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Abstract

This paper introduces an adaptive growing and pruning radial basis function (GAP-RBF) neural network for on-line identification of hybrid systems. The main idea is to identify a global nonlinear model that can predict the continuous outputs of hybrid systems. In the proposed approach, GAP-RBF neural network uses a modified unscented kalman filter (UKF) with forgetting factor scheme as the required on-line learning algorithm. The effectiveness of the resulting identification approach is tested and evaluated on a simulated benchmark hybrid system.

1. Introduction

In recent years, the interest in hybrid systems has grown widely. Hybrid systems arise when the continuous dynamics, driven by physical laws, and discrete dynamics, derived by logical rules, coexist and interact with each other. Thus, hybrid models describe processes that evolve according to dynamic equations and logic rules. Most of the literature on hybrid systems has dealt with control, stability analysis, verification and fault detection based on the availability of a model for the hybrid system. Getting such a model from a given input-output data is an identification problem, which does not seem to have enough attention in the hybrid system research community.

The existing identification approaches for the hybrid systems can generally be classified into the variants of the Mixed-Integer Programming (MIP) approach [1], the clustering-based approach [2], the bounded-error approach [3], the Bayesian approach [4] and the algebraic approach [5].

[1] reformulates the identification problem into subclasses of PieceWise Affine (PWA) systems which require highly computational complexity of Mixed-Integer Linear or Quadratic Programming algorithms to be solved.

Other identification approaches rely on different model classes of PieceWise Auto-regressive eXogenous (PWARX) systems. PWARX models are suitable when dealing with input-output data, since they provide an input-output description of PWA systems. This model structure partitions the regressor space into polyhedral with ARX sub-models. Thus, it involves the estimation of the number of discrete modes, the parameters of the affine sub-models and the coefficients of the hyperplanes defining the partitions of the regressor set. However the simultaneous optimal estimation of all the quantities mentioned is a very hard and computationally intractable problem. Hence, the available approaches are heuristic and suboptimal based which in most cases either assume a fixed number of discrete modes, or adjust it iteratively. For instance, both the clustering-based approach and the Bayesian approach require a priori knowledge of the discrete modes number. Moreover, some approaches, such as the clustering-based approach, the bounded-error approach and the Bayesian approach require the ARX sub-models orders to be fixed. This can arise two significant limitations. First, all the existing identification algorithms concern only the classes of switched linear and PWA model while the actual hybrid systems are dynamically non-linear. Second, it can be practically unacceptable to assume a priori knowledge about the hybrid system dynamics such as modes and the model orders.

To the best of our knowledge, [6] is the first contribution paper which addresses the more identification challenging case in which the hybrid system dynamics are assumed to be non-linear without the need to consider a priori assumptions about the number of modes, the model parameters and the switching sequence. For the first time, they used a feed-forward neural network to obtain global parametric models for the considered class of hybrid

systems without the need to know the number of modes and the current mode. However this approach works in an off-line mode and results in a black-box model with a large number of parameters.

In this paper, an on-line identification approach based on an adaptive GAP-RBF neural network is presented. The UKF learning algorithm is utilized to estimate the network parameters during the identification phase.

The paper is organized as follows. The proposed on-line identification approach is introduced briefly in section 2. In section 3, performance of the developed identification approach is illustrated on a simulated benchmark hybrid system and a well known PWA system. Section 4 summarizes the conclusions from this research study.

2. Adaptive GAP-RBF Neural Network

The ability of Neural Networks to identify a global parametric model for a class of hybrid systems has been demonstrated in [6] for the first time in the literature. In this contributed paper, two feed-forward neural networks were used to model a hybrid benchmark system consisting of two interconnected tanks with 64 different number of modes. In this approach, each neural network was associated with each continuous tank level as the corresponding system outputs. However the proposed identification approach was off-line based which required some dedicated methods to choose the optimal structure of the neural networks involving the choice of inputs (i.e. the regression vector) and the number of hidden neurons.

