

Reliability and Robustness Evaluation of Timber Structures

Short Term Scientific Mission : COST E55 Action

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Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Cizmar, D., Sørensen, J. D., & Kirkegaard, P. H. (2009). *Reliability and Robustness Evaluation of Timber Structures: Short Term Scientific Mission : COST E55 Action*. Department of Civil Engineering, Aalborg University. DCE Technical reports No. 58

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Reliability and robustness evaluation of timber structures

Short Term Scientific Mission, COST E55 Action

**Dean Čizmar, John Dalsgaard Sørensen,
Poul Henning Kirkegaard**



Department of Civil Engineering

**ISSN 1901-726X
DCE Technical Report No. 58**

Aalborg University
Department of Civil Engineering
Group Name

DCE Technical Report No. 58

Reliability and robustness evaluation of timber structures

by

Dean Čizmar, John Dalsgaard Sørensen, Poul Henning Kirkegaard

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Published 2008 by
Aalborg University
Department of Civil Engineering
Sohngaardsholmsvej 57,
DK-9000 Aalborg, Denmark

Printed in Aalborg at Aalborg University

ISSN 1901-726X
DCE Technical Report No. 58

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1 SYSTEM MODELING OF TIMBER STRUCTURES

1.1 INTRODUCTION

In the last few decades there have been intensely research concerning reliability of timber structures. This is primarily because there is an increased focus in society on sustainability and environmental aspects. Modern timber as a building material is also being competitive compared to concrete and steel. However, reliability models applied to timber were always related to individual components but not the systems. As any real structure is a complex system, system behaviour must be of a particular interest. In the chapter 1 of this document an overview of stochastic models for strength and load parameters is given. System models (series and parallel) are discussed and methods for reliability calculation are given. Special attention is drawn upon brittle/ductile modelling of timber and connections. In chapter 2 robustness requirements implemented in codes are presented. State of the art definitions (deterministic, probabilistic and risk based approaches) of the robustness are given. Chapter 3 deals more detailed with the robustness of timber structures.

1.2 STOCHASTIC MODELS FOR STRENGTH AND LOADS PARAMETERS

1.2.1 STOCHASTIC MODEL FOR STRENGTH

During evolution trees have specialized in resisting their natural environment. As a result they have special material properties like significant variability, anisotropy and interaction between moisture content and duration of a load to mechanical properties.

Timber is an orthotropic material consisting of “high strength” fibers (grains) oriented along the longitudinal axis of a timber log and packed together within a “low strength” matrix. Material properties depend upon the orientation of the moment axis to the fiber direction. Irregularities in regard to grain direction, knots and fissures are decisive for the load bearing capacity of a structural timber [1].

The reference properties of structural timber are:

the bending strength $r_{m,s}$ in [MPa]

bending modulus of elasticity $moe_{m,s}$ in [MPa], both measured on short-term standard test specimens

timber density $q_{den,s}$ in [kg/m³]

Relation reference properties – other properties is defined in table 1.

Table 1. Relation to other properties and reference properties [1]

Property	Expected Values $E[X]$	Coefficient of variation $COV[X]$
Tension strength par. to the grain, $r_{t,0}$:	$E[R_{t,0}] = 0.6 E[R_m]$	$COV[R_{t,0}] = 1.2 COV[R_m]$
Tension strength perp. to the grain, $r_{t,90}$:	$E[R_{t,90}] = 0.015 E[P_{den}]$	$COV[R_{t,90}] = 2.5 COV[P_{den}]$
MOE - tension par. to the grain, $MOE_{t,0}$:	$E[MOE_{t,0}] = E[MOE_m]$	$COV[MOE_{t,0}] = COV[MOE_m]$
MOE - tension perp. to the grain, $MOE_{t,90}$:	$E[MOE_{t,90}] = \frac{E[MOE_m]}{30}$	$COV[MOE_{t,90}] = COV[MOE_m]$
Compression strength par. to the grain, $r_{c,0}$:	$E[R_{c,0}] = 5 E[R_m]^{0.45}$	$COV[R_c] = 0.8 COV[R_m]$
Compression strength perp. to the grain, $r_{c,90}$:	$E[R_{c,90}] = 0.008 E[P_{den}]$	$COV[R_{c,90}] = COV[P_{den}]$
Shear modulus, mog_v :	$E[MOG_v] = \frac{E[MOE_m]}{16}$	$COV[MOG_v] = COV[MOE_m]$

Shear strength, r_v :	$E[R_v] = 0.2 E[R_m]^{0.8}$	$COV[R_v] = COV[R_m]$
-------------------------	-----------------------------	-----------------------

Typical ultimate limit state equation should be formed according to [1]. The ultimate limit state equation for a cross section subjected to combined bending and tension parallel to grain is given as:

$$g_i(X) = 1 - \left(\frac{\sum_i S_{t,i}}{z_{d,A} \cdot R_{t,0}} + \frac{\sum_i S_{m,i}}{z_{d,M} \cdot R_m} \right) \cdot X_M = 0$$

where $z_{d,A}$ and $z_{d,M}$ are design variables, $R_{t,0}$ and R_m resistances (tension strength and bending moment capacity), $\sum_i S_{t,i}$ and $\sum_i S_{m,i}$ are the sum of load effects (axial forces and bending moments) and X_M model uncertainty.

Typical serviceability limit state equation [1] can be expressed as:

$$g(t) = \delta_L - W_\Delta(\sum_i S_i, E_{0,mean}, t) \cdot X_M = 0$$

where δ_L is allowable deflection limit, $W_\Delta(\sum_i S_i, E_{0,mean}, t)$ is the deflection in time t , dependant on load effects $\sum_i S_i$ and modulus of elasticity ($E_{0,mean}$).

In tables 2 and 3 [1], distribution functions for reference properties and other material properties are given respectively. Correlation coefficient matrix is given in table 4.

Table 2. Probabilistic variables for reference properties

	Distribution	COV
Bending strength $R_m = R_{m,s}$	Lognormal	0.25
Bending MOE $MOE_m = MOE_{m,s}$	Lognormal	0.13
Density $P_{den} = P_{den,s}$	Normal	0.1

Table 3. Probabilistic variables and distribution functions for other material properties

Property	Distribution Function
Tension strength par. to the grain, $R_{t,0}$:	Lognormal
Tension strength perp. to the grain, $R_{t,90}$:	2-p Weibull
MOE - tension par. to the grain, $MOE_{t,0}$:	Lognormal
MOE - tension perp. to the grain, $MOE_{t,90}$:	Lognormal
Compression strength par. to the grain, $R_{c,0}$:	Lognormal
Compression strength perp. to the grain, $R_{c,90}$:	Normal
Shear modulus, MOG_v :	Lognormal
Shear strength, R_v :	Lognormal

Table 4. Correlation coefficient matrix

	MOE_m	P_{den}	$R_{t,0}$	$R_{t,90}$	$MOE_{t,0}$	$MOE_{t,90}$	$R_{c,0}$	$R_{c,90}$	MOG_v	R_v
r_m	0.8	0.6	0.8	0.4	0.6	0.6	0.8	0.6	0.4	0.4
MOE_m		0.6	0.6	0.4	0.8	0.4	0.6	0.4	0.6	0.4
P_{den}			0.4	0.4	0.6	0.6	0.8	0.8	0.6	0.6
$R_{t,0}$				0.2	0.8	0.2	0.5	0.4	0.4	0.6
$R_{t,90}$					0.4	0.4	0.2	0.4	0.4	0.6
$MOE_{t,0}$						0.4	0.4	0.4	0.6	0.4
$MOE_{t,90}$							0.6	0.2	0.6	0.6
$R_{c,0}$								0.6	0.4	0.4
$R_{c,90}$									0.4	0.4
MOG_v										0.6

The reference properties in situ (bending moment capacity, bending MOE and density in situ) can be estimated as follows [1]:

$$r_{m,\alpha} = \alpha(Ex) \cdot r_m$$

$$moe_{m,\alpha} = \frac{moe_{m,0}}{1 + \delta(Ex)}$$

$$q_{den,\alpha} = q_{den,s}$$

where reference properties in situ have index α , $\alpha(Ex)$ is a strength modification function (dependent upon loads, humidity and temperature) and $\delta(Ex)$ is a stiffness modification function. Both these functions are, in general, defined for a particular set of exposures [1]. Tables 5 and 6 represent strength modification function and stiffness modification function table.

