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Accuracy assessment of digital elevation models by means of robust statistical methods

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ABSTRACT

Measures for the accuracy assessment of Digital Elevation Models (DEMs) are discussed and characteristics of DEMs derived from laser scanning and automated photogrammetry are presented. Such DEMs are very dense and relatively accurate in open terrain. Built-up and wooded areas, however, need automated filtering and classification in order to generate terrain (bare earth) data when Digital Terrain Models (DTMs) have to be produced. Automated processing of the raw data is not always successful. Systematic errors and many outliers at both methods (laser scanning and digital photogrammetry) may therefore be present in the data sets. We discuss requirements for the reference data with respect to accuracy and propose robust statistical methods as accuracy measures. Their use is illustrated by application at four practical examples. It is concluded that measures such as median, normalized median absolute deviation, and sample quantiles should be used in the accuracy assessment of such DEMs. Furthermore, the question is discussed how large a sample size is needed in order to obtain sufficiently precise estimates of the new accuracy measures and relevant formulae are presented.

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1. Introduction

Digital Elevation Models are today produced by digital photogrammetry or by laser scanning. Both methods are very efficient and accurate; the density of the elevations is very high. However, blunders may occur at both methods. From the raw data a Digital Terrain Model (DTM) and a Digital Surface Model (DSM) are generated by means of filtering (for classifying into ground and off terrain points) and interpolation (for filling gaps). Errors may also occur during such a post-processing. The quality control should detect errors and outliers in order to eliminate them. As a final step it has to be checked, whether the edited DTM and DSM achieve the accuracy of the specification. For this purpose, accurate reference values are required, and accuracy measures like the Root Mean Square Error (RMSE), mean error and the standard deviation have to be derived. The amount of data is huge, but the accuracy assessment has to be made with few check points only as it is very labour intensive to obtain them. However, the sample size should be large enough to guarantee reliable accuracy measures, which are valid for the whole DTM or DSM. Usually, the specification of accuracy measures is based on the assumption that the errors follow a Gaussian distribution and that no outliers exist. But all too often this is not the case, because objects above the terrain like vegetation, buildings and unwanted objects (cars, people, animals, etc.) are present, and the filtering program may not label all ground elevations correctly. Also system errors will occur: Photogrammetry needs structure and texture in the images and not all of the image parts fulfil this requirement. Laser light is not always reflected directly by the points to be measured and the position and altitude of the sensor may be in error. Positional errors will cause vertical errors at terrain with steep slopes and buildings. Altogether, editing of the data has to detect and correct such errors, but even with the most careful editing errors will remain. The number or percentage of outliers should be documented, for example in metadata, so that one can judge whether the derived DTM is usable for the intended application (“fit for purpose”).

The derivation of accuracy measures has to adapt to the fact that outliers may exist and that the distribution of the errors might not be normal. There is thus a need for accuracy measures, which are reliable without being influenced by outliers or a skew distribution of the errors.

These facts are well known and mentioned in recently published textbooks and manuals, for example in Li et al. (2005) and Maune (2007). Recent publications, which deal in detail with
accuracy assessment, are for example Carlisle (2005), Höhle and Potuckova (2006), Fisher and Tate (2007), Aguilar et al. (2007a) and Zandbergen (2008).

Our approach continues along these lines as we focus on vertical accuracy assessment in the light of outliers and non-normal distributions. It is the objective of this article to advocate robust statistical methods for the derivation of vertical accuracy measures for digital elevation models. Robust approaches handling outliers and detaching accuracy measures from the assumption of an underlying normal distribution have increasingly been suggested in the literature (e.g. Atkinson et al., 2005 and Aguilar et al., 2007b). It is a topic of discussion how national and international standards for DEMs should cope with these matters. So far, only the Lidar committee of the American Society for Photogrammetry and Remote Sensing deals with it and requires non-parametric accuracy measures for non-open terrain (ASPRS Lidar Committee, 2004). With this article focusing on robust estimation of variance and the estimation of sample quantiles as measures for vertical accuracy, we want to contribute to this discussion of DEMs produced either by airborne laser scanning or automated digital photogrammetry. In our approach we interpret accuracy measures directly as quantities of the error distribution — alternatives are more indirect measures such as e.g. the coefficient of variation of the sample variance (Aguilar et al., 2007a).

When validating DEMs, accurate reference data have to be available in a sufficiently large number. The question how many check points are needed can be treated within the statistical context of sample size computation. We show how required sample sizes can be calculated for the suggested quantile approach to accuracy.

