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By bringing our research work together, we are able to discuss the potential of combining the notions of the *learning landscape* and *school mathematical discourse*. We do so in a search for concepts and methodological tools to challenge the simplification of issues in regard to mathematics learning in multicultural settings, when adopting restricted perspectives on issues of bilingualism. In the paper we discuss the relationship between the learning landscape and school mathematical discourse. We then use these notions to analyse two case studies in Danish and New Zealand schools. Our conclusion raises possibilities about how these notions can be used when researching mathematics education in multicultural settings.

Generating theoretical and methodological tools to study and understand the complexity of mathematics education practices in multicultural settings is a current challenge for researchers in many countries. Multicultural classrooms include cultural, ethnic, religious, linguistic and socio-economic diversity among students and teachers (Hodge, 2006; Nieto, 2002). Bringing together our research in multicultural classrooms in Denmark and New Zealand has enabled us to explore socio-cultural-political perspectives on language and school mathematics practices. In this paper, we discuss two central notions emerging from our research, and the relationships between them. *Learning landscape* interprets

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(mathematics) education as a complex network of social practices that is constituted by different interrelated dimensions (Alrø, Skovsmose & Valero, 2007). *School mathematical discourse* is understood to be anything that is involved in the communication and the construction of meaning within the mathematics classroom and can include the mathematics register, the interaction patterns between teachers and students and between students and students, the textbook format, and the more general classroom language (Sfard, 2001; Meaney, 2005).

Language as a dominant approach to difficulties

For many teachers and researchers one of the most evident problems when meeting minority students in their classrooms is the students' lack of competence in the language of instruction. Consequently, there is a belief that if students cannot participate effectively in schooling and in school mathematics it is because they have not mastered the language of instruction. Such a view is challenged in reports such as "The teaching of bilingual students", on the instruction of minority students in Denmark, produced by the Danish Evaluation Institute:

> The prevailing view on language appears static in more of the schools. The schools focus primarily on the students' linguistic skills as a condition for learning and do not sufficiently see language as something that is created in a context. As a consequence second language acquisition pedagogy recedes into the background, and a focus on pedagogy and learning is replaced by a focus on the students' deficiencies. Therefore, a central challenge to the schools is to develop their view on language. Another important challenge is to develop their view on inclusion. This evaluation emphasises that the education of bilingual students often requires a difficult reorganization from handling a group of students that require inclusion in one way to a group of students that as a whole require more kinds of inclusion.
>
> (Danmarks Evalueringsinstitut, 2007, p. 8, Troels Lange's translation)

The report emphasises the need for schools to increase their views of both language and inclusion. Language can be simultaneously a broad and also a very narrow construct depending upon how it is perceived. It is broad when it is considered the vehicle for communicating meaning between people (or with one's self) and includes a variety of forms of communication (words, symbols, gestures) as well as all the meanings that can be conveyed (Bower, 2005). This broad construct is related to

beliefs about language as a thinking tool (Sfard, 2001; Langer, Applebee & Nystrand, 1995; Vygotsky, 1962). A wide view of language includes the contexts in which it is used and learnt, such as the society as a whole, but also the immediate school and classroom contexts. However, as a cultural artefact, language has been developed with and in response to cultural demands. It is not a simple, unproblematic construct that can be learnt without also absorbing cultural baggage (Holmen, 2008).

If language is restricted to just being fluent in the language of instruction, then language becomes a narrow construct. When this occurs, students who are learning the language of instruction as an additional language have been labelled as learning disabled (Naudé, Pretorius, Vandeyar, 2002). Contrary to this perception, research has shown that students who are taught bilingually not only achieve better results in content areas such as mathematics but that they also achieve better results in the second language (Saunders, 2001).

In the same way that language can be interpreted in multiple ways, inclusion also has equally distinct definitions depending on the kinds of relationships established among minority groups and the dominant culture with which they interact. These can be seen in the table 1.

Table 1: Linguistic and cultural ideologies

¿Se promovueren las relaciones intergrupales?	¿Se facilita el mantenimiento de la identidad y las características culturales y lingüísticas minoritarias?	
	Sí	No
Sí	**Integración**	**Asimilación**
No	**Segregación**	**Marginación**

Note. From Martín-Rojo (2003, p. 33)

In describing different linguistic and cultural ideologies, Martín-Rojo brings together two dimensions: whether there is a promotion of relationships between groups, and whether the maintenance of minorities' cultural identity and cultural characteristics is promoted. Different intersections of these two variables produce four types of ideologies: *Integration*, promoting inter-group relationships and allowing the maintenance of identity and cultural characteristics of minorities; *assimilation*, promoting inter-group relationships but denying the maintenance of cultural and linguistic identity; *segregation*, not promoting inter-group relationships but allowing minorities' identities; and *marginalisation*, denying both dimensions. The way in which a society deals with minorities has clear consequences for schools and mathematics classrooms. In a parallel way to Martín-Rojo's typology, one could ask whether

minority students' identity and cultural characteristics have been recognised and their retention supported in relation to the students' understandings of schooling and school mathematics. In this way we could talk about inclusion not only in terms of having minority students succeeding in mathematics performance tests – which unfortunately is not the case in many countries (OECD, 2006) – but also in terms of bringing diversity to the way in which school mathematics is done and understood in each national and local context. As a result of attending mathematics classes, students should neither be alienated from their home culture or from the dominant school culture (Cantoni, 1991).