Table 5. Strength modification function table

sc	Permanent ($t > 10$ years)	Long term ($0.5 < t < 10$ years)	Medium term ($0.25 < t < 6$ month)	Short term ($t < 1$ week)	Instantaneous
1/2	$\alpha = 0.6$	$\alpha = 0.70$	$\alpha = 0.80$	$\alpha = 0.9$	$\alpha = 1.1$
3	$\alpha = 0.5$	$\alpha = 0.55$	$\alpha = 0.65$	$\alpha = 0.7$	$\alpha = 0.9$

Table 6. Stiffness modification function table

sc	Permanent ($t > 10$ years)	Long term ($0.5 < t < 10$ years)	Medium term ($0.25 < t < 6$ month)	Short term ($t < 1$ week)	Instantaneous
1	$\delta = 0.6$	$\delta = 0.5$	$\delta = 0.25$	$\delta = 0.0$	$\delta = 0.0$
2	$\delta = 0.8$	$\delta = 0.5$	$\delta = 0.25$	$\delta = 0.0$	$\delta = 0.0$
3	$\delta = 2.0$	$\delta = 1.5$	$\delta = 0.75$	$\delta = 0.3$	$\delta = 0.0$

Model uncertainties

The model uncertainties account for random effect neglected in models and mathematical simplifications. Model uncertainties can be subdivided into:

- 1) load calculations models
- 2) load effect calculation models
- 3) local stiffness and resistance models

In order to calculate the response of the structure Y with random actions X_1, X_2, \dots, X_n (variables) model function f is used:

$$Y = f(X_1, X_2, \dots, X_n)$$

As the model function is not complete and exact, so the response cannot be predicted with error. If Y' is the real response than variable X_M accounts for the uncertainties:

$$Y' = f(X_1, X_2, \dots, X_n, X_M)$$

There are other ways of introducing the model uncertainties into calculation [2], but to avoid dependencies of statistical properties of the model uncertainties upon response is given in:

$$X_M = \frac{Y'}{Y}$$

Table 7. Model uncertainties X_M

Component	Long term	mean	st.dev.	Distribution
		1	0.05-0.10	Lognormal

Spatial variability

Material properties vary randomly in space: the strength in one point of a structure is not the same as the strength in another point of the same structure or another one. Koehler et al [1] propose a bending moment capacity approach where the bending strength $r_{m,ij}$ at a particular point j in the component i of a structure/batch is given as:

$$r_{m,ij} = \exp(\nu + \varpi_i + \chi_{ij})$$

where ν is the unknown logarithm of the mean strength of all sections in all components (see figure 1), ϖ_i is the difference between the logarithm of the mean strength of the sections within a component i and ν , ϖ_i is normal distributed with mean value equal to zero and standard deviation σ_{ϖ} , χ_{ij} is the difference between the strength weak section j in the beam i and the value $\nu + \varpi_i$. χ_{ij} is normal distributed with mean value equal to zero and standard deviation σ_{χ} . ϖ_i and χ_{ij} are statistically independent.

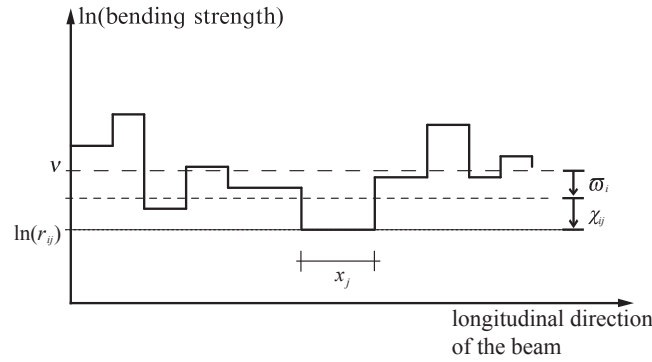


Figure 1. Section model for the longitudinal variation of bending strength.

Size effect

The dimensions of the beam affect the strength, since there is higher probability of having a weaker section in a longer beam (or generally with any increase of cross section). When the strength parameter is Weibull distributed the probability of failure becomes:

$$p_f = 1 - e^{-\left(\frac{a-b}{a}\right)^{1/k}}$$

where a is the scale factor, b location factor and k shape factor.

Generally it can be shown that the following relationship will apply between two volumes V if the location factor is set to zero:

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{V_1}{V_2}\right)^k$$

where σ_1 and σ_2 are the stresses causing failures for volumes V_1 and V_2 .

Design codes (Eurocode 5) use, however, the depth of a timber beam as a parameter to account for a size effect. For the beams smaller in size than 150 mm size effect is calculated as (it must be noted that maximal value of a size factor $k_h=1,3$):

$$k_h = \left(\frac{150}{h} \right)^{0.2}$$

1.2.2 STOCHASTIC MODEL FOR LOAD PARAMETERS

Loads (actions) can be classified with respect to time variations as:

Permanent loads whose variation in time is small and slow (selfweight, soil pressure etc) or the loads that have a limiting value (prestressing, shrinkage, creep etc).

Variable actions, whose variations in time and space are frequent and large

Exceptional actions, whose magnitude can be considerable but with a low probability of occurrence

Selfweight and volume

The weight density is assumed to have Gaussian distribution. Indicative values are given in following table.

Table 8. Expected value and coefficient of variation of timber strength [kN/m³]

	E[X]	COV
Spurce, Fir (Picea)	4.4	0.10
Pine (Pinus)	5.1	0.10
Larch (Larix)	6.6	0.10
Beech (Fagus)	6.8	0.10
Oak (Quercus)	6.5	0.10

If weight density is unknown, the mean value of 5 kN/m³ and standard deviation of 0.5 kN/m³ can be assumed.

Volume of element (cross section dimensions) is Gaussian distributed. Standard deviations of cross sections are given in table 9.

Table 9. Cross section dimensions

	E[X]	Standard dev.
Sawn beam or strut	1.05 · a _{nom}	2 mm
Laminated beam	a _{nom}	1 mm

Snow load

Snow load codes presented in Eurocode1 [25] assume that the snow load on the roof, if all other conditions are kept constant, is proportional to the snow load on the ground. A stochastic model of snow load is based on meteorological data. Using the annual maximum values, probabilistic analysis allows defining characteristic values of the snow load, having a certain probability of being exceeded, in any year which is directly associated to a certain mean recurrence interval. National codes have been based on different mean recurrence intervals (MRI) of 5, 20 or 50 years. In Eurocodes and National annexes, the characteristic value is the snow load which has a probability of only 0,02 of being exceeded within any one year. This corresponds to a MRI of 50 years.

Based on this, snow load Q_{gk} on roof is determined as:

$$Q_{gk} = S_g \cdot C$$

where S_g refers to snow on ground and C is the roof snow load shape factor. It is assumed that snow on ground is Gumbel distributed and the shape factor C is assumed Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.35$ [26]. As a snow load on the ground S_g is usually given as a characteristic value corresponding to a 98% quantile in an annual maximum distribution, following equations are given in order to calculate mean value. If COV for ground snow load is assumed to be V_{Qg} , then the expected value μ_{Qs} can be determined from the Gumbel cumulative distribution function $F_{Qg}(\cdot)$ as:

$$F_{Q_g}(Q_{gk}) = \exp(-\exp(-\alpha(Q_{gk} - \beta)))$$

$$\mu_{Q_g} \approx \beta + \frac{0.577216}{\alpha}, \quad \sigma_{Q_g} = \frac{\pi}{\alpha \cdot \sqrt{6}}, \quad V_{Q_g} = \frac{\sigma_{Q_g}}{\mu_{Q_g}}$$

Wind load

The annual maximum wind load on a structure can be determined from:

$$Q_w = C \cdot P_{w,\max}$$

where $P_{w,\max}$ is the annual maximum wind pressure (Gumbel distributed with COV=0.25) and C is a shape factor (modelled as Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.215$) [26].

1.3 SYSTEM MODELS

A mechanical system is defined as a combination of individual elements that are synthesized to perform a dedicated mechanical function [14]. Any mechanical system may be assigned to one of the following three categories: series systems, parallel systems or combination of series and parallel system (also referred as hybrid systems) (figure 2). In series systems failure of any element leads to the failure of the system. Parallel systems are those systems in which the combined failure of each and every element of the system results in the failure of the system [14]. If a system does not satisfy these strict definitions of “series” or “parallel” systems, the system is classified as a hybrid system mode.



Fig. 1. Series system.

