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Critical mathematics education for the future

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Abstract

Mathematics education can mean disempowerment or empowerment. It does not contain any strong 'spine', but could collapse into rigid forms and support problematic features of any social development. However, mathematics education can also contribute to the creation of a critical citizenship and support democratic ideals. The socio-political roles of mathematics are neither fixed nor determined. In this sense I talk about mathematics education as being critical.

I see critical mathematics education as a preoccupation with challenges emerging from the critical nature of mathematics education. Critical mathematics education refers to concerns which have to do with both research and practice, and a concern for equity and social justice being one of them. Here I want to refer to the following challenges: (1) How do processes of globalisation and ghettoising frame mathematics education? (2) What does it mean to go beyond the assumptions of Modernity? (3) How should 'mathematics in action', including a mixing of power and mathematics, be interpreted? (4) What forms of suppression can be exercised through mathematics education? (5) How could mathematics education provide empowerment?

Such questions reflect an uncertainty with respect to the possible socio-political functions of mathematics education. Facing this uncertainty, this aporia, is a characteristic of critical mathematics education. This cannot be based on any political or epistemological foundation. Our situation is similar to that of those who need to construct a ship while swimming around in the open sea.

Introduction

Does critical mathematics education embody an obsolete line of thought? Is it just a leftover from an outdated leftist educational movement? If not, what could critical mathematics education mean today and for the future?

I see critical mathematics education as an expression of concerns for what socio-political roles mathematics education might play. Critical mathematics education has many roots, one of which is found in Critical Theory that also nourished critical education in general. Sources of inspiration can, however, also bring about presumptions, which can obstruct further development. I suggest there is a need for critical mathematics education to become re-conceptualised, and developed with new references. Roots are important, but an uprooting can sometimes be necessary.

There is an ongoing discussion in education, especially amongst critical educators, about the relationship between research and practice. One could expect a

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‘resonance’ between, on the one hand, theoretical and methodological considerations, and, on the other hand, priorities and approaches within educational practice. Although I might articulate issues with reference sometimes to research and sometimes to practice, I intend, in what follows, not to address research and practice as separate entities. I want to express some of the concerns of critical mathematics education that have significance for both research and practice, i.e. concerns which could bring about the resonance to which I have just referred.²

I have already used the notion ‘critical mathematics education’ several times. It is not clear to what that might refer, in particular when I have a kind of uprooting in mind. In order to provide some initial clarifications I consider the claim that ‘mathematics education is critical’, before I try to clarify the notion of ‘critical mathematics education’. This brings me to the concerns of critical mathematics education, and I want to address the following questions: (1) How do processes of globalisation and ghettoising frame mathematics education? (2) What does it mean to go beyond the assumptions of Modernity? (3) How should ‘mathematics in action’, including a mixing of power and mathematics, be interpreted? (4) What forms of suppression can be exercised through mathematics education? (5) How could mathematics education provide empowerment? Finally, I will sum up the basic uncertainty in addressing such concerns by referring to the notion of *aporia*.³

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Mathematics education is critical

Mathematics education could mean empowerment, but also suppression. It could mean inclusion, but also exclusion and discrimination:

“Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an ‘objective’ judge, in order to decide who in the society ‘can’ and who ‘cannot’. It therefore serves as the gate keeper to participation in the decision making processes of society. To deny some access to participation in mathematics is then also to determine, a priori, who will move ahead and who will stay behind.” (Volmink, 1994: 51-52)

This statement by John Volmink can be read as a dramatic description of the role of mathematics education in marking a division between those who become included in and those who become excluded from society. (I do not propose that mathematics education, or education in general, provides the main cause for social inclusion and exclusion. Many causes come in play together, but mathematics classrooms are important sites to consider.)

There is no lack of suggestions of how to interpret mathematics education as serving questionable socio-political roles. Besides operating as a gate keeper, it can ensure the social order in such a ‘smart’ form that ‘rational’ citizens, by using their own free will, accept an imposed order.⁴ Mathematics education can support the development of an ideology of certainty (Borba and Skovsmose, 1997); it can provide

² For a more explicit discussion of methodological issues related to critical mathematics education, see Vithal (2003), in particular Chapter 2; Vithal and Valero (2003); Valero (2004); Valero and Zevenbergen (Eds.) (2004); and Skovsmose and Borba (2004).

³ For a more detailed exposition of the concerns of critical mathematics education, see Skovsmose (2003).

⁴ This is how Walkerdine (1989), inspired by Foucault (1977), interprets the possible functioning of mathematics education.



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an unjustified ‘trust in numbers’ (Porter, 1995). Paul Dowling (1998) has emphasised how mathematics education establishes different curricula for different groups of students, and in this way it influences what opportunities become available (or not available) for different groups of students. Generally speaking, mathematics education and social stratification interact.