Opening the perspective

Understanding the relationship between language and mathematics education needs to be done by considering issues of inclusion. This cannot be achieved in a restrictive manner because it is within the complexity of the situation that the impact of different factors can be seen. Therefore, any analytical tool should not only provide insights from focussing attention on specific aspects but should do this in a way that does not ignore other equally relevant factors. In order to overcome the shortcomings of a restricted understanding of school mathematics practices in situations of diversity that concentrate only on students' fluency in the language of instruction, we have devised the model seen in figure 1.

Rather than talking about language in the model, we use the term *school mathematical discourse* to refer to the use of a natural language and the mathematics register as a part of the practices of the teaching and learning of mathematics. This includes the interaction among the participants in that practice and the establishment of patterns of communication for expressing mathematical meaning (Herbel-Eisenmann, 2003). Therefore, the discourse is the set of uses of language that have to do with all the implicated actors and factors associated with school mathematics. In this model, school mathematical discourse is at the centre of the learning landscape as we see it as something that permeates and is permeated by all other aspects.

Based on recent research literature on multiculturalism and mathematics education as well as on our empirical work, we have selected nine dimensions – represented by the petals of the model – that have been considered to be influential for learning possibilities in multicultural settings (Alrø, Skovsmose & Valero, 2007). We bring the nine dimensions together in what we call a *learning landscape*. The notion of learning landscape is a tool to guide us in exploring the empirical field, and it has a double meaning. First, it represents an interpretation of (mathematics)

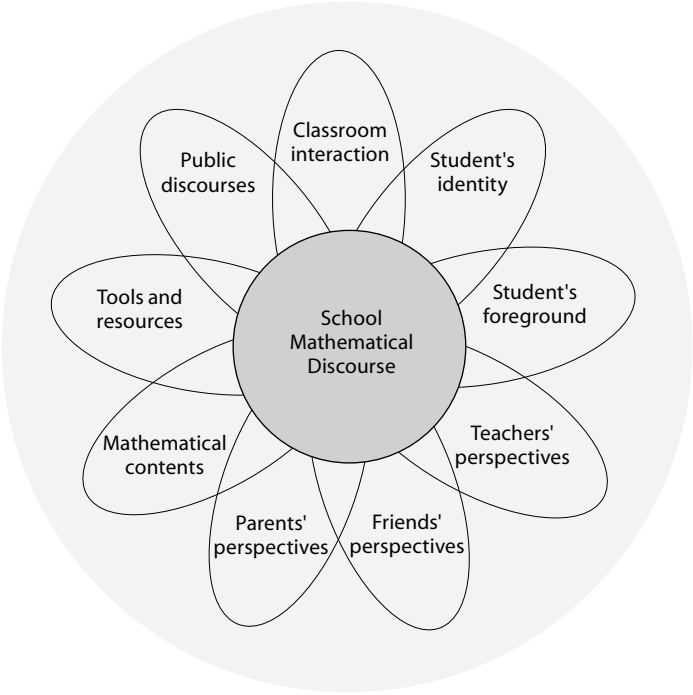

Figure 1. *School Mathematical Discourse in a Learning Landscape*

education as a complex network of social practices that is constituted by different interrelated dimensions (Valero, 2007). Second, it makes it possible to identify specific – but interrelated – dimensions of an empirical field to do research. Thus, it brings together a research perspective and a research field. The nine dimensions selected are: (a) Students' foregrounds as an experienced socio-political reality (Skovsmose, 2005); (b) Students' construction of identity and of cultural diversities (Sfard & Prusak, 2006); (c) Teacher's perspectives, opinions and priorities of teaching (César & Favilli, 2005); (d) The content of learning, in our case the mathematical content for classroom interaction (Powel, 2002); (e) Tools or resources for learning that students might have available; (f) Classroom interaction among students and between students and teacher (Alrø & Skovsmose, 2002); (g) Family and parents who influence students' priorities (Gutstein, 2003); (h) Friends, who are reference groups for the students (Bishop, 2002); and (i) Public discourses about immigrants, schooling and multiculturalism (Martín-Rojo, 2003).

We contend that the combination of school mathematical discourse within the learning landscape provides a powerful tool for researching

the complexity of school mathematics practices in settings of diversity. In what follows, we illustrate how we have used this tool in researching schools and classrooms in two very different contexts and two quite distinct situations. We provide case studies from our research, one of a second-generation immigrant student in Denmark and one of a Mäori immersion school in New Zealand. The Danish situation illustrates how the model can be used to identify how students' opportunities for their future can be restricted, whilst the New Zealand situation shows how the interaction of the same dimensions opens up possibilities. In using an analytical tool it is important to understand how it operates in a variety of different contexts. The differences between these situations certainly enable the usefulness of the tool to be investigated.

In both cases, large amounts of data were collected of which only a small part is shared here. Given that our assumption for both projects was that the complexity within each context was important, the data collection needed to be substantial and robust. After briefly describing each case, we analyse them using our theoretical tool to illustrate the insights that can emerge from bringing it into operation. Finally, we draw conclusions by using our tool to analyse the two cases together.