Radial basis functions (RBF) neural networks have been popularly used in many identification applications in recent years due to their ability to approximate complex non-linear mappings directly from the input-output data with a simple topological dynamic structure. Combining this network with self-organizing network learning algorithms offers an attractive approach to make efficient adaptive RBF neural network which can adjust its dynamic structure complexity and parameters to varying non-linear system dynamics without requiring a priori knowledge. Huang et al. [7] have recently proposed a simple sequential learning algorithm with network growing and pruning (GAP) capabilities. The original GAP-RBF neural network algorithm has been modified in [8] to enhance it's capabilities for on-line system identification applications as follows:

a) Enhancing the smooth criterion of the neurons.
b) Enhancing the pruning criterion to prevent probable oscillation in the number of created neurons.
c) Utilization of the unscented kalman filter (UKF) estimation algorithm to adjust free network parameters.

d) Utilization of a time-varying forgetting factor scheme in the UKF learning algorithm to maintain a desired parameter tracking capability.

2.1. Modified GAP-RBF (MGAP-RBF) Neural Network

The complete description of the MGAP-RBF learning algorithm can be summarized as follows:

The learning algorithm begins with no initial hidden neurons. As each new observation data (x_n, y_n), where $x_n \in R^l$, are received the following steps are performed:

1. **Compute** the overall network output:

$$f(x_n) = \sum_{k=1}^{K} \alpha_k \phi_k(x_n) \qquad (1)$$

where K is the number of hidden neurons, α_k is the connecting weight of the kth hidden neuron to the output neuron. $\phi_k(x_n)$ denotes response of the kth hidden unit to the input vector x_n, defined by the following Gaussian function:

$$\phi_k(x_n) = \exp\left(-\frac{\|x_n - \mu_n\|^2}{\sigma_k^2}\right) \qquad (2)$$

where μ_k and σ_k refer to the center and width of the kth hidden neuron respectively and $\|\cdot\|$ indicates the Euclidean distance.

2. **Calculate** the parameters required in the modified growth criterion:

$$\varepsilon_n = \varepsilon_{min} + (\varepsilon_{max} - \varepsilon_{min})(1 - e^{-n/\tau}) \qquad (3)$$

where τ is the time constant parameter that can be used to control the time rate evolution of ε_n. ε_{min} and ε_{max} are minimum and maximum distance thresholds, respectively.

$$e_n = y_n - f(x_n) \qquad (4)$$

3. **Apply** the growth criterion for adding neurons:

$$\text{If} |e_n| > e_{min} \text{ and } \|x_n - \mu_{nr}\| > \varepsilon_n$$
$$\text{and } \left(1.8\kappa \|x_n - \mu_{nr}\|^l\right) e_n| / S(X) > e_{min} \qquad (5)$$

(where e_{min} is the expected desired accuracy and μ_{nr} is the center of the nearest neuron to x_n and $S(X)$ is the estimated size of range (X) where the training samples are drawn from)
Allocate a new hidden K+1 with:

258

$$\alpha_{K+1} = e_{min}$$

$$\mu_{K+1} = x_n \qquad (6)$$

$$\sigma_{K+1} = \kappa \|x_n - \mu_{nr}\|.$$

Else

Adjust the network parameters $\alpha_{nr}, \mu_{nr}, \sigma_{nr}$ for the nearest (nrth) neuron only, using the UKF algorithm.

Check the modified pruning criterion for the nearest (nrth) hidden neuron:

If $\left|(1.8\sigma_{nr})^l \alpha_{nr}/S(X)\right| < \beta\, e_{min}$, (in which a new pruning factor $0 < \beta \le 1$ has been added), remove the nearest (nrth) hidden neuron and do the necessary changes in the UKF algorithm.

Endif

Endif

2.2. The UKF Modified Learning Algorithm

The original GAP-RBF algorithm uses the extended kalman filter (EKF) as its network parameter estimation algorithm. In practice, however, the use of the EKF has two well-known drawbacks:

a. Linearization can provide highly unstable filter if the local linearity assumption is violated.

b. The derivations of the Jacobian matrices are nontrivial inn most applications and often lead to significant implementation difficulties.

