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Estimation of Stator Winding Faults in Induction Motors Using an Adaptive Observer Scheme

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***Abstract*— This paper addresses the subject of inter-turn short circuit estimation in the stator of an induction motor. In the paper an adaptive observer scheme is proposed. The proposed observer is capable of simultaneously estimating the speed of the motor, the amount turns involved in the short circuit and an expression of the current in the short circuit. Moreover the states of the motor are estimated, meaning that the magnetizing currents are made available even though a fault has happened in the motor. To be able to develop this observer, a model particular suitable for the chosen observer design, is also derived. The efficiency of the proposed observer is demonstrated by tests performed on a test setup with a customized designed induction motor. With this motor it is possible to simulate inter-turn short circuit faults.**

I. INTRODUCTION

Stator faults are according to [1] the most common electrical faults in electrical motors. Moreover according to [2] most of these faults start as an inter-turn short circuit in one of the stator coils. The increased heat due to this short circuit will eventually cause turn to turn and turn to ground faults and finally lead to a break down of the stator.

The inter-turn short circuits are caused by several different influences on the stator. For example mechanical stress during assembling or during operation can create scratches in the insulation and cause short circuits. Especially if the motor is placed in a moist environment. Moisture can cause flow of current from scratch to scratch, which can make a hot spot and thereby destroy the insulation. Partial discharges due to very high alternating voltage between turns, when the stator is supplied by a PWM voltage source, can also over time degrade the insulation and cause a short circuit.

In the literature different approaches are proposed for detection of inter-turn faults. In [3] the stator currents are transformed using the Park transformation. Second order harmonics in the length of the transformed current vector is then used for fault detection. In [4] oscillations in the voltage between the line neutral and the star point of the motor is used as a fault indicator. This is also shown in [5] using a model of a faulty motor. In [6] estimation of the negative impedance of the motor is used as a fault indicator, and in [7] the negative sequence current is used for the same purpose. In [8] high frequency voltage injection in the supply voltage is utilized to create a response on the motor current. This response contains information of the inter-turn short circuit fault.

In this paper a model-based approach is proposed. The proposed approach is based on a model of the induction motor including an inter-turn fault in the stator. Different approaches for modeling inter-turn short circuits in the stator windings are found in the literature. In [9] a higher order model is used. This model is an extension of the model presented in [10]. This type of model is used for simulating higher order effects in the motor, but the obtained model is of high order. The inter-turn short circuit fault has it main harmonics in the lower frequency range. Therefore observers designed on the basis of this model will be of unnecessary high order for this kind of fault.

In [11] a steady state model of both inter-turn and turn-turn faults in an induction motor is developed using a low order model. In [5] a transient model of the same order as the one presented in [11] is developed. This model is similar to the one used in the observer design presented in this paper.

In this paper an adaptive observer is proposed for estimation of the inter-turn short circuit fault. Theoretical considerations on adaptive observers can for example be found in [12], [13]. The proposed observer is capable of simultaneously estimating the speed of the motor, the amount of turns involved in the short circuit, and an expression of the current in the short circuit. The observer is based on a model, developed particular for this purpose. This model is similar to the model described in [5]. As the proposed observer estimates the impact of the inter-turn short circuit on the induction motor, it can be used for fault robust control of the motor. Thereby it is possible to obtain control in the case of a inter-turn short circuit, meaning that it is possible to control the process, driven by the motor, to a fail-safe mode.

As a model based approach for fault estimation is proposed in this paper, the paper starts by deriving a model of the induction motor with an inter-turn short circuit in section III. This model is in section IV used in the design of the proposed adaptive observer. In section V test results from tests on a customized designed motor are presented. Finally concluding remarks ends the paper.

II. Nomenclature

In this paper large bold letters denote matrices. Small bold letters denotes matrices in the motor model when described in abc-coordinats and vectors respectively. The parameters in the model presented in section III is decribed in the following.

- $\mathbf{v}_{sabc}$ The terminal voltage of three phase induction motor, $\mathbf{v}_{sabc} = (v_{sa} \;\; v_{sb} \;\; v_{sc})^T$.
- $\mathbf{i}_{sabc}$ The current at the terminals of the three phase induction motor, $\mathbf{i}_{sabc} = (i_{sa} \;\; i_{sb} \;\; i_{sc})^T$.
- $\boldsymbol{\psi}_{sabc}$ The flux linkage in the stator phases of the induction motor, $\boldsymbol{\psi}_{sabc} = (\psi_{sa} \;\; \psi_{sb} \;\; \psi_{sc})^T$.
- $\mathbf{i}_{rabc}$ The current in the three equivalent phases of the rotor circuit in the induction motor, $\mathbf{i}_{rabc} = (i_{ra} \;\; i_{rb} \;\; i_{rc})^T$.
- $\boldsymbol{\psi}_{rabc}$ The flux linkage in the three equivalent phases of the rotor circuit in the induction motor, $\boldsymbol{\psi}_{rabc} = (\psi_{ra} \;\; \psi_{rb} \;\; \psi_{rc})^T$.
- i_f The current in the short circuits of the stator.
- $\mathbf{x}_{sdq0}$ The transformed stator variable vectors of the induction motor, e.i. $\mathbf{x}_{sdq0} = \mathbf{T}_{dq0}\mathbf{x}_{sabc}$, where $\mathbf{x}_{sdq0} = (x_{sd} \;\; x_{sq} \;\; x_{s0})^T$.
- $\mathbf{x}_{sdq0}$ The transformed rotor variable vectors of the induction motor, e.i. $\mathbf{x}_{rdq0} = \mathbf{T}_{dq0}(\theta)\mathbf{x}_{rabc}$, where $\mathbf{x}_{sdq0} = (x_{rd} \;\; x_{rq} \;\; x_{r0})^T$.
- $\mathbf{T}_{dq0}(\theta)$ A transformation matrix given by $\mathbf{T}_{dq0}(\theta) = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos(\theta + \frac{2}{3\pi}) & \cos(\theta + \frac{4}{3\pi}) \\ \sin(\theta) & \sin(\theta + \frac{2}{3\pi}) & \sin(\theta + \frac{4}{3\pi}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$.
- $\mathbf{T}_{dq0}$ A transformation matrix given by $\mathbf{T}_{dq0} = \mathbf{T}_{dq0}(0)$.