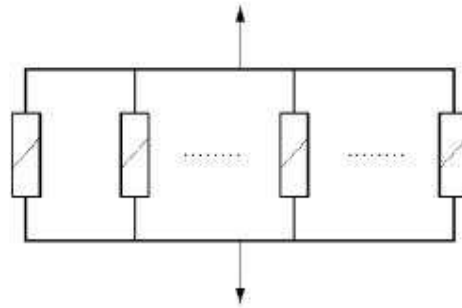


Fig. 2. Parallel system.

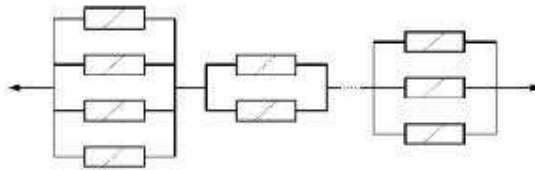


Fig. 3. Hybrid system.

Figure 2. Different systems

1.3.1 SERIES MODELS

We consider a structural system where the system reliability model is a series system of m failure elements. Each of the failure elements is modelled with a safety margin:

$$M_i = g_i(\mathbf{X}) \quad , \quad 1, 2, \dots, m$$

The probability of failure of a single element can be written as:

$$\begin{aligned} P_{f_i} &= P(M_i \leq 0) = P(g_i(\mathbf{X}) \leq 0) = P(g_i(\mathbf{T}(\mathbf{U})) \leq 0) \\ &\approx P(\beta_i - \boldsymbol{\alpha}_i^T \mathbf{U} \leq 0) = \Phi(-\beta_i) \end{aligned}$$

The probability failure of system is:

$$P_f^S = P\left(\bigcup_{i=1}^m \{M_i \leq 0\}\right) = P\left(\bigcup_{i=1}^m \{g_i(\mathbf{X}) \leq 0\}\right) = P\left(\bigcup_{i=1}^m \{g_i(\mathbf{T}(\mathbf{U})) \leq 0\}\right)$$

If all the failure functions are linearized at their respective β -points the FORM approximation of probability of failure (P_f) of a series system can be written:

$$P_f^S \approx P\left(\bigcup_{i=1}^m \{-\boldsymbol{\alpha}_i^T \mathbf{U} \leq -\beta_i\}\right)$$

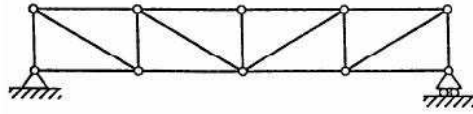


Figure 3. Example of series system

By DeMorgan law it can be written:

$$P_f^S \approx 1 - P\left(\bigcap_{i=1}^m \{-\boldsymbol{\alpha}_i^T \mathbf{U} > -\beta_i\}\right) = 1 - P\left(\bigcap_{i=1}^m \{\boldsymbol{\alpha}_i^T \mathbf{U} < -\beta_i\}\right) = 1 - \Phi_m(\boldsymbol{\beta}; \boldsymbol{\rho})$$

where Φ_m is the m -dimensional normal distribution function. Correlation coefficient ρ_{ij} between two linearized safety margins is given as:

$$\rho_{ij} = \boldsymbol{\alpha}_i^T \cdot \boldsymbol{\alpha}_j$$

From previous equations formal or so-called generalized series systems reliability index β_S can be introduced as:

$$P_f^S = 1 - \Phi_m(\boldsymbol{\beta}, \boldsymbol{\rho}) = \Phi(-\beta^S)$$

or:

$$\beta^S = -\Phi^{-1}(P_f^S) = -\Phi^{-1}(1 - \Phi_m(\boldsymbol{\beta}; \boldsymbol{\rho}))$$

As Φ_m is very computational costly to solve analytical usually numerical bounds methods are used (simple bounds and parallel bounds).

Simple bounds

Simple bounds can be introduced as:

$$\max_{i=1}^m P(M_i \leq 0) \leq P_f^S \leq \sum_{i=1}^m (P(M_i \leq 0))$$

The lower bound corresponds to the exact value of system reliability if all the elements are fully correlated. In the term of reliability indices this can be written:

$$-\Phi^{-1}\left(\sum_{i=1}^m \Phi(-\beta_i)\right) \leq \beta^S \leq \min_{i=1}^m \beta_i$$

Ditlevesen bounds

Ditlevesen bounds are usually much tighter. In the terms of reliability indices it can be written:

$$\Phi(-\beta^S) \geq \Phi(-\beta_1) + \sum_{i=2}^m \max\left\{\Phi(-\beta_i) - \sum_{j=1}^{i-1} \Phi_2(-\beta_i, -\beta_j; \rho_{ij}), 0\right\}$$

$$\Phi(-\beta^S) \leq \sum_{i=1}^m \Phi(-\beta_i) - \sum_{i=2}^m \max_{j < i} \{\Phi_2(-\beta_i, -\beta_j; \rho_{ij})\}$$

1.3.2 PARALEL MODELS

We consider a system shown in figure 4. In this case mechanical system will not fail as soon as one of structural elements fails. After failure of one element, the load carrying capacity of a structure is obtained after redistribution of load effects in the structure has taken place. Since this redistribution of load effect has to take place it is very important to describe/model the behaviour of failed element after the redistribution has taken place.

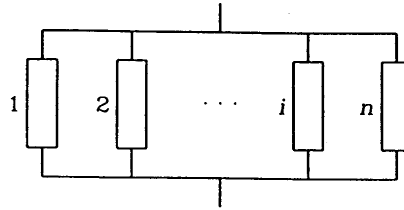


Figure 4. Parallel system

If we consider a parallel system of m failure elements as for the series models, than the probability of failure of the parallel system is defined as the intersection of the individual failure events:

$$p_f = P\left(\bigcap_{i=1}^m \{M_i \leq 0\}\right) = P\left(\bigcap_{i=1}^m \{g_i(X) \leq 0\}\right)$$

The FORM approximation of a parallel system can be written:

$$p_f \approx P\left(\bigcap_{i=1}^{m_o} \{\beta_i^J - \alpha_i^T \cdot U \leq 0\}\right) = \Phi_{m,A}(-\beta^J, \rho)$$

where $\Phi_{m,A}$ is the m -dimensional normal distribution function and ρ_{ij} correlation coefficient.

From previous equation formal generalized parallel systems reliability β^P can be introduced by:

$$P_f^P = \Phi_{n_A}(-\beta^J; \rho) = \Phi(-\beta^P)$$

In the terms of reliability indices it can be written:

$$\beta^P = -\Phi^{-1}(P_f^P) = -\Phi^{-1}(\Phi_{n_A}(-\beta^J; \rho))$$

This is very computational costly and instead bounds methods are also used.

Simple bounds

The simple bounds can be introduced as:

$$0 \leq P_f^P \leq \min_{i=1}^{n_A} (P(M_i^J \leq 0))$$

where M_i^J , $i = 1, \dots, n_A$ are the linearized safety margins at the joint β -point. The upper bound corresponds to the exact value of P_f^P if all the n_A elements are fully correlated with $\rho_{ij} = 1$.

In the terms of reliability indices β^J :

$$\max_{i=1}^{n_A} \beta_i^J \leq \beta^P \leq \infty$$

If all correlation coefficients ρ_{ij} between the n_A elements are higher than zero, the following simple bounds are obtained:

$$\prod_{i=1}^{n_A} P(M_i^J \leq 0) \leq P_f^P \leq \min_{i=1}^{n_A} P(M_i^J \leq 0)$$

where the lower bound corresponds to uncorrelated elements ($\rho_{ij} = 0$), $i \neq j$. In terms of β^J previous equation can be formulated:

$$\max_{i=1}^{n_A} \beta_i^J \leq \beta^P \leq -\Phi^{-1}\left(\prod_{i=1}^{n_A} \Phi(-\beta_i^J)\right)$$

Second order upper bounds

A second-order upper bound of P_f^P can be derived as:

$$P_f^P \leq \min_{i,j=1}^{n_A} P(M_i^J \leq 0 \cap M_j^J \leq 0)$$

The corresponding lower bound of β^P is:

$$\beta^P \geq -\Phi^{-1}\left(\max_{i,j=1}^{n_A} \Phi_2(-\beta_i^J, -\beta_j^J, \rho_{ij})\right)$$

1.3.3 MODELLING OF DUCTILE / BRITTLE MATERIALS

As stated before it is very important for calculation of a parallel system reliability to describe the behavior of the failed element after the failure has taken place. For the series system this is not very significant because when one element fails the failure of system is inevitable. In following figure perfectly brittle and perfectly ductile elements are shown.