When being good at maths is not an advantage

Minh was 15 and like many of his 9th grade school mates at Mælkevejen Skole[1,2], he seemed to be more interested in anything except school. Showing signs of manhood, his body had changed dramatically in the last two years: man's voice, long hair and one of the tallest bodies in the class. From being an annoying trouble-maker together with some of his Vietnamese friends, Minh had now become a popular boy in the class. He was not only good at sports – and was therefore ready to take all kinds of physical challenges – he was also good at mathematics and gave a helping hand to those classmates in need of a push for finishing the set of exercises for homework.

When Helle first met Minh, he was in 8th grade. In an inter-view[3] with him about how he saw his future life and the role of mathematics in it, he stated that he liked to go to school. He considered school and mathematics to be very important for his future life, as at that time he wanted to become a banker or estate agent because he considered being good at mathematics to be his major strength. Therefore, he could imagine in the future to do "something with mathematics – I love numbers". However, Minh's appreciation seemed not to coincide with his parents' views. He did not think that his parents considered him to be good at mathematics as they had never told him so explicitly. Consequently, sometimes he

doubted his ability and felt that he was not good enough. He seemed to be upset about that. As Minh's teacher expressed when commenting on the boy's possibilities for the future, Minh's parents were very ambitious on behalf of Minh. The father had a grill-bar, and he wanted his son to get a well-paid job, so he wanted him to become a medical doctor, a very respected and socially valued profession in the eyes of Vietnamese immigrants. Minh himself would rather get a job that he was interested in and happy about. Nevertheless, he knew that his father would be disappointed if he chose "only" to become a bank clerk. Considerations about being a medical doctor had also passed through Minh's mind.

One year later, Paola met Minh again. At that time his engagement in school had changed and teenage priorities were more clearly present in his mind. In the setting of the computer room where the teacher had invited students to work on an open problem – imagining themselves in a profession and making a budget for monthly expenses – Minh was quite engaged in the task but seemed very hesitant and troubled about what to choose. On the web, he looked for information about a career in one of the local banks. "Hmm ... were those the numbers he should enter in his Excel spreadsheet to calculate a budget?". He also looked for information about the monthly income of medical doctors. "But ... what would it take him to be a doctor?"

M: Paola, if I want to become a medical doctor, how many years do I have to study?

P: Well, after you finish school you will have to go to high school which is 3 years, then you have to go to university for 5 years more, then you have an internship of around 1 year more, then you have to choose an specialization line of about ...

M: Stop, stop, stop. That is a lot. Many years. I cannot imagine that. I rather become a bank assistant.

For his young age, 3 + 5 + 1 + ... extra years seemed like an eternity. He expressed his doubt about whether he could engage in studies for such a long time. Despite expressing his wish to become a medical doctor, he used the data for a life in a bank. "Finances could not be that bad, after all ..."

In a later interview Minh expressed his confusion. His parents had great expectations and showed great appreciation of the path taken by his cousin. She was doing really well, they thought. Minh felt annoyed by being measured against one of his relatives. As far as he was concerned, he would make a good career ... in a bank.

P: And what about medical studies?

M: Well, my teacher says that I can't be a medical doctor because I am not good enough at Danish

P: What do you mean?

M: My Danish teacher says that since I am not very good at writing essays and that stuff, I won't become a medical doctor

P: But you are pretty good at Danish!

M: She really doesn't think so. I could do well studying something with mathematics. Banking is fine for me.

The mathematics teacher appreciated Minh's ability in mathematics and thought he could almost study whatever he wanted – if he overcame his laziness. In the class, Minh was a relatively active participant. He did the tasks that the teacher gave him and worked with the exercises in the textbook. The teacher's style at that time of 9th grade was preparing kids for the National Test at the end of the academic year, through giving students open, word problems from the main textbook, from other newer textbooks with problems that seemed more attractive to 15 year-old youngsters, and activities that he himself had designed. Minh was also known for doing the weekly assignment consisting of a set of old national tests, which he responded to thoroughly and in an organised fashion. He would explain the problems to some of his mates that they could not understand, and sometimes even let them copy from his own work sheets. In an environment where there was great freedom for the students to work in groups or individually, Minh engaged in the tasks as much as he allowed himself to be distracted, just like all the other students in the class.

In contrast, Minh's Danish teacher saw obstacles. Having been the class teacher for Minh all the way from 1st grade to 9th grade, she felt she knew well the boy's strengths and weaknesses. He was clever at maths; that was true. But it was difficult to know whether studying medicine was the boy's real wish or his parents' desire for him – "Vietnamese parents are always quite ambitious on behalf of their children ... probably all too unrealistically ambitious sometimes", the Danish teacher thought. Advising him – and his parents – about how to continue with his studies after compulsory school was a complicated issue. The teacher said:

> So when he shows hesitation, I think: If he takes an education in commerce, a three-year study, he will be in school three years and one in practical training in a bank, that is something he can complete. Then he can build on top of that if he wishes. But if we push him to go to high school and get a certification in mathematics, and later on six or seven years at the university, then I become more hesitant about whether he will be able to cope with that and succeed

> in finishing. And then he would drop out of school and will be left without any education at all. I would worry about him not having any certificate.