III. Mathematical Model of Short Circuits in the Stator of an Induction Motor

An inter-turn short circuit denotes a short circuit between two windings in the same phase of the stator, see Fig. 1. Here the electrical circuit of an $\mathbf{Y}$-connected stator is shown.

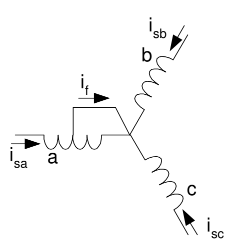


Fig. 1. Simplified electrical diagram of a three phase $\mathbf{Y}$-connected stator with an inter-turn short circuit in phase a.

In this figure a short circuit between the star point of the motor and an arbitrary point of the coil is shown. This seems as a rather limited model assumption, but if the electrical circuit is assumed linear, all short circuits in the given coil can be represented as a short circuit connected to the star point with the same amount of turns as in the real case.

In the following, a model of an induction motor, including an inter-turn short circuit in phase a, is developed. The model is developed under the assumption that the short circuit does not affect the overall angular position of the coil in the motor.

A. The Induction Motor Model in abc-Coordinats

Setting up the mesh equations for the circuit in figure 1 and rearranging these equations, a model describing a motor with one short circuit in phase a is found. Using the matrix notation presented in [14] this model is given by the following set of equations,

$$\mathbf{v}_{sabc} = \mathbf{r}_s(\mathbf{i}_{sabc} - \boldsymbol{\gamma} i_f) + \frac{d\boldsymbol{\psi}_{sabc}}{dt} + \mathbf{1}v_0 \qquad (1)$$

$$0 = \mathbf{r}_r\mathbf{i}_{rabc} + \frac{d\boldsymbol{\psi}_{rabc}}{dt} \qquad (2)$$

$$\boldsymbol{\psi}_{sabc} = \mathbf{l}_s(\mathbf{i}_{sabc} - \boldsymbol{\gamma} i_f) + \mathbf{l}_m(\theta)i_{rabc} \qquad (3)$$

$$\boldsymbol{\psi}_{rabc} = \mathbf{l}_r\mathbf{i}_{rabc} + \mathbf{l}_m(\theta)(\mathbf{i}_{sabc} - \boldsymbol{\gamma} i_f) \qquad (4)$$

$$l_f\frac{di_f}{dt} = -r_f i_f + \boldsymbol{\gamma}^T\mathbf{v}_{sabc} \qquad (5)$$

where (1) and (3) describe the currents and the flux linkages in each stator phase and (2) and (4) describe the currents and the flux linkages in each rotor phase. Finally (5) describes the current in the short circuit. In (1) $\mathbf{v}_{sabc}$ is the terminal voltage and v_0 is the star point voltage. The matrices $\mathbf{r}_s$, $\mathbf{r}_r$, $\mathbf{l}_s$ and $\mathbf{l}_r$ are defined by,

$$\begin{aligned} \mathbf{r}_s &= r_s\mathbf{I} & \mathbf{r}_r &= r_r\mathbf{I} \\ \mathbf{l}_s &= l_{ls}\mathbf{I} + \mathbf{l}_m(0) & \mathbf{l}_r &= l_{lr}\mathbf{I} + \mathbf{l}_m(0) \end{aligned}$$

where r_s and l_{ls} are the resistance and the leakage inductance in the stator windings respectively, and r_r and l_{lr} are the resistance and leakage inductance in the rotor windings respectively. $\mathbf{I}$ is the identity matrix and finally $\mathbf{l}_m$ is the mutual inductance and is given by,

$$\mathbf{l}_m(\theta) = l_m \begin{bmatrix} \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{4\pi}{3}) \\ \cos(\theta + \frac{4\pi}{3}) & \cos(\theta) & \cos(\theta + \frac{2\pi}{3}) \\ \cos(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{4\pi}{3}) & \cos(\theta) \end{bmatrix} \qquad (6)$$

where l_m is a constant and θ is the angle between the stator and rotor phases.

The vector $\boldsymbol{\gamma}$ in (1) to (5) represents the position and the amount of turns in the short circuit. The vector is, in the case of a short circuit in phase a, given by,

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_a & 0 & 0 \end{bmatrix}^T$$

where γ_a is the amount of turns involved in the short circuit. The inductor and the resistor in (5) are given by,

$$l_f = \gamma_a(1 - \gamma_a)l_{ls} \qquad r_f = \gamma_a(1 - \gamma_a)r_s + r_i$$

where r_s is the stator resistance, l_{ls} is the leakage inductance of the stator and r_i is the resistance in the insulation break. $r_i = \infty$ means that no short circuit has occurred and $r_i \neq \infty$ means that some leakage current is flowing. The evolution from $r_i = \infty$ to $r_i = 0$ is very fast in most insulating materials, meaning the value r_i can be assumed to either equal ∞ or 0.

