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# Robust Feedback Linearization-based Control Design for a Wheeled Mobile Robot

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This paper considers the trajectory tracking problem for a four-wheel driven, four-wheel steered mobile robot moving in outdoor terrain. The robot is modeled as a non-holonomic dynamic system subject to pure rolling, no-slip constraints. A nonlinear trajectory tracking feedback control law based on dynamic feedback linearization is designed for this model. Since several parameters in the model, in particular the ground-wheel contact friction, are not well known a priori, a robustness analysis is carried out for bounded uncertainties. It is demonstrated that uncertainties can render the closed-loop system unstable, and two approaches to avoid this are suggested.

Keywords: Autonomous Vehicles, Feedback Linearization, Robust Control, Vehicle Dynamics and Control, Wheeled Mobile Robots

## 1. INTRODUCTION

The work presented in this paper is motivated by a project currently in progress, where an autonomous four-wheel driven, four-wheel steered robot is under construction. The purpose of the project, which is a collaboration between the Danish Agricultural Research Center and Aalborg University, Denmark, is to construct a robot that is able to survey an agricultural field autonomously. The vehicle has to navigate to certain waypoints (measurement locations), where digital images of the crops, weeds, etc. can be taken. Image analysis will be used in order to obtain estimates of the crop and weed density at each measurement location. This information will be combined for each location to yield a digitized weed map of the field, opening up opportunities for the farmer to adjust the application of fertilizer and pesticides according to the state of the field. The robot will be equipped with GPS, magnetometer and odometer sensors, which will not only help in the exact determination of the location where each image is taken, but also provide measurements for an estimation of the robot's position and orientation for a tracking algorithm.

The robot is equipped with independent steering and drive motors (8 DC motors in total), whose indi-

vidual controllers are connected to a main computer via a fieldbus. It is thus possible at any given time to set rotation speed or torque references for each motor.

As stated above, the robot needs to navigate from waypoint to waypoint, and in order to minimize the damage to the crop rows, there will be significant portion of the operation where it is not convenient to follow straight lines between the waypoints. Rather, the robot needs to track a smooth, spline-type trajectory between the waypoints. To address the tracking problem, which is the main subject of this paper, we will therefore need to consider not only the kinematics of the robot, but also the dynamics. Following the approach taken in [1] and [2], we present a dynamic model of the robot containing a kinematic sub-model describing the geometric aspects of the robot's trajectory tracking and a dynamic sub-model describing the dynamics from input torques to resulting velocities. It is assumed in the modeling that there is neither slip nor skidding.

As the model is highly nonlinear and involves non-holonomic constraints, it is clear that a nonlinear control scheme is more suited than a linear one. We therefore design an input-output feedback linearization-based control law to solve the trajectory tracking problem, considering both the kinematics and dynamics in the design. The steering dynamics will be dealt

with by closing local loops around each steering motor; the presence of large friction forces makes it a necessity to apply servo control to the steering. The driving dynamics can be partly linearized by computing a total torque that negates the nonlinearities, but an interesting point arises when this torque has to be distributed to the four driving motors, as the system is mechanically over-determined. We choose one of several solutions, which minimizes the maximum torque applied to any one wheel at a given time.

After the partial linearization of the dynamics, we design a path tracking control law, also based on feedback linearization. Feedback linearization designs have the potential of reaching a low degree of conservativeness, since they rely on explicit canceling of nonlinearities. However, such designs can also be quite sensitive to noise, modeling errors, actuator saturation, etc. In case of this robot, there are several parameters in the model that are not known well, in particular the friction disturbances from the ground, but also motor time constants and inertia distributions. As a consequence hereof, we will conduct a robustness analysis of the feedback linearization design. We show that even quite limited uncertainties can cause instability under normal driving conditions, and we suggest two approaches to deal with this problem.

## 2. DYNAMIC MODEL AND LINEARIZATION

We consider a four-wheel driven, four-wheel steered robot moving on a horizontal plane, constructed from a rigid frame with four identical wheels. Each wheel can turn freely around its horizontal axis, and its direction can be freely controlled as well. The contact points between each of the wheels and the ground must satisfy pure rolling and non-slip conditions.

