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Uncertainty on Fatigue Damage Accumulation for Composite Materials

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Summary In the present paper stochastic models for fatigue damage accumulation for composite materials are presented based on public available constant and variable amplitude fatigue tests. The methods used for estimating the SN-curve and accumulated fatigue damage are presented.

Introduction

Damage accumulation models for composite materials exposed to fatigue loading have been widely considered in the literature, see e.g. [1] for a review. However, even though new empirical and physical models for the accumulation of damage are proposed, these models do not seem to perform much better than the linear damage accumulation proposed by Miner [2]. For this reason the accumulated damage is normally determined by Miners rule as recommended in [3] for wind turbine blades.

The uncertainties in damage accumulation based on Miners rule can in general be divided into three parts:

- Physical uncertainty on the SN-curves
- Statistical uncertainty on the SN-curves due to a limited number of tests
- Model uncertainty on Miners rule

The physical uncertainty on the SN-curves is due to the natural inherent uncertainty in the material and can not be reduced. The statistical uncertainty can be reduced by performing additional fatigue tests and the model uncertainty can in principle be reduced by adopting a better model.

In the present paper is the physical and statistical uncertainty on SN-curves for composite material determined based on a number of constant amplitude fatigue tests performed with different mean stresses. Based on variable amplitude fatigue tests for the same material using a standard load spectrum and the estimated SN-curves based on constant amplitude fatigue tests is the model uncertainty on Miners rule determined.

In most standards and regulations including [3] are fatigue design performed by using a deterministic design approach. However, calculation of the accumulated fatigue damage includes significant uncertainties for which reason a probabilistic design approach should be adopted in order to take the individual uncertainties into account in a rational manner. The stochastic models determined in the present paper forms the basis for a probabilistic modeling of the considered class of composite materials in fatigue loading.

Constant amplitude fatigue tests

The constant amplitude and variable amplitude fatigue tests used in the present paper are given in the OptiDAT database [4] for geometry R04 MD (MultiDirectional laminate). This geometry has been selected due to the many fatigue tests performed with this geometry. For composite materials the mean stress can have a significant influence on the fatigue properties which can be taken into account by calculation of an SN-curve for different R-ratios and arranging these in a constant life diagram. The R-ratio is defined by:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{1}$$

where $\sigma_{\min}$ and $\sigma_{\max}$ are the minimum and maximum stress in a stress cycle respectively. Different types of SN-curves have been used for composite material, but no specific SN-curve have been recommended in [3]. In the present study is a log-log SN-curve used:

$$\log N = \log K - m \log \Delta\sigma + \varepsilon \tag{2}$$

where N is the number of cycles to failure, $\Delta\sigma$ is the stress range and ε is parameter which model the lack of fit and is assumed normal distributed with mean value zero and standard deviation σ_ε. The constants K and m are material parameters. By assuming that the residuals are normal distributed on a log-log scale the likelihood function in case of n constant amplitude tests and n_0 run-outs is given by:

$$L(\log K, \sigma_\varepsilon) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} \exp\left(-\frac{1}{2}\left(\frac{\log N_i - (\log K - m \log \Delta\sigma_i)}{\sigma_\varepsilon} \right)^2 \right) \cdot \prod_{i=n+1}^{n+n_0} \Phi\left(\frac{\log N_i - (\log K - m \log \Delta\sigma_i)}{\sigma_\varepsilon} \right) \tag{3}$$

where N_i and $\Delta\sigma_i$ is the number of cycles to failure and stress range for test specimen number i respectively. The parameter m is determined by least square method and the parameters $\log K$ and σ_ε are estimated using the Maximum-Likelihood Method where the optimization problem $\max\limits_{\log K, \sigma_\varepsilon} L(\log K, \sigma_\varepsilon)$ is solved using a standard nonlinear optimizer, e.g. the NLPQL algorithm [5]. In this paper is m assumed fixed determined by the least square method, but this parameter could also be included in the optimization. Since the parameters $\log K$ and σ_ε are estimated by the Maximum-Likelihood technique they become asymptotically Normal distributed stochastic variables with expected values equal to the Maximum-Likelihood estimators and covariance equal to, see e.g. [6]:

$$C_{\log K, \sigma_\varepsilon} = \left[-H_{\log K, \sigma_\varepsilon} \right]^{-1} = \begin{bmatrix} \sigma_{\log K}^2 & \rho_{\log K, \sigma_\varepsilon} \sigma_{\log K} \sigma_{\sigma_\varepsilon} \\ \rho_{\log K, \sigma_\varepsilon} \sigma_{\log K} \sigma_{\sigma_\varepsilon} & \sigma_{\sigma_\varepsilon}^2 \end{bmatrix} \tag{4}$$

where $H_{\log K, \sigma_\varepsilon}$ is the Hessian matrix with second order derivatives of the log-Likelihood function. $\sigma_{\log K}$ and $\sigma_{\sigma_\varepsilon}$ denote the standard deviation on $\log K$ and σ_ε respectively. $\rho_{\log K, \sigma_\varepsilon}$ is the correlation coefficient between $\log K$ and σ_ε. The Hessian matrix is estimated by numerical differentiation.

In table 1 are the estimated parameters shown and the SN-curves are fitted using all valid constant amplitude fatigue tests for the particular R-ratio and runouts are taken into account. The parameters $\log K$ and σ_ε can be assumed uncorrelated. It is noted that σ_ε represents the physical uncertainty and that $\sigma_{\log K}$ and $\sigma_{\sigma_\varepsilon}$ represents the statistical uncertainty. In table 2 are the static tension and compression strength given. In figure 1 (left) is the constant life diagram containing the SN-curves and static strengths shown.