To address these limitations, Julier and Uhlmann [9] developed the UKF algorithm. Consider the non-linear system, described by the following equations:

$$x_{k+1} = f(x_k, u_k) + w_k \qquad (7)$$

$$y_k = h(x_k) + v_k \qquad (8)$$

In the UKF algorithm, instead of linearizing the above non-linear system model equations using Jacobian matrices in the EKF, a "deterministic sampling" approach is used to calculate the mean and variance estimates of Gaussian random state variables x_n with a minimal set of $2L+1$ sample points (L is the state dimension), called as sigma points. The results are accurate to the third-order (Taylor series expansion) for Gaussian inputs for all nonlinearities.

The UKF learning algorithm can be summarized as follows:

1. Initialize with some initial guesses for the state estimate (x_n) and the error covariance matrix ($P_\circ$), defined as:

$$\hat{x}_0 = E[x_0] \qquad (9)$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] \qquad (10)$$

for $k \in \{1,\ldots,\infty\}$, ($E[\cdot]$ denotes the expected value).

2. Calculate the sigma points:

$$\chi_{k-1} = [\hat{x}_{k-1} \quad \hat{x}_{k-1} + \gamma\sqrt{P_{k-1}} \quad \hat{x}_{k-1} - \gamma\sqrt{P_{k-1}}] \quad (11)$$

where $\lambda = \alpha^2(L+k) - L$ and $\gamma = \sqrt{L+\lambda}$ are scaling parameters.

3. Time update equations:

$$\chi_{k|k-1}^* = f[\chi_{k-1}, u_{k-1}] \qquad (12)$$

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \chi_{i,k|k-1}^* \qquad (13)$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1}^* - \hat{x}_k^-][\chi_{i,k|k-1}^* - \hat{x}_k^-]^T + R^w$$
$$(14)$$

where $\{W_i^{(m)}\}$ and $\{W_i^{(c)}\}$ are sets of scalar weights, R^w is process noise covariance, and

$$\chi_{k|k-1} = [\hat{x}_k^- \quad \hat{x}_k^- + \gamma\sqrt{P_k^-} \quad \hat{x}_k^- - \gamma\sqrt{P_k^-}] \quad (15)$$

$$Y_{k|k-1} = h[\chi_{k|k-1}] \qquad (16)$$

$$y_k^- = \sum_{i=0}^{2L} W_i^{(m)} Y_{i,k|k-1} \qquad (17)$$

4. Measurement update equations:

$$P_{\bar{y}_k \bar{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [Y_{i,k|k-1} - \hat{y}_k^-][Y_{i,k|k-1} - \hat{y}_k^-]^T + R^v$$
$$(18)$$

$$P_{\bar{x}_k \bar{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1} - \hat{x}_k^-][Y_{i,k|k-1} - \hat{y}_k^-]^T \quad (19)$$

$$K_k = P_{\bar{x}_k \bar{y}_k} P_{\bar{y}_k \bar{y}_k}^{-1} \qquad (20)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - \hat{y}_k^-) \qquad (21)$$

$$P_k = (P_k^- - K_k P_{\bar{y}_k \bar{y}_k} K_k^T)/\eta_k \qquad (22)$$

where η_k behaves as the forgetting factor concept in the usual recursive least-squares (RLS) algorithm which undergoes the following time-varying evolution.

$$\eta_k = \eta_{k-1} + (1 - \eta_{k-1})(1 - e^{-t/\tau}), 0 \prec \eta_k \le 1 \quad (23)$$

where t is the recursive time interval that is spent in the UKF learning algorithm to estimate the MGAP-RBF neural network free parameters with fixed structure. Thus, t is reset to zero when any network structural change, i.e. neuron creation or pruning occurs. This scheme maintains a desired parameter adaptive capability in the UKF algorithm whenever process dynamics undergo a time-varying change.

3. Identification case studies

3.1. Identification of a two tanks flow hybrid benchmark system

The proposed on-line identification approach has been applied to the two tanks flow hybrid benchmark system introduced in [6] which has been depicted in

259

figure 1. This system is considered as a black-box system equipped with some sensors that provide the required I/O data for identification purpose. The dynamics of the system can be described by the following equations:
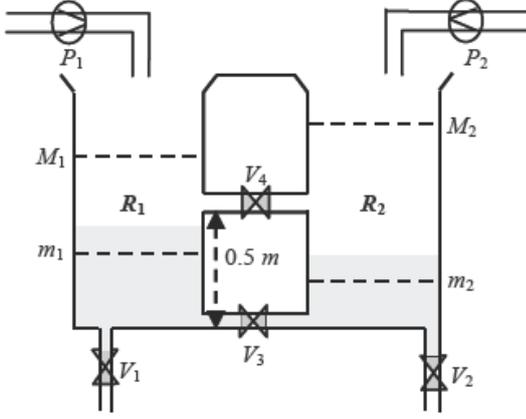

