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Wireless Positioning Based on a Segment-Wise Linear Approach for Modeling the Target Trajectory

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***Abstract*— Positioning solutions in infrastructure-based wireless networks generally operate by exploiting the channel information of the links between the Wireless Devices and fixed networking Access Points. The major challenge of such solutions is the modeling of both the noise properties of the channel measurements and the user mobility patterns. One class of typical human being movement patterns is the segment-wise linear approach, which is studied in this paper. Current tracking solutions, such as the Constant Velocity model, hardly handle such segment-wise linear patterns. In this paper we propose a segment-wise linear model, called the Drifting Points model. The model results in an increased performance when compared with traditional solutions.**

I. Introduction

During the last decades, location information has been considered an interesting and intensively investigated topic of research. Primarily considered as a vital information for vehicle tracking and military strategies [1], [2], [3], it has nowadays been introduced in common wireless communication networks [4], [5], [6]. Typically, radio-localization solutions for wireless networks rely on robust algorithms that estimate information of position based on indirect measurements of the physical length of the communication links. Since these solutions do not require integration of additional hardware into the mobile nodes, they are cheap and simple to implement. As a price to pay, accuracy is typically lower in comparison to dedicated systems such as the Global Positioning System (GPS).

The general problem of tracking mobile devices can be described as the problem of estimating a quantity X, traditionally a vector of position and its derivatives, based on measurements of a communication channel dependent vectorial quantity Z.

$$\dot{X} = f\big(X, U, w\big) \qquad w \sim \mathrm{Norm}\big(0, Q\big) \tag{1}$$

$$Z = h\big(X, v\big) \qquad v \sim \mathrm{Norm}\big(0, R\big) \tag{2}$$

In eq.(1) and eq.(2), the functions f and h are respectively the evolution and the observation models. The variable U represents and external excitation of the system, not necessarily known, and w and v represent random noise components commonly approximated by zero mean Gaussian random variables with covariance matrices Q and R respectively.

Particular movement models for tracking mobile users can be either simple as non-maneuvering models [1], [7] or more complex with multiple models [8], maneuver detections [3] or variable dimension state estimation [9]. The choice of the positioning method depends on the properties of the target to be tracked and in particular on the characteristics of U in eq.(1). Concerning the uncertainty in the model, i.e. the noise component w, it can be either uncorrelated or correlated noise [10], [11]. Regarding the user mobility, it is common to model user position and its derivatives. A typical example is the Constant Velocity (CV) model [1]. In this paper we propose a different approach for modeling the user trajectory. We assume that the user moves according to a segment-wise linear trajectory (Fig. 1) at constant velocity. The main difference is in the state space X and subsequent model of eq.(1). Practically, instead of estimating at every time step the kinetics of the user, as it is done in the CV model, in this model, called the Drifting Points (DP) model, we attempt to localize segments of movement as a whole. To do that we assume the position derivatives constant and not corrupted by noise along each entire segment. Subsequently, the current position of the user has a well defined relation with its position at the time when the segment has started, i.e. when the last maneuver has occurred. The noise is instead only assumed in the beginning of the segment and the current position. Thus, we estimate both the current position of the user and the position when the segment started, given the measurements until present time. The start of new segments is discovered using a maneuver detector. By comparing the DP model with the standard CV model [1], we could see a considerable higher performance for the proposed solution. An advantage of this model is that it does not increase the computational complexity.

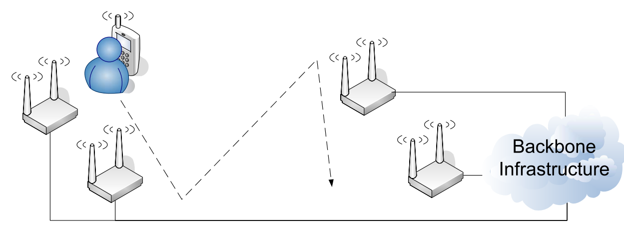

Fig. 1. Infrastructure-base network of wireless Access Points.

The paper is organized as follows: in Section II, the problem is formulated and the scenario is defined. Then, in Section III, the common CV model is presented and in Section IV the proposed approach is introduced. Finally, performance is compared in Section V and the final conclusion is drawn in Section VI.