B. Transformation to a Stator fixed $dq0$-frame

Using the $dq0$-transformation $\mathbf{T}_{dq0}(\theta)$ presented in [14] the model described in section III-A is transformed into $dq0$-coordinats fixed to the stator. Doing this the following model is obtained,

$$\mathbf{v}_{sdq0} = \mathbf{R}_s(\mathbf{i}_{sdq0} - \mathbf{T}_{dq0}\boldsymbol{\gamma} i_f) + \frac{d\boldsymbol{\psi}_{sdq0}}{dt} + \mathbf{v}_0 \tag{7}$$

$$0 = \mathbf{R}_r \mathbf{i}_{rdq0} + \frac{d\boldsymbol{\psi}_{rdq0}}{dt} - z_p\omega_r \mathbf{J}\boldsymbol{\psi}_{rdq0} \tag{8}$$

$$\boldsymbol{\psi}_{sdq0} = \mathbf{L}_s(\mathbf{i}_{sdq0} - \mathbf{T}_{dq0}\boldsymbol{\gamma} i_f) + \mathbf{L}_m \mathbf{i}_{rdq0} \tag{9}$$

$$\boldsymbol{\psi}_{rdq0} = \mathbf{L}_r \mathbf{i}_{rdq0} + \mathbf{L}_m(\mathbf{i}_{sdq0} - \mathbf{T}_{dq0}\boldsymbol{\gamma} i_f) \tag{10}$$

$$l_f \frac{di_f}{dt} = -r_f i_f + \boldsymbol{\gamma}^T \mathbf{T}_{dq0}^{-1}(\mathbf{v}_{sdq0} - \mathbf{v}_0) \tag{11}$$

where $\mathbf{v}_{sdq0}$ is the terminal voltage and $\mathbf{v}_0 = \begin{bmatrix} 0 & 0 & v_0 \end{bmatrix}^T$. Using the $dq0$-transformation all matrices in the model have a diagonal structure i.e. they are given by,

$$\mathbf{R}_s = \text{diag}\{r_s,\ r_s,\ r_s\} \qquad \mathbf{R}_r = \text{diag}\{r_r,\ r_r,\ r_r\}$$
$$\mathbf{L}_s = \text{diag}\{\tfrac{3}{2}l_m + l_{ls},\ \tfrac{3}{2}l_m + l_{ls},\ l_{ls}\}$$
$$\mathbf{L}_r = \text{diag}\{\tfrac{3}{2}l_m + l_{lr},\ \tfrac{3}{2}l_m + l_{lr},\ l_{lr}\}$$
$$\mathbf{L}_m = \text{diag}\{\tfrac{3}{2}l_m,\ \tfrac{3}{2}l_m,\ 0\}$$
$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From (7) to (11) it is seen that it is convenient to define a new current vector $\mathbf{i}'_{sdq0} = \mathbf{i}_{sdq0} - \mathbf{T}_{dq0}\boldsymbol{\gamma} i_f$. This current equals the amount of the stator current, which generates air gab flux. Rewriting (7) and (9) and introducing the current $\mathbf{i}'_{sdq0}$ the stator model becomes,

$$\mathbf{v}_{sdq} = \mathbf{R}_s \mathbf{i}'_{sdq} + \mathbf{L}_s \frac{d\mathbf{i}'_{sdq}}{dt} + \mathbf{L}_m \frac{d\mathbf{i}_{rdq}}{dt} + \mathbf{v}_0$$

$$v_{s0} = r_s i'_{s0} + l_{ls}\frac{di'_{s0}}{dt} + v_0 \tag{12}$$

$$\mathbf{i}_{sdq} = \mathbf{i}'_{sdq} + \mathbf{T}_{dq}\boldsymbol{\gamma} i_f$$

$$0 = i'_{s0} + \frac{1}{3}\gamma_a i_f \tag{13}$$

Here the two equations are divided into four equations, where vectors with subscribe dq and variables with subscribe 0 contains respectively the two first components and the last component of vectors with subscribe $dq0$.

In (7) and (9) the current $\mathbf{i}_{sdq0}$ is the current in the terminals of the motor. Therefore the current $i_{s0} = 0$ in the case of a $\mathbf{Y}$-connected stator. This is utilized to obtain (13).

The last row of the voltage $\mathbf{v}_{sdq0} - \mathbf{v}_0$ in (11) equals $v_{s0} - v_0$ and is unknown because the star point voltage v_0 is assumed unknown. From (11) it is seen that the current in the short circuit depends upon this voltage. Therefore it is necessary to find an expression for it. Equation (12), describing the zero current in the stator, can be used for this. Including the expression of the voltage $v_{s0} - v_0$ obtained from (12) in (11) and using (13) to describe i'_{s0} the following expression is obtained,

$$L_f \frac{di_f}{dt} = -R_f i_f + \gamma_a v_{sd} \tag{14}$$

where L_f and R_f respectively are given by,

$$L_f = l_f + \frac{1}{3}\gamma_a^2 l_{ls} \qquad R_f = r_f + \frac{1}{3}\gamma_a^2 r_s$$

Equation (8) and (10) describes the rotor circuit. Rewriting these expressions and using that $\mathbf{i}'_{sdq0} = \mathbf{i}_{sdq0} - \mathbf{T}_{dq0}\boldsymbol{\gamma} i_f$, the model of the rotor circuit becomes,

$$\frac{d\psi_{rdq}}{dt} = -\left(\mathbf{R}_r\mathbf{L}_r^{-1} - z_p\omega_r\mathbf{J}\right)\psi_{rdq} + \mathbf{R}_r\mathbf{L}_r^{-1}\mathbf{L}_m\mathbf{i}'_{sdq} \tag{15}$$

$$\frac{d\psi_{r0}}{dt} = -\frac{r_r}{l_r}\psi_{r0} \tag{16}$$

Equation (16) shows that $\lim_{t\to\infty}\psi_{r0} = 0$ despite of a short circuit in the stator. Moreover ψ_{r0} does not appear in the remaining model equations, i.e. it is not necessary to include (16) in the final model.