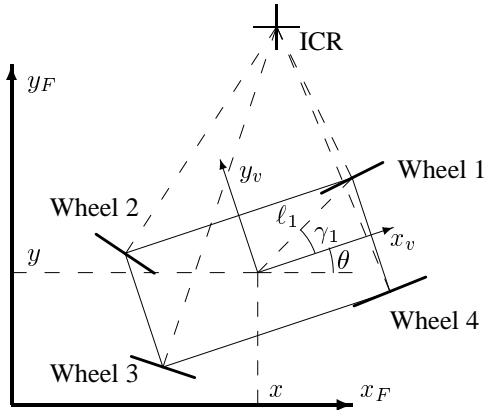


Figure 1. Definition of the field coordinate system \$(x\_F, y\_F)\$, vehicle coordinate system \$(x\_v, y\_v)\$, vehicle orientation \$\theta\$, and the distance \$l\_1\$ and direction \$\gamma\_1\$ from the center of mass \$(x, y)\$ to wheel 1. Each wheel plane is perpendicular to the ICR.

Consider a reference ('field') coordinate system \$(x\_F, y\_F)\$ in the plane of motion. The position of the robot is then completely described by the coordinates \$(x, y)\$ of a reference point within the robot frame, which without loss of generality can be chosen as the center of mass, and the orientation \$\theta\$ relative to the field coordinate system of a ('vehicle') coordinate system \$(x\_v, y\_v)\$ fixed to the robot frame. These coordinates are collected in the *posture* vector \$\xi = [x \ y \ \theta]^T \in \mathbb{R}^3\$. The position of each wheel within the vehicle coordinate system will be described by a set of vectors from the center of mass to the point of rotation of each wheel. The position of the \$i\$'th wheel, \$1 \leq i \leq 4\$, is thus given by a constant angle relative to the \$x\_v\$ axis, denoted \$\gamma\_i\$, and the constant distance from the center of mass, denoted \$l\_i\$. Because the wheels are not allowed to slip, the planes of each of the wheels must at all times be tangential to concentric circles with the center in the instantaneous center of rotation (ICR). The angle between the wheel plane of the \$i\$'th wheel and the \$x\_v\$ direction is denoted \$\beta\_i\$. The wheels are placed in a rectangular configuration, as indicated in Figure 1. Denoting the distances between wheels 1 and 2 by \$d\_{12}\$ and between wheels 1 and 4 by \$d\_{14}\$, the following two auxiliary equations describing the last two wheel orientations can be obtained:

$$\beta_3 = \tan^{-1} \left( \frac{\cos \beta_1 \sin \beta_2}{\cos \beta_1 \cos \beta_2 + \frac{d_{14}}{d_{12}} \sin(\beta_2 - \beta_1)} \right) \quad (1)$$

$$\beta_4 = \tan^{-1} \left( \frac{\sin \beta_1 \cos \beta_2}{\cos \beta_1 \cos \beta_2 + \frac{d_{14}}{d_{12}} \sin(\beta_2 - \beta_1)} \right) \quad (2)$$

Define \$\beta = [\beta\_1 \ \beta\_2 \ \beta\_3 \ \beta\_4]^T\$ and \$\phi = [\phi\_1 \ \phi\_2 \ \phi\_3 \ \phi\_4]^T\$. The motion of the four-wheel drive, four-wheel steered robot is then completely described by the following 11 generalized coordinates:

$$q = [x \ y \ \theta \ \beta^T \ \phi^T]^T = [\xi^T \ \beta^T \ \phi^T]^T \quad (3)$$

and we can write the pure rolling, no slip constraints on the compact matrix form

$$A(q)\dot{q} = \begin{bmatrix} J_1(\beta)R_\theta & 0 & J_2 \\ C_1(\beta)R_\theta & 0 & 0 \end{bmatrix} \dot{q} = 0 \quad (4)$$

in which

$$J_1(\beta) = \begin{bmatrix} \cos \beta_1 & \sin \beta_1 & l_1 \sin(\beta_1 - \gamma_1) \\ \cos \beta_2 & \sin \beta_2 & l_2 \sin(\beta_2 - \gamma_2) \\ \cos \beta_3 & \sin \beta_3 & l_3 \sin(\beta_3 - \gamma_3) \\ \cos \beta_4 & \sin \beta_4 & l_4 \sin(\beta_4 - \gamma_4) \end{bmatrix}$$