The paper is organized as follows: Section 2 discusses accuracy requirements for DEMs, Sections 3–5 deal with vertical errors and provide ordinary and robust accuracy measures. Section 6 discusses how these can be used to assess fulfillment of a specification using statistical tests, and Section 7 illustrates the methods using four examples of DTM accuracy. A discussion of the results completes the paper.

2. Requirements regarding the reference data

Accuracy assessment of the DEM is carried out by means of reference data called checkpoints. Because their position does not coincide with the points of the DEM, an interpolation is necessary. For a DEM with a grid structure a bilinear interpolation is normally used. The accuracy of the checkpoints should be at least three times more accurate than the DEM elevations being evaluated (Höhle, 2007, pp. 407). By using the formula for error propagation the influence on the DEM accuracy can be estimated:

\[
\sigma_{\text{DEM−REF}}^2 = \sigma_{\text{DEM}}^2 + \sigma_{\text{REF}}^2 \leq \sigma_{\text{DEM}}^2 + \left( \frac{1}{3} \cdot \sigma_{\text{DEM}} \right)^2
\]

\[
= \frac{10}{9} \cdot \sigma_{\text{DEM}}^2.
\]

Hence, \(\sigma_{\text{DEM−REF}} \leq 1.05 \cdot \sigma_{\text{DEM}}\). The derived DEM accuracy is then incorrect by 5%, which is acceptable. For example, if the accuracy of a DEM is specified with \(\sigma = 10\) cm then the checkpoints should have an accuracy of \(\sigma \leq 3.3\) cm. The DEM accuracy would then amount to 10.5 cm only.

An important issue is the spatial distribution of the checkpoints: they should be distributed randomly. If checkpoints are positioned along break lines, at steep slopes, at positions of sudden slope change, close to buildings, etc., large errors may be found. On the other hand, the number of checkpoints (aka. sample size) should be sufficiently large in order to obtain reliable accuracy measures.

In Section 6 we return to the issue of how to compute the sample size in order to prove the compliance with accuracy specifications.

3. The distribution of errors

The distribution of errors can be visualized by a histogram of the sampled errors, where the number of errors (frequency) within certain predefined intervals is plotted. Such a histogram gives a first impression of the normality of the error distribution. Fig. 1 depicts a histogram of the error distribution for photogrammetric measurements compared with checkpoints. In order to compare with normality, the figure contains a superimposed curve for a normal distribution (Gaussian bell curve) obtained by ordinary estimation of mean error and variance. Because outliers are present in the data, the estimated curve does not match the data very well. Reasons could be that the errors are not originating from a normal distribution, e.g. because a skew distribution exists which is not symmetric around its mean or because the distribution is more peaked around its mean than the normal distribution while having heavier tails. The latter effect is measured by the kurtosis of the distribution, which in this situation is bigger than zero.

A better diagnostic plot for checking a deviation from the normal distribution is the so-called quantile–quantile (Q–Q) plot. The quantiles of the empirical distribution function are plotted against the theoretical quantiles of the normal distribution. If the actual distribution is normal, the Q–Q plot should yield a straight line. Fig. 2 shows the Q–Q plot for the distribution of \(\Delta h\) in the same example as Fig. 1. A strong deviation from a straight line is obvious, which indicates that the distribution of the \(\Delta h\) is not normal.

It is also possible to use statistical tests to investigate whether data originate from a normal distribution, but these tests are often rather sensitive in case of large data sets or outliers. We, therefore, prefer the visual methods presented above. More details about the mentioned statistical tests for normality can be taken from e.g. D’Agostino et al. (1990), who also recommend visual methods as a component of good data analysis for investigating normality.
4. Accuracy measures for the normal distribution

If a normal distribution can be assumed and no outliers are present in the data set, the following accuracy measures for DEMs can be applied (cf. Table 1).

In the table \( \Delta h_i \) denotes the difference from reference data for a point \( i \), and \( n \) is the number of tested points in the sample (sample size). When an underlying normal distribution of the errors can be assumed, it is well known from the theory of errors that 68.3% of the data will fall within the interval \( \mu \pm \sigma \), where \( \mu \) is the systematic error (aka. bias) and \( \sigma \) is the standard deviation, see e.g. Mikhail and Ackermann (1976). If the accuracy measure should be based on a 95% confidence level (aka. a 95% tolerance interval), the interval is \( \mu \pm 1.96 \cdot \sigma \) instead. However, for DEMs derived by laser scanning or digital photogrammetry a normal distribution of the errors is seldom due to e.g. filtering and interpolation errors in non-opent terrain. In this work we describe and compare several approaches how to deal with this situation.