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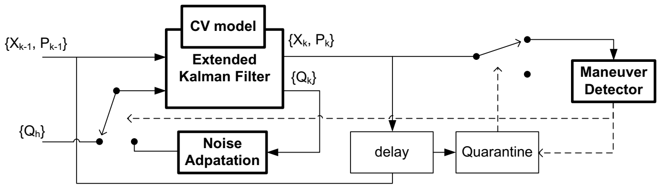

Fig. 2. Estimation algorithm using the Constant Velocity model.

II. Problem and Scenario Definition

Consider a wireless network, as the one in Fig 1, composed of N Access Points (APs) at known geographic positions $\{\Lambda_n = [a_n, b_n]^T : n = 1, \ldots, N\}$, and a single Wireless Device (WD), in a 2-dimensional scenario. Let the position and velocity of the WD at time t be denoted by the 2-dimensional column vectors $s(t) \in \mathbb{R}^2$ and $\dot{s}(t) \in \mathbb{R}^2$, respectively. The velocity vector is assumed to be a piece-wise constant function of t. The maneuvers U, defined as jumps experienced in $\dot{s}$, are assumed to happen at unknown times. The result is a segment-wise trajectory defined by the following differential equation.

$$\dot{X} = f\big(X, U\big) \tag{3}$$

where X is a vector of positions and velocities, commonly considered as $X = [s^T, \dot{s}^T]^T$. Note that in eq.(3) it is assumed that the movement is not corrupted by noise, so that the only uncertainly in the model is when maneuvers U happen.

We assume that the WD is able to respond to communication requests from only one AP at the time. Let $n_k \in \{1, \ldots, N\}$ be the index of the AP active at time t_k, $k = 0, 1, \ldots$. Without loss of generality, we use the convention $t_0 = 0$. We assume at time t_k the AP index n_k is picked at random and that the APs are equiprobable.

During the WD-AP communication, the AP is able to obtain measurements of signal strength $\{Z_k\}$ on the incoming packets. Thus, it is possible to relate $\{Z_k\}$ with the distance AP-WD, at the instant when the WD moves along the position $s_k = s(t_k)$. Note that, since only a single WD-AP link is available at each time t_k, only one measurement is available, and hence Z_k is scalar. Finally, Z_k is corrupted by a noise component v_k. We model $v_k, k = 0, 1, 2, \ldots$ as independent Gaussian random variables with zero mean and standard deviation σ_Z:

$$Z_k = \alpha - 10\beta \log\Big(\|s_k, \Lambda_{n_k}\|\Big) + v_k \qquad v_k \sim \mathrm{Norm}\big(0, \sigma_Z\big) \tag{4}$$

where α and β are constant propagation parameters (determined through a calibration phase) and $\|s_k, \Lambda_{n_k}\|$ is the Euclidean distance between s_k and Λ_{n_k}.

III. The Standard Constant Velocity Model

For performance comparison we introduce in this section the typical solution for the application in Section II: the Constant Velocity (CV) model (Section III-A) applied in the prediction phase of an Extended Kalman Filter (Section III-B). The unknown maneuvers are estimated using a maneuver detector (Section III-C), which continuously evaluates the innovation process in order to adapt the system. The uncertainty in the estimated position is externally controlled by a noise adaptation module (Section III-D). Figure 2 shows the algorithm.

A. Constant Velocity Model

Since we are interested to localize devices in a 2 dimensional plane, moving at a constant speed, we can define the hidden state X as the position s as well as the speed $\dot{s}$. Thus, at time t_k:

$$X_k = \begin{bmatrix} s_k^T & \dot{s}_k^T \end{bmatrix}^T \tag{5}$$

The evolution of the system is modeled according to eq.(3). Although the movement model as defined in eq.(3) considers no noise in the trajectory, the estimation algorithm has to assume noise in order to account with some uncertainty on that models. Thus, disregarding maneuvers (treated in Section III-C) at this point:

$$\dot{X} = f\big(X\big) + w \tag{6}$$

Given $X_0 = X(t_0)$, determining the first moment of the differential equation of eq.(6) with state space given by eq.(5), the evolution model can be calculated as:

$$X_k = A_k X_{k-1}, \quad A_k = \begin{bmatrix} I_2 & T_k I_2 \\ 0 & I_2 \end{bmatrix} \tag{7}$$

where $T_k = t_k - t_{k-1}$ and I_2 is the 2×2 identity matrix.