The final model describing the electrical part of the induction motor is given by (12), (15) and (14). These equations respectively describe the stator circuit, the rotor circuit, and the short circuit. Defining the magnetizing current $\mathbf{i}_{mdq}$ such that it fulfills the equation $\psi_{rdq} = \mathbf{L}_m\mathbf{i}_{mdq}$ the model of the induction motor with a short circuit in the stator becomes,

$$\mathbf{L}'_s\frac{d\mathbf{i}'_{sdq}}{dt} = -\left(\mathbf{R}_s + \mathbf{R}'_r\right)\mathbf{i}'_{sdq} + \left(\mathbf{R}'_r - z_p\omega_r\mathbf{J}\mathbf{L}'_m\right)\mathbf{i}_{mdq} + \mathbf{v}_{sdq}$$

$$\mathbf{L}'_m\frac{d\mathbf{i}_{mdq}}{dt} = \mathbf{R}'_r\mathbf{i}'_{sdq} - \left(\mathbf{R}'_r - z_p\omega_r\mathbf{J}\mathbf{L}'_m\right)\mathbf{i}_{mdq}$$

$$L_f\frac{di_f}{dt} = -R_f i_f + \gamma_a v_{sd}$$

where the current measurable at the terminals of the motor is given by $\mathbf{i}_{sdq} = \mathbf{i}'_{sdq} + \mathbf{T}_{dq}\boldsymbol{\gamma} i_f$. In these equations the matrices $\mathbf{R}'_r$, $\mathbf{L}'_s$ and $\mathbf{L}'_m$ are given by,

$$\mathbf{R}'_r = \mathbf{L}_m\mathbf{L}_r^{-1}\mathbf{R}_r\mathbf{L}_r^{-1}\mathbf{L}_m$$
$$\mathbf{L}'_s = \mathbf{L}_s - \mathbf{L}_m\mathbf{L}_r^{-1}\mathbf{L}_m \qquad \mathbf{L}'_m = \mathbf{L}_m\mathbf{L}_r^{-1}\mathbf{L}_m$$

meaning that the new matrices retain the diagonal structure.

IV. An Adaptive Observer for Inter-turn Fault Detection

In the previous section a model of an induction motor with an inter-turn short circuit is developed. This model will in this section be used in the development of an adaptive observer. This observer is based on only the electrical quantities available at the terminals of the motor.

The model developed in the previous section can be put on matrix form resulting in the following description,

$$\begin{aligned}\dot{\mathbf{x}} &= (\mathbf{A}_0 + \omega_r\mathbf{A}_{\omega_r})\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned} \tag{17}$$

where,

$$\mathbf{x} = \begin{bmatrix} \mathbf{i}'^{\,T}_{sdq} & \boldsymbol{i}^T_{mdq} & i_f \end{bmatrix}^T \qquad \mathbf{u} = \mathbf{v}_{sdq} \qquad \mathbf{y} = \mathbf{i}_{sdq} \tag{18}$$

The matrices in this model are given by,

$$\mathbf{A}_0 = \begin{bmatrix} -\mathbf{L}'^{-1}_s(\mathbf{R}_s + \mathbf{R}'_r) & \mathbf{L}'^{-1}_s\mathbf{R}'_r & 0 \\ \mathbf{L}_m'^{-1}\mathbf{R}'_r & -\mathbf{L}_m'^{-1}\mathbf{R}'_r & 0 \\ 0 & 0 & -\frac{R_f}{L_f} \end{bmatrix}$$

$$\mathbf{A}_{\omega_r} = \begin{bmatrix} 0 & -z_p\mathbf{J}\mathbf{L}_s'^{-1}\mathbf{L}_m' & 0 \\ 0 & z_p\mathbf{J} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \mathbf{L}_s'^{-1} \\ 0 \\ \begin{bmatrix} \frac{\gamma_a}{L_f} & 0 \end{bmatrix} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{I} & 0 & \mathbf{T}_{dq}\boldsymbol{\gamma} \end{bmatrix}$$

All parameters of the matrices are known except for the vector $\boldsymbol{\gamma}$ and the parameters R_f and L_f. The last two appear in the matrices in a fraction, which can be rewriten as shown below,

$$\frac{R_f}{L_f} = \frac{r_f + \frac{1}{3}\gamma_a^2 r_s}{l_f + \frac{1}{3}\gamma_a^2 l_{ls}} = \frac{\gamma_a(1-\frac{2}{3}\gamma_a)r_s + r_i}{\gamma_a(1-\frac{2}{3}\gamma_a)l_{ls}}$$

If it is assumed that $r_i = 0$ this fraction equals $\frac{r_s}{l_{ls}}$, meaning that the fraction is known. r_i is the resistance of the insulation in the short circuit. In section III-A it is argued that this resistance almost always is either ∞ or 0. Therefore the assumption that $r_i = 0$ is almost always true if a short circuit has occurred.

Defining a linear state transformation $\mathbf{x} = \mathbf{T}\mathbf{z}$ as shown below,

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & 0 & \begin{bmatrix} -1 \\ 0 \end{bmatrix} \\ -\mathbf{L}_m'^{-1}\mathbf{L}_s' & \mathbf{I} & 0 \\ 0 & 0 & \frac{3}{2\gamma_a} \end{bmatrix} \tag{19}$$

and defining a fault signal f as

$$f = \frac{\gamma_a}{1-\frac{2}{3}\gamma_a} \tag{20}$$

the system described by (17) is transformed into a bilinear system on the form,

$$\begin{aligned} \dot{\mathbf{z}} &= (\mathbf{A}_0 + \omega_r\mathbf{A}_{\omega_r} + v_{sd}\mathbf{A}_{v_{sd}})\mathbf{z} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{z} \end{aligned} \tag{21}$$

where the state vector is extended with a state describing the fault signal, e.i. $\mathbf{z} = \begin{bmatrix} (\mathbf{T}^{-1}\mathbf{x})^T & f \end{bmatrix}^T$. The matrices in (21) are given by,