Figure 1. The schematic diagram of two tanks system

$$Q_{P1} = D.P_1, \qquad P_1 \in \{0,1\} \tag{24}$$

$$Q_{P2} = D.P_2, \qquad P_2 \in \{0,1\} \tag{25}$$

$$Q_1 = A.\sqrt{2.g.h_1}V_1, \qquad V_1 \in \{0,1\} \tag{26}$$

$$Q_2 = A.\sqrt{2.g.h_2}V_2, \qquad V_2 \in \{0,1\} \tag{27}$$

$$Q_3 = \alpha.\sqrt{|h_1 - h_2|}V_3, \qquad V_3 \in \{0,1\} \tag{28}$$

$$Q_4 = \alpha.\sqrt{|\sup(h_1,0.5) - \sup(h_2,0.5)|}V, \tag{29}$$
$$V_4 \in \{0,1\}$$

$$S\dot{h}_1 = Q_1 - Q_3 - Q_4 \tag{30}$$

$$S\dot{h}_2 = -Q_2 + Q_3 + Q_4 \tag{31}$$

where $\alpha = A \cdot \sqrt{2g}\,sign(h_1 - h_2)$, D is the constant input flow of both pumps, $Q_{Pi,i \in \{1,2\}}$ is the input flow of tank i, $Q_{i,i \in \{1,2\}}$ is the output flow of tank i, Q_3 is the flow in the pipe C_1 and Q_4 is the flow in the pipe C_2.

In order to avoid either the draining or the overflow of the tanks, the electro valves V_1, V_3 and V_4 as well as the pump P_1 are driven by an algorithm which guarantees the following levels of the fluids in the tanks R_1 and R_2:

$$m_1 \le h_1 \le M_1 \tag{32}$$

$$m_2 \le h_2 \le M_2 \tag{33}$$

with

$$M_1 = 0.6m, M_2 = 0.75m,$$
$$m_1 = 0.4m \text{ and } m_2 = 0.2m.$$

The pump P_2 and the electro valve V_2 are considered as perturbations which cannot be controlled. Hence P_2 and V_2 were opened and closed according to two Nearly Random Binary Sequences of length: $l_{P_2} \in [10, 30]$ and $l_{V_2} \in [30, 50]$.

The simulated hybrid system was identified by two MGAP-RBF neural networks using the proposed adaptive approach presented in section 2. Each neural network was used to identify the behavior of the liquid level in each individual tank by considering the following Nonlinear Auto-Regressive with eXegenous (NARX) input model structure:

$$y_i(t) = f_i(x_n(t)); \quad i = 1,2 \tag{34}$$

where

$$x_n(t) = [P_1(t-1), P_1(t), P_2(t-1), P_2(t),$$
$$P_3(t-1), P_3(t), V_1(t-1), V_1(t),$$
$$V_2(t-1), V_2(t), V_3(t-1), V_3(t), \tag{35}$$
$$y_i(t-1), y_i(t-2)]$$

Thus, each neural network has 14 inputs which are associated with 6 binary inputs at two latest time instants of $t-1$ and t and one continuous liquid level output recorded at of $t-2$ and $t-1$ time instants.

During the identification, the first data set called as the identification data set, containing N=5000 data, generated through the simulation experiment done by N. Messai et al.[6] was used in a sequential manner to emulate a real-time identification experiment. As a consequence, each MGAP-RBF neural network started without a priori knowledge about the network structure by setting the free-algorithm parameters at $\varepsilon_{min} = 9$, $\varepsilon_{max} = 20$, $k = 50.9$, $e_{min} = 0.001$.