Concerning the process noise, it is considered as uniquely introduced in the acceleration. Calculating the second moment of eq.(6), it is possible to obtain the covariance matrix:

$$Q_k = \delta_k T_k \sigma_{\ddot{s}}^2 \begin{bmatrix} T_k^2/3 I_2 & T_k/2 I_2 \\ T_k/2 I_2 & I_2 \end{bmatrix} \tag{8}$$

where δ_k is an adaptation parameters defined in Section III-D. Additionally, in eq.(8), the parameter $\sigma_{\ddot{s}}$ corresponds to the standard deviation of the zero mean gaussian noise in any of the acceleration coordinates. Finally, the observation model is given by eq.(4). Note here that since Z_k is a single entry vector, the covariance matrix R equals the variance of the measurements σ_Z^2.

B. Extended Kalman Filter

The EKF is an extension of the optimal Kalman filter that approximates nonlinear models by their Taylor series expansion. In particular for the presented scenario, the observation model of eq.(4) is the subject of this linearization. The filter presents a recursive sequence of consecutive predictions and corrections. The prediction is based on the evolution model and the correction is based on the observation model. We refer to [12] for the details and the mathematical formulation.

C. The Maneuver Detector

As mentioned in [3], a possible maneuver detection scheme for the present tracking problem is the so called White Noise Model with Adjustable Level. The detection of a maneuver is verified when a sequence of "large" innovations happen,

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what can be discovered by using the normalized innovations squared:

$$\varepsilon_k = \tilde{Z}_k^T S_k^{-1} \tilde{Z}_k \tag{9}$$

where $\tilde{Z}_k = Z_k - h(X_k, 0)$ is the innovation process and S_k its covariance matrix. The maneuver detector then considers an exponential discounted average with forgetting factor γ:

$$\varepsilon_k^\gamma = \gamma \varepsilon_{k-1}^\gamma + \varepsilon_k, \quad 0 < \gamma < 1, \quad \varepsilon_0^\gamma = 0 \tag{10}$$

where ε_k^γ is, by first moment approximation, chi-squared distributed with $l/(1-\gamma)$ degrees of freedom (l is the dimension of the observation vector Z). For additional details see [3].

When a maneuver is detected, i.e., ε_k^γ has exceeded a predefined threshold Γ, the level of process noise is momentarily increased (to δ_{high} as mentioned in Section III-D). Inversely, when ε_k^γ gets below the threshold Γ, the noise adaptation schemes in Section III-D starts operating. Although this scheme of adapting the system is commonly accepted as standard, we have decided to introduce an additional quarantine period once a maneuver is detected. The quarantine block disables the maneuver detector right after one detection and reactivates it once the system regains observability.

D. Noise Adaptation

The noise adaptation method used in the current system defines an upper bound threshold δ_{high}, a lower bound threshold δ_{low} and a decay parameter $0 < \varsigma < 1$. The value δ_{high} represents the maximum value of δ_k used in eq.(8), the value δ_{low} is the lowest value allowed for δ_k and ς dictates how fast δ_k decays from δ_{high} to δ_{low} along time. Thus:

$$\delta_k = \max\left(\delta_{\text{low}}, \varsigma \delta_{k-1}\right) \qquad \text{where} \qquad \delta_0 = \delta_{\text{high}} \tag{11}$$

In order to obtain the process noise Q_k, the value of eq.(11) has to be included in eq.(8). In eq.(11), the initial threshold identifies the initial value for the process noise. Then, at every time step, by using eq.(11), the noise is gradually reduced according to the decay parameter ς. Once the process noise reaches the threshold σ_{low}, it keeps that value for the rest of the estimation procedure. Additionally, everytime a maneuver is detected by the module in Section III-C, the value δ_k is reset to δ_{high}.