But why would he drop out? In reality, Minh's weakness was his lack of mastery of the Danish language. "He is not very articulated, he ... hum ... he is not very good to formulate himself in written form. He is not a catastrophe, but he is not good. We have given him special support in the past but it has not really helped much," the Danish teacher said. In general, the Danish teacher considered that it is difficult for students who do not speak Danish at home to get acquainted with many words and with the right syntactic structure of Danish, particularly if they use bad oral Danish or if they do not communicate in writing at home. And even if they wrote, their parents cannot really correct them. Minh is not an exception. "Students can get better, Minh has in fact improved. But he will never have a fully developed language like many others in the classroom", the Danish teacher added. In a school such as Mælkevejen Skole, such a problem is common to many bilingual students, as well as to many students coming from "weak homes" or "homes without many resources", according to the Danish teacher.

A few months later, Ole met Minh again. Ole had agreed with the teachers to do a sequence of activities with the students to explore how they saw mathematics in their everyday activities. Minh was enthusiastic and, in the same way he had welcomed Helle and Paola, showed interest in Ole being in the school. He was ready to engage in Ole's proposal. Unfortunately, during the very first session of the project-work with Ole, Minh was called out of the classroom by the school doctor for a routine check. The second session, Minh was absent because he had to do some sports. The third session ... well, then it was too late to get engaged. Sometimes other school priorities interfere with school mathematics.

An interpretation of Minh's story

Minh's conflicts are the ones that many minority students may experience when giving meaning to the learning of mathematics in relation to their future life possibilities. They also reveal the role given to mathematical learning as just one factor for success in life and how such a role is constructed in school practice, in the intersection between teachers' priorities and views of their immigrant students' futures and the students' views of their own wishes and hopes in life (and the parents' expectations). In Minh's case, being good at mathematics as perceived through his use of school mathematical discourse did not open up the same possibilities for his future as it may have for many non-immigrant students. The following analysis uses the model in figure 1 to explore

the intersections of the multiple factors that impact on the learning possibilities in this multicultural setting.

When first reading Minh's story, it seems that he has little difficulty participating in the school mathematical discourse. It is his written Danish that is "the problem". Yet, as more of his story is unfolded, connections can be made between these two aspects because of the different perceptions about the importance of his grasp of school mathematical discourse. For example, being good at mathematics as illustrated through his fluency with school mathematical discourse in the classroom was initially something that Minh did not feel that his parents recognised. Later, although his fluency in this was acknowledged by the mathematics teacher, his Danish teacher did not see it as showing the necessary knowledge of the Danish language for surviving in higher education. In many ways Minh is a typical Danish youngster in 9th grade with similar concerns to other Danish youngsters. Minh's story is about his foreground, i.e. about his hopes, dreams and more or less realistic expectations for the future. But it is also about his current and future identities which are very much affected by his parents and teachers' views of who he is and what he is capable of becoming.

Minh is in conflict about the choices he will have to make soon about his life – what he perceives as his foreground is not stable. For many Danish students the end of 9th grade and the possibility of leaving the compulsory school represents the beginning of a life of further study, career and life choices, that may not represent problematic challenges. However, as a Danish, second generation Vietnamese youngster, his story reveals interesting aspects of second-generation immigrants and their experiences in schools and in mathematics classrooms. For Minh, this moment seemed to be decisive in dealing with what he wanted, what his teachers showed as possibilities for him, and what his family expected from him. There seemed to be little agreement between the people who were the most influence in helping him make a choice and he was left to steer his own course that would involve not fulfilling someone's expectations. The role of his Danish teacher, however, appeared to have the greatest impact on how he would be guided at school.

Minh's mathematics teacher's appreciation about Minh's being good at mathematics through his command of school mathematical discourse provided Minh with one view of his foreground, that of having a good future. He could study almost whatever he wanted. His competence in mathematics opened the doors of an education in banking and finances, not at the university though, but as a middle vocational education. This possibility, although being expressed strongly by Minh when we first met him, seemed to generate conflict one year later because of his parents'

expectations that he would gain a high status job such as a medical doctor. However, being good at mathematics may not necessarily be so advantageous in obtaining such a career since it was not estimated good enough to succeed in high school and to make it through university. For a possible future as a medical doctor, mathematical competence seemed to be subordinated to his mastery of the Danish language, according to the Danish teacher, his class teacher, a person who plays an important role in guiding Minh in his further choices at the end of the compulsory school. For Minh, what was being discussed was not only the teachers' and parents' perceptions of his possible foregrounds but also his identity. His construction of himself as "being good at maths" was now being constructed as "not being good enough at mathematics" to overcome perceptions of his lack of competency with the Danish language. It is the complex interconnections, rather than just his achievement in mathematics, that contributed to construction of such an identity. Consequently the opportunities that would be open to someone who was good at mathematics were now restricted to the opportunities for someone who was not good enough at mathematics.

The mathematics teacher did not express any particular concern about Minh's competence in the Danish language. In the context of the mathematics classroom, this did not seem to be an issue that represented an obstacle for Minh. He was competent in school mathematical discourse. However, in a Danish context, there is a tradition of collaboration among teachers in interdisciplinary teams organised around a class teacher. The Danish teacher, had a great influence on assessing students' strengths and guiding students and parents with advice about possible future study paths. This teacher's assessment of Minh's lack of mastery of the Danish language was decisive for how he was perceived in general in the school and how different schoolteachers' practices contributed to create an image of Minh's perceptions of his foreground.