The identification results have been illustrated in figures 2 and 3. Figures 2(a) and 3(a) show the measured and identified liquid levels. The analysis of these figures demonstrates the high accuracy of the identified models. Figures 2(b) and 3(b) show the error residuals which do not exceed 4% of the measured levels. Figures 2(c) and 3(c) illustrate the distributions of the resulting error residuals which can infer the high accuracy of the identified models. Figures 2(d) and 3(d) show the time history of the hidden neuron evolution during the on-line identification for the two MGAP-RBF neural networks. As illustrated, the resulting network structures are quite simple and compact with 2 and maximum 4 neurons respectively.

Comparing the obtained results with those recorded in [6] for the same hybrid system (shown in figures 4 and 5 [10]), demonstrates the superiority of the proposed identification approach based on the following general observations:

    a. The proposed algorithm is simple and on-line without a priori knowledge while the approach

proposed in [6] is off-line and require dedicated methods to extract the required model structure

b. The resulting models are very simple and compact. Each network uses 14 inputs with maximum 2 and 4 hidden neurons while the identified networks in [6] uses 21 input with 8 hidden neurons.

c. The obtained models have better accuracy in terms of error residuals or residual distributions.
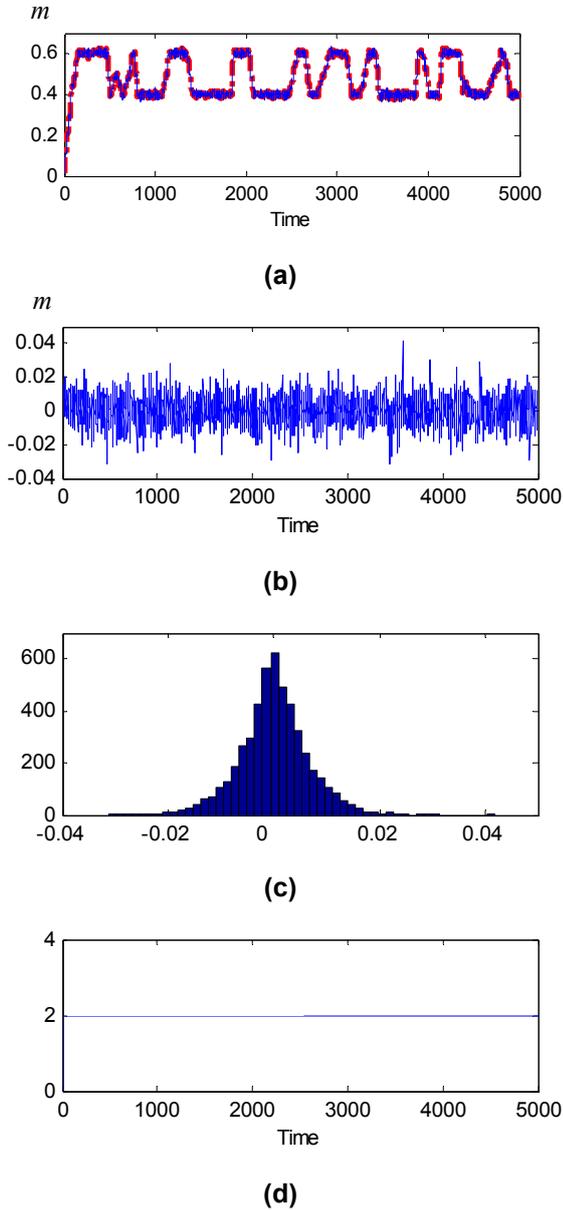

m

(a)

m

(b)

(c)

(d)

