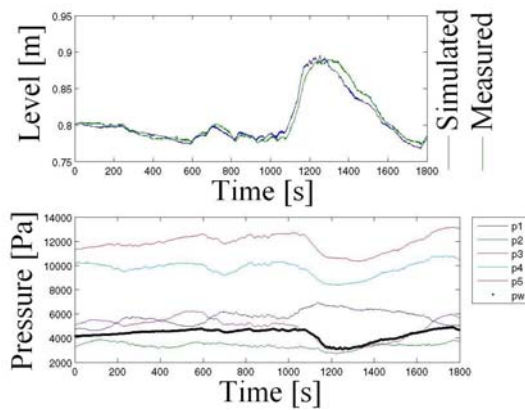


Figure 7: Level simulated and measured.

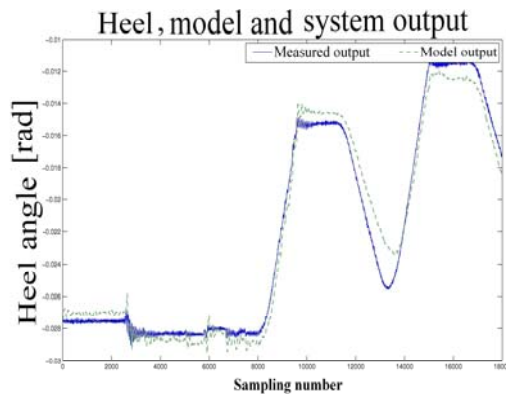


Figure 8: Heel simulated and measured (Andersen, 2007).

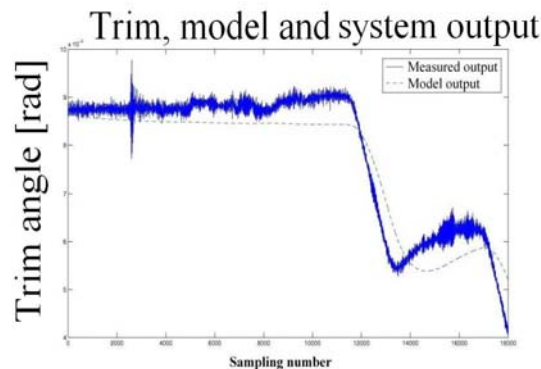


Figure 9: Trim simulated and measured (Andersen, 2007).

The variations on the heel and trim angles are small. As seen on the figures the behaviour is captured by the model. All the data were recorded prior to this project and not prepared for this study, unfortunately the WD ran severely aground during the project meaning that controlled input signal could not be tested. Regardless of this the model performed well on the recorded data.

## 6 CONCLUSION

A nonlinear physical model with a complexity that is suitable for model based control has been presented. The model is partly based on physical parameters for the Wave Dragon and may be scaled to a future larger version. The model has four main equations, one describing the state of the air in a chamber, and three accounting for the level, trim and heel motion of the WD. The model has been validated against measured WD data, where it captures the gross behaviour of the Wave Dragon. In particular it describes the response of the level very well. The model does, however, have one serious deficiency because it does not capture the distribution and movement of water in the water reservoir. A comprehensive study of this is outside the scope of this paper.

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