However, mathematics education can also serve to empower students. Thus, Renuka Vithal (2003) outlines what mathematics education for empowerment could mean in a South African context, and Eric Gutstein (2003a) discusses empowerment with reference to a Mexican community in the USA. Ethnomathematical studies have discussed what empowerment might mean in different cultural settings (see, for instance, D’Ambrosio, 2001; Knijnik, 1999, 2002; and Ribeiro, Domite and Ferreira, 2004); and Arthur Powell (2002) and Marilyn Frankenstein (1989, 1995, 1998) have discussed what empowerment could mean for marginalised adults in a USA metropolis.⁵ Helle Alrø and I have discussed empowerment, in terms of “mathemacy”, with reference to students in a Danish context (Alrø and Skovsmose, 2002). All such references indicate that it is possible to relate the learning of mathematics to empowerment, and also that an interpretation of empowerment depends on the particular contexts of the learners.

Mathematics education can mean disempowerment or empowerment. Mathematics education does not contain any strong ‘spine’, but could collapse into dictatorial forms and support the most problematic features of any social development, exemplified by the adaptation during the 1930s of mathematics education in Germany to Nazi-friendly forms (Mehrtens, 1993). However, mathematics education can also contribute to the creation of a critical citizenship and support democratic ideals. The socio-political roles of mathematics are neither fixed nor determined. Both roles, of being a hero or a scoundrel, are available to being enacted through mathematics education. In this sense I talk about mathematics education as being *critical*.⁶ I do not relate mathematics education to any optimistic position claiming the existence of an intrinsic connection between mathematics education and, say, democratic values. Nor do I claim that mathematics education *per se* will serve anti-democratic interests. Instead, I simply claim that no actual functions of mathematics education represent its essence. There is no such essence.

The critical nature of mathematics education represents a great uncertainty. Naturally, it is possible to try to ignore this uncertainty. This can, for instance, be done by assuming that mathematics education can somehow become ‘determined’ to serve some attractive social functions when organised in, say, a national curriculum crowned by some nice-looking aims and objectives. But I find this an illusion. The functions of mathematics education cannot be determined (or re-determined) by some positive guiding principles in the introduction sections of the curriculum. There are no straightforward procedures for ‘determining’ the functions of mathematics education, as they might depend on many different particulars of the context in which the curriculum is acted out. To acknowledge the critical nature of mathematics education, including all the uncertainties related to this subject, is a characteristic of *critical mathematics education*.⁷

⁵ See also Frankenstein (1987) and Powell and Frankenstein (Eds.) (1997).

⁶ For a discussion of the ‘critical’ position of mathematics education see also Skovsmose and Valero (2001), where the relationship between mathematics education and democracy is presented as being critical. See also Valero (2002).

⁷ Both ‘crisis’ and ‘critique’ are derived from the Greek word *krinein*, which refers to ‘separating’, ‘judging’ and ‘deciding’. A ‘critical situation’ or a ‘crisis’ brings about a need for action and involvement, i.e. a need for critique.



Critical mathematics education is not to be understood as a special branch of mathematics education. It cannot be identified with a certain classroom methodology, nor can it be constituted by a specific curriculum. Instead, I see critical mathematics education as being characterised through concerns emerging from the critical nature of mathematics education. These concerns have to do with both research and practice. And, as we shall see in what follows, many share these concerns.

First concern: Globalisation and ghettoising

Globalisation is not a new phenomenon. During the explorations of the 15th and 16th centuries globalisation was accompanied by suppression and by a cultural invasion that interpreted the existing culture as inferior, and later, maybe, as fascinating and picturesque.

Today globalisation refers to the facts that new connections are established between previously unconnected social entities and that what is happening to and done by one group of people may affect, for good or bad, a completely different group of people, even people unaware of the nature of the effect. Thus, globalisation can mean a creation of interrelations accompanied by a loss of transparency. The concept of globalisation contains both positive and negative connotations: “For some, ‘globalization’ is what we are bound to do if we wish to be happy; for others ‘globalization’ is the cause of our unhappiness. For everybody, though, ‘globalization’ is the intractable fate of the world, an irreversible process ...” (Bauman, 1998: 1).

We might think of ghettoising as a kind of side effect of globalisation, but it could also be seen as an integral part of it: Globalisation also means ghettoising. Zygmunt Bauman makes the following observation: “Globalization divides as much as it unites; it divides as it unites – the causes of division being identical with those which promote the uniformity of the globe.” (Bauman, 1998: 2) To me, globalisation and ghettoising represent different aspects of (or different perspectives on) our society. Manuel Castells (1998) refers to the *Fourth World*, which is created through globalised processes of social, economic and political exclusion. The Fourth World includes large parts of what is traditionally referred to as the ‘Third World’. However, the Fourth World is not defined simply in geographical terms, as it spreads around the world. It is present in any metropolis, in any country. It includes those very many and large groups of people who are not operating ‘functionally’ within the globalised economy.

Globalisation and ghettoising have to do with schooling, education and learning in general. Many schools operate on the borderline between the Fourth World and the rest of society. Schooling can be seen as a support for entering the social order, but it also becomes a gate keeper and an ‘excluder’ from global ‘networking’.⁸ Much statistics illustrates the universal extent of inequality in education.⁹ Thus, while 16% of the total number of children in this world do not go to school, the total number of children in the so-called developed world (including North America, Western Europe, Japan, Australia and New Zealand) is only 10 % of the world’s population of children. If we consider the paradigmatic priorities in educational research, however, the learning conditions of the 10% are extremely over-exposed compared to the learning conditions of the 16%. In fact conceptual development in education, as well as the development of learning theories, appear extremely biased.