$$\mathbf{A}_0 = \begin{bmatrix} \mathbf{A}_{0,11} & \mathbf{L}_s'^{-1}\mathbf{R}_r' & \begin{bmatrix} \frac{R_s+R_r'}{L_s'} - \frac{r_s}{l_{ls}} \\ 0 \end{bmatrix} & 0 \\ \mathbf{L}_m'^{-1}\mathbf{R}_s & 0 & \begin{bmatrix} \frac{R_s}{L_m'} - \frac{r_s}{l_{ls}} \\ 0 \end{bmatrix} & 0 \\ 0 & 0 & -\frac{r_s}{l_{ls}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_{\omega_r} = \begin{bmatrix} z_p\mathbf{J} & -z_p\mathbf{J}\mathbf{L}_s'^{-1}\mathbf{L}_m' & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_{v_{sd}} = \begin{bmatrix} 0 & 0 & 0 & \begin{bmatrix} \frac{2}{3l_{ls}} \\ 0 \end{bmatrix} \\ 0 & 0 & 0 & \begin{bmatrix} \frac{2(l_m+l_{ls})}{3L_m l_{ls}} \\ 0 \end{bmatrix} \\ 0 & 0 & 0 & \frac{2}{3l_{ls}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} \mathbf{L}_s'^{-1} \\ \mathbf{L}_m'^{-1} \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 \end{bmatrix}$$

where $\mathbf{A}_{0,11} = -\mathbf{L}_s'^{-1}(\mathbf{R}_s + \mathbf{R}_r') - \mathbf{L}_m'^{-1}\mathbf{R}_r'$ in matrix $\mathbf{A}_0$.

If the speed is assumed constant, this system is a bilinear system with one unknown but constant parameter, namely the speed. For such a system an adaptive observer can be used for simultaneous estimation of the states and the unknown parameter. This is possible if the system fulfills some demands presented in definition 1 below.

In [12] a definition of a system on nonlinear adaptive form is given. The corresponding definition for the bilinear case is presented below,

Definition 1 *A system on the form*

$$\dot{\mathbf{z}} = \mathbf{A}(\mathbf{u}, \boldsymbol{\theta})\mathbf{z} + \mathbf{B}\mathbf{u} \tag{22}$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{y} & \boldsymbol{\zeta} \end{bmatrix}$ *and* $\mathbf{y}$ *is the measured output, is said to be on bilinear adaptive form if,*

- $\mathbf{A}(\mathbf{u}, \boldsymbol{\theta})$ *is bounded for all* $\boldsymbol{\theta} \in \mathcal{D}_\theta$ *and* $\mathbf{u} \in \mathcal{U}$*, where* $\mathcal{D}_\theta$ *is the parameter space and* $\mathbf{u} \in \mathcal{U}$ *is the input space.*
- *the set* $(\mathbf{A}(\mathbf{u}, \boldsymbol{\theta}), \mathbf{C})$ *is observable for every* $\boldsymbol{\theta} \in \mathcal{D}_\theta$ *and* $\mathbf{u} \in \mathcal{U}$*, where* $\mathbf{C} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix}$.
- $\mathbf{A}_{\theta_1}(\mathbf{u})$ *to* $\mathbf{A}_{\theta_n}(\mathbf{u})$ *are linear independent matrices for every* $\mathbf{u} \in \mathcal{U}$.

where,

$$\mathbf{A}(\boldsymbol{\theta}) = \mathbf{A}_0(\mathbf{u}) + \theta_1 \begin{bmatrix} \mathbf{A}_{\theta_1}(\mathbf{u}) \\ \mathbf{0} \end{bmatrix} + \cdots + \theta_n \begin{bmatrix} \mathbf{A}_{\theta_n}(\mathbf{u}) \\ \mathbf{0} \end{bmatrix}$$

Remark 1 *Definition 3.1 in [12] requires existence of a Lyapunov function and Lipschitz conditions on the nonlinearities. These are fulfilled in this case as the system is state affine.*

The system described by (21) fulfills definition 1 except for the observability condition when $v_{sd} = 0$. It can be argued that when the induction motor is running, the fraction of the time where $v_{sd} = 0$ is almost equal to zero. Therefore the system described by (21) is observable almost all the time and therefore fulfills definition 1 almost all the time.

For a system on the form defined by definition 1 an adaptive observer exists according to the following lemma.

Lemma 1 *For a system of the form defined in definition 1 an adaptive observer exists and has the following form,*

$$\dot{\hat{\mathbf{z}}} = \mathbf{A}(\mathbf{u}, \hat{\boldsymbol{\theta}})\hat{\mathbf{z}} + \mathbf{B}\mathbf{u} + \mathbf{K}(\mathbf{y} - \hat{\mathbf{y}}) \tag{23}$$

$$\dot{\hat{\theta}}_i = \kappa(\mathbf{y} - \hat{\mathbf{y}})^T\mathbf{P}_1\mathbf{A}_{\theta_i}(\mathbf{u})\hat{\mathbf{z}} \qquad \forall\, i \in \{1, \cdots, n\} \tag{24}$$

where $\mathbf{P}_1$ *is a positive definite matrix,* $\hat{\mathbf{z}} = \begin{bmatrix}\hat{\mathbf{y}} & \hat{\boldsymbol{\zeta}}\end{bmatrix}^T$ *and*

$$\mathbf{A}(\mathbf{u},\hat{\boldsymbol{\theta}}) = \mathbf{A}_0(\mathbf{u}) + \hat{\theta}_1 \begin{bmatrix}\mathbf{A}_{\theta_1}(\mathbf{u}) \\ \mathbf{0}\end{bmatrix} + \cdots + \hat{\theta}_n \begin{bmatrix}\mathbf{A}_{\theta_n}(\mathbf{u}) \\ \mathbf{0}\end{bmatrix}$$

If there exists a $\mathbf{K}(\mathbf{u})$ *stabilizing the system for everly* $\boldsymbol{\theta}$ *and* $\mathbf{u}$*, i.e. making the matrix* $\mathbf{A}(\mathbf{u},\boldsymbol{\theta}) - \mathbf{K}(\mathbf{u})\mathbf{C}$ *Hurwitz for every* $\boldsymbol{\theta} \in \mathcal{D}_\theta$ *and* $\mathbf{u} \in \mathcal{U}$*, where* $\mathbf{C} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$.