$$J_2 = rI_{4 \times 4}$$

$$C_1(\beta) = \begin{bmatrix} -\sin \beta_1 & \cos \beta_1 & l_1 \cos(\beta_1 - \gamma_1) \\ -\sin \beta_2 & \cos \beta_2 & l_2 \cos(\beta_2 - \gamma_2) \\ -\sin \beta_3 & \cos \beta_3 & l_3 \cos(\beta_3 - \gamma_3) \\ -\sin \beta_4 & \cos \beta_4 & l_4 \cos(\beta_4 - \gamma_4) \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

[2] defines two characteristic numbers, *degree of mobility*  $\delta_m$  and *degree of steerability*  $\delta_s$ , which express how the kinematic constraints restrict the movement of the wheeled mobile robot.  $\delta_m$  is defined as the dimension of the null space of  $C_1$  and expresses the number of degrees of freedom that can be manipulated directly from the inputs to the kinematic model (velocities) without reorientation of the wheels. The degree of steerability is defined as  $\delta_s = \text{rank}\{C_1(\beta)\} = 3 - \delta_m$  and expresses the number of wheel orientations that can be oriented independently when steering the robot. It can be deduced from equations (1) and (2) that  $\delta_m = \text{rank}\{C_1(\beta)\} = 2$  and  $\delta_s = 3 - 2 = 1$ . Then, following the argumentation in [2], it can be deduced that the posture velocity of the wheeled mobile robot  $\dot{\xi}$  is constrained to belong to a one-dimensional distribution parametrized by the orientation angles of two wheels, say,  $\beta_1$  and  $\beta_2$ . Thus,

$$\dot{\xi} \in \text{span}\{\text{col}\{R_\theta^T \Sigma(\beta)\}\}$$

where  $\Sigma(\beta) \in \mathbb{R}^3$  is perpendicular to the space spanned by the columns of  $C_1$ , i.e.,  $C_1(\beta)\Sigma(\beta) \equiv 0 \forall \beta$ .  $\Sigma$  can be found by combining the expression for  $C_1(\beta)$  with equations (1) and (2) to

$$\Sigma = \begin{bmatrix} \ell_1 \cos \beta_2 \cos(\beta_1 - \gamma_1) - \ell_2 \cos \beta_1 \cos(\beta_2 - \gamma_2) \\ \ell_1 \sin \beta_2 \cos(\beta_1 - \gamma_1) - \ell_2 \sin \beta_1 \cos(\beta_2 - \gamma_2) \\ \sin(\beta_1 - \beta_2) \end{bmatrix}.$$

The discussion above implies that the robot posture can be manipulated via a velocity input  $\eta(t) \in \mathbb{R}$  in the instantaneous direction of  $\Sigma$ , that is,  $R_\theta \dot{\xi}(t) = \Sigma \eta(t) \forall t$ . Similarly, it is possible to manipulate the orientations of the wheels via an orientation velocity input  $\zeta(t) = [\dot{\beta}_1 \ \dot{\beta}_2]^T \in \mathbb{R}^2$ . This allows us to arrive at the so-called *posture kinematic model*:

$$\begin{bmatrix} \dot{\xi} \\ \dot{\beta}_c \end{bmatrix} = \begin{bmatrix} R_\theta^T \Sigma & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \eta \\ \zeta \end{bmatrix}. \quad (5)$$

Due to large friction effects in gears and contact friction between the ground and wheels, it has been decided to apply local servo loops to control  $\dot{\beta}$ , yielding approximately linear dynamics without overshoot.

The dynamics of  $\eta$  will be dealt with according to the approach suggested in [1] and [2], which is to apply the Lagrange formalism to the problem. The Lagrange equations for non-holonomic systems are written on the form

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = c_k(q)^T \lambda + Q_k$$

in which  $T$  is the total kinetic energy of the system and  $q_k$  is the  $k$ 'th generalized coordinate. On the left-hand side,  $c_k(q)$  is the  $k$ 'th column in the kinematic constraint matrix  $\mathcal{A}(q)$  defined in (4),  $\lambda$  is a vector

of so-called *Lagrange undetermined coefficients*, and  $Q_k$  is a generalized force (or torque) acting on the  $k$ 'th generalized coordinate.