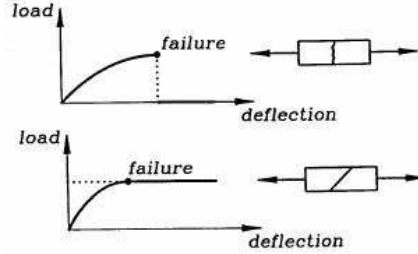


Figure 5. Brittle and ductile material behaviour

If we assume static load S and a parallel system consisting of m independently distributed element strengths X_i , a constant modulus of elasticity and perfect equal load sharing among ideally brittle elements [6]. If element strength is set in a decreasing order, the system strength R_m can be calculated as:

$$R_m = \max_{i=1}^n \{(m-i+1) \cdot X_i\}$$

The corresponding system probability of failure is :

$$p_f = P(R_m \leq S) = P\left(\bigcap_{i=1}^m \{(m-i+1) \cdot X_i - S \leq 0\}\right) \leq \min_{i=1}^m P(\{(m-i+1) \cdot X_i - S \leq 0\})$$

For the arbitrary force-deformation curve, the element failure event for a given imposed deformation δ is:

$$F(\delta) = \left\{ \sum_{i=1}^m R_i(\delta) - S \leq 0 \right\}$$

where S is uncertain load and R denotes the uncertain element (component) force at deformation δ . System failure occurs if the maximum system resistance is exceeded by the load:

$$F_{sys} = \max_{\delta} \left(\sum_{i=1}^m R_i(\delta) - S \leq 0 \right) = \bigcap_{\delta} \left(\sum_{i=1}^m R_i(\delta) - S \leq 0 \right)$$

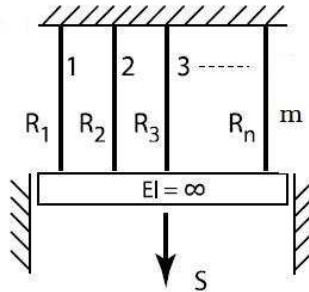


Figure 6. Mechanical model [7]

In paper [7] numerical investigation concerning parallel/serial systems and ideal ductile/brittle elements is conducted. The components of the system were designed for a reliability index $\beta_k=2$ as if no system effect exists. The following figure, in which the system reliability index versus number of elements is given, demonstrates the influence of the mechanical behavior of the elements (components) on system reliability. In this figure it can be seen that for a small number of elements the

brittle system behaves much like the series system. As number of elements is increased the reliability of parallel system is increased significantly (and vice-versa for the series system).

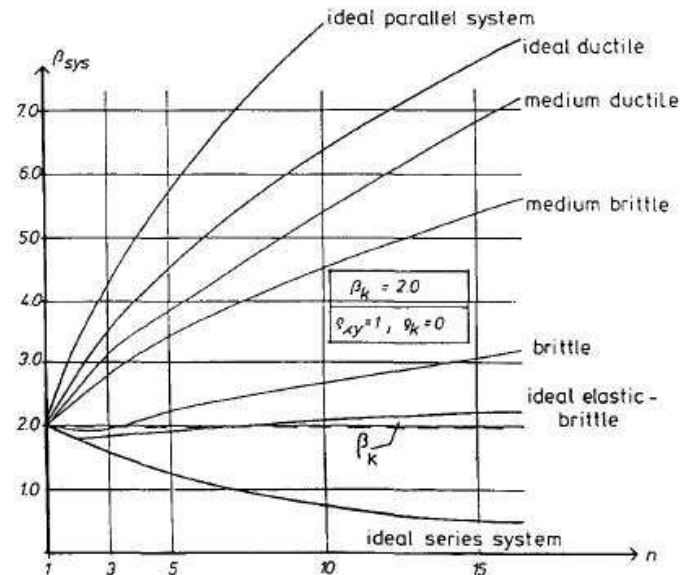


Figure 7. System reliability vs. numbers of elements

Ductility/brittleness of components

Figure 8 represents an influence of ductility on five elements system. As ductility increases linearly the reliability of the system increases much steeper (exponentially), so a relatively little ductility accounts for a considerable extra reliability.

Stochastic dependencies

Figure 9 represents system reliability vs. correlation between element strength. It can be easily noticed that reliability is largest for ideal ductility and zero correlation. As correlation increases so the reliability decreases. For the medium correlation between element strength brittle systems experience decrease in reliability.

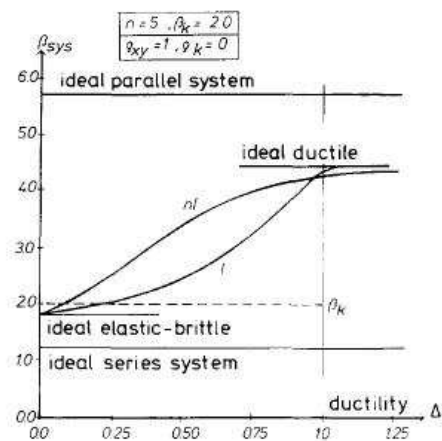


Figure 8. System reliability vs. ductility

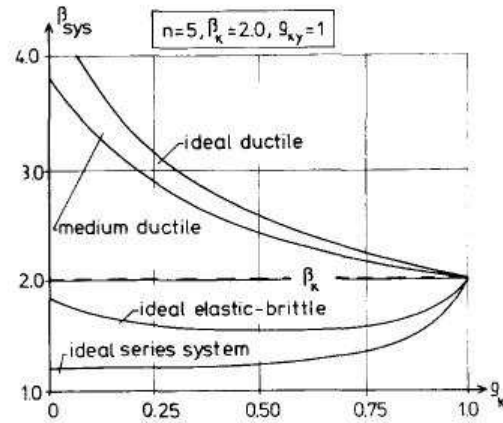


Figure 9. System reliability vs. correlation of strength between elements

Load and strength variability

For the load and strength variability ratio ranging from 0 to 5 reliability of the system is calculated (figure 10). Much influence upon reliability is in the range 0 to 2 where reliability decreases exponentially. For the brittle systems this effect is not so pronounced as for ideal ductile elements. On the other hand if only element strength is varied (figure 11), one can notice a positive effect on the reliability of the system (for both ductile and brittle systems). This, of course, should not imply that in structural design high variability in strength is better than a small one.

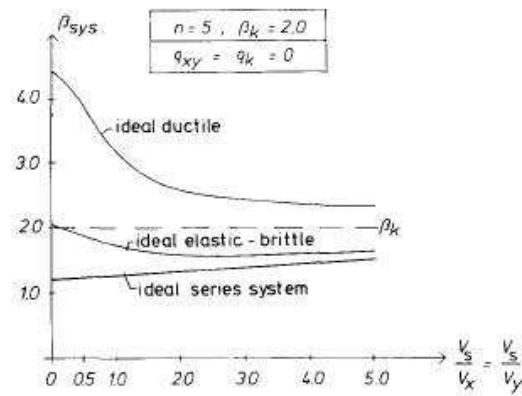


Figure 10. System reliability vs. ratio of load/strength variability

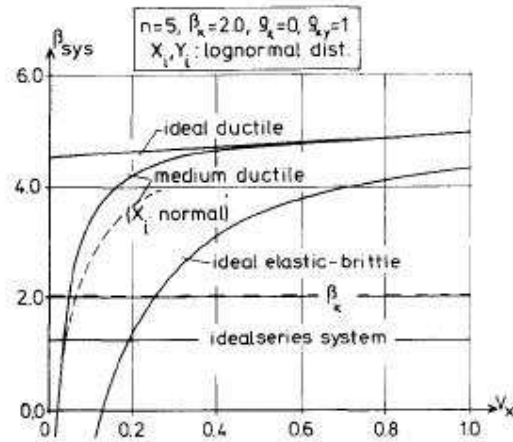


Figure 11. System reliability vs. element strength variability

In summary, if there is a moderate degree of ductility, ductile systems will provide significant extra reliability only if elements are low correlated or with no correlation at all and if the load variability is not high. On the other hand, if there is a brittle behaviour, there is a relatively little effect of the system (especially for the small systems). There is even a small negative effect for medium coefficients of strength variation.