Table 1: SN-curves for different R-ratios for geometry R04 MD.

R-ratio	Tests *n*	Runouts n_0	*m*	log *K*	σ_ε	$\sigma_{\log K}$	$\sigma_{\sigma\varepsilon}$
0.5	15	0	10.541	27.768	0.358	0.092	0.065
0.1	45	2	9.508	27.191	0.259	0.039	0.027
-0.4	28	0	7.582	23.398	0.435	0.082	0.058
-1.0	84	3	6.719	21.359	0.878	0.094	0.068
-2.5	10	2	11.983	35.231	0.633	0.197	0.143
10.0	34	0	22.211	58.664	0.644	0.110	0.078
2.0	6	3	29.686	73.780	0.354	0.143	0.103

Table 2: Static tension strength (STT) and static compression strength (STC) for geometry R04 MD.

Test-type	Tests *n*	Mean [MPa]	Std. [MPa]
STT	66	556.5	64.2
STC	55	-458.6	33.2

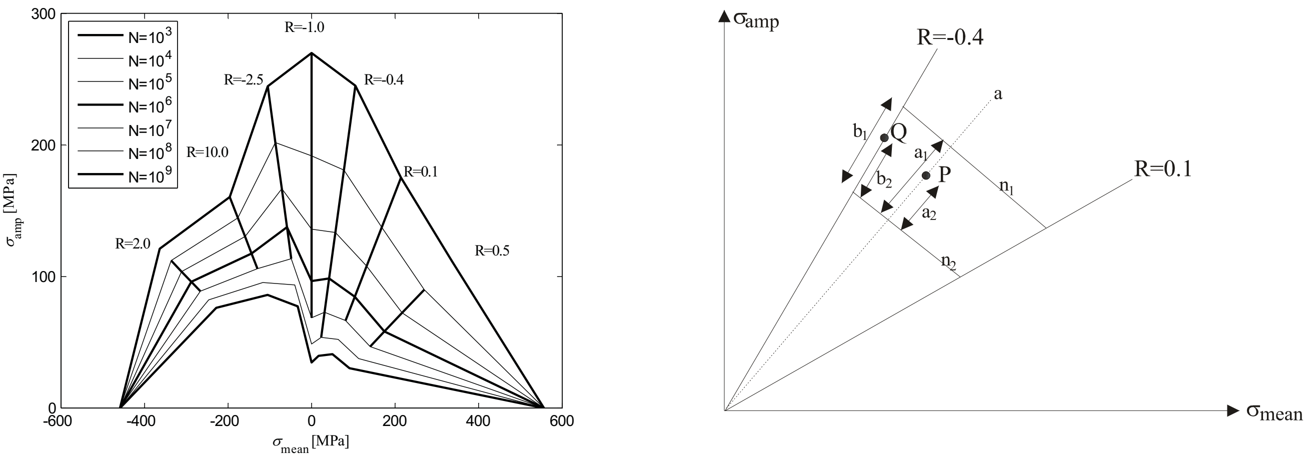

Figure 1. Left: Constant life diagram for geometry R04 MD. Right: Linear interpolation.

Variable amplitude fatigue tests

Variable amplitude fatigue tests are also performed with geometry R04 MD. The load spectrum used is the wisper and wisperx spectra developed for representing the flap bending moment of a wind turbine blade. In order to calculate the accumulated damage D Miners rule for linear damage accumulation is used:

$$D = \sum_{i=1}^{n} \frac{1}{N\left(\Delta\sigma_i\right)} \tag{5}$$

In order to calculate the accumulated damage at failure the number of cycles to failure N has to be determined for an arbitrary R-ratio. This is in the present paper done by linear interpolation in the constant life diagram using the procedure given in the following, see also figure 1 (right).

- The stress cycle P is located in the constant life diagram
- Draw a line a from the origin through and beyond the point P
- Identify the constant life lines closest to P, denoted n_1 and n_2
- Calculate the length a_1 on line a between the two constant life lines n_1 and n_2

- Calculate the length a_2 on line a between point P and the constant life line n_2
- Find the R-ratio closest to P and calculate the length b_1 between n_1 and n_2
- Calculate $b_2 = \frac{b_1 a_2}{a_1}$
- Determine the stress amplitude σ_{CLD} corresponding to point Q
- Determine the expected number of cycles to failure N using the SN-curve for the R-ratio

In table 3 are the conducted variable fatigue tests listed together with the mean and standard deviation on the accumulated damage at failure using the material parameters given in table 1 and 2. It is noted that COV_D represents the model uncertainty.

Table 3: Mean and standard deviation for estimated damage at failure for variable amplitude tests.

Spectrum	Tests N	Mean	Std.	COV_D
Wisper	10	0.9000	0.5355	0.595
Wisperx	13	0.2763	0.1982	0.717
Reverse Wisper	2	0.2020	-	-
Reverse Wisperx	10	0.3179	0.1576	0.496
Wisper, Wisperx	23	0.5474	0.4886	0.893
All	35	0.4621	0.4196	0.908

From table 3 it is seen that except for the wisper spectrum the estimated accumulated damage at failure is significantly below one. The uncertainty for fatigue damage accumulation are often modelled by a lognormal distribution in order two avoid negative values of Miners rule which are physical impossible. The mean and standard deviations given in table 3 can be used in a lognormal distribution.

Concluding remarks

In the present paper are stochastic models for the uncertainty related to fatigue damage accumulation for composite materials presented. The stochastic models are based on public available constant and variable amplitude fatigue tests and the procedure used for estimating the stochastic models are presented.

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