Moreover, the issue of the lack of competence in the Danish language seemed to be connected not to the fact that Minh may have lacked linguistic competence, but that Minh and his family were lacking the cultural capital associated with adequate linguistic fluency. Here, we find how public discourses around multiculturality and immigrants' possibilities and limitations enter the school scene: second generation immigrants such as Minh and his family lack the cultural (and even also the social) capital needed to succeed in higher education. Competency and fluency in the language of mathematics do not seem to be enough to succeed and climb up the social ladder. The view expressed by the Danish teacher seems to resonate with the idea that many immigrant families have weak cultural resources and therefore are unable to help their children with the

linguistic habits that are necessary to succeed in school as a whole (not only in mathematics). When such a perception is part of the teachers' views and perspectives, then the perception that immigrant parents have extremely high (and unrealistic) ambitions for their children emerges as a kind of explanation among teachers for the mismatch between the school expectations and children and parents' wishes for their future.

The analytical tool when applied to Minh's story shows how it can highlight certain dimensions without loosing sight of the interconnections to others, especially when the connection is not immediately obvious. It shows how Minh's competency in regard to school mathematical discourse through his participation in class was of only limited value because it was not combined with cultural baggage about Danish that could only be provided by the family. The tool highlights these aspects by first enabling them to be discussed separately and then for the connections between them to be described.

Te reo tätaitai: developing rich mathematical language

For many years, we – Tamsin, Tony and Uenuku – have worked with teachers and others at a Mäori immersion school, *kura kaupapa Mäori*, in New Zealand. In the first project, 1998–1999, we worked on developing a culturally appropriate mathematics curriculum. School mathematical discourse was just one issue that was considered. In the second project, which is ongoing from 2005, we are investigating issues to do with using the Mäori language, *te reo Mäori*, in the teaching of mathematics. This section then draws upon the case study of a particular school, Te Kura Kaupapa Mäori o Te Koutu, or Te Koutu for short. Students at this school achieve well in mathematics, not just compared to other Mäori students in mainstream school but also when compared to all New Zealand students.

Te Kura Kaupapa Mäori were instituted by Mäori communities, initially without support from the state education system, in order to maintain te reo Mäori and to improve educational outcomes for Mäori children (May, 2003). Consequently, issues to do with language were never far away from teachers' and parents' minds, but on the whole being multilingual was something to be celebrated. Teachers at Te Koutu continue to discus how to increase their students' fluency in te reo Mäori as well as in *te reo Tätaitai*, the mathematics register (Meaney, Fairhall & Trinick, 2007). However, this is not the only conversation about school mathematical discourse.

For this school, the parents and teachers had decided that the "language of instruction for mathematics is Mäori, language adapts with every new situation that it meets, rich language leads to a fuller understanding and

discussion" (Minutes from previous meeting read out at Meeting held 15/8/99). With any language of instruction, they felt that there was a need to specifically teach the features of te reo Tätaitai because these may not be shared across a community. The parents may have learnt a different version of the mathematics register than that learnt by their children. For example, the parents discussed how the way that multi-digit additions and subtractions had been done when they were at school was different to how their children were being taught.

T4: And we talked about the different languages that we use, Parent3 found out about the way that we were saying our times was totally different to how she was understanding it, you know. Yeah.

T3: Oh and then we got into decomposition as opposed to abracadabra.

T4: Talking about that, yeah, how the language is different because we're talking about maths and that. (Meeting 15/8/99)

If the parents did not understand the mathematics including the school mathematical discourse, it was difficult for them to talk with their children and help them with their homework. Parental involvement is something that marks Kura Kaupapa Mäori as being very different from other New Zealand schools and contributes to a shared objective of supporting students to do well. Therefore, finding ways to support this occurring was an aim for the school.

In the following extract from a lesson in 2006, a Year 12 class discussed the r and t co-ordinates and the t-intercept (k) in regard to the equation for a straight line, $t = pr + k$. This discussion began with ideas about the terms for a particular line.

S: Äe, tërä mea i konei kei runga.

T7: Nä, ko te kotinga 't' he törunga, he töraro ränei?

S: He törunga.

T7: Törunga. Engari käore tätou i te möhio, nö reira, täpiri.

S: Täpiritia te 'r', oh, he 'k'.

S: He aha?

T7: Äe, täpiritia te 'k' i tënei wä.

S: 'W', oh, rima e (?)

T7: Arohamai, i hoatu i tëtahi mea uaua ki a koutou. Kei te tere whakaatu pëhea te kimi i te 'k'.

S: Ka taea te whaka, ah, (?) atu i ngä rärangi mö te mahi tukutuku. I te mea he tino uaua, äe, ki ngä mätua.

S: Äe. Nä, te mea käore e taea te kite.

The transcript above illustrates a situation when the students showed confusion over the meaning of the different terms used in a lesson and the teacher (T7) acknowledged that it was difficult to learn all the words. One student followed this up by saying that it would be very difficult to explain it to parents and this was confirmed by another student.