2 ROBUSTNESS OF STRUCTURES

2.1 INTRODUCTION

A progressive collapse of a building is defined as a catastrophic partial or total failure that starts from local damage, caused by a certain event, that can't be absorbed by the structural system itself [10]. The "normal" or "usual" structural design usually provides a certain amount of additional strength and ductility that is available to withstand abnormal loads and progressive collapse. But, due to "structural revolution" (use of computers, high performance materials and modern building systems) much of the inherent strength was taken out [4, 10]. Progressive collapse is characterised by disproportion between the magnitude of a triggering event and resulting in collapse of large part or the entire structure [20].

Robustness of structures has been recognized as a desirable property because of a several high system failures, such as the Ronan Point Apartment Building in 1968, where the consequences were deemed unacceptable relative to the initiating damage [21]. After the collapse of the World Trade Center, the robustness has obtained a renewed interest, primarily because of the serious consequences related to failure of the advanced types of structures. In order to minimize the likelihood of such disproportional structural failures many modern building codes consider the need for robustness in structures and provide strategies and methods to obtain robustness. In fact, in all modern building codes, one can find a statement (in a slightly different form): "total damage (or risk) resulting from an action should not be greater than the initial damage caused by this action".

2.2 ROBUSTNESS IN BUILDING CODES

The requirement of robustness exists in two European documents: Eurocode EN 1990: Basis of Structural Design [11] and EN 1991-1-7 Eurocode 1: Part 1-7 Accidental Actions [12]. The first document provides principles, e.g. it is stated that a structure shall be "designed in such a way that it will not be damaged by events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the original cause." It also states that potential damage shall be avoided by "avoiding, eliminating or reducing the hazards to which the structure can be subjected; selecting a structural form which has low sensitivity to the hazards considered; selecting a structural form and design that can survive adequately the accidental removal of an individual member or a limited part of the structure, or the occurrence of acceptable localized damage; avoiding as far as possible structural systems that can collapse without warning; tying the structural members together".

The EN 1991-1-7 document provides strategies and methods to obtain robustness, actions that should be considered and different design situations: 1) designing against identified accidental actions, and 2) designing unidentified actions (where designing against disproportionate collapse, or for robustness, is important). The methods used to design for robustness of a structure are divided into several levels based on potential consequences of structural failure (Consequence Class). CC1 represents low consequence class with no special requirements, CC2 are structures with medium consequences that can be handled using simplified analysis, while CC3 stands for high consequence class where a reliability or risk analysis must be conducted [13]. However, there is no specific criteria which could be used to quantify the level of robustness of a structure which could have a benefit for design and analysis of structures.

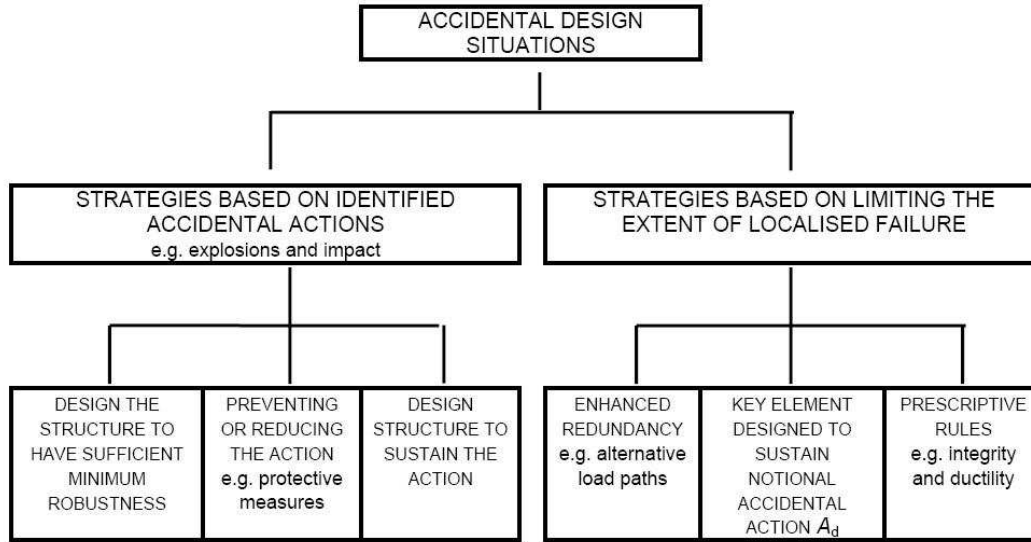


Figure 12. Design situations according to EN 1991-1-7

In the Probabilistic model code [2] robustness requirement is also formulated as: “A structure shall not be damaged by events like fire, explosions or consequences of human errors, deterioration effects, etc. to an extent disproportionate to the severeness of the triggering event”. In order to attain adequate safety in relation with accidental loads, two basic strategies are proposed: non-structural (prevention, protection and mitigation) and structural measures (making the structure strong enough to withstand the loads limiting the amount of structural damage or limiting the amount of structural damage).

According to Danish design rules robustness shall be documented for all structures where consequences of failure are serious. The requirements regarding structural robustness could also be to reduce the sensitivity of a structure with respect to unintentional loads and defects that are not included in the codes and design requirements. Such a robustness analysis framework is introduced in the Danish Code of Practice for the Safety of Structures [16, 17].

2.3 DEFINITION OF A ROBUSTNESS

During the last decades there has been a significant effort to quantify aspects of robustness. In the [12] general definition of robustness is given: “robustness is the ability of a structure to withstand events like fire, explosion, impact or the consequences of human error without being damaged to an extent disproportionate to the original cause”. When modelling robustness, system effects are very important, however building code criteria are primary related to design of components. It must also be noted that redundancy in systems is closely related to robustness. In principle redundant system are believed to be more robust. Approaches to define robustness index can be divided with the respect of the procedure used into:

1. Deterministic approach
2. Probabilistic approach
3. Risk based approach

2.3.1 DETERMINISTIC DEFINITION OF A ROBUSTNESS

Simply and practical definition is given in [28]. The reserve strength ratio (RSR) is defined as:

$$RSR = \frac{R_c}{S_c}$$

where R_c denotes characteristic value of the base shear capacity of the platform and S_c design load. If we consider a limit state function:

$$g(X) = R - S$$

where S is the base shear load and R is the base shear capacity and suppose that the load S can be expressed in term of a maximum annual value of wave height H :

$$S = b \cdot H^\delta$$

where b and δ are determined by means of structural analysis. The limit state equation can be solved so the relation between the probability of failure and RSR value can be obtained.

In order to measure the effect of full damage or full loss of functionality of structural member i on the structural capacity the following equation is given:

$$RIF_i = \frac{RSR_{Fi}}{RSR_{int\ act}}$$

where RIF denotes Residual Influence Factor (sometimes referred as a Damaged Strength Ratio). The RIF can vary between 0 and 1, where the larger RIF stand for a more robust structure. Value $RSR_{int\ act}$ is constant for the same structure.

Simply defined measure of robustness is proposed in [20]. R_s denote stiffness based robustness measure defined as:

$$R_s = \min_j \frac{\det K_j}{\det K_0}$$

where K_j and K_0 are system stiffness matrix of the intact structure and stiffness matrix after the removal a structural element or a connection j , respectively. However, it seems that this robustness measure is not sufficient in this form [20]. Same authors also proposed an energy based measure of robustness and damage based measure of robustness. Energy based measure is defined as:

$$R_s = 1 - \max_j \frac{E_{r,j}}{E_{s,k}}$$

where $E_{r,j}$ is amount of energy released by the initial failure of a structural element j and available energy for the damage of the next structural element k , while $E_{s,k}$ is the energy required for the failure of the next structural element.

Damage based measure of robustness is defined as:

$$R_d = 1 - \frac{p}{p_{lim}}$$

where p is maximum extent of the damage caused by initial damage i_{lim} and p_{lim} is acceptable damage progression [20].

2.3.2 PROBABILISTIC DEFINITION OF A ROBUSTNESS

In the early 90's Frangopol and Curley [22] proposed probabilistic indices to measure structural redundancy index (RI):

$$RI = \frac{P_{f(dm)} - P_{f(sys)}}{P_{f(sys)}}$$

where $P_{f(dm)}$ is the probability of damage occurrence to the system and $P_{f(sys)}$ is the system failure probability. Redundancy index as defined above provides the residual strength of a damaged system. They also considered the following redundancy factor:

$$\beta_R = \frac{\beta_{intact}}{\beta_{intact} - \beta_{damaged}}$$

where β_{intact} is the reliability index of the intact system and $\beta_{damaged}$ is the reliability index of the damaged system.