Figure 2. Predicted and measured liquid level (a), identification residuals (b), distribution of the error residuals (c) and hidden neuron evolution (d) for 1st tank
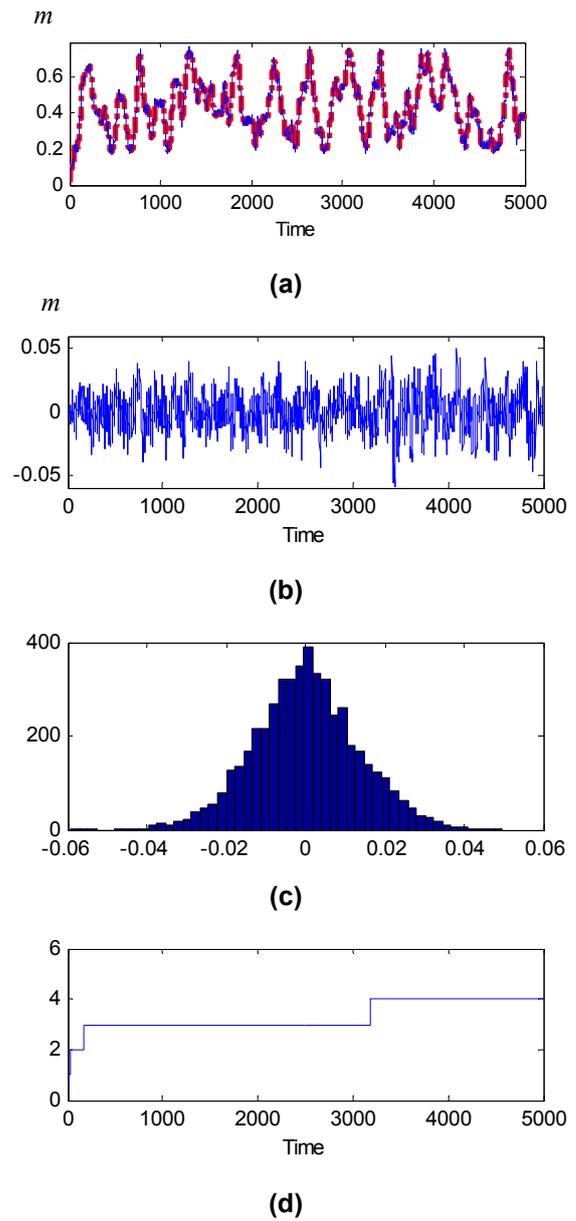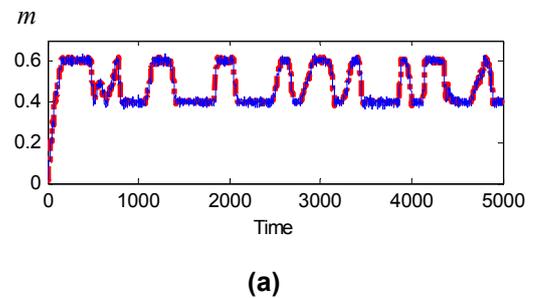

m

(a)

m

(b)

(c)

(d)

Figure 3. Predicted and measured liquid level (a), identification residuals (b), distribution of the error residuals (c) and hidden neuron evolution (d) for 2nd tank
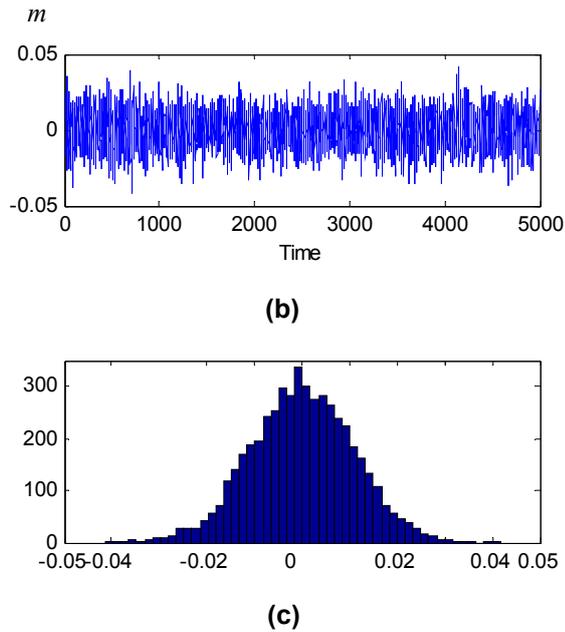

m

(a)

261

(b)

(c)

Figure 4. Predicted and measured liquid level (a), identification residuals (b) and distribution of the error residuals (c) for 1st tank using feed forward NN