⁸ See, for instance, Bigelow and Petersen (2002).

⁹ See, for instance, UNESCO (2000).

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Mathematics education is part of the processes of globalisation and ghettoising. It is not easy to identify how mathematics operates/might operate in this context. However, some paradigmatic assumptions appear to guide the mathematics education research community, and they might make such identification even more difficult. One such assumption establishes the dominance of the prototype mathematics classroom in mathematics education research (although not in mathematics education reality). By prototype mathematics classroom I am referring to a classroom that is well-resourced, in particular with a computer with adequate network facilities if necessary. It has an adequately educated teacher. There is no obvious problem of motivating the students. There is no violence, and no police positioned on the school premises. There are no hungry students. A whole discourse in mathematics education has been developed around such a prototype mathematics classroom. When we consider all transcripts presented in mathematics education over the years, the prototype mathematic classroom is the one most elaborately presented.¹⁰ This classroom might be difficult to find in the Fourth World. It might also be difficult to locate it in the rest of the world. Nevertheless, it operates as a research construct in mathematics education.

To critical mathematics education, the discursive dominance of the prototype mathematics classroom is a problem to the extent that it hides how mathematics education operates with respect to inclusion and exclusion on a global scale. Critical mathematics education tries to question the dominance of any 'prototype thinking'.¹¹ Many studies have in fact moved beyond the prototype mathematics classroom and in this way demonstrated concerns of critical mathematics education.¹²

Second concern: Beyond the assumptions of Modernity

Modernity can be characterised in many ways, but I limit myself to emphasising two assumptions, concerning (1) the existence of an intimate connection between scientific and social progress, and (2) the possibility of establishing epistemic transparency. Modernity stretches back in time and includes the period of the Enlightenment that celebrated the importance of knowledge, but the extent to which Modernity might stretch into the future is questionable, as notions like post-modernity, risk society, liquid modernity, hyper-complex society have gained applicability.

One characteristic assumption of Modernity is that the path of science, as initiated by the scientific revolution of the 17th century, is always in the direction of progress. However, it is not only science, but also society in general that is considered to be progressing, meaning that we can work our way towards utopia. The principal claim is that scientific progress provides a motor for 'true' socio-political progress (indicated by the fact that the Scientific Revolution was followed by the Industrial Revolution).

¹⁰ 'Proto-typing' in mathematics education is discussed in Skovsmose (2004a).

¹¹ Let me add that my concern about the discursive dominance of the prototype mathematics classroom is 'paradigmatic'. Thus, it might be very interesting and important to study this classroom (and in fact many of my own studies have done so). Studies of the prototypical mathematics classroom can well form part of critical mathematics education. The problems emerge when a preoccupation with the prototype mathematic classroom turns into a neglect of fundamental processes of inclusion and exclusion.

¹² Such studies are presented in, for instance, the proceedings of the Mathematics Education and Society conferences, the latest edited by Valero and Skovsmose (2002). Notions like 'schizo-mathematicslearner' or the 'supermathematicsteacher'. as developed by Valero (2002, 2004), serve also to counteract the prototyping in mathematics education research and to sharpen the critical self-reflection of mathematics education research..

Knowledge represents epistemic welfare, and epistemic optimism became a characteristic of Modernity.¹³

A second assumption of Modernity claims the possibility of establishing epistemic transparency, meaning that it is possible to specify, in a simple and clear-cut way, what counts as knowledge and what does not, and how knowledge can be pursued. This transparency was pointed out in René Descartes's rationalism and John Locke's empiricism. Epistemic transparency is also an integral part of the outlook of logical positivism with the suggestion that although it might be a complex task to identify specific bits and pieces of scientific knowledge, it would be simple, logically speaking, to provide an overall philosophical characterisation of knowledge.

In *Democracy and Education*, first published in 1916, John Dewey expressed the assumptions of Modernity in the following way:

“The coincidence of the ideal of progress with the advance of science is not a mere coincidence. Before this advance men placed the golden age in remote antiquity. Now they face the future with a firm belief that intelligence properly used can do away with evils once thought inevitable. ... Science has familiarized men with the ideas of development ...” (1966: 224-225).

Dewey saw science as “an indispensable factor in social progress” (1966: 226) and as “the sole instrument of conscious, as distinct from accidental, progress” (1966: 228).

The assumption of Modernity, which also has to do with mathematics, has been challenged in the articulation, by D'Ambrosio (1994), of the following paradox:

“In the last 100 years, we have seen enormous advances in our knowledge of nature and in the development of new technologies. ... And yet, this same century has shown us a despicable human behaviour. Unprecedented means of mass destruction, of insecurity, new terrible diseases, unjustified famine, drug abuse, and moral decay are matched only by an irreversible destruction of the environment. Much of this paradox has to do with an absence of reflections and considerations of values in academics, particularly in the scientific disciplines, both in research and in education. Most of the means to achieve these wonders and also these horrors of science and technology have to do with advances in mathematics.” (D'Ambrosio, 1994: 443)