Proof of the lemma is given in appendix I. This lemma is a simplification of proposition 3.1 in [12].

Lemma 1 states that a $\mathbf{K}(\mathbf{u})$ must exist, which garanties that the real part of the eigenvalues of,

$$\mathbf{A}(\mathbf{u},\boldsymbol{\theta}) - \mathbf{K}(\mathbf{u})\mathbf{C} \tag{25}$$

are less than zero for all $\mathbf{u} \in \mathcal{U}$ and $\boldsymbol{\theta} \in \mathcal{D}_\theta$. In the induction motor case this is the same as saying that the eigenvalues must be less than zero for all possible values of v_{sd} and ω_r.

Choosing $\mathbf{K}(\mathbf{u}) = \mathbf{K}_0 + v_{sd}\mathbf{K}_1$ the expression in (25) will in the induction motor case become,

$$(\mathbf{A}_0 - \mathbf{K}_0\mathbf{C}) + \omega_r \mathbf{A}_{\omega_r} + v_{sd}(\mathbf{A}_{v_{sd}} - \mathbf{K}_1\mathbf{C})$$

A common way to treat a bilinear system is to use $\mathbf{K}_1$ for cancelation of the bilinear terms [15]. This is not possible in this case due to the structure of the matrices. Therefore, to avoid dependency of the sign of the voltage v_{sd}, $\mathbf{K}_1$ is chosen such that the eigenvalues of $\mathbf{A}_{v_{sd}} - \mathbf{K}_1\mathbf{C}$ is placed on the imaginary axis. Doing this the behaviour of the system is the same for positive and negative values of v_{sd}.

In Fig. 2 the locus of the roots for a given motor and a given choise of $\mathbf{K}_0$ and $\mathbf{K}_1$ is shown. The presented locuses are functions of speed at four different values of v_{sd}.

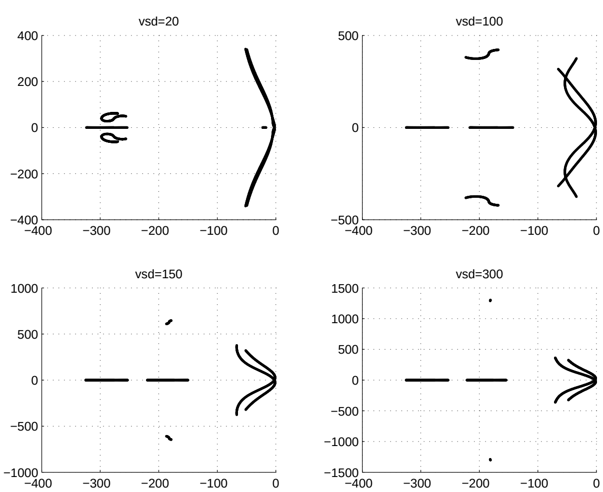


Fig. 2. Root locus plot of the observer eigenvalues for four different supply voltages v_{sd} and speed values from -400 to 400 $[rad/sec]$.

From Fig. 2 it is seen that in the presented cases the system is stable. Numerical analysis has shown that this is the case for all values of v_{sd} from -400 $[V]$ to 400 $[V]$ except for $v_{sd} = 0$.

Remark 2 *In the text above it is argued that the observer is stable for all values of the voltage* v_{sd} *between -400 and 400* $[V]$ *except for* $v_{sd} = 0$*, and all speeds between -400 and 400* $[rad/sec]$*. This is not the same as saying that it is possible to estimate the fault and speed at zero speed due to demands for persistence of excitation.*

Remark 3 *Fault tolerant control can be obtained using the current vector* $\mathbf{i}'_{sdq}$*, estimated by the proposed observer, as input to the current controllers. This current is the part of the stator current producing air gab flux. Therefore, by using this current the control is not affected by the short circuit.*

This current vector is given by the two first terms of the state vector $\mathbf{x}$ *in (18), which is calculated using* $\mathbf{x} = \mathbf{T}\mathbf{z}$*, where* $\mathbf{T}$ *is defined in (19).*

Remark 4 *According to [16] isolation between different faults can be obtained using a set of adaptive observers. This idea can, in the case of the induction motor, be used if three identical observers are designed, each detecting a stator winding fault in one of the three phases.*

V. Test Results

In this section the adaptive observer is tested on an induction motor setup where inter-turn stator faults can be simulated. The electrical circuit of the stator is shown in Fig. 3. The motor

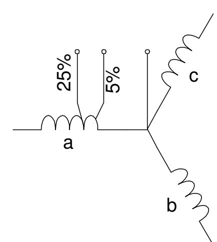
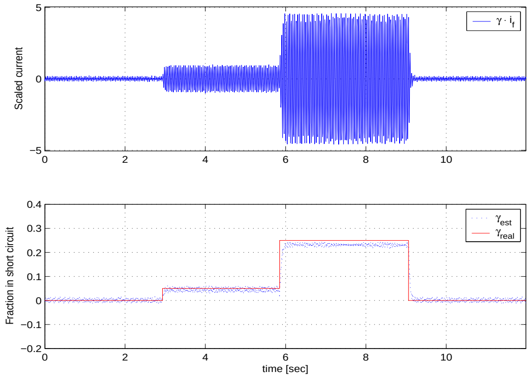