One approach to deal with outliers is to remove them by applying a threshold. For example, the threshold can be selected from an initial calculation of the accuracy measures. In the DEM tests described in Höhle and Potuckova (2006), the threshold for eliminating outliers was selected as three times the Root Mean Square Error (RMSE), i.e. an error was classified as outlier if \(| \Delta h_i | > 3 \cdot \text{RMSE} \). Another approach is to use \( 3 \cdot \sigma \) as the threshold for the outlier detection (aka. 3 sigma rule), where \( \sigma \) is the specified vertical accuracy or a preliminary value for the standard deviation which is derived from the original data set (Daniel and Tennant, 2001). But not all of the outliers can be detected in this way, and the DEM accuracy measures (mean error and standard deviation) will therefore be wrong or inaccurate. Furthermore, it can be shown that even the best methods based on outlier removal do not achieve the performance of robust methods, because the latter are able to apply a more smooth transition between accepting and rejecting an observation (Atkinson et al., 2005). Robust methods for the derivation of accuracy measures should therefore be applied for the assessment of DEM accuracy.

5. Robust accuracy measures suited for non-normal error distributions

If the histogram of the errors reveals skewness, kurtosis or an excessive amount of outliers, another approach for deriving accuracy measures has to be taken. Such an approach has to be resistant to outliers, and the probability assumptions to be made should not assume normality of the error distribution. Our suggestion in this case is to use the sample quantiles of the error distribution.

The quantile of a distribution is defined by the inverse of its cumulative distribution function (CDF), \( F \), i.e.
\[
Q(p) = F^{-1}(p)
\]
for \( 0 < p < 1 \). For example, the 50% quantile, \( Q(0.5) \), is the median of the distribution. An alternative definition using the minimum is necessary in cases where \( F \) is a step function and thus no unique definition of the inverse exists:
\[
Q(p) = \min\{x : F(x) \geq p\}.
\]

Sample quantiles are non-parametric estimators of the distributional counterparts based on a sample of independent observations \( \{x_1, \ldots, x_n\} \) from the distribution. We use the so-called order statistic of the sample \( \{x_1, \ldots, x_n\} \), where the order statistic is the minimum and the order statistic is the maximum of \( \{x_1, \ldots, x_n\} \). Thus, for a sample \( \{0.1, -0.3, -0.5, 0.4, 0.1\} \) of size \( n = 5 \), the order statistic is \( \{-0.5, -0.3, 0.1, 0.1, 0.4\} \).

A simple definition of the sample quantile is now \( \hat{Q}(p) = x_{j(\cdot)} \) where \( j = \lfloor p \cdot n \rfloor \) and \( [p \cdot n] \) denotes rounding up to the smallest integer not less than \( p \cdot n \).

If an interest exists in the 10%, 20%, 50% and 90% quantile, the following values for \( j \) are obtained:
\[
j = \{0.1 \cdot 5\} = 1, \quad j = \{0.2 \cdot 5\} = 2, \quad j = \{0.5 \cdot 5\} = 3, \quad j = \{0.9 \cdot 5\} = 5.
\]

The corresponding sample quantiles of the distribution are then:
\[
\hat{Q}(0.1) = x_{(1)} = -0.5, \quad \hat{Q}(0.2) = x_{(2)} = -0.3, \quad \hat{Q}(0.5) = x_{(3)} = 0.1, \quad \text{and} \quad \hat{Q}(0.9) = x_{(5)} = 0.4.
\]

In other words, the 50% quantile or median of this sample is 0.1 and the 90% quantile equals 0.4.

An often desired property of \( \hat{Q}(p) \) is that it should be a continuous function of \( p \). To obtain this and other desirable properties, one can extend the above definition by using a linear interpolation between the two relevant successive data points (Hyndman and Fan, 1996). The calculation for the practical examples of Section 6 will be carried out by the software “R” — a free software environment for statistical computing and graphics (R Development Core Team, 2008).

With respect to the application of accuracy measures of DEMs we use the distribution of \( \Delta h \) and \( | \Delta h \| \). One robust quality measure is the median \( \hat{Q}_{\Delta h}(0.5) \), also denoted \( m_{\Delta h} \), which is a robust estimator for a systematic shift of the DEM. It is less sensitive to outliers in the data than the mean error and provides a better distributional summary for skew distributions.

A robust and distribution free description of the measurement accuracy is obtained by reporting sample quantiles of the distribution of the absolute differences, i.e. \( | \Delta h \| \). Absolute errors are used, because we are interested in the magnitude of the errors and not their sign. Furthermore, absolute errors allow us to make probability statements without having to assume a symmetric distribution.