IV. A Novel Approach on the Motion Model

In this section we introduce a new model which complies with the main idea of estimating a segment-wise linear trajectory. The algorithm, shown in Fig. 3, differs from the CV algorithm shown in Fig. 2 on the DP model (Section IV-A) and the "Transformation Model" (Section IV-B). Regarding the remaining blocks in Fig. 3, the Filter, the Noise Adaptation and the Maneuver Detector are exactly the same as those used in Sec. III. Note that though the filter is the same for both CV and DP model, the state space and the external adaptation of the filter are different.

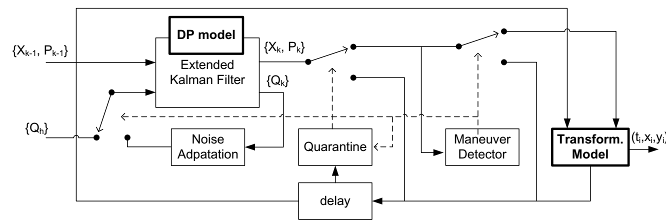

Fig. 3. Estimation algorithm using the Drifting Points model.

A. Drifting Points Model

Before starting to define the model we introduce the definition of Drifting Point: a Drifting Point (DP) is the position where the target was placed at the precise moment when a maneuver occurs. The process noise w between maneuvers is assumed to be zero. For this paper, we assume that the maneuvers occur only at a subset of the observation times t_j, $j = 0, 1, ...$, and the very first observation at t_0 is assumed to be a maneuver point.

The main idea of the following model is to estimate both the position s_k at time t_k and the position $s_{i|k}$ of the previous DP detected at time t_i ($i < k$), given measurements until time t_k. In order to not complicate the presentation with technicalities, the following explanation of the approach is referring to the very first segment starting at $t_0 = 0$.

As the velocity vector is constant and not corrupted by noise in between maneuvers, it is possible to relate this velocity with the current position at time t_k and the initial position at time t_0 by:

$$\begin{bmatrix} s_k \\ \dot{s} \end{bmatrix} = \Xi_k \begin{bmatrix} s_k \\ s_{0|k} \end{bmatrix}, \quad \Xi_k = \begin{bmatrix} I_2 & 0 \\ \frac{1}{t_k} I_2 & -\frac{1}{t_k} I_2 \end{bmatrix} \tag{12}$$

By applying eq.(12) in eq.(7), it is possible to obtain a state space that models at time k, the position of the previous DP and the current position of the target at time k.

$$\begin{bmatrix} s_{k+1} \\ s_{0|k+1} \end{bmatrix} = \Phi_{k+1} \begin{bmatrix} s_k \\ s_{0|k} \end{bmatrix}, \quad \Phi_{k+1} = \Xi_{k+1}^{-1} A_{k+1} \Xi_k \tag{13}$$

Thus, one can see that for the DP model the state space is now defined as:

$$X_k = \begin{bmatrix} s_k^T & s_{0|k}^T \end{bmatrix}^T \tag{14}$$

while the transition matrix is given by:

$$\Phi_k = \begin{bmatrix} \phi_k I_2 & (1-\phi_k) I_2 \\ 0 & I_2 \end{bmatrix}, \quad \phi_k = \frac{t_k}{t_{k-1}} \tag{15}$$

Note that if the last DP has occurred at time t_i with $t_i < t_{k-1}$, the quantity ϕ_k shall be replaced by $\phi_k = (t_k - t_i)/(t_{k-1} - t_i)$ and $s_{0|k}$ defined as $s_{i|k}$.

Concerning the covariance of the uncertainty in the movement we simply define a diagonal matrix with an exponential decay factor.

$$Q_k = \delta_k T_k \sigma_s^2 I_4 \tag{16}$$

where, δ_k, similarly to the CV model, is obtained from Section III-D.

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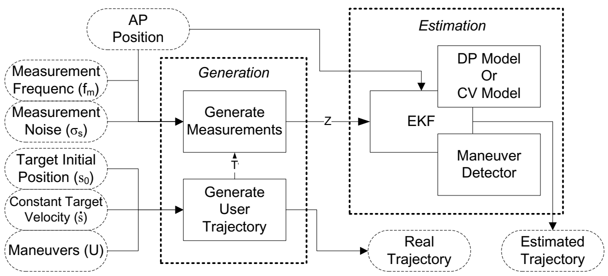

Fig. 4. Block diagram of the simulation framework.