Lind [23] proposed a generic measure of system damage tolerance, based on the increase in failure probability resulting from the occurrence of damage. The vulnerability (V) of a system is defined as:

$$V = \frac{P(r_d, S)}{P(r_o, S)}$$

where r_d is the resistance of the damaged system, r_o is the resistance of the undamaged system, and S is the prospective loading on the system $P(\cdot)$ is the probability of failure of the system, as a function of the load and resistance of the system. Vulnerability parameter indicates the loss of system reliability due to damage.

As progressive collapse is characterised by disproportion between the magnitude of a triggering event and resulting in collapse of large part or the entire structure [20], Ellingwood and Leyendecker [19] defined the probability of such collapse as a chain of partial probabilities:

$$P(F) = P(F|DH) \cdot P(D|H) \cdot P(H)$$

where $P(H)$ denotes the probability of an abnormal event that threatens the structure (generally hazard H), $P(F|DH)$ is the probability of local damage D as a result of event H and $P(D|H)$ is the probability of failure F of the structure as a result of local damage D or H .

$$P(F) = \underbrace{\overbrace{P(F|D \cap H)}^{\text{collapse resistance}} \cdot \underbrace{P(D|H)}_{\text{vulnerability}} \cdot \underbrace{P(H)}_{\text{hazard}}}_{\text{robustness element event behaviour control}}$$

Figure 13. Terms in context regarding progressive failure [20]

Term hazard refers to abnormal loads or load effects [27]. Abnormal loads can be grouped as pressure loads (e.g., explosions, detonations, tornado wind pressures), impact (e.g., vehicular collision, aircraft or missile impact, debris, swinging objects during construction or demolition), deformation-related (softening of steel in fire, foundation subsidence), or as faulty practice. These loads usually act over a relatively short period of time in comparison with ordinary design loads. The loads generally are time-varying, but may be static or dynamic in their structural action [27].

Another approach to assess robustness is the robustness analysis framework proposed and introduced in the Danish Code of Practice for the Safety of Structures [16, 17]. It is based on progressive collapse concept [19,20]. Robustness is related to scenarios where exposures result in damage to

structural system. This means that a robust structure can be achieved by means of suitable choices of materials, general static layout and structural composition, and by suitable design of key elements. Robustness should be distinguished from accidental loads although some of the design procedures and measures are similar; structures should be robust regardless of the likelihood of accidental loads. A key element is defined as a limited part of the structure, which has an essential importance for the robustness of the structure such that any possible failure of the key element implies a failure of the entire structure or significant parts of it [4, 16, 17]. Examples of unintentional loads and defects are e.g. unforeseen load effects, geometrical imperfections, settlements and deterioration, unintentional deviations between the actual function of the structure and the applied computational models and between the executed project and the project material. The requirements to robustness of a structure should be related to the consequences of a failure of the structure. Therefore documentation of robustness is only required for structures in high safety class.

Robustness shall be assessed by preparation of a technical review where at least one of the following criteria shall be fulfilled:

- a) by demonstrating that those parts of the structure essential for the safety only have little sensitivity with respect to unintentional loads and defects
- b) by demonstrating a load case with 'removal of a limited part of the structure' in order to document that an extensive failure of the structure will not occur if a limited part of the structure fails
- c) by demonstrating sufficient safety of key elements, such that the entire structure with one or more key elements has the same reliability as a structure where robustness is documented by b

The design procedure to document sufficient robustness can be summarized in the following steps:

1. Review of loads and possible failure modes/scenarios and determination of acceptable collapse extent
2. Review of the structural systems and identification of key elements
3. Evaluation of the sensitivity of essential parts of the structure to unintentional loads and defects
4. Documentation of robustness by 'failure of key element' analysis
5. Documentation of robustness by increasing the strength of key elements if Step 4 is not possible.

This framework where robustness is related to an extensive failure of the structure due to unintentional loads and defects subjected to a limited part of the structure can be formulated in a probabilistic format [10, 17, 19]. Assume a structural damage D_j among j different types resulting from a number of exposures, i.e. unintentional loads and defects. If each of these i distinct exposures is represented by an event E_i then the total probability of structural collapse with the consequence C can be written as:

$$P(C) = \sum_i \sum_j P(C|E_i \cap D_j) \cdot P(D_j|E_i) \cdot P(E_i)$$

where $P(D_j|E_i)$ is the probability of damage type j given exposure type i and $P(C|E_i \cap D_j)$ is the probability of collapse given exposure type i and damage type j . For damages related to key elements the probability of collapse is $P(C|E_i \cap D_j) \approx 1$. From previous equation it is obvious that the probability of collapse can be reduced (and robustness can be increased) by:

- Reducing one or more of the probabilities of exposures $P(E_i)$

- Reducing one or more of the probabilities of damages $P(D_j|E_i)$ or reducing the extent of the damage
- Reducing one or more of the probabilities $P(C|E_i \cap D_j)$

Increasing the robustness at the design stage will in many cases only increase the cost of the structural system marginally – the key point is often to use a reasonable combination of a suitable structural system and materials with a ductile behaviour. In other cases increased robustness will influence the cost of the structural system. If more alternatives to increase the robustness are considered, then from a decision theoretical point of view, the optimal alternative is that which results in the smallest expected total costs.

2.3.3 RISK BASED DEFINITION OF A ROBUSTNESS

Baker, Schubert and Faber [21] proposed their definition of a robustness index. The approach divides consequences into direct consequences associated with local component damage (that might be considered proportional to the initiating damage) and indirect consequences associated with subsequent system failure (that might be considered disproportional to the initiating damage) [13]. An index is formulated by comparing the risk associated with direct and indirect consequences. The index of robustness (I_{Rob}) is defined as

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}}$$

where R_{Dir} and R_{Ind} are the direct and indirect risks. The index takes values between zero and one, with larger values indicating larger robustness. This method for assessing robustness is based on a risk assessment framework proposed by Joint Committee on Structural Safety. The assessment begins with the consideration and modelling of exposures (EX) that can cause damage to the components of the structural system. Term “exposures” refers on extreme values of design loads, accidental loads and deterioration processes but also includes human errors in the design, execution and use of the structure. Term “damage” refers to reduced performance or failure of individual components of the structural system. After the exposure event occurs, the components of the structural system either remain in an undamaged state (\bar{D}) as before or change to a damage state (D). Each damage state can then either lead to the failure of the structure (F) or no failure (\bar{F}).

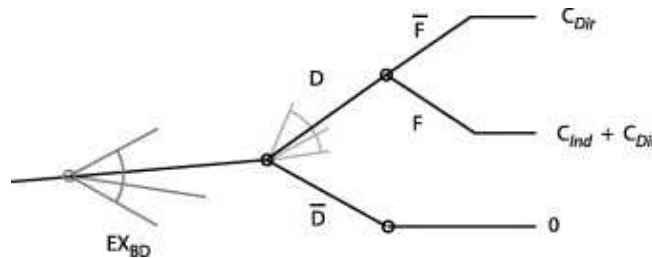


Figure 14. An event tree for robustness quantification [21]

As stated before, consequences are associated with each of the possible damage and failure scenarios, and are classified as either direct (C_{Dir}) indirect (C_{Ind}). Direct consequences are considered to result from damage states of individual component(s). Indirect consequences are incurred due to loss of system functionality or failure and can be attributed to lack of robustness [21].

In paper [21] example systems are also considered in order to provide an insight regarding system properties affecting robustness. The system consisting of one to ten components is examined (figure 19). Loads and resistances are modelled according to the PMC [2]. Individual components are assumed to be lognormal distributed with a COV=0.07. Exposures are defined as events which have

the ability to cause damage to the system. The applied load is assumed to be Weibull distributed. The mean value of the load is chosen to equal one, and various levels of COV are considered. The mean component resistance is selected so that each element has a specified probability of damage, given the distribution of applied loads [21]. The components are assumed to be either perfectly ductile or brittle with random resistances and each carries an equal portion of the applied load. When a component's resistance is exceeded, the additional load not carried by that component is either redistributed equally to the other components (figure 19a) or not redistributed (figure 19b). Damage is considered to have occurred when the load exceeds the resistance of at least one component. Failure occurs when the resistance of all components is exceeded. When abnormal loads causing the loss of one or more components are considered, damage will represent the loss of a component due to either the abnormal load or the applied load, and failure will indicate that the remaining components are not able to resist the applied load.