For example, the 95% sample quantile of \( | \Delta h \| \) literally means that 95% of the errors have a magnitude within the interval \( [0, \hat{Q}_{\Delta h}(0.95)] \). The remaining 5% of the errors can be of any value making the measure robust against up to 5% blunders. Such probability statements about a certain proportion of the errors falling within a given range are usually obtained by assuming a normal distribution. For example, one assumes that the symmetric interval of \( \pm 1.96 \cdot \hat{\sigma} \) (where \( \hat{\sigma} \) is the estimated standard deviation) contains 95% of the errors (assuming no systematic error). Equivalently, this means that 95% of the absolute errors are within \( [0, 1.96 \cdot \hat{\sigma}] \). Thus, if the distribution of \( \Delta h \) is really normal then \( \hat{Q}_{\Delta h}(0.95) \) converges to the estimator of \( 1.96 \cdot \hat{\sigma} \).

If the problem is the heavy tails of the error distribution due to a large amount of outliers, an alternative approach to estimate the scale of the \( \Delta h \) distribution is to use a robust scale estimator such as the Normalized Median Absolute Deviation (NMAD):
\[
\text{NMAD} = 1.4826 \cdot \text{median}_i (| \Delta h_i | - m_{\Delta h}),
\]
where \( \Delta h_i \) denotes the individual errors \( j = 1, \ldots, n \) and \( m_{\Delta h} \) is the median of the errors. The NMAD is thus proportional to the median of the absolute differences between errors and the median error. It can be considered as an estimate for the standard deviation more resilient to outliers in the dataset. In
case of an underlying normal distribution this value is equivalent to
the standard deviation if the number of checkpoints (i.e. \( n \)) is
sufficiently large. More details about such robust estimation can
be taken from Hoaglin et al. (1983).

In summary, as a robust and distribution free approach handling
outliers and non-normal distribution we suggest the following
accuracy measures given in Table 2.

One could furthermore assess non-normality using estimators
for the skewness and kurtosis of the distribution. However,
to be consistent, robust estimators such as the Bowley coefficient
of skewness and a standardized kurtosis measure suggested by
Moors (1988) should be used. However, we will in this text use the
more intuitive visual inspection using QQ-plots and histograms.

In a statistical context all estimates of population quantities
should be supplied by measures quantifying the uncertainty
of the estimator due to estimation from a finite sample. One way to
achieve this is to supply with each point estimator a confidence
interval (CI) with a certain coverage probability. For example, a
95% CI \([c_1, c_2] \) for the sample median says that in 95% of the errors
the interval \([c_1, c_2] \) contains the true but unknown median of the
error distribution.

Using the bootstrap is one approach to assess the uncertainty
of the above sample quantiles as estimators of the true quantiles
of the underlying distribution (Davidson and Hinkley, 1997). Here
one draws a sample of size \( n \) with replacement from the available
data \( \{x_1, \ldots, x_n\} \) and then uses this new sample to compute
the desired \( \hat{Q}(p) \). This procedure is repeated a sufficiently large
number of \( m \) times; in our case we choose \( m = 999 \), which yields
999 values \( \hat{Q}(p, 1), \ldots, \hat{Q}(p, 999) \). A bootstrap based 95% confidence
interval of \( Q(p) \) can then be obtained as the interval from the
2.5% to the 97.5% sample quantiles of the 1000 available values
\( \{\hat{Q}(p, 1), \hat{Q}(p, 2), \ldots, \hat{Q}(p, 999)\} \).

We prefer such a bootstrap approach over classical large sample
arguments to construct confidence intervals (as e.g. in Desu and
Raghavarao (2003)), because the bootstrap is more robust to the
small number of check points used and can be extended to handle
autocorrelated data.

So far all calculations in our work are based on the assumption
that errors are independent and identically distributed. However,
as Fig. 7 shows, there is substantial spatial autocorrelation present
in the data, which will make the proposed bootstrap estimated
confidence intervals too narrow. One approach to treat this
problem is to modify the above bootstrap procedure to take the
dependence of data into account using e.g. a block bootstrap
(Lahiri, 2003). Another approach would be dividing data into a
number of terrain classes and compute sample quantiles within
each class. A statistical framework for this task is quantile
regression (Koenker, 2005). In its simplest form a quantile
regression model for the \( p \)-th quantile of the absolute error
distribution is

\[
|\Delta h_i| = \beta_0 + \varepsilon_i,
\]

where the \( \varepsilon_i \) are independent realisations of a random variable
having a CDF \( F \) which is completely unspecified having the only
requirement that \( F(0) = p \). The value of \( \beta_0 \) depends on the quantile
\( p \), but for ease of exposition we have omitted this from the notation.