The kinetic energy of the robot is calculated as

$$T = \frac{1}{2} \dot{q}^T \begin{bmatrix} R_\theta^T M R_\theta & R_\theta^T V & 0 \\ V^T R_\theta & J_\beta & 0 \\ 0 & 0 & J_\phi \end{bmatrix} \dot{q} \quad (6)$$

with appropriate choices of  $M$ ,  $J_\beta$  and  $J_\phi$ . In the case of the wheeled mobile robot we can derive the following expressions:

$$M = \begin{bmatrix} m & 0 & \mu_s \\ 0 & m & \mu_c \\ \mu_s & \mu_c & \mathcal{I}_f + m_w \sum_{i=1}^4 \gamma_i^2 \end{bmatrix}. \quad (7)$$

Here,  $\mathcal{I}_f$  is the moment of inertia of the frame around the center of mass. Furthermore,  $m = m_f + 4m_w$ ,  $\mu_s = -m_w \sum_{i=1}^4 \ell_i \sin \gamma_i$ , and  $\mu_c = m_w \sum_{i=1}^4 \ell_i \cos \gamma_i$ , where  $m_f$  and  $m_w$  are the masses of the robot frame and each wheel, respectively. We note that since the wheels are placed symmetrically around the  $x_v$  and  $y_v$  axes,  $\mu_s$  and  $\mu_c$  should vanish. However, this may not be possible to achieve completely in practice, due to uneven distribution of equipment within the robot.

Turning to the wheels, we denote the moment of inertia of each wheel by  $\mathcal{I}_w$  and find

$$J_\beta = \frac{1}{2} \mathcal{I}_w I_{4 \times 4} \quad \text{and} \quad J_\phi = \mathcal{I}_w I_{4 \times 4} \quad (8)$$

and

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathcal{I}_w & \mathcal{I}_w & \mathcal{I}_w & \mathcal{I}_w \end{bmatrix}. \quad (9)$$

The Lagrange undetermined coefficients are then eliminated in order to arrive at the following dynamics:

$$h_1(\beta) \dot{\eta} + \Phi_1(\beta) \zeta \eta = \Sigma^T E \tau_\phi \quad (10)$$

in which  $E = J_1^T J_2^{-1} \in \mathbb{R}^{3 \times 4}$  and  $\tau_\phi \in \mathbb{R}^4$  is a vector of torques applied to rotate (drive) the wheels. The quadratic function  $h_1(\beta)$  is given by

$$h_1(\beta) = \Sigma^T (M + E^T J_\phi E) \Sigma > 0 \quad (11)$$

and  $\Phi_1(\beta) \in \mathbb{R}$  is given by

$$\Phi_1(\beta) = \Sigma^T (M + E^T J_\phi E) N(\beta) \quad (12)$$

$N(\beta) = [N_1 \ N_2]$ , where

$$N_1 = \begin{bmatrix} -\ell_1 \cos \beta_2 \sin(\beta_1 - \gamma_1) + \ell_2 \sin \beta_1 \cos(\beta_2 - \gamma_2) \\ -\ell_1 \sin \beta_2 \sin(\beta_1 - \gamma_1) - \ell_2 \cos \beta_1 \cos(\beta_2 - \gamma_2) \\ \cos(\beta_1 - \beta_2) \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -\ell_1 \sin \beta_2 \cos(\beta_1 - \gamma_1) + \ell_2 \cos \beta_1 \sin(\beta_2 - \gamma_2) \\ \ell_1 \cos \beta_2 \cos(\beta_1 - \gamma_1) + \ell_2 \sin \beta_1 \sin(\beta_2 - \gamma_2) \\ -\cos(\beta_1 - \beta_2) \end{bmatrix}$$

Equation (10) can be partially linearized by choosing  $\tau_\phi$  appropriately. The torques are simply distributed evenly to each wheel; we observe that

$$E\Sigma\tau_\phi = [a_1 \ a_2 \ a_3 \ a_4] \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} = L$$

where  $L$  is the left-hand side of (10). Then we set  $\tau_\phi = \Psi\tau_0$ ,  $\Psi \in \mathbb{R}^4$  and choose  $\Psi_i = L\text{sign}(a_i)/\sigma$ , where  $\sigma$  is the sum of the four entries in the vector  $\Sigma^T E$ . This distribution policy ensures that the largest torque applied to the individual wheels is as small as possible.