In kura kaupapa Mäori, parents are an integral part of their children's education process (Meaney & Fairhall, 2003). As a result, various activities have been instigated over the years to increase parents' knowledge about using te reo Mäori to talk with their children about mathematics. This has been integrated with discussions about current trends in the teaching of mathematics. Parents, thus, had opportunities to discuss with children their mathematics learning, even if the parents did not speak te reo Mäori.

The school also recognised the students' future needs when learning mathematics. Mathematics symbolism was seen as one way to support students switching between different languages of instruction.

> T3: I think a really important issue in Kura Kaupapa is that mathematical symbolic language has to be really emphasised because it's one of the things, that's the crutch, which let's them move from language to language. They've got to see the mathematics there and written in mathematical notation, if they don't get on top of that then they are, they will have those problems when they shift over. (Meeting 15/8/99)

At Te Koutu, English, the dominant language in mainstream New Zealand, is not taught formally until students begin Year 7. However, Spanish is introduced as an additional language when children start at the kura at the age of five. In mathematics lessons, both English and Spanish can occur with te reo Mäori. Figure 2 shows posters of 3-D shapes, some giving the title in te reo Mäori, *Ahua 3-D*, and some in Spanish, *Formas de 3D*. It also shows a copy made by a student of his teacher's notes about the features of a triangle. *Koki* has the English translation, angle, underneath.

In kura kaupapa Mäori, teaching mathematics is done in te reo Mäori. Although this is the second language of most students, the school community considered this essential if te reo Mäori was to be pulled back from the brink of extinction. Nevertheless, the choice to teach in te reo Mäori has not been without its challenges, but through ongoing discussion these challenges are being overcome. Unlike the Danish situation, in kura kaupapa Mäori being bilingual was seen as being useful and this shared belief contributes to students achieving in mathematics.

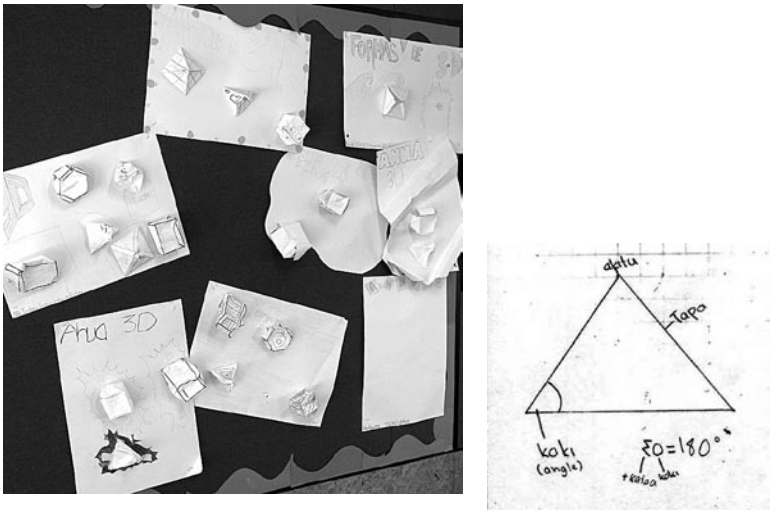

Figure 2. *Student work from Year 7*

A learning landscape in a kura kaupapa Mäori

In some ways, the case study about learning mathematics in Te Kura Kaupapa Mäori o Te Koutu is disjointed as we had to make choices about what to include and what to leave out from our longitudinal project. What we wanted to illustrate was the complexity in considering how learning of mathematics occurs in this situation and at the same time the centrality of school mathematical discourse. We also chose aspects that were related to Minh's story. However, as a result of trying to stay in contact with the complexity and keep a broad view of language, the connections between different aspects of the story are blurred. Applying the model in figure 1 as an analytical tool to the story allows some of the connections to become clearer.

The model has no set starting point for the analysis but rather allows understanding to develop from any position. This means that we could start from the centre, school mathematics discourse, and then work out, or we can begin from the petals of the flower and see what they say about the centre. In the story of Minh, the centrality of school mathematical discourse becomes clearer by first considering the petals. In the story of Te Kura Kaupapa Mäori, the reverse is the case. It is from understanding the role of language, in particular school mathematical discourse, that the complexity of influences on the learning processes becomes clearer.

In considering the centrality of school mathematical discourse, the first question to arise could be about why should a community choose to

use te reo Mäori in schools when it involved engineering a mathematics register? One response comes from looking at the operation of public discourse at the time that Kura Kaupapa Mäori were established. The recognition of the loss of te reo Mäori and the poor academic outcomes for Mäori students in mainstream schools was very much part of the public discourse for the Mäori community in the 1980s (Bishop & Glynn, 1999). This discourse contributed to the identification of alternatives for Mäori students. Rather than constrain the discussion about what was normal/possible, it broadened many parents' perceptions. The open discussion about the reality that schooling was unlikely to improve Mäori students' academic outcomes or revive the dying language, te reo Mäori, meant that it was possible to envisage a completely different schooling system. There was a perception by the Mäori community that the schooling system at that time was so dire that any alternative had to be an improvement and this has indeed proved to be the case. However, the recognition that many parents were themselves products of the limited mainstream education meant that the kura had to provide opportunities for parents to learn about mathematics if they were to use a shared school mathematical discourse with their children.