It is then possible to show that with this formulation we have
\( \hat{\beta}_0 = Q(p) \). An estimator for \( \beta_0 \) is obtained from the \( n \) observed
absolute errors by solving the following minimization problem:

\[
\hat{\beta}_0 = \arg\min_{\beta_0} \sum_{i=1}^{n} \rho_\beta(|\Delta h_i| - \beta_0)
\]

where \( \rho_\beta(u) \) is the so-called check function for the \( p \)-th quantile
defined as \( \rho_\beta(u) = u - (p - I(u < 0)) \) with \( I(u < 0) \) being one
if \( u < 0 \) and zero otherwise. The above minimization problem
can be solved by linear programming and is implemented in the R
add-on package quantreg, see Koenker (2005). It can be shown that \( \hat{\beta}_0 \) is equivalent to the previous definition of the \( p \)-th sample quantile, i.e. \( \hat{\beta}_0 = Q(p) \).

General quantile regression for a specific quantile \( p \) proceeds
by replacing the single parameter model \( Q(p) = \beta_0 \) with a linear
predictor \( Q(p) = x^T \beta \) as in ordinary linear regression. This allows
e.g. modelling the \( p \)-quantile as a function of terrain classes as in
Carlisle (2005), who used ordinary least squares regression for this
task. Modelling autocorrelation in an even better way would be to
model the \( p \)-quantile as a function of the \((x, y)\) position in the
plane, i.e. \( Q(p) = f(x, y) \), where the function \( f \) could for example
be a tensor product of univariate basis functions or a triogram
function as described in Koenker and Mizera (2004). In both
cases, penalization is used to ensure an appropriate smoothness
of the function. Using the Koenker and Mizera (2004) approach
a continuous, piecewise linear function over an adaptively selected
triangulation over the \((x, y)\) plane for e.g. the 95% quantile of the
absolute error distribution can thus be obtained.

6. Statistical tests and sample size calculations

It becomes obvious from the confidence intervals of the preceding
section that an important issue in the above quality control is the question
of how large a sample size is needed in order to obtain sufficiently precise estimates of \( \sigma \) and the sample quantities. One approach to this problem is to solve it by means
of a sample size methodology for statistical tests, which will be
described in the following for the case where errors are assumed
to have no autocorrelation.

Considering a normal distribution of the errors, a typical
specification would be to demand \( \sigma < \sigma_{\text{spec}} \) with e.g. \( \sigma_{\text{spec}} =
10 \text{ cm} \), i.e. the true (but unknown) standard deviation of the error
distribution is smaller than 10 cm. Thus, we may formulate a
statistical test with null hypothesis

\[
H_0 : \sigma^2 = \sigma_{\text{spec}}^2
\]

and an alternative hypothesis

\[
H_a : \sigma^2 < \sigma_{\text{spec}}^2.
\]

A sample of size \( n \) is now drawn and the hope is to be able to reject the null hypothesis (thus
proving the desired specification). This is done (see e.g. Desu and
Raghavarao (1990)) if

\[
\hat{\sigma}^2 < \frac{\sigma_{\text{spec}}^2 \cdot \chi_{a,n-1}^2}{n - 1},
\]

(3)

where \( \alpha \) is the pre-specified type 1 error probability, i.e. the
probability of erroneously rejecting \( H_0 \) when \( \sigma = \sigma_{\text{spec}} \) and
\( \chi_{a,n-1}^2 \) denotes the \( \alpha \) quantile of the \( \chi^2 \) distribution with \( n - 1 \) degrees
of freedom. The parameter \( \alpha \) is also called the level of the test —
in our work we shall use \( \alpha = 0.05 \). In other words: To check if
the DEM specification \( \sigma < 10 \text{ cm} \) is fulfilled we specify that at
the extreme setting where the specification is not fulfilled, i.e. at
\( \sigma = 10 \text{ cm} \), we want to detect this lack of compliance based on (3)
with a probability of 95%.