Hence, by applying the torque

$$\tau_0 = \frac{1}{E\Sigma\Psi} (h_1(\beta)\nu + \Phi_1(\beta)\zeta\eta) \quad (13)$$

we arrive at the model

$$\dot{\chi} = \begin{bmatrix} \dot{\xi} \\ \dot{\eta} \\ \dot{\beta}_c \end{bmatrix} = \begin{bmatrix} 0 & R_\theta^T \Sigma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \chi + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \nu \\ \zeta \end{bmatrix} \quad (14)$$

where it is assumed that the  $\beta$  dynamics can be controlled via local servo loops, such that we can manipulate  $\dot{\beta}$  as an exogenous input to the model. The standard approach from here would then be to transform the states into a nonlinear input equation followed by a feedback linearization of the nonlinearities and a standard linear control design. We choose the new states  $z = T(\chi) = [\xi_{ref} - \xi^T \ \dot{\xi}_{ref} - \dot{\xi}^T]$ , which yields the following dynamics:

$$\begin{aligned} \dot{z} &= \begin{bmatrix} \dot{\xi} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \dot{\xi} \\ \frac{d}{dt}(R_\theta^T \Sigma \eta) \end{bmatrix} \\ &= Az + B\delta(\chi)\nu - \alpha(\chi), \quad (15) \\ A &= \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix} \end{aligned}$$

$\delta(\chi)$  and  $\alpha(\chi)$  are found by calculating  $\frac{d}{dt}(R_\theta^T \Sigma \eta)$  to

$$\delta(\chi) = R_\theta^T [\Sigma(\beta) \ N(\beta)\eta] \quad (16)$$

and

$$\begin{aligned} \alpha(\chi) &= \sin(\beta_1 - \beta_2)\eta^2 \times \\ &\begin{bmatrix} -\ell_1 \sin \beta_2 \cos(\beta_1 - \gamma_1) + \ell_2 \sin \beta_1 \sin(\beta_2 - \gamma_2) \\ \ell_1 \cos \beta_2 \cos(\beta_1 - \gamma_1) - \ell_2 \cos \beta_1 \cos(\beta_2 - \gamma_2) \\ 0 \end{bmatrix}. \quad (17) \end{aligned}$$

If we then apply the control law  $[\nu \ \zeta^T]^T = \delta(\chi)^{-1}(\alpha(\chi) - K\tilde{z})$ , we obtain the closed-loop dynamics  $\dot{z} = (A - BK)z$ , which can easily be made to tend to 0 as  $t \rightarrow \infty$ , assuming of course that we avoid situations where  $\delta(\chi)$  becomes singular. Figure 2 shows a situation where the robot follows a pre-set path.

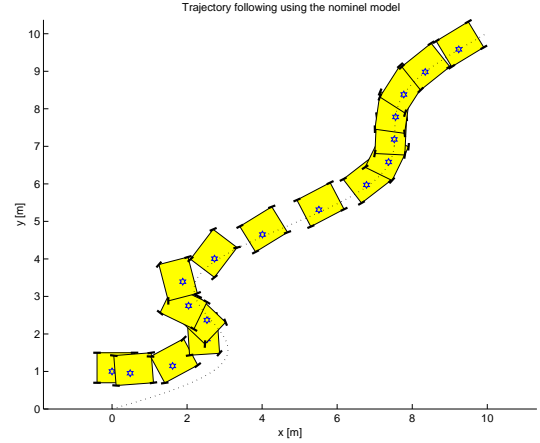


Figure 2. Path tracking with the nominal model and state feedback linearization.