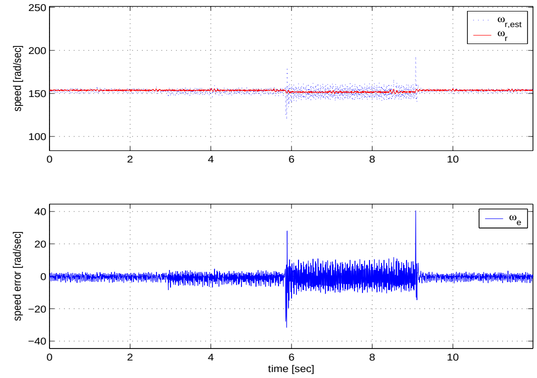
Fig. 3. The electrical circuit of the stator in the test setup. Two points of phase a of the stator and the star point are available at the terminal box.

used in the test is a 1.5 [KW] customized Grundfos motor, supplied by a Danfoss frequency converter. The speed, the three phase currents, and the three phase voltages are available at the test setup. The voltage to the motor is controlled using a linear voltage to frequency relation, with a voltage boost at low frequencies. All tests are preformed at supply frequencies between 10 and 30 $[Hz]$ to avoid too large short circuit currents and thereby burnout of the motor during the tests.

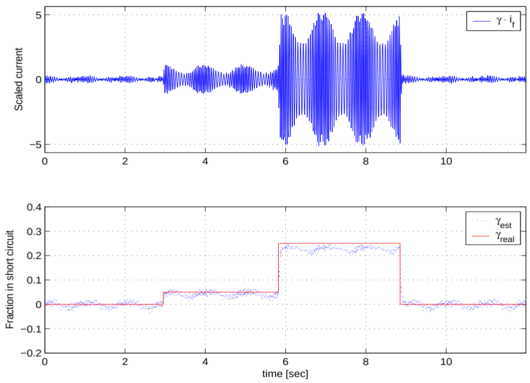
Three tests are performed, showing the estimation capability of the algorithm under three different conditions. In each of the tests the algorithm is tested with no short circuit, 5% of the windings short circuited, and 25% of the windings short circuited. In the first test the motor is running at constant speed with a supply frequency of 25 $[Hz]$. The results from this test are shown in Fig. 4(a) and 4(b). In the second test the supply frequency of the motor is changed each second between 15 and 30 $[Hz]$. The results from this test are shown in Fig. 4(c)



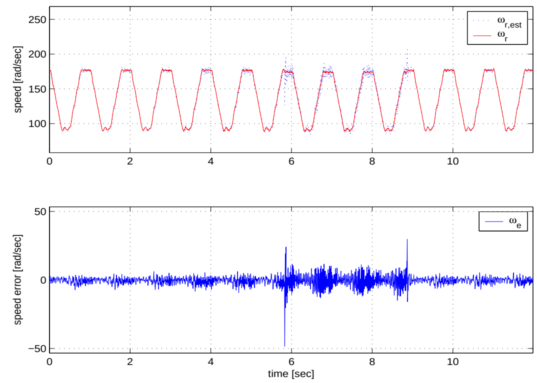
(a) The top figure shows the estimation of the scaled current $\gamma_a i_f$ and the bottom figure shows the estimated and real amount of windings affected by the short circuit.



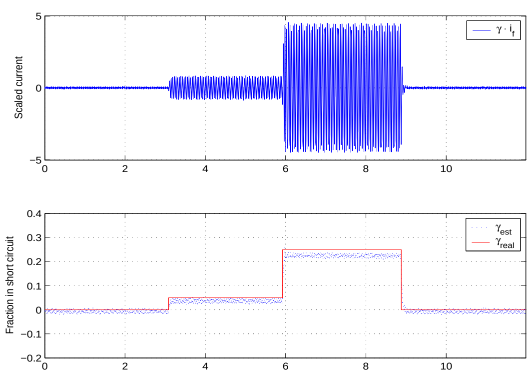
(b) The top figure shows the estimated and the measured speed and the bottom figure shows the error between the estimated and measured speed ω_e.



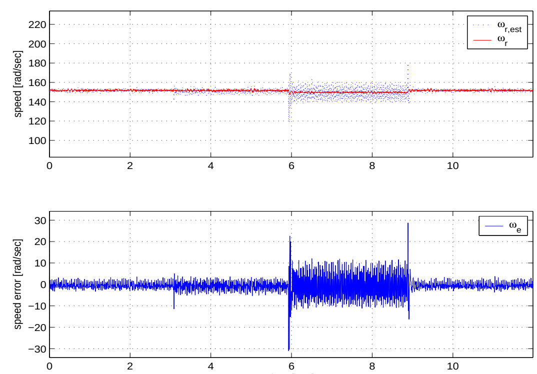
(c) The top figure shows the estimation of the scaled current $\gamma_a i_f$ and the bottom figure shows the estimated and real amount of windings affected by the short circuit.



(d) The top figure shows the estimated and the measured speed and the bottom figure shows the error between the estimated and measured speed ω_e.



(e) The top figure shows the estimation of the scaled current $\gamma_a i_f$ and the bottom figure shows the estimated and real amount of windings affected by the short circuit.



(f) The top figure shows the estimated and the measured speed and the bottom figure shows the error between the estimated and measured speed ω_e.

Fig. 4. The results form tests with the proposed algorithm. Figure 4(a) and 4(b) show results at constant speed and balanced supply voltage, figure 4(c) and 4(d) shows results with changing speed, and figure 4(e) and 4(f) shows results with unbalanced supply voltage.

and 4(d). In the last test the amplitude of voltage supplying phase a is decreased with 5%, meaning that the supply voltage is unbalanced. The results from this test are shown in Fig. 4(e) and 4(f).