Let \( \sigma_1 < \sigma_{\text{spec}} \) and \( \beta \) be two predefined constants, for example
\( \sigma_1 = 7.5 \text{ cm} \) and \( \beta = 0.05 \). If we require that the probability
of correctly rejecting \( H_0 \) when \( \sigma = \sigma_1 \) is equal to \( 1 - \beta \), the


Table 2

<table>
<thead>
<tr>
<th>Proposed accuracy measures for DEMs.</th>
<th>Error type</th>
<th>Notational expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy measure</td>
<td>Error type</td>
<td>Notational expression</td>
</tr>
<tr>
<td>Median (50% quantile) ( \Delta h )</td>
<td>( \hat{Q}<em>{\Delta h}(0.5) = m</em>{\Delta h} )</td>
<td>NMAD = 1.4826 \cdot \text{median}(</td>
</tr>
<tr>
<td>Normalized median absolute deviation ( \Delta h )</td>
<td>( \hat{Q}_{\Delta h}(0.683) )</td>
<td>MADM = 0.68268 \cdot \text{median}(</td>
</tr>
<tr>
<td>68.3% quantile</td>
<td>( \hat{Q}_{\Delta h}(0.68) )</td>
<td></td>
</tr>
<tr>
<td>95% quantile</td>
<td>( \hat{Q}_{\Delta h}(0.95) )</td>
<td></td>
</tr>
</tbody>
</table>
necessary sample size is found as the positive smallest integer \( n \), which satisfies

\[
\frac{\sigma_{\text{spec}}}{\sqrt{n}} - \frac{1}{1 - \beta} \cdot \frac{1}{n - 1} \geq 0.
\]

Here, we say that the test has a power of \( 1 - \beta \) at \( \sigma_1 \), i.e. \( \beta \) is the probability of erroneously keeping \( H_0 \) even though \( \sigma = \sigma_1 \) and thus \( H_1 \) applies because \( \sigma < \sigma_{\text{spec}} \). For the above selection of values for \( \sigma_{\text{spec}}, \sigma_1, \alpha \) and \( \beta \) we obtain \( n = 68 \). Fig. 3 shows the involved quantities and how different sample sizes \( n \) lead to different powers at \( \sigma_1 \). With \( n = 68 \) the desired power of 0.95 is achieved.

Because \( (10\, \text{cm})^2 \cdot X_{0.05,67} / 67 = 73.8 \, \text{cm}^2 \) we have that \( \sigma^2 < 73.38 \, \text{cm}^2 \) in 95\% of the cases when \( \sigma^2 \) is computed from 68 normally distributed random variables having \( \sigma = 10 \, \text{cm} \). Similarly, at \( \sigma = 7.5 \, \text{cm} \), where the specification is fulfilled, we want to be sure to detect this compliance based on (3) with a probability of 95\% — for \( \sigma \) greater than 7.5 \, \text{cm} \) this probability will be smaller as shown in Fig. 3.

For comparison, the American Society of Photogrammetry and Remote Sensing (ASPRS) recommends a minimum of 20 checkpoints in each of the major land cover categories. In the case of three landcover classes (e.g. open terrain, forested areas, and urban areas) a minimum of \( n = 60 \) checkpoints are required (ASPRS Lidar Committee, 2004).

The above test is, however, very sensitive to deviations from the normal distribution. Here our suggestion was to use the quantiles of the absolute error distribution. Similarly, we suggest proving compliance with a specification using statistical tests for the quantiles of the error distribution. To test whether the 68.3\% quantile of the absolute error distribution is below 10 \, \text{cm}, for each observation \( \Delta h_i \), a zero-one variable \( Y_i \) is defined as

\[
Y_i = I(|\Delta h_i| < 10 \, \text{cm})
\]

where \( I \) is the indicator function and the constant \( c \) is chosen as the smallest integer so that

\[
F(c - 1; \, n, \, p_0) = \sum_{i=1}^{c-1} \binom{n}{i} p_0^i (1 - p_0)^{n-i} \leq \alpha,
\]

with \( F \) the cumulative distribution function of the binomial distribution with parameters \( n \) and \( p_0 \). The test is performed with the alternative hypothesis \( H_1: \, p > p_0 \). In our example of the 68.3\% quantile \( p_{0.05} = 0.683 \), i.e. we want to investigate if more than 68.3\% of the absolute errors are smaller than 10 \, \text{cm}. If so, the desired accuracy specification is fulfilled. The test statistic is defined as

\[
T = \sum_{i=1}^{n} Y_i
\]

where \( T \) is normal with mean zero and variance \( \sigma^2 \). From the Q–Q plot in Fig. 4, the test statistic is defined as

\[
|\Delta h_i| < 10 \, \text{cm} \quad \text{if } T > c, \quad \text{where } c = \alpha / \sqrt{n}
\]

which leads to a relatively large sample size. To test whether the 68.3\% quantile of the absolute error distribution is below 10 \, \text{cm}, for each observation \( \Delta h_i \), a zero-one variable \( Y_i \) is defined as

\[
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\]