From the insights gained from considering the influence of public discourse, the next question could be about the impact of teaching and learning mathematics in a language where the mathematics register had to be engineered (see Meaney, Fairhall & Trinick, 2008). Insights on this issue can be gleamed from considering three different dimensions of the model: classroom interaction; teachers' perspectives; and parents' perspectives. The discussion from 1999 between parents and teachers showed an awareness that there might be difficulties in learning mathematics in te reo Mäori. As well, the discussion illustrated that the parents believed that these difficulties were similar to those faced by students in learning the mathematics register in any language. They also felt that because of the changes that occurred in approaches to teaching mathematics, the mathematics register that their children learnt was likely to be different to the one that they had learnt at school. The parents and teachers' perceptions showed an awareness of the potential difficulties faced by students in using the relevant school mathematical discourse. However, no one believed that these were insurmountable difficulties.

The classroom interaction illustrated the difficulties students faced in learning the vocabulary and grammatical expressions of the mathematics register. However, the fact that it was openly discussed meant that the teacher could empathise with the students' difficulties whilst also providing specific activities to support their learning. The strong belief in students' capabilities contributed to the ways that the teachers support

students' learning. The strategies that teachers felt were valuable to their students in acquiring the mathematics register included ones that supported their students becoming self-regulatory in their learning. Over the course of our 2005–2006 project, teachers adopted strategies where they "provide students with and have them describe a rationale for their learning" (Meaney, Fairhall & Trinick, 2007, p. 78).

By considering the different petals of the model, it is possible to see the relationship between the choice of the language of instruction made as a result of public discourse and the consequences for students in learning mathematics through using the mathematics register in te reo Mäori. It also shows how parent and teacher perceptions' of the importance of using te reo Mäori meant that they were able to consider the difficulties that students faced as being no more problematic than difficulties faced by students who learnt mathematics in an international language such as English. As a consequence, teachers were able to empathise with students but then look for ways to move them forward in their learning because the teacher also believed that this difficulty could be overcome. Students could not be excused poor achievement because they were learning in a second language, rather all students were expected to achieve in mathematics whilst at the same time improving their fluency in te reo Mäori.

The model has a petal for tools and resources that has an interesting connection to school mathematical discourse. The teachers not only drew on the students' knowledge about purposes for learning but also their knowledge of te reo Mäori, English and Spanish. The teachers recognised and made use of these linguistic resources because they perceived them as being valuable. Another resource that they considered valuable was that of mathematical symbolism. They felt that this would contribute to students' opportunities to continue to learn mathematics in the future and, thereby, generally broadening students' life possibilities of further schooling and education. This view can be related to how the teachers see mathematics learning as part of the students' foregrounds, which is another petal on the model. x and y as algebraic variables were not used in te reo Tätaitai because these letters are not part of the Mäori alphabet. However, the mathematical reasons behind using letters in algebraic reasoning was considered important and this is what was concentrated on in class. The concept of variables was a transferable concept that students could make use of in any language. If students moved on to situations where English was the language of instruction then their understanding of algebra was not specific to te reo Mäori. Thus, applying the model to the situation in this Kura Kaupapa Mäori meant that two more petals, tools and resources and students' foregrounds can be seen to be connected through school mathematical discourse.

The model in figure 1 can be used to show how different aspects of a learning landscape are related in multicultural settings. Starting from the perspective of school mathematical discourse it is possible to move outwards to see how the different petals are related. In our analysis we did not find connections to all the petals of the learning landscape. However, it was possible to see connections between different aspects such as tools and resources and students' foregrounds that were not clear from just looking at the data. This means that particular aspects that connect with school mathematical discourse can be identified without isolating them away from other aspects with which they interact. Consequently, the complexity of the learning landscape can be shown in a number of different ways.

Bringing the cases together: the power of the petals

Learning is considered as "the process in which persons make the decision of engaging in getting to know" (Alrø, Skovsmose & Valero, 2007, p. 2). In multicultural settings as with other settings, this process is not achieved in isolation. Consequently, the learning landscape provides information about the influences on this decision making act, and the model in figure 1 provides an important tool in this analysis. However, like any research tool, considerations about *school mathematical discourse in a learning landscape* will bring some aspects of the phenomena being studied into focus whilst blurring others (Schoenfeld, 2008). In considering how useful this tool is we need to identify what becomes the focus whilst also considering what becomes out of focus.

Our reasons for using this model is that it enables complexity involved in the interaction between learning landscapes and school mathematical discourse to be explored but without it being simplified. As is seen in the two case studies, it is the complexity that results in some students gaining rich and rewarding opportunities for their futures, whilst for others it produces restricted and regimented ones. For Minh, the complex interplay of the different dimension was resulting in him having restricted opportunities for his future. On the other hand, students at Te Koutu had a greater likelihood of a range of foregrounds because their learning of mathematics in a second language was not perceived in anything but a positive light. In trying to understand how this differentiating process occurs, we acknowledge that we are dealing with issues of inclusion/exclusion and social justice. The importance of these issues requires us to employ an analytical tool that works with rather than against the complexity. Simplifying what happens in classrooms has often resulted in intervention programmes that focus on one aspect such as fluency in

the language of instruction. Quite often, as was the case for the special language support provided to Minh in the past, the results are seen as having little residual value. It is by seeing how the different petals of the model are connected that a better understanding of the situation can be gained.