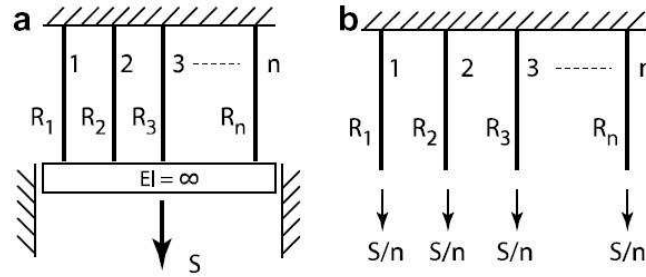


Figure 15. (a) A parallel system with load redistribution after component damage. (b) A parallel system with no load redistribution after component damage [21]

The results in the next figures illustrate the effect of varying coefficients of variation of the loading. In figure 20, results are shown for ductile systems with a varying number of components, and in figure 21 the same result is shown for brittle systems. In this paper it is proven that increasing the number of components increases a system's robustness. The robustness of brittle systems is very low (nearly zero) in all cases. Increasing the correlation among resistances has the same effect as reducing the number of components in the system. For systems in figure 21, the index of robustness (I_{Rob}) takes values that are very near to 0. This implies that the risk to a system primarily comes from indirect risks due to system failure.

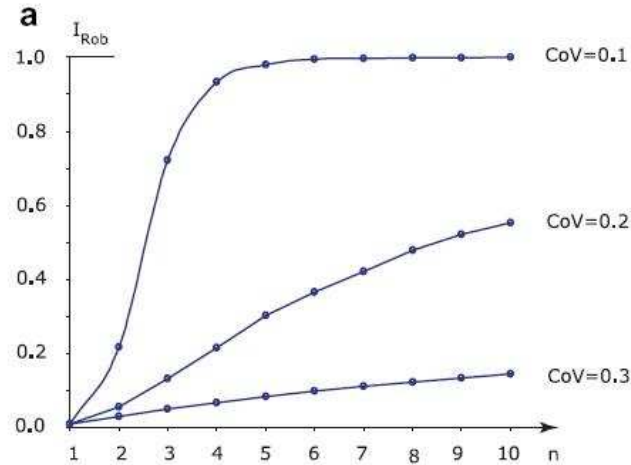


Figure 16. Ideal ductile parallel system

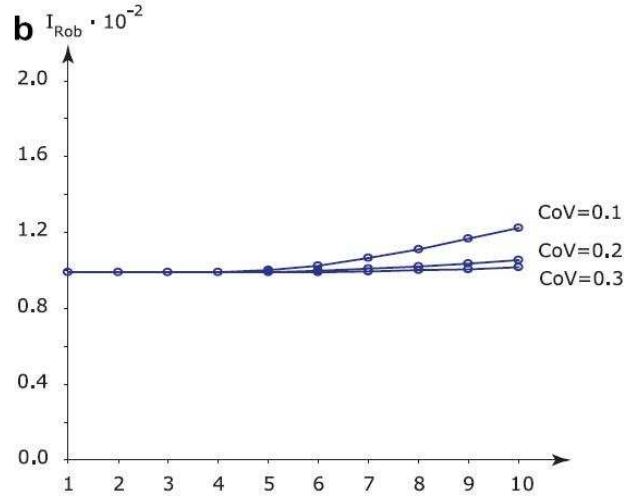


Figure 17. Ideal brittle parallel system

2.4 DESIGN PRINCIPLES FOR ROBUSTNESS

Because of many potential means by which a local collapse in a specific structure may propagate from its initial extent to its final state, there is no universal approach for evaluating the potential for progressive collapse [27].

For reduction of the risk of progressive collapse in the event of loss of structural element(s), the following structural traits should be incorporated in the design: (according to [27]):

- **Redundancy:** incorporation of redundant load paths in the vertical load carrying system
- **Ties:** using an integrated system of ties in three directions along the principal lines of structural framing (figure 22)
- **Ductility:** structural members and member connections have to maintain their strength through large deformations (deflections and rotations) so the load redistribution(s) may take place
- **Adequate shear strength:** as shear is considered as a brittle failure, structural elements in vulnerable locations should be designed to withstand shear load in excess of that associated with the ultimate bending moment in the event of loss of an element
- **Capacity for resisting load reversals:** the primary structural elements (columns, girders, roof beams, and lateral load resisting system) and secondary structural elements (floor beams and slabs) should be designed to resist reversals in load direction at vulnerable locations
- **Connections (connection strength):** connections should be designed in such way that it will allow uniform and smooth load redistribution during local collapse
- **Key elements:** exterior columns and walls should be capable of spanning two or more stories without bucking, columns should be designed to withstand blast pressure etc [27].
- **Alternate load path(s):** after the basic design of structure is done, a review of the strength and ductility of key structural elements is required to determine whether the structure is able to “bridge” over the initial damage [27].

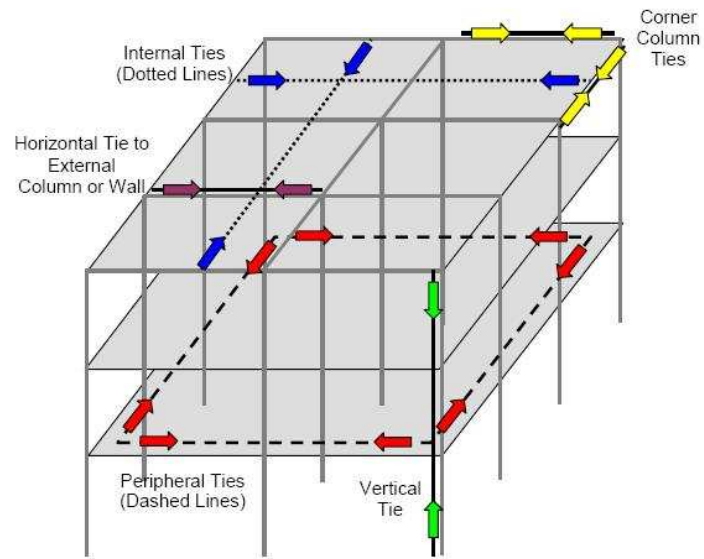


Figure 18. Different types of ties incorporated to provide structural integrity [27]

3 ROBUSTNESS OF TIMBER STRUCTURES

In the last few decades there have been intensely research concerning reliability of timber structures but robustness of timber structures has not been evaluated yet. One of the reasons for lacking information about robustness of timber structures is that a unified approach for assessing robustness of any material is not defined yet. As timber is a complex building material, assessment of robustness is very hard to conduct. As there is obvious correlation between the redundancy and robustness, redundant structures will, in principle, be a more robust than statically determinate. However, in respect to timber structures, there are not many redundant systems, and the obvious way to asses a robustness of such structures is to demonstrate that the part(s) of the structure essential for the reliability have little sensitivity with respect to unintentional loads and defects. Other approach is to increase redundancy, but this will definitely affect the cost of the structure.

For the purpose of the project „Timber Frame 2000” [24] a six-storey experimental timber frame building was erected, in order to investigate the performance and economic prospects of medium-rise timber frame buildings in the UK. As a part of a testing programme the investigation of disproportionate collapse (robustness) was conducted. This evaluation is to verify that the inherent stiffness of cellular platform timber frame construction can provide the necessary robustness so that, in the event of an accident, the building will not suffer collapse to an extent disproportionate to the cause [24]. This is achieved by designing in such a way that a beam, column or section of wall can be removed without the structure above collapsing (although damage to the building is allowed). To achieve this, beams are incorporated within floor depths over external walls, or the walls themselves are made to act as beams. The building was loaded with sandbags positioned on each floor. Based on an analytical review of the building, agreed serviceability requirements and defined rules 'worst case scenario' is chosen for the test. Result obtained show that this kind of timber frame system is very robust.



Figure 19. Test of timber frame

Table 10. Summary of disproportional collapse results [24]

Duration	Vertical deflection (mm)	
	Floor	Wall above
For internal wall removed		
30 minutes	13	-
4 hours	19	-
20 hours	26	-
For external wall removed		
30 minutes	2.5	1.2
4 hours	3.7	2.4
20 hours	4.0	2.6

3.1 DUCTILITY OF A TIMBER AND TIMBER CONNECTIONS (JOINTS)

Ductility aspects of a timber

Timber is considered to be a brittle material, because failure occurs suddenly, without any warning. This can be considered as an obstacle when comparing to other materials like steel. It has no or a very little ductility in the tensile area, while in compressive area linear elastic-plastic behaviour can be assumed. [7]

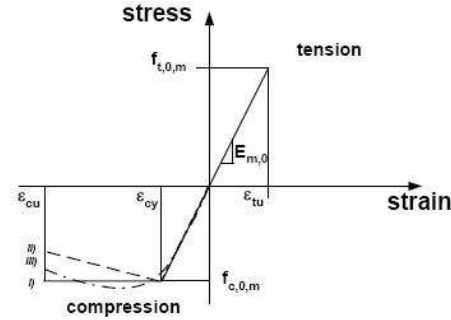


Figure 20. Typical stress strain curve of timber

Typical stress redistribution for different states (during bending) along the cross section of the beam is given in following figure. In stage I purely elastic behaviour is observed. With the increase of a load the part of a beam in compression behaves plastic and neutral axis shifts along the tensile side of a cross section. Stage III represents stresses prior to failure of a beam.