with \( F \) the cumulative distribution function of the binomial distribution with parameters \( n \) and \( p_0 \). The test is performed with the alternative hypothesis \( H_1: \, p > p_0 \). In our example of the 68.3\% quantile \( p_{0.05} = 0.683 \), i.e. we want to investigate if more than 68.3\% of the absolute errors are smaller than 10 \, \text{cm}. If so, the desired accuracy specification is fulfilled. The test statistic is defined as

\[
T = \sum_{i=1}^{n} Y_i
\]

where \( T \) is normal with mean zero and variance \( \sigma^2 \). From the Q–Q plot in Fig. 4, the test statistic is defined as

\[
|\Delta h_i| < 10 \, \text{cm} \quad \text{if } T > c, \quad \text{where } c = \alpha / \sqrt{n}
\]

which leads to a relatively large sample size. To test whether the 68.3\% quantile of the absolute error distribution is below 10 \, \text{cm}, for each observation \( \Delta h_i \), a zero-one variable \( Y_i \) is defined as

\[
Y_i = I(|\Delta h_i| < 10 \, \text{cm})
\]

where \( I \) is the indicator function and the constant \( c \) is chosen as the smallest integer so that

\[
F(c - 1; \, n, \, p_0) = \sum_{i=1}^{c-1} \binom{n}{i} p_0^i (1 - p_0)^{n-i} \leq \alpha,
\]

with \( F \) the cumulative distribution function of the binomial distribution with parameters \( n \) and \( p_0 \). The test is performed with the alternative hypothesis \( H_1: \, p > p_0 \). In our example of the 68.3\% quantile \( p_{0.05} = 0.683 \), i.e. we want to investigate if more than 68.3\% of the absolute errors are smaller than 10 \, \text{cm}. If so, the desired accuracy specification is fulfilled. The test statistic is defined as

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\[
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\]

with \( F \) the cumulative distribution function of the binomial distribution with parameters \( n \) and \( p_0 \). The test is performed with the alternative hypothesis \( H_1: \, p > p_0 \). In our example of the 68.3\% quantile \( p_{0.05} = 0.683 \), i.e. we want to investigate if more than 68.3\% of the absolute errors are smaller than 10 \, \text{cm}. If so, the desired accuracy specification is fulfilled. The test statistic is defined as

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where \( T \) is normal with mean zero and variance \( \sigma^2 \). From the Q–Q plot in Fig. 4, the test statistic is defined as

\[
|\Delta h_i| < 10 \, \text{cm} \quad \text{if } T > c, \quad \text{where } c = \alpha / \sqrt{n}
\]
7.1.1. Table 6

| Mean (after removal of outliers) ($\hat{\mu}^*$) | 88 |
| Standard deviation (after removal of outliers) ($\hat{\sigma}^*$) | 14 |

8. Confidence intervals are relatively large at such a small sample size. Note also the extreme value of the estimated 95% quantile: the computations are robust against 5% outliers, but with one out of 19 points being an outlier this value enters the calculations.

7.2. Test of the DTM derived by laser scanning

In this section the test area is identical with the one described in Section 7.1.2. Elevations of the checkpoints were derived by ground surveying and of the DTM derived by laser scanning. The checkpoints are randomly distributed. For reasons of space we do not report histograms and Q–Q plots for this example, but refer to Tables 7 and 8 for the results.

The standard deviation after removal of one outlier is much lower as with the outlier included.

The NMAD value and the 68.3% quantile are nearly the same (6 and 7 cm, respectively). The achieved standard deviation (after removal of outliers) is $15/7 = 2.1$ times better than at the DEM derived by digital photogrammetry.

The same improvement can be found at the NMAD value and the 68.3% quantile. The condition of a three times higher accuracy is not completely fulfilled but nevertheless we will use the DTM of the whole model area as reference data for the DEM derived by photogrammetry (cf. next section). The derived accuracy measures are then relative errors.

7.3. Test of photogrammetric data by means of laser scanned data

The availability of an accurate and very dense DTM, which was derived from airborne laser scanning (ALS) and automatic labelling of ground points, gives the possibility of checking the...
elevations of automated photogrammetry thoroughly. The applied filter is the two step approach described by Axelsson (2000), where a temporary TIN model is created first and then densified with new points by checking distance and angle parameters. Both DEMs were determined with a high density (grid spacing \( \approx 3 \text{ m} \) for photogrammetry, grid spacing \( \approx 2 \text{ m} \) for ALS); the sample size is therefore very high. No editing occurred at the sample size is therefore very high. No editing occurred at the top right corner of the figure. The large amount of blunders in the data is due to the fact that no editing of the photogrammetrically derived DEM occurred.