An analysis of these case studies using the model presented in figure 1 highlights the connections between different aspects of the stories. School mathematical discourse is a thread that runs through any learning landscape. However, in the situation where the students are learning mathematics in a second language, the linkages that it provides between other aspects of the learning landscape may be more obvious than when the students are using their first language as the language of instruction. By using aspects of the learning landscape as an analysis tool, it is possible to not only identify challenges but also to see how these challenges can be successfully managed in practice.

May, Hill & Tiakiwai (2004, p. 49) made the following statement about different approaches to bilingualism:

> Māori-medium contexts foster *additive* bilingualism – that is, their goal is to acknowledge and promote bilingualism, and its attendant advantages. In contrast, English-only submersion programmes are invariably subtractive bilingual contexts – that is, they specifically devalue and exclude the child's bilingualism, and thus also the potential resource such bilingualism is to learning.

The two case studies described in this paper show how beliefs about being bilingual can contribute to perceptions by students, teachers and family members of success or failure within a learning landscape. When bilingualism is seen as something valuable a range of opportunities are presented to support students in making decisions to engage in getting to know. When bilingualism is seen as problematic, then opportunities can be restricted to gaining fluency in the language of instruction. How these opportunities and restrictions are put in place is not always obvious and applying the model does provide insights into this process.

The model provides multiple entry points in order to understand the combination of factors that contribute to the different outcomes experienced by students in multicultural situations. In Minh's story, the model gave us insights into perceptions of the value of school mathematical discourse by starting our investigation with some of the dimensions. These dimensions illuminated how being fluent in school mathematical discourse was not enough to lead to a university education. In the story of Te Kura Kaupapa Mäori o Te Koutu, we began at the centre of the model with school mathematical discourse and then moved out to see how it

was related to the different dimensions. This exploration showed the importance of the connections between the dimensions, sometimes in unexpected ways such as in regard to the tool of mathematical symbolism being perceived as a resource by teachers and parents for students' foregrounds. In both stories, the petals have the power to provide insights into the situation, individually but also by looking at the connections that existed between them. However, without the anchor of the school mathematical discourse, either as a starting or finishing point for the analysis, then it would be difficult to see how the myriad of different aspects were connected.

For us, this analytical tool provided insights by embracing the complexity of the situation. Nevertheless, other aspects of the situation were blurred by adopting this tool. Using this tool could mean that we "find" connections between petals which are in reality of little value in trying to understand what contributes to the differentiating of outcomes for students. The tool could allow too much to be focused on, thus making it difficult in any specific situation to know what value should be given to different aspects or connections between the dimensions.

Another aspect that is blurred by looking at the different petals and the connections between them is the way that power is distributed between the different participants. For example, the process by which knowledge, such as the usefulness of being bilingual, becomes valuable may be clearer if school mathematical discourse is replaced by power at the centre of the flower. In order to highlight how power operates in this process, it may need to be the centre of the model with school mathematical discourse being subsumed into the tools and resources petal. Thus, although the model with school mathematical discourse in the centre can be useful in unfurling the complexity of the learning landscape of multicultural classrooms, it is not able to show the interrelatedness of all the many different aspects of the mathematics classroom. Without a clearer and more explicit focus on issues of power within the flower, the understanding of the relationship between language, mathematics education and inclusion it allows is not yet optimal.

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Notes

1 This story is based on our empirical investigations in the project "Learning From Diversity" (Alrø, Skovsmose and Valero, 2003, 2005). In the study, the research team (Helle Alrø, Ole Skovsmose and Paola Valero) followed for a period of two school years, two classes of students and their teachers. We have carried out empirical foregrounds investigations (see Alrø, Skovsmose & Valero, 2009), a participant observation study in the school, and a study based on a classroom intervention run by the researchers. The case of the student presented here has emerged from an analysis of a combination of information from the different studies in the two years period.

2 Mælkevejen Skole is a primary and lower-secondary school situated in a low-income, difficult suburb of a bigger Danish city. This school hosts students from 29 countries from all over the world. We established collaboration with two 8th grade classes of 24 and 21 students respectively. Half of them come from countries such as Iceland, Ireland, Iraq, Greenland, Lebanon, Malaysia, Somalia, Turkey and Vietnam. The two teachers in the classes we followed are a young man and a very experienced man, both Danish.

3 Following Kvale (1996), we use inter-views as a form of inquiry that allows the inter-viewer and inter-viewed to "see together" and explore together the object of research, in this case, the students' foregrounds. For more details see Alrø, Skovsmose and Valero (2009).

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Sammendrag

I denne artikel bringer vi vore respektive forskningsperspektiver i spil ved at diskutere forholdet mellem begreberne *læringslandskab* og *skolematematisk diskurs*. Formålet er at udvikle teoretiske begreber og metodologiske redskaber, som kan udfordre faren for en forenkling af de forhold, som har indflydelse på matematiklæring i multikulturelle kontekster. En sådan forenkling kan f.eks. ses i begrebet om "de tosprogede". Vi diskuterer læringslandskab og skolematematisk diskurs i forbindelse med en analyse af to case studier fra hhv. Danmark og New Zealand. I konklusionen peger vi på de muligheder, som disse begreber kan tilføre forskningen inden for matematikundervisning i det multikulturelle klasseværelse.