### 3. ROBUSTNESS ANALYSIS

However, the aforementioned standard approach relies on the assumption that the model is perfectly known. If this is not the case, we may risk that the closed loop becomes unstable. Motivated by this consideration, we present the main contribution of this paper: a robustness analysis of the state feedback linearization control design, leading to the identification of problematic issues for stability. By including these issues in a robust control design, the closed-loop system can be guaranteed to be capable of dealing with uncertain parameters in the model (14) such as friction, gravity, etc. Firstly, we augment (10) with uncertain terms related to friction losses and uncertainties on the parameters in the expression for  $T$ :

$$(h_1(\beta) + \Delta h)\dot{\eta} + (\Phi_1(\beta)\zeta + \Delta\Phi)\eta = \Sigma^T E\tau_\phi$$

The nominal values used for computing the torque in (13) are adjusted accordingly, such that the acceleration of the vehicle after application of the computed torque can be written as

$$\dot{\eta} = \Delta_f \eta + (1 + \Delta_{g1})\nu$$

where  $\Delta_f \in [-\bar{f}_1; \bar{f}_1]$ ,  $\Delta_{g1} \in [-\bar{g}_1; \bar{g}_1]$  are bounded uncertainties. Similarly, we can expect that the local controllers governing the steering angles are not able to follow the reference values for  $\dot{\beta}$  as a perfectly known first-order linear system, resulting in uncertainties on  $\zeta$ . These considerations give rise to the following uncertain version of (14):

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \\ \dot{\beta}_c \end{bmatrix} = \begin{bmatrix} R_\theta^T \Sigma \eta \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \Delta_f \eta \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ (1 + \Delta_{g1})\nu \\ (I + \Delta_{g2})\zeta \end{bmatrix} \quad (18)$$

If we now apply the state transformation  $z = T(\chi)$  as above and the nominal control law  $[\nu \ \zeta^T]^T =$

$\delta(\chi)^{-1}(\alpha(\chi) - Kz)$ , we obtain the following closed-loop dynamics:

$$\dot{z} = (A - BK)z + B(\delta(\chi)\Delta_g\delta(\chi)^{-1}(-Kz) + \delta(\chi)\Delta_g\delta(\chi)^{-1}\alpha(\chi) + R_\theta^T\Delta_f\eta) \quad (19)$$

This closed-loop system can be rendered unstable, if the term  $\delta(\chi)\Delta_g\delta(\chi)^{-1}$  is large. Figure 3 shows a simulation made under the same conditions as in Figure 2, except that  $\Delta_g$  has been set to  $\Delta_g = \text{diag}\{1.1, 0.98, 0.98\}$ , i.e., 10 % uncertainty on the robot velocity  $\eta$  and 2 % uncertainty on the steering velocities  $\zeta$ . In this case, the robot loses stability in the sharp curve, not because of the uncertainties  $\Delta_g$  *per se*, but rather because  $\delta(\chi)\Delta_g\delta(\chi)^{-1}$  becomes large.

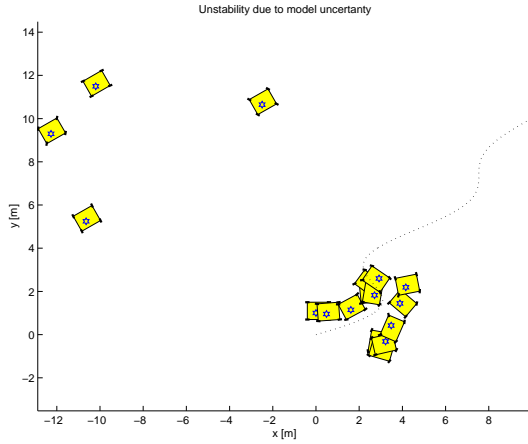


Figure 3. Loss of stability due to uncertainties in the feedback linearization.