On the one hand, our knowledge of nature and the development of new powerful knowledge-based technologies have surpassed any possible expectation, and, on the other hand, we witness a ‘despicable human behaviour’ resourced directly by this very same knowledge of nature and of knowledge-based technologies. Scientific progress does not simply bring about ‘wonders’. Progress is also accompanied by ‘horrors’, implying that the very meaning of ‘progress’ becomes obscure. Such a paradox seems incomprehensible, if we consider the two assumptions of Modernity. D'Ambrosio's paradox annihilates the assumption of the intrinsic connection between scientific and socio-political progress. And, according to D'Ambrosio, mathematics is positioned in the middle of this paradox. There are no epistemic transparent qualities in mathematics to ensure that mathematics in action (a notion I explore further in the following section) brings about ‘wonders’. Although it has been cultivated and organised in

¹³ For a discussion of the notion of progress see Bury (1955) and Nisbeth (1980).



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formal structures, and although mathematics in this way appears pure, mathematics in action can include implications of all different sorts. We can observe horrors and wonders in an unpredictable mix. In fact, the situation might be even more confused and uncertain: We might be lacking any reasonable standards for distinguishing between ‘horrors’ and ‘wonders’. Transparency is lost.

When the assumptions of Modernity are substituted by a paradox, mathematics education faces a new challenge. It seems impossible that mathematics educators could any longer serve as ambassadors for mathematics, implying that the primary task of mathematics education is to bring students into mathematics. The assumptions of Modernity hide any position in which it is important to be critical of mathematics and mathematics education. I find that much mathematics education has enjoyed the protective shield established by the assumptions of Modernity. But if the intrinsic link between mathematics and socio-political progress cannot be maintained, mathematics education must address the uncertainty with respect to what might be done through mathematics.

Third concern: Mathematics in action

Mathematics has been described as the sublime and purified language of science. However, speech act theory and discourse theory have brought about radically different ideas as to what language might be doing. When we see language and also the language of mathematics from the perspective of discourse theory, the questions become: What might be done through mathematics? How is the world organised according to mathematics? Mathematics becomes related to action, and I talk about ‘mathematics in action’. Michel Foucault related knowledge and power.¹⁴ He exemplified this broadly, but he did not include reference to how scientific and mathematical knowledge could be related to power. I find, however, that with mathematics we find a paradigmatic case of interaction between knowledge and power. By delving into this issue, we see the doubtfulness of maintaining the existence of an intrinsic connection between scientific and socio-political progress. A study of mathematics in action brings us beyond the assumptions of Modernity.

Mathematics is brought into operation in very many different contexts, and in daily-life practices we find interesting ‘meetings’ of different forms of mathematics in action. A product has been fabricated according to particular schemes; it has, to some extent, been tested (and risks connected with the use of the product have been estimated). The product has been advertised. The consumer’s interest in buying the product has been estimated, and this interest depends on the marketing of the product. The consumers, in particular their propensities to buy, have to be represented in a marketing model. The cost of distributing the product on the market must be estimated. And the whole set of estimations has to be condensed into the price of the product. A huge variety of mathematical modelling is included in all procedures and decision-making that brings the product onto the market. Here the product meets the real consumer who is a living organism for mathematics in action. Mathematical reasoning, included in common sense, contributes to the decision-making: to buy or not to buy.

Almost everything can turn into products for sale, seats in airplanes providing only one example. And also in this case mathematics is brought in action; such action includes highly sophisticated sales promotions. Airlines deliberately overbook in order

¹⁴ See, for instance, Foucault (1989, 1994).



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to maximise profit.¹⁵ It is essential to try to prevent departures with empty seats, as the costs associated with flying a full airplane or one with empty seats are approximately the same. For every departure, it is most likely that some of the passengers who have already booked will fail to turn up, the so-called ‘no shows’. As a consequence, it seems economically advisable for airline companies to overbook flights, but, certainly, there is an upper limit to this, as the company compensates those passengers who might be refused, or ‘bumped’. The probability of a passenger being a ‘no show’ depends on, for instance, the destination, the time of the day, the day of the week, and the type of his or her ticket. All this can be incorporated into a mathematical model containing parameters such as the cost of providing a flight, the fare paid by each passenger, the capacity of the airline, the number of passengers booked on a flight, the costs of refusing a passenger who has booked a ticket, the probability of a booked passenger being a ‘no show’, the surplus generated by a flight, etc. With reference to the model, it becomes possible to plan the overbooking in such a way that revenue is maximised.

This example illustrates that mathematics may serve as a basis for planning and decision-making. The traditional principle: ‘Do not sell any more tickets than there are seats’ becomes substituted with the much more complex principle: ‘Overbook, but do it in such a way that revenue is maximised, considering the amount of money to be paid as compensation, the destination, the time of departure, the day of the week, as well as the long term effects of having sometimes to bump passengers who in fact have made valid bookings.’ This new principle cannot be created or come into operation without mathematics. It’s complexity presupposes that applications of mathematical techniques are ‘condensed’ into a booking programme. The principle illustrates what, in general, can be called mathematics-based action design.

Several observations can be made with reference to mathematics in action. I will highlight seven of them in order to illustrate how mathematics may operate in technologies, in production, in schemes for management and in decision-making. In such cases a separation of power and knowledge seems impossible. I find that mathematics in action is a paradigmatic site for discussing knowledge-power structures in today’s society.