3.2. Identification of a PWARX benchmark model

The following PWARX model is considered [11]:

$$
y_k = \begin{cases}
-0.4y_{k-1} + u_{k-1} + 1.5 + e_k, \\
\qquad if\ 4y_{k-1} + u_{k-1} + 10 \prec 0 \\
0.5y_{k-1} - u_{k-1} - 0.5 + e_k, \\
\qquad if\ 4y_{k-1} + u_{k-1} + 10 \geq 0 \quad (36) \\
\qquad and\ 5y_{k-1} + u_{k-1} - 6 \leq 0 \\
-0.3y_{k-1} + 0.5u_{k-1} - 1.7 + e_k, \\
\qquad if\ 5y_{k-1} + u_{k-1} - 6 \geq 0
\end{cases}
$$

The input u_k and the noise e_k are white noises generated from uniform distributions over the intervals $[-5, 5]$ and $[-0.1, 0.1]$, respectively. 200 data points are available as depicted in figure 6 [11].

We applied the bounded-error clustering-based identification algorithm [12] on this example. In order to compare the results, the predicted output of the identification together with error residuals are presented in figures 7a-7c. The predicted output and actual data are depicted in figure 7a, and the identification error with its distribution is shown in figures 7b and 7c, respectively.
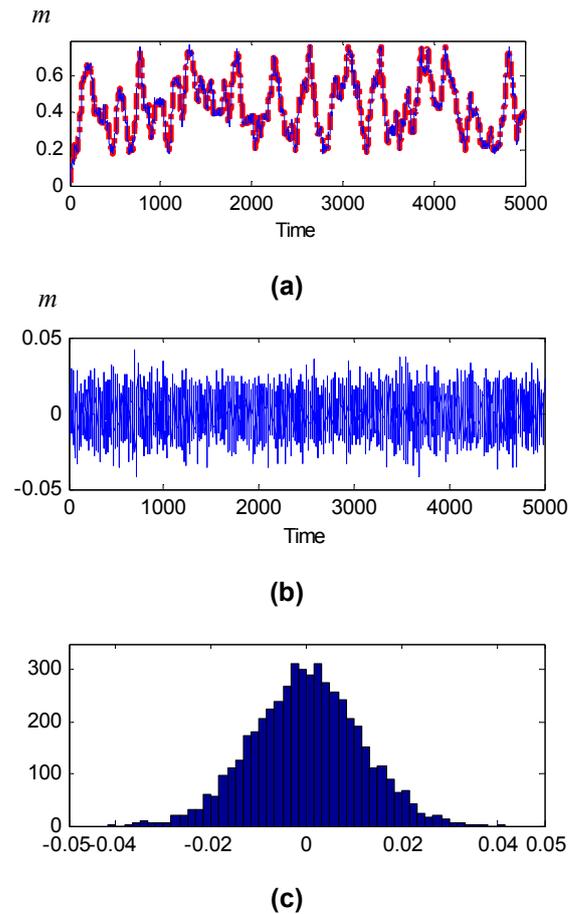

(a)

(b)

(c)

Figure 5. Predicted and measured liquid level (a), identification residuals (b) and distribution of the error residuals (c) for 2nd tank using feed forward NN
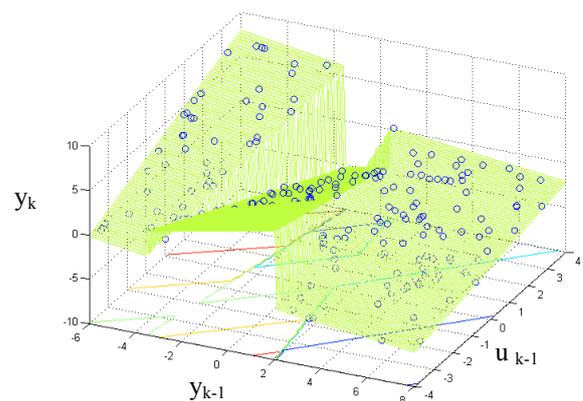