One circumstance under which this phenomenon may occur, is when the robot is driving at low speed. This is the standard difficulty encountered at low vehicle speeds in non-holonomic systems due to Brockett's Obstruction, cf. [6]. Obviously, this is not the reason why we lose stability in the simulation shown in Figure 3, however. This instability is caused by the wheel configurations getting close to the singularities, that is, the ICR depicted in Figure 1 gets close to one of the wheels. This causes  $\delta(\chi)\Delta_g\delta(\chi)^{-1}$  to grow without bounds, since

$$\begin{aligned} \|\delta(\chi)\Delta_g\delta(\chi)^{-1}\| &\leq \bar{\sigma}(\delta(\chi)\Delta_g\delta(\chi)^{-1}) \\ &\leq \bar{\sigma}(\delta(\chi))\bar{\sigma}(\Delta_g)\frac{1}{\underline{\sigma}(\delta(\chi))} \\ &= \kappa(\delta(\chi))\bar{\sigma}(\Delta_g). \end{aligned} \quad (20)$$

Here,  $\bar{\sigma}(\cdot)$ ,  $\underline{\sigma}(\cdot)$ , and  $\kappa(\cdot)$  are the largest and smallest singular values and the condition number of a matrix, respectively.

The inequality (20) shows that it is possible to decrease the bound on the norm on the left-hand side

either by decreasing the uncertainties or by ensuring that the condition number of  $\delta$  is bounded. Decreasing the first term can for instance be done by designing a local torque feedback controller, such that the uncertainties on  $\eta$  are suppressed. By applying this approach in our model, the robot could be stabilized such that it was able to follow the trajectory. The disturbances were decreased to  $\Delta_g = \text{diag}\{1.02, 0.98, 0.98\}$ , i.e., the uncertainty on the velocity was decreased to 2%. This simulation is shown in Figure 4.

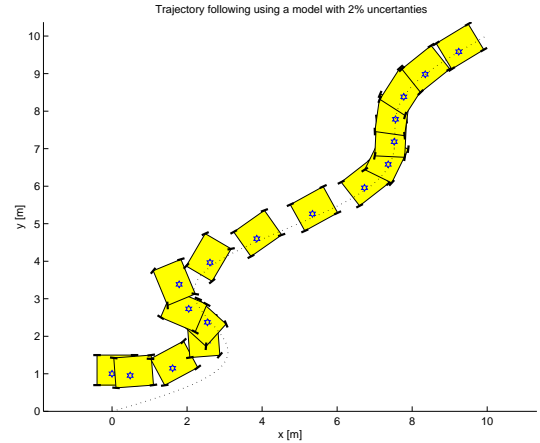


Figure 4. Stable tracking caused by decrease of uncertainties in feedback linearization.

The second approach is to avoid large condition numbers of  $\delta(\chi)$ . Since  $\delta$  depends only on  $\beta$  and  $\eta$ , this can only be ensured by choosing trajectories where  $\kappa(\delta)$  is bounded. The obvious means of doing that, would be to include the calculation of  $\kappa(\delta)$  in the path planning algorithm, and discard paths that tends to destabilize the robot.

#### 4. CONCLUSION

In this paper, we have considered the path-tracking problem for a four-wheel driven, four-wheel steered autonomous robot. The robot needs to navigate between waypoints on agricultural fields, and in order to minimize the damage to the crop rows, there will be significant portion of the operation where it is not convenient to follow straight lines between the waypoints. Rather, the robot needs to track a smooth, spline-type trajectory between the waypoints.

Taking the starting point in standard non-slipping and pure rolling conditions, a kinematic-dynamical model was established via the Lagrange formalism. The main purpose of deriving this model, was to establish how the driving and steering torques affect the robot motion.

Based on this, two feedback linearization-based control loops were devised. A partial linearization of

the dynamics was achieved using computed torques and local servo loops around the steering motors. Then, a path tracking controller was designed according to the feedback linearization method.

Robustness analysis showed, however, that if parameters are not fully known, or there are unmodeled friction effects etc., instability may occur. This was demonstrated using a simulation example, where a small deviation from the nominal model caused the robot to become unstable. The main reason for this phenomenon was found to be imperfect cancelation of certain nonlinear terms. Two ways to avoid instability were then suggested. Firstly, it is possible to minimize the effects of the uncertainties in the model using local feedback loops. The applicability of this approach was demonstrated on the same simulation example as mentioned above. The second approach would be to obtain the condition number of the aforementioned nonlinear term and use this in the planning algorithm, ensuring an upper bound on the perturbations to the nominal linearized system. Lyapunov-like arguments can then be used to prove that the closed loop system is stable in the presence of bounded disturbances and/or parametric uncertainties along the lines of for instance [4] or [5].

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