(1) By means of mathematics, we can represent something not yet realised and therefore identify technological alternatives to a given situation. Mathematics provides a form of technological freedom by opening a space of hypothetical situations. In this sense, mathematics becomes a resource for *technological imagination* and, therefore, for technological planning processes including mathematically based action-design. A mathematical framework provides us with new alternatives. Here we witness a tremendously creative element that can be exercised through mathematics. There are technological devices and products which would be impossible to imagine and identify without the use of the extreme sophistication of mathematics-based technological imagination. A paradigmatic example is the computer, including its schemes for cryptography (see, for instance, Skovsmose and Yasukawa, 2004).

(2) Mathematics provides the possibility of *hypothetical reasoning*, by which I refer to analysing the consequences of an imaginary scenario. By means of mathematics we are able to investigate particular details of a not-yet-realised design. Thus, mathematics constitutes an important instrument for carrying out detailed thought experiments. However, mathematics also places severe limitations on hypothetical reasoning, as any technological design has implications not identified by

¹⁵ See Skovsmose (2004b) for further references and for a more detailed presentation of the example.



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hypothetical reasoning. The implications of the realised situation might be very different from the calculated implications of a mathematically described hypothetical situation. The ‘Star Wars’ systems were supposed to include a computer system that would work, for the first time, if and when the U.S. was faced with a full-scale nuclear attack. As any ‘real’ testing of the computer was out of the question, software-verification techniques had to be applied formally to prove that the software would work. But in this case the hyper-complexity of the system made any adequate hypothetical reasoning impossible. The organization Computer Professionals for Social Responsibility argued that, from a mathematical standpoint, it was impossible to provide any adequate verification. Instead risks could be produced because of the inadequacy of hypothetical reasoning.¹⁶

(3) Mathematics can help to construct *justification* as well as (false) *legitimation* of certain actions. The Danish report, Teknologirådet (1995), discusses the increasing use of computer-based models in political decision-making. The report refers to 60 models, covering areas such as economics, the environment, traffic, fishing, defence, population. The models are developed and used by public as well as private institutions in Denmark. The report emphasises that political decision-making concerning a wide range of social affairs is closely linked to the application of such models, and that this development may erode conditions for democratic life: Who constructs the models? What aspects of reality are included in the models? Who has access to the models? Who is able to control the models? In what sense is it possible to falsify a model? If such questions are not adequately clarified, traditional democratic values may be hampered. The report emphasises, in particular, that models related to traffic and environmental issues, such as the construction of a bridge, are often used in support of decisions that cannot be changed. In several cases it appears that models are used in order to legitimate *de facto* decisions, as a model-construction may provide numbers and figures that justify a decision already made. So, mathematics operates in the space between establishing justification and dubious forms of legitimation of decisions and actions.

(4) When an alternative is chosen and *realised*, our environment changes. As already emphasised, the model that structures airline bookings is certainly not simply a description of what takes place when tickets are booked and sold. When introduced, the model becomes part of the passengers’ reality. As part of the realisation of technologies, mathematics itself becomes part of that reality.

(5) The model for airline booking provides uniformity. A grand scale of *routinisation* is set in operation. This is simply one of the basic reasons for the success of a booking model and other similar schemes of management. Just imagine what mess there might be if, when travelling, there was no simple procedure for handling large scale booking procedures. Certainly, the grandiose task of travelling has been normalised and ‘serialised’ by means of a huge conglomerate of mathematical models, the booking model being only one of them. Mathematics brought into action reconstructs the whole travel business. And this is just an exemplification of a much grander change bringing about routinisation: much business can now be carried out with a mouse and a credit card at hand. A new range of selling and buying procedures are implemented by means of all those ‘packages’ that materialise mathematical procedures. Mathematical algorithms provide raw material for routinisation.

(6) Another aspect of mathematically based actions is *authorisation*. It is possible to refer to some calculations (which ‘obviously’ cannot be different) for carrying out certain tasks or for justifying some decisions. When a certain passenger is bumped,

¹⁶ Gutstein has drawn my attention to this example. See: <http://www.cpsr.org/>.



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nothing can be done about it. The person behind the desk can only express professionally measured feelings and sympathy for the bumped passenger, but he or she has no responsibility for the bumping. A quite different example of the complicity of authorisation through numbers has been addressed through the project 'Mortgage Loans: Is Racism a Factor?' described by Gutstein (2003b, 2007). The initial observation of the project was that African Americans were denied 5 times more often than whites when applying for mortgages in the Chicago area in 2001. Racism appears to be a factor. Usually a mathematical algorithm determines whether or not one gets a mortgage, and individual racism should be less and less of a factor. However, through a comparison of numbers, the projects revealed that the disparity between African Americans and whites in getting a mortgage gets *worse* as you go *up* the income ladder. Making sense of such data entails a digging into the complex history of racism, accumulated wealth, property, ownership, etc. The point is that 'authorisation' with reference to numbers could also mean a pseudo-authorisation, where reference to numbers only serve to 'sanitise' a decision that includes other factors.