Figure 6. Data points used for identification
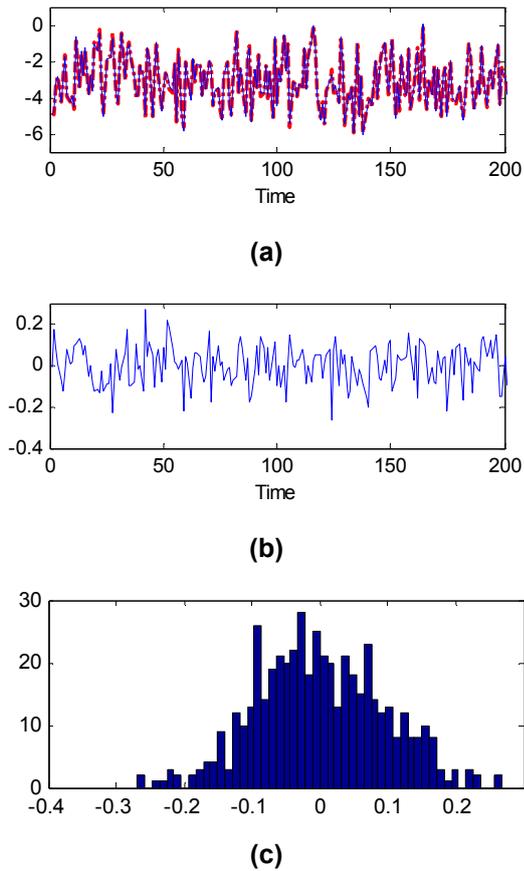
262

(a)

(b)

(c)

Figure 7. Predicted and measured outputs (a), identification residuals (b) and distribution of the error residuals (c) using Clustering-Based Bounded-Error approach.

u_{k-1} and y_{k-1} are used as inputs of the MGAP-RBF neural network. Identification is performed using just one hidden neuron and the identification error is very small compared to previous results (neglecting a few points, identification error is between -0.007 and 0.007), as shown in figures 8a-8d.

4. CONCLUSION

In this paper, a new method for on-line identification of hybrid system by using the MGAP-RBF neural network has been proposed. The proposed approach tested on two examples and the results are compared with some existing approaches which leads to the following observations:

1. The proposed approach is an on-line identification method, while the considered approaches are off-line.
2. The MGAP-RBF neural network has an adaptive dynamic structure.

3. The results show better accuracy in the identified models with simple network structures.
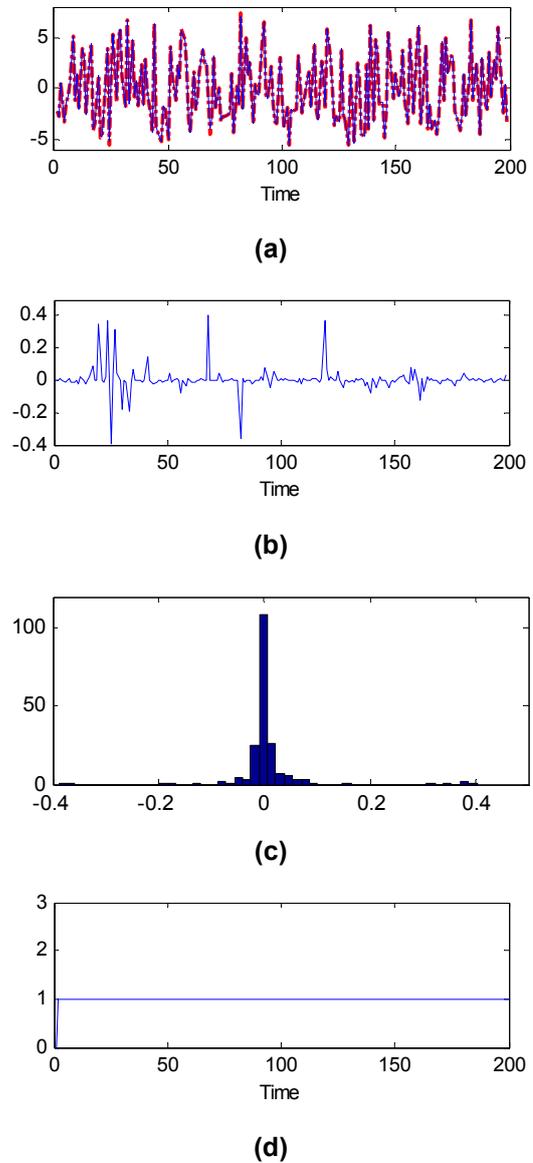

(a)

(b)

(c)

(d)

Figure 8. Predicted and measured outputs (a), identification residuals (b), distribution of the error residuals (c) and hidden neuron evolution (d).

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