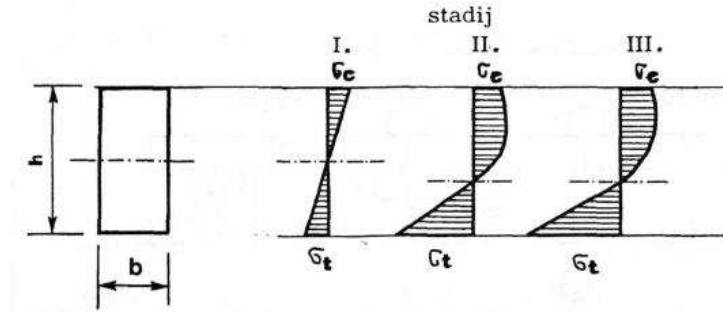


Figure 21. Stress stages in bending

There are many models for plastic distribution of the stresses in the beam. One of the simplest is given in figure 14 [15]. The tension zone is characterized by a linear, brittle model, but the compression zone is elastic-plastic. The modulus of elasticity for both tension and compression are the same [15].

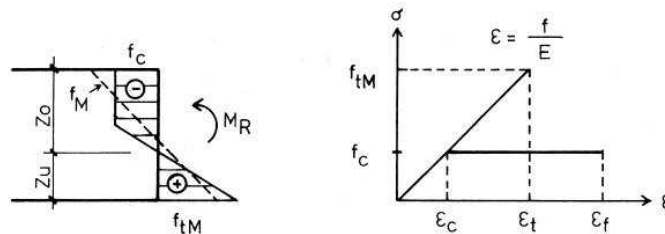


Figure 22. Stress stages in bending

The value f_{tM} can be determined by comparing the equations for the bending resistance MR of beam with a rectangular cross-section, calculated according to [15]) and then according to the Eurocode.

$$MR = f_c \cdot (b \cdot h^2 / 6) \cdot c, \quad c = [3 + 8 \cdot m + 6 \cdot m^2 - m^4] / (1 + m)^4$$

$$m = f_c / f_{tM}$$

$$MR = f_M \cdot (b \cdot h^2 / 6)$$

Ductility of joints

In the aspect of timber joints all agree that the way to achieve high ductility is to take advantage of the plasticity of mechanical connectors (nails, dowels, bolts, etc.) The only certain way to create ductile structure is design in which collapse of a structure is governed by failures of mechanical joints [8]. This is especially important for the seismic behaviour of a timber structure. The definition of ductility, with respect to behaviour in joints, is:

$$D_f = \frac{u_f}{u_y}$$

where u_f denotes the deformation at which the connection loses stability and u_y is the elastic deformation [9].

It must be noted that there are many different approaches to quantify ductility in joints but all of them incorporate relationships between the elastic displacements, displacements at maximum load and ultimate displacements [8]. Another very important issue is that the joint ductility, elastic displacements, displacements at maximum load and ultimate displacements depend much upon the type of the connections used (dowel type fasteners, tooth plates and punched metal plates). There are also significant differences between different dowel type fasteners (bolts, dowels, nails, etc.)

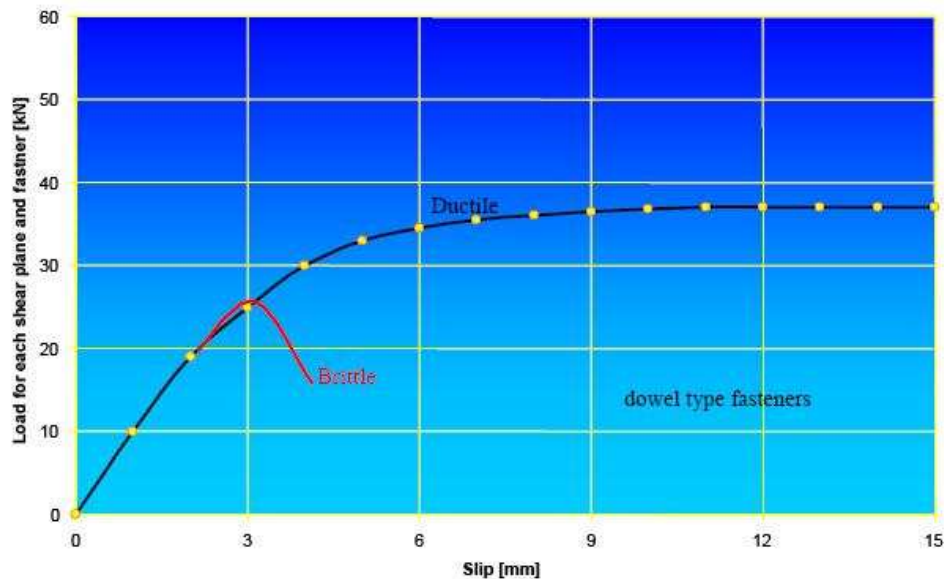


Figure 23. Ductile and brittle behaviour of a joint

3.2 METHODS OF ROBUSTNESS ASSESSMENT

Following the definition of a key element it is possible to propose general concept for the robustness assessment. First, there should be distinction between the redundant and non redundant structures. One certain way to increase robustness is to induce redundancy but this should not be the general concept because this affects the overall cost of a structure and on the other hand, potentially induces stresses due to temperature, creep etc.

3.2.1 REDUNDANT STRUCTURES

For redundant structures it is necessary to identify key element(s) and document that after the local failure or a collapse, the system has an ability to redistribute forces (alternative path method) and that the local failure won't result as a total collapse. Based on these scenarios, the robustness of a system can be calculated. If the systems robustness is not adequate, the modification / strengthening of the structural system can be made.

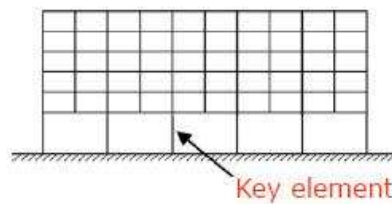


Figure 24. Identification of key element

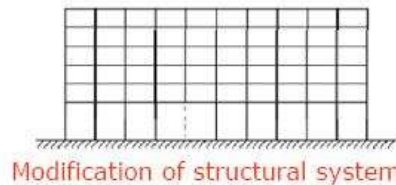


Figure 25. Modification of structural system

3.2.2 NON REDUNDANT STRUCTURES

For non redundant structures sensitivity of a key element(s) upon increased (abnormal, extreme) load must be verified. It should be documented that for a given abnormal load, the load effect will not be disproportionate to the given load. Similar to redundant systems, the robustness of the system can be calculated.

4 CONCLUSION

Purpose of this STSM was to provide “state of the art” on the existing methods for reliability and robustness assessment of structures in general and a possibility for their application to timber structures. Special attention is drawn upon the system reliability and the modelling of timber as a material and timber joints. It can be concluded that both behaviours (the possible ductile behaviour of a timber and ductility in joints) may result in significant increase of the robustness. For the joints, ductility is necessary as they can constitute weak components in structure. Ductility is also very important during seismic excitations. For the redundant structures behaviour of the joints will play a significant role. Material ductility is also desired. However, for the material behaviour no specific conclusion can be given, as timber is a complex material with different (ductile/brittle) behaviour in relation to type of the load effect (compression/tension). Additional investigation must be made to in order to quantify for what kind of structures, and more importantly, in which extent, can material ductility influence the robustness. Detailed 3D FEM analysis as input for robustness assessment could provide better and more realistic behaviour of the structure than usual plane models.

ACKNOWLEDGEMENTS

The work described in this report was conducted at Aalborg University, Denmark as a part of the COST Action E55 “Modelling of the performance of timber structures”. Financial support provided by COST Action E55 is gratefully acknowledged.

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