(7) *Filters for ethical responsibility* have been put in place. Responsibility for model-based actions becomes distributed among different actors, and this could mean that the person who is dealing with, say, a consumers or a patient has only a limited and well-defined responsibility for what is done. The operation of the model can be kept at a convenient distance from the implications of the model-based actions.

Are these aspects of mathematics in action 'good' or 'bad'? Are we having to do with 'horrors' or 'wonders'? My point is that mathematics in action operates in the middle of the complexities of human action in general. As with any other human action, mathematics in action can be reasonable, horrible, doubtful, suspicious, risky, wonderful, etc. Mathematics is critical in the same way as referred to previously. Its way of functioning is not pre-determined by any essence. There is no transparency left around mathematics in action. There is no assurance of 'progress'. Instead, we find an integration of power in the presented elements of action. To me this brings forward an important concern in mathematics education: Mathematics must be reflected on and criticised in its variety of forms of action.

Fourth concern: Disempowerment through mathematics education

Mathematics operates in a global setting, and so does mathematics education. If mathematics and power operate together, the issues of empowerment or disempowerment become crucial to mathematics education. In this section, I point out some of the ways in which mathematics education might operate as disempowering, for instance through forms of discrimination and exclusion. I will make remarks about: (1) discrimination in terms of (lack of) resources, (2) racism, (3) sexism, (4) discrimination in terms of language, (5) discrimination in terms of what is referred to as 'ability', and (6) disempowerment through decomposition.

(1) Mathematics education presupposes investment. Computers enter the classroom, and often they are celebrated as ensuring a new powerful learning environment. The computer with the proper software can engage students in mathematical activities, through which they can develop their creativity; they can allow experiment and exploration; and they can construct mathematics knowledge. Computers can ensure motivation as well as 'learning efficiency'. So goes the celebration. What is seldom under discussion, however, is the implication of this observation for the majority of the world's children, who learn mathematics in non-prototype classrooms without any computer in sight. Are they left behind? Is this a



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new form of social exclusion?¹⁷ A taken-for-granted perspective pervades in the discussion of technology in mathematics education. Naturally, there is no problem that particular studies of the use of technology in mathematics education assume that this technology is available, but the technology-can-be-taken-for-granted perspective turns into a problem for the research paradigm, if the majority of studies assume the perspective as a natural given.¹⁸ To make the story brief: Mathematics education and poverty is not a topic explored widely in research in mathematics education. But it is an essential topic as the discrimination of learning opportunities is caused by an unequal distribution of resources.¹⁹

(2) It is not difficult to find examples of racism exercised through mathematics education, especially when we consider the role of education through the apartheid era in South Africa. Other, possibly more indirect expressions of racism, are revealed when, as suggested by Wenda Bauchspies (2005), we consider to what degree learning, and in particular the learning of mathematics, can mean colonisation. Many studies, also outside of the ethnomathematical paradigm, have emphasised that learning mathematics in a particular form could serve as a suppression of an existing form of thinking.²⁰ Munir Fasheh (1993, 1997) has talked about the occupation of the mind and related issues, and Herbert Khuzwayo (2000) has developed further what this could mean for interpreting the operation of mathematics education during the apartheid era in South Africa.

(3) Sexism, or the issue of gender, has been addressed in mathematics education for a longer period.²¹ Mathematics can be interpreted as a language giving access to power, to technology, to job opportunities. Statistics have documented the unequal distribution of men and women with respect to mathematics-dense studies and later jobs. It is apparent that mathematics education includes or 'materialises' discrimination, and this could take place in all groupings of society. With reference to the Movimento Sem Terra (The Movement of Landless People) in Brazil, Gelsa Knijnik (1998) raises the gender issue with respect to mathematics within a group of people suffering from poverty and social exclusion. Sexism is also an issue in university studies. It could be claimed with reference to statistics, that university studies in mathematics are more gender balanced now than in the past.²² So, apparently, women have better access to the 'powerful' mathematics. However, if we consider the areas such as computer science, programming, systems development, etc. which are mathematics-dense fields, there we see few women. The gender balance is only to be found (to some extent) in pure mathematics. This is an important issue, when we reconsider the mathematics-power relationship.²³

¹⁷ For reflections on access and non-access to computers, see Borba and Penteado (2001), and Borba and Villarreal (2005).

¹⁸ To me the extensive discussion of responses in mathematics education to technological development in Bishop, Clements, Keitel, Kilpatrick, and Leung (Eds.) (2003) suffers from this limited perspective, as does the section 'Influences of Advanced Technologies' in English (Ed.) (2002).

¹⁹ For a discussion of mathematics education and poverty see, for instance, Kitchen (2002, 2003).

²⁰ See, for instance, Frankenstein (1990).

²¹ See, for instance, Burton (1990, 2003); Rogers and Kaiser (1995); Grevholm and Hanna (1995); Secada, Fennema and Adajian (1995); and Keitel (1998).

²² The imbalance is, however, still huge when we look at the academic staff in mathematics departments. Figures illustrating this point were presented by Gila Hanna at a plenary interview session at the International Congress on Mathematics Education in Copenhagen, July 2004

²³ See also Burton (2004), who discusses how mathematics becomes constructed as male community of practice.