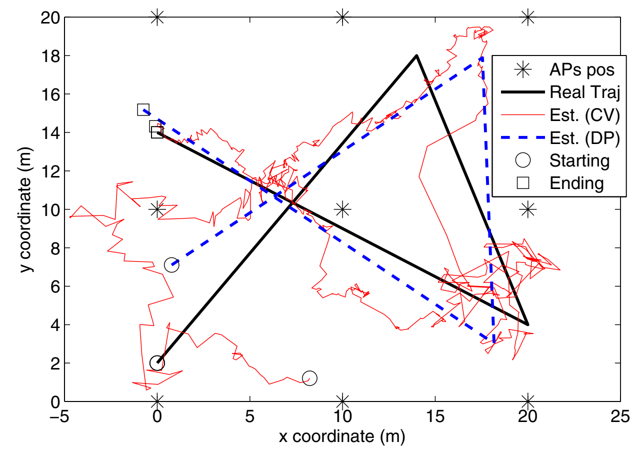

Fig. 5. Example of a maneuvering trajectory estimation for a single run.

B. Transformation of Movement Moments

Similarly to the CV model, in the present case, the filtering method is an EKF. Based on DP model just presented in Section IV-A, we can see that the state space is composed by the previous DP and the location of the target at time k. For this reason, when a change of direction (maneuver) happens, a new DP is created and thus the state space has to be manipulated. Additionally, the covariance matrix P has to be redesigned as well. As we can see in Fig. 3, this manipulation is made in the block "Transformation Model". Assuming that at time $t_i = t_k$, a maneuver is detected, a new DP is added to the system and the estimated moments are changed such that:

$$X_k = \begin{bmatrix} 0 & 0 \\ I_2 & 0 \end{bmatrix} X_{k-1} + \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} X_k \tag{17}$$

$$P_k = \begin{bmatrix} 0 & 0 \\ I_2 & 0 \end{bmatrix} P_{k-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} \tag{18}$$

Regarding the uncertainty in the user mobility, the transformation is:

$$Q_k = \begin{bmatrix} 0 & 0 \\ I_2 & 0 \end{bmatrix} Q_{k-1} \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \delta_{\text{high}} T_k \sigma_s^2 I_2 & 0 \\ 0 & 0 \end{bmatrix} \tag{19}$$

When a maneuver is detected, the knowledge of the second moment (i.e. covariance) of the current position is erased. This is because at time t_i when the maneuver is detected, we do not know the direction chosen by the target.

It is important to mention here that the final trajectory estimation will be given by $s_{i|l}$ $(i < l)$, where $s_{i|l}$ is the drifting point estimated at time t_i, given the time until the subsequent drifting point at time t_l. To determine the position of the target at different times than t_i, it is only necessary to interpolate the several estimated drifting points. This means that the trajectory is uniquely defined by the points where maneuvers were estimated.

V. Performance Comparison

For performance comparison we have developed a simulation framework in MATLAB, in which the estimated target trajectories of the DP model and CV model were compared in terms of Root Mean Square Error (RMSE).

A. The Simulation Framework

The simulation framework was designed according to the structure shown in Fig. 4. The generation block is responsible for generating the measurements Z_k at deterministic frequency $f_m \equiv 1/T_k$ according to the previously defined models. Additionally, it is responsible for determining the real position of the target at the times $\{t_k\}$ equivalent to the timestamps of the measurements. Though the maneuvers U are set deterministically in the generation block, they are not known at the estimation block. The estimation block is responsible for running the EKF coupled with the maneuver detector and one of the movement models (CV or DP). The APs were placed in a grid with edges length of 10 m. The remaining parameters used in the generation block are stated in Table I and the ones used in the estimation block are stated in Table II. For trajectory estimation, the parameters α, β and σ_Z are assumed known. In practice these values could be determined in a pre-calibration phase.