All the tests have shown that the observer is stable. From the first test, presented in Fig. 4(a) and 4(b), it is seen that the speed is estimated without any bias. It is also seen that there is a bias on the estimated fraction of turns in the short circuit. This bias is partly due to noise on the measurements, and partly due to mismatch between the real motor parameters and the motor parameters used in the observer. This bias is repeated in each of the three tests.

Results from the second test, presented in 4(c) and 4(d), shows that the observer is capable of estimating the wanted quantaties despite of speed changes. Still it is seen that the speed changes affect the estimated amount of turns in the short circuit. This is because of the constant speed assumption used in the design. It is, however, still possible to use the estimate of the fault.

From the results of the last test, presented in 4(e) and 4(f), it is seen that an unbalanced supply of 5% is not affecting the performance of the observer.

VI. Conclusion

An adaptive observer for simultaneous estimation of the motor states, the speed, and the amount of turns in an inter-turn short circuit is proposed. The observer is tested on a customized designed induction motor. The tests have shown that the observer can estimate an inter-turn fault despite of speed changes and unbalanced supply conditions. This makes the estimation scheme usable in inverter feed induction motor drives, or in motor applications supplied by a bad grid.

As both the short circuit and the states of the motor are estimated, the proposed observer might be used for fault tolerant control. Meaning that torque control can be obtained despite of an inter-turn short circuit in the stator.

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Appendix I
Proof of Lemma 1

This appendix contains the proof of lemma 1. The proof is quite simple and is based on a Lyapunov analysis of the error equation of the observer. The error equation of the adaptive observer is given by,

$$\begin{aligned}\dot{\tilde{z}} &= \mathbf{A}(\boldsymbol{\theta},\mathbf{u})\mathbf{z} + \mathbf{B}\mathbf{u} - \left(\mathbf{A}(\hat{\boldsymbol{\theta}},\mathbf{u})\hat{\mathbf{z}} + \mathbf{B}\mathbf{u} + \mathbf{K}(\mathbf{u})(\mathbf{y}-\hat{\mathbf{y}})\right)\\ &= (\mathbf{A}(\boldsymbol{\theta},\mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C})\tilde{\mathbf{z}} + \mathbf{A}_{\boldsymbol{\theta}}(\tilde{\boldsymbol{\theta}},\mathbf{u})\hat{\mathbf{z}}\end{aligned} \quad (26)$$

where $\tilde{z} = \mathbf{z} - \hat{\mathbf{z}}$, $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ and,

$$\mathbf{A}_\theta(\tilde{\boldsymbol{\theta}},\mathbf{u}) = \sum_{i=1}^{n} \tilde{\theta}_i \begin{bmatrix}\mathbf{A}_{\theta_i}(\mathbf{u})\\ \mathbf{0}\end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}$$

The Lyapunov function presented below is used for the stability analysis of the error equation,

$$V = \tilde{\mathbf{z}}^T\mathbf{P}\tilde{\mathbf{z}} + \frac{1}{2\kappa}\sum_i \tilde{\theta}_i^2 > 0$$

where $\mathbf{P}$ is a positive definite matrix of the following form,

$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{P}_2\end{bmatrix}$$

The derivative of this Lyapunov function along the trajectory of the error system presented in equation 26 is given by,

$$\begin{aligned}\dot{V} =& \tilde{\mathbf{z}}^T\left((\mathbf{A}(\boldsymbol{\theta},\mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C})^T\mathbf{P} + \mathbf{P}(\mathbf{A}(\boldsymbol{\theta},\mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C})\right)\tilde{\mathbf{z}}\\ &+ \sum_i \tilde{\theta}_i\left(\tilde{\mathbf{z}}^T\mathbf{P}\begin{bmatrix}\mathbf{A}_{\theta_i}(\mathbf{u})\\ \mathbf{0}\end{bmatrix}\hat{\mathbf{z}} + \frac{1}{\kappa}\dot{\tilde{\theta}}_i\right)\end{aligned}$$

The adaptation law is given by setting the terms inside the sum equal to zero and using the assumption the $\boldsymbol{\theta}$ is constant meaning that $\dot{\tilde{\boldsymbol{\theta}}} = -\dot{\hat{\boldsymbol{\theta}}}$. From this the following adaptation law is obtained,

$$\dot{\hat{\theta}}_i = \kappa(\mathbf{y} - \hat{\mathbf{y}})^T \mathbf{P}_1 \mathbf{A}_{\theta_i}(\mathbf{u})\hat{\mathbf{z}} \qquad \forall i \in \{1, \cdots, n\}$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{y} & \boldsymbol{\zeta} \end{bmatrix}$ from definition 1 is used to interchange $\tilde{\mathbf{z}}$ and $\mathbf{y} - \hat{\mathbf{y}}$. Using the adaptation law, the derivative of the Lyapunov function along the trajectory of the error equation reduces to,

$$\dot{V} = \tilde{\mathbf{z}}^T \left((\mathbf{A}(\boldsymbol{\theta}, \mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C})^T \mathbf{P} + \mathbf{P}(\mathbf{A}(\boldsymbol{\theta}, \mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C}) \right) \tilde{\mathbf{z}}$$

which is smaller than zero when $\tilde{\mathbf{z}} \neq \mathbf{0}$ if,

$$\Re e\{\mathrm{eig}(\mathbf{A}(\boldsymbol{\theta}, \mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C})\} < 0 \qquad \forall\, \boldsymbol{\theta} \in \mathcal{D}_\theta, \mathbf{u} \in \mathcal{U}$$

where $\mathcal{D}_\theta$ and $\mathcal{U}$ are a bounded sets. or in other words the matrix $\mathbf{A}(\boldsymbol{\theta}, \mathbf{u}) - \mathbf{K}(\mathbf{u})\mathbf{C}$ is Hurwitz for all $\boldsymbol{\theta} \in \mathcal{D}_\theta$ and for all $\mathbf{u} \in \mathcal{U}$.