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(4) The language issue includes many socio-political controversies. On the one hand it seems disempowering, if children have to learn in a language different from their mother tongue. It can be claimed that to learn in one's own language is a human right. This point has been emphasised strongly, not least by mathematics educators from South Africa, where, during the apartheid era, suppression effected through language domination and all the related inequities were experienced. Such suppression can be observed in many other cases. Thus, in Denmark the general approach to immigrant students is to teach them Danish as soon as possible. The issue of possible cultural suppression is not dealt with in any proper way.²⁴

(5) Discrimination in terms of ability can take the form of elitism in mathematics education. In many cases it has been argued that it is important to differentiate between students according to their so-called 'abilities' (certainly the notion of 'ability' is a problematic one).²⁵ Differentiation can turn into elitism when groups of students are treated differently according to their seemingly different capacities for learning mathematics and when the 'best' are best resourced. Elitism might be 'justified' in economic terms by claiming it to be profitable to invest in the seemingly better students. But if we consider education as a human right, then this appeal to economic productivity as an underlying principle for an unequal distribution of learning possibilities appears absurd.

(6) Disempowerment is often associated with the teaching of marginalised groups, and this is in fact one important aspect. But disempowerment can also occur in a quite different form. In what sense could students in mathematics, computer science, or in technology be disempowered, seen from the perspective of critical mathematics education? Such students experience a strongly decomposed field. During their study they may concentrate on one topic, then on another topic, and so on. They may acquire a particular set of techniques. As an example: they can develop expertise in designing high-speed algorithms, important to establishing pattern-recognition. After graduation, they can get jobs in companies, and again come to work with well-defined sets of problems. However, an institutionally-defined preoccupation with a particular set of technological issues makes it difficult, if not impossible, to establish an overview of the whole enterprise. Through such decomposition, filters for ethical responsibility are brought into operation. To me this is form of disempowerment.

One important preoccupation of critical mathematics education is to struggle against any form of disempowerment that might have to do with (lack of) resources, racism, sexism, language, 'ability', and decomposition. Disempowerment can occur in all forms of mathematics education, from primary school to university level as well as in adult and vocational education.

Fifth concern: Empowerment through mathematics education

Mathematics education could involve discrimination, but also has the potential for empowerment. To identify such potential is an important concern for critical mathematics education. Several notions, like 'mathematical literacy', 'mathematics for social justice' and 'numeracy', have been coined in order to highlight such potential. I

²⁴ The complexities of such multilingual issues are addressed by Adler (2001); Gorgorió and Planas (2000, 2001); Gorgorió, Planas and Vilella, X. (2002); Moschkovich (2002); and Setati, Adler, Reed and Bapoo (2002).

²⁵ For a critical examination of the notion of ability see, for instance, Lynch and Lodge (2002).

prefer, however, to talk about *mathemacy* in order to signal a ‘critical’ content of mathematics education.²⁶

Very many proposals for what mathematical literacy, numeracy, mathemacy, etc. could mean in different contexts have been developed. One could think of students from a Latino school in the USA; from a provincial town in Denmark; township students from South Africa; students from a wealthy neighbourhood in São Paulo; engineering students at universities, etc. Mathemacy means different things depending on where one is positioned in the multitude of globalising-ghettoising processes. I will restrict myself to illustrate one meaning of mathemacy by indicating how reliability and responsibility could be addressed within mathematics education.

The project ‘Terrible small numbers’ had been tried out in Danish classrooms with 13 to 16 year old students. The project is described in Alrø and Skovsmose (2002), but it has also been presented in other contexts.²⁷ The idea of the project was to address figures and numbers that refer to risks that are part of our daily-life conditions. The project discussed risks with reference to salmonella infected eggs. A black film case represented an egg. If it contained a yellow piece of plastic, the egg was a healthy one, while a blue piece of plastic indicated that the egg was infected with salmonella. In the first activity 500 eggs were mixed in a trolley, and every student knew that 50 of these eggs were infected. They selected samples of 10 eggs, and checked the number of salmonella infected eggs. Contrary to what some might have expected, the number of salmonella infected eggs in a sample often differed from 1. Could it be that the eggs in the trolley had not been properly mixed? Could it be that if they were perfectly mixed, the samples would contain one and only one infected egg? The activity was carried out in order to address the problem of *reliability*. A sample tells ‘something’ about the totality (the ‘population’) from which it is selected, but not necessarily anything trustworthy. How, then, to operate in situations where samples provide the only access to information about a population? In most cases, quality control of a product is based on samples. How reliable could such tests be? And more generally, reliability is an issue each and every time numbers are brought into operation. This is the first observation of which we wanted the students to become aware.