B. Accuracy Results

For comparing both models we have started by considering a straight trajectory as the simplest case. The measurements were simulated according to the simulation framework previously presented. We have first compared the estimation result for a single run of the two models, in order to have practical insight regarding the behavior of each model. We saw that the DP model clearly outperformed the CV model. For evaluating a more complex trajectory with actual maneuvers, we have defined a second trajectory, named cross trajectory. For a single

TABLE I

Generation step parameters for the simulation framework

α	β	σ_Z	f_m	s_0	$\|\dot{s}_0\|$
–50 dBm	2.5	6 dBm	15 Hz	See Fig. 5	1.4 m/s

TABLE II

Parameters for the estimation algorithms. The settings of α, β and σ_Z are the same as in Table I

σ_s	$\sigma_{\ddot{s}}$	δ_{high}	δ_{low}	ς	Γ	γ
12 m	5 m/s^2	1	0	0.95	95 %	0.9
EKF/DP	EKF/CV	Noise Adaptation			Maneuver	

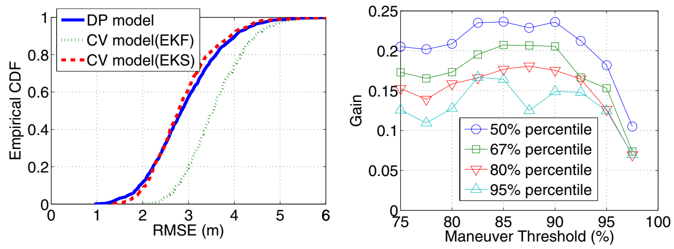

Fig. 6. Empirical CDF of the RMSE of both models (left) and the gain of the DP versus the CV model (right). The results regard 1000 runs of the estimation algorithms performed for the cross trajectory depicted in Fig. 5.

run, the results are shown in Fig. 5. We can see that the trajectory estimated using the DP model is considerably less noisier and it shows a smaller error than the one produced by the CV model. Additionally, in Fig. 5 we can also see that, while the initial guess in the CV model has a clear impact in the final trajectory estimation, for the DP model, that influence is dramatically reduced.

After analyzing performance for a single run, 1000 different runs of the same simulation were executed in order to evaluate the statistical behavior of the RMSE metric. The RMSE metric is calculated upon the timewise pair of real and estimated position of the user. After plotting the empirical CDF of the RMSE in Fig. 6 (left) we have confirmed that for the tested scenarios, the performance is considerably higher for the DP model than for the CV model. In Fig. 6 (left), results for an Extended Kalman Smoother (EKS) are also presented as a comparison case where tracking is performed in an offline manner and all data is used for estimating each point of the trajectory. A great advantage of the DP model with EKF upon the CV model with the EKS is that computational complexity is considerably lower, i.e. linear in contrast to quadratic.

To study the influence of the threshold level, we plot the threshold level vs. the gain of the DP model with respect to the CV model. The gain is calculated as $(z_h^{dp} - z_h^{cv})/z_h^{cv}$, where z_h^{dp} (z_h^{cv}) is the inverse of the empirical RMSE cumulative density function at $h\%$ for the DP (the CV) model. As we can see in Fig. 6 (right), the DP model outperforms the CV model for several different threshold levels. We can see that there is a trend for lower gains, the higher the maneuver threshold is. This is because maneuvers do exist, so when the threshold is increased above a certain level, the number of detected maneuvers decreases and from some threshold value onwards, a single segment results from the estimation. On the other hand, for the straight trajectory, we have seen an inverse trend.

To understand performance beyond segment-wise linear trajectories, the two models were applied to a circle and a zigzag trajectory. The results have shown that the DP model outperformed the CV model. In this case, curves were estimated as sequences of linear segments.

VI. Conclusion

Regarding wireless positioning, this paper has studied segment-wise linear patterns to model user mobility. Since current positioning/tracking solutions hardly handle this segment-wise linear approach, we have introduced the Drifting Points model. The proposed Drifting Points solution, based on a Bayesian filter and a maneuver detector, assumes that the user moves in a segment-wise linear fashion and that noise is only existent in the positions where maneuvers occur. Contrarily to traditional solution, which estimate the user kinetics at each time step, the DP model estimates segments of movement as a whole. The solution presents an increased performance of about 20% or 30% according to the RMSE metric. Additionally, the computational complexity of the DP model equals the one of the CV model, however, it cannot be used in real-time estimation since the final estimation of the DP model is based on all observations in a segment. One drawback is that the model requires a transformation of the state space and covariance matrix, what provokes a discontinuity in the propagation of the measurement history when maneuvers are detected.

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