Finally, Fig. 8 shows a contour plot of the 95% quantile of the absolute error distribution as a function of location using the Koenker and Mizera (2004) approach, i.e. a quantile regression using \( Q(0.95) = f(x, y) \). For computational reasons the plot is only based on a subset of 12,000 points. The plot provides an overview for direct accuracy checking using quantiles, which better takes autocorrelation into account.

7.4. Summarizing results with the proposed accuracy measures

Several examples of DEM quality control were presented with different numbers of outliers and checkpoints. The presented histograms revealed skewness and kurtosis, which should be taken into account when deriving accuracy measures. In order to derive reliable values for the systematic error and the standard deviation three different approaches have been used: Estimation using all data, blunders removed using a RMSE based threshold, and estimation of location and scale using a robust method. Fig. 9 depicts the differences between the different approaches for the example of Section 7.3.

From the graph it is obvious that the robust approach fits the histogram best. The removal of outliers by a threshold \( T \geq 3 \text{ RMSE} \) does not eliminate all of the outliers. Therefore, the use of

\[
\text{Table 9} \\
\begin{array}{ccc}
\text{Accuracy measures} & \text{Value (cm)} \\
\hline
\text{RMSE} & 106 \\
\text{Mean} (\hat{\mu}) & 13 \\
\text{Standard deviation} (\hat{\sigma}) & 105 \\
\text{Mean (after removal of outliers)} (\hat{\mu}^*) & 2 \\
\text{Standard deviation (after removal of outliers)} (\hat{\sigma}^*) & 34 \\
\end{array}
\]

\[
\text{Table 10} \\
\begin{array}{ccc}
\text{Accuracy measure} & \text{Error type} & \text{Value (cm)} & \text{95% confidence interval (cm)} \\
\hline
\text{50% quantile (median)} & \Delta h & 2 & [1, 2] \\
\text{NMAD} & \Delta h & 12 & [12, 13] \\
\text{68.3% quantile} & |\Delta h| & 13 & [13, 14] \\
\text{95% quantile} & |\Delta h| & 68 & [62, 88] \\
\end{array}
\]
Fig. 7. Spatial distribution of differences between elevations from laser scanning and automated photogrammetry. The coloured areas have differences above 1 m (≈ 3 times standard deviation). The white areas are large buildings and elevations have been removed there by a filter program.

Fig. 8. Contour plot of the triogram surface describing the 95% quantile of the absolute error distribution. The iso-lines illustrate how the 95% quantile differs as a function of measurement location.

Fig. 9. Histogram of the differences $\Delta h$ between two DEMs in metres truncated to the range $[-2 \text{ m}, 2 \text{ m}]$. Superimposed on the histogram are the corresponding normal distribution curves when estimating parameters 'mean' and 'variance' through one of the three approaches.

new quality measures (median, NMAD) is more adequate for DEMs derived by means of digital photogrammetry or laser scanning. A distribution-free and non-parametric alternative is the use of quantiles, therefore, we suggest computing the 68.3% and 95% quantile of the absolute errors additionally. The use of histogram and Q–Q plot provide a visual tool to help decide which accuracy measures are more appropriate for the tested DEM.

8. Discussion

The accuracy measures (systematic shift and standard deviation) should not be influenced from outliers and non-normality of the error distribution. Therefore, we suggest applying robust statistical methods in the assessment of DEM accuracy. Confidence intervals for the various quantiles are only small if the number of checkpoints is large. It is possible to treat the issue of sample size within a statistical context. Guidelines on how large the sample should be are given in Section 7.

Quality control by means of visual inspection and photogrammetric measurements has to detect as many outliers as possible and to remove them. Stereo measurements and other editing by an operator may have to be added. The DTM is then cleaner, but some of the outliers may remain undetected. Use of robust methods is, therefore, highly recommended for DEMs derived by digital photogrammetry or laser scanning.
Our method for accuracy assessment can be summarized as follows: Compute vertical errors with all points in the sample. Then generate histograms and Q–Q plots to visualize the error distribution and to assess non-normality. Thereafter, compute mean error and standard deviation as well as median and NMAD together with confidence intervals. In case of big discrepancies assess whether outliers in the data are an issue. Also compute the 68.3% quantile and compare it with the NMAD value. In case of discrepancy decide (based on the histogram and Q–Q plot) if non-normality is an issue. If non-normality is an issue use the more robust and conservative quantile measures supplemented by a quantile surface plot. In case compliance with a specification has to be proven, use the appropriate statistical test procedure as described in Section 6.

The proposed methodology adapts to the specialities of laser scanning and digital photogrammetry, where blunders and non-normal distribution are often present, especially in non-open terrain. DEM standards have to take this into account and non-parametric and distribution free methods should be calculated in the assessment of accuracy.

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