Second, we wanted students to consider issues of *responsibility*. They were presented with two trolleys containing eggs, referred to as “Spanish” and “Greek” eggs. The qualities of the two collections, in terms of the number of their salmonella infected eggs, were not known by the students, nor by the teacher, who had just mixed a number of infected eggs in each collection. The students, working in groups, had to imagine themselves as a company that imported eggs. Should they import from Spain

²⁶ Different interpretations of mathematical literacy have been discussed by Jablonka (2003). In Alrø and Skovsmose (2002) *mathemacy* is related to the notions of dialogue, intention, reflection and critique. See also Yasukawa (2002) with respect to investigating the relationship between mathematics and technological literacy; Johnston and Yasukawa (2001) discuss numeracy; and Gutstein (2003) provides a definition of ‘reading the world with mathematics’. See also Frankenstein (1998) and Powell (2002)

Apple (1992) distinguish between ‘functional’ and ‘critical’ literacy which highlights the difficulties in operating with a clear distinction between empowerment and disempowerment. To develop a literacy, also a functional one, might in many cases be seen as an empowerment, and at the same time such a functional ‘regimentation for the labour market’ might be seen as an disempowerment. Adult mathematics education for numeracy or mathematical literacy might include both these aspects. See, for instance, Hoyles, Wolf, Molyneux-Hodson and Kent (2002); and FitzSimons (2004).

²⁷ See, for instance, Alrø, Blomhøj, Bødtker, Skovsmose and Skånstrøm (2003).



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or from Greece? The students were provided with different sort of information: the prices at which they could buy the eggs; the possible selling price of the eggs; and the price of having an egg checked for salmonella. Furthermore, an egg 'opened' at the salmonella control was destroyed; it could not be sold. In order to make a qualified choice between the Greek and the Spanish eggs, samples had to be selected (and paid for) and checked for salmonella (also to be paid for). It was also clear that a complete check was not possible. As a checked egg became a spoiled egg, perfect control would imply that there were none to sell. The financial dilemma was apparent. Making a qualified choice between the two types of eggs could turn into a too high-priced business, while saving money in this procedure of quality control could hamper making a qualified choice. In general: What would it mean to act in a responsible way, when actions are based on numbers and figures?

While the issue of reliability concerns the 'adequacy of numbers', the issue of responsibility addresses the actions that are carried out with reference to numbers and figures that are more or less reliable. I find that such issues could be raised with respect to any form of mathematics in action. (That there are many more aspects to address is clear from the variety of previously mentioned aspects of mathematics in action.) I find that an educational practice that includes considerations of reliability and responsibility exemplifies one way of developing a mathemacy. And it must be noted that these two issues are relevant both in everyday practices as well as in advanced technological practices.

The notion of mathemacy is complex. It cannot be depicted within a well-elaborated definition. As a consequence, there is no recipe for how to organise a practice that might support the development of mathemacy. Addressing reliability and responsibility is only one suggestion for how to articulate a concern for empowerment and develop a practice of mathematics education with a critical dimension.

Summing up: An aporia

An aporetic situation where it seems that reason cannot be applied can be produced by a paradox. Zenon's paradoxes are classic highly provocative examples, as they reveal that the conceptual framework in which philosophy at that time tried to grasp basic natural phenomena like movement, was trapped in paradoxes. In Ancient Greek the word *aporia* could refer to 'a question for discussion', 'a difficulty' or 'a puzzle'. According to *The Cambridge Dictionary of Philosophy*, *aporia* means 'puzzle', 'question for discussion' or 'state of perplexity'. An aporia refers to a difficulty that remains open.²⁸

Bertrand Russell identified a classic paradox with respect to the foundation of mathematics. To address this paradox became almost an obsession for him. And with good reason as the paradox was not just a logical puzzle. It revealed a huge gap in the basic conceptual framework of mathematics. How could the very foundation of mathematics include contradictions? A paradox can strike a blow to an accepted foundation of knowledge, and the whole epistemic foundation of mathematics seems to shake. Put generally: an aporetic situation can destroy foundations. It can mean a questioning of paradigmatic assumptions. It can mean that, using a terminology from Michel Foucault, we have to move from one discourse into another.

I also interpret D'Ambrosio's paradox as a challenge to reason. But while the Russell paradox indicates a logical problem, the D'Ambrosio paradox indicates a socio-logical problem. It strikes a blow at the very assumptions of Modernity: it is not

²⁸ See also Skovsmose (1998, 2000, 2003) for a discussion of the notion.



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possible to assume an intrinsic connection between scientific progress and socio-political progress in general. It becomes problematic to assume epistemic transparency. Knowledge and power mix, and in the nucleus of this mix we find mathematics in action. The notion of progress, both with respect to science and socio-political affairs becomes problematic. As the Russell paradox struck a blow to the foundations of mathematics, so does the D'Ambrosio paradox strike a blow to the assumptions of Modernity. This moves any foundation out of sight. On the one hand, we find that mathematics plays a role in all forms of technological enterprises. We cannot eliminate 'mathematics in action', which propels our rapid socio-technological development. On the other hand, we find it extremely difficult to form any opinion about the way mathematics is playing its role. Mathematics in action can produce horrors as well as wonders.

This aporia forms part of the concerns of critical mathematics education. If one wants to address globalisation-ghettoising; to go beyond the assumptions of Modernity; to analyse mathematics and power in the form of mathematics in action; and to address disempowerment and empowerment, then one is unable to do so on any firm theoretical or epistemological foundation. Addressing such concerns means accepting a basic uncertainty. There is no underpinning to be recaptured from Critical Theory, nor from any other theory claiming to stand on some firm ground. Uncertainty accompanies critical mathematics education into the future.

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