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Brincker, Rune

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RUNE BRINCKER
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CRACK TIP PARAMETERS FOR GROWING CRACKS IN LINEAR VISCOELASTIC MATERIALS

Rune Brincker
University of Aalborg, Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

Abstract

In this paper the problem of describing the asymptotic fields around a slowly growing crack in a linearly viscoelastic material is considered. It is shown that for plane mixed mode problems the asymptotic fields must be described by 6 parameters: 2 stress intensity factors and 4 deformation intensity factors. In the special case of a constant Poisson ratio only 2 deformation intensity factors are needed. Closed form solutions are given both for a slowly growing crack and for a crack that is suddenly arrested at a point at the crack extension path. Two examples are studied; a stress boundary value problem, and a displacement boundary value problem. The results show that the stress intensity factors and the displacement intensity factors do not depend explicitly upon the velocity of the crack tip.

1. Introduction

Most of the engineering materials suffer from rate sensitivity. This means that the engineering strength will depend on the strain rate or on the duration of the load. For polymers this rate sensitivity is strong and of great importance when the material strength becomes an important design parameter, for instance when polymers are used in composite structures and adhesive joints.

However, also tradional structural materials like wood and concrete show a significant rate sensivity that cannot be explained by dynamic effects alone. For instance it is well known that the long-term strenght of wood in codes is usually prescibed as about 60 % of the short-term strength.

Rate sensitivity problems have been treated by modelling the dynamic fracture process, or by modelling the time dependency of the material on a microscopic level or by continuum mechanical theories, Knauss [1].

In this paper we will restrict ourselves to the quasi-stationary case (dynamic effect can

be ignored) and to a continuum mechanical description of the fracture process. This means that the fracture process is modeled by voids or cracks growing in the body when external forces are applied.

Since the late sixties much work has been done to model the time dependent fracture by slow crack growth in a viscoelastic solid, Schapery, [2 - 4], Knauss et al, [5,6], McCartney [7 - 9] and Christensen [10,11]. However, the theories were applied to simple problems where the solutions for stress and strain were easy to obtain. Some solutions have been published for a single crack growing or closing in an infinite sheet, Graham [12,13], and a solution for stresses and displacements around a crack growing in an infinite strip has been published by Mueller [15]. However, only a few solutions are awailable, and general solution techniques do not seem to be developed.

In this paper a new technique is outlined for the determination of the stress and deformation field around a crack growing slowly in a linearly viscoelastic body. The solutions are general with respect to boundary conditions and material proporties (general isotropic linear viscoelasticity), but quasi-static and isothermal conditions are assumed.

In the first section it is shown that in the case af a plane mixed mode crack problem, six parameters are needed to descibe the stress and displacement fields around the crack tip. The crack tip parameters are defined, and solutions for the stationary crack problems are given.

In the next section general solutions are obtained, both for growing cracks and cracks that are suddenly arrested at a point on the crack extension path.

Finally, in the last sections two examples are given. The examples are illustrating the use of the technique, and the obtained solutions are compared to solutions from the literature.

2. Definition of the Crack Tip Parameters

In fracture mechanics the state of stress and strain around a crack tip is described by a limited number of constants. In the case of a crack in a linear elastic material it can be shown that the state of stress and strain in the vicinity of the crack is completely determined by two constants, the so-called stress intensity factors K_1^e and K_2^e, Williams [16], Rice [17].

In general the stress intensity factors will be functions of the elastic constants and of the time

$$K_\gamma^e = K_\gamma^e(2\mu, 3\kappa, t) \tag{1}$$

where μ is the shear modulus and κ is the modulus of compression, Sokolnikoff, [18]. The most common way to obtain viscoelastic solutions is to derive the desired solutions from the corresponding elastic problem by the so-called correspondence principle,

Lee [19]. The correspondence principle is based on the fact that the equations for the viscoelastic and the corresponding elastic problem (same geometry, same boundary conditions) become identical if the viscoleastic equations are Laplace transformed and the elastic constants 2μ and 3κ are replaced by the complex functions $sR_1^*(s)$ and $sR_2^*(s)$, where $R_1^*(s)$ and $R_2^*(s)$ are the Laplacetranformed isotropic relaxation functions, Christensen [20]. Using the correspondence principle, the viscoelastic stress intensity faktors K_γ for a stationary crack are given by

$$K_\gamma(t) \;=\; \mathcal{L}_{s\to t}^{-1}\{K_\gamma^{e*}(sR_1^*(s), sR_2^*(s), s)\} \tag{2}$$

where

$$K_\gamma^{e*}(2\mu, 3\kappa, s) \;=\; \mathcal{L}_{t\to s}\{K_\gamma^e(2\mu, 3\kappa, t)\}\;;\;\; R_\gamma^{e*}(s) \;=\; \mathcal{L}_{t\to s}\{R_\gamma^e(t)\} \tag{3}$$

and where $\mathcal{L}_{t\to s}\{.\}$ and $\mathcal{L}_{t\to s}^{-1}\{.\}$ is the Laplace transformation and the inverse Laplace transformation, respectively.

These viscoelastic stress intensity factors, however, do not uniqely define the crack tip fields, since in the viscoelastic case there is not a one-to-one relationship between the stress field and the strain field. In this case the strain field or the displacement field must be described by its own state variables. In the elastic case the displacement field is given by

$$u_\alpha^e(r,\theta) \;=\; \sqrt{\frac{r}{2\pi}}\{\frac{1}{2\mu}\, g_{\alpha\beta}(\theta)K_\beta^e + \frac{\lambda(2\mu,3\kappa)}{2\mu} h_{\alpha\beta}(\theta)K_\beta^e\} \tag{4}$$

where (r,θ) is the polar coordinates, $g_{\alpha\beta}(\theta)$ and $h_{\alpha\beta}(\theta)$ are well-known angular functions and $\lambda(2\mu,3\kappa) = (3-4\nu)$ for plane strain and $\lambda(2\mu,3\kappa) = (3-\nu)/(1+\nu)$ for plane stress, ν being the Poisson ratio, Rice [17]. From eq. (4) it is seen, that if the following quantities are defined

$$\begin{aligned} C_\alpha(t) \;&=\; \mathcal{L}_{s\to t}^{-1}\{\frac{1}{sR_1^*(s)}\; K_\alpha^{e*}(sR_1^*(s), sR_2^*(s), s)\} \\ D_\alpha(t) \;&=\; \mathcal{L}_{s\to t}^{-1}\{\frac{\lambda(sR_1^*(s), sR_2^*(s))}{sR_1^*(s)}\; K_\alpha^{e*}(sR_1^*(s), sR_2^*(s), s)\} \end{aligned} \tag{5}$$

then the viscoeleastic displacement field - and therefore also the strain field - is uniquely defined. The parameters C_1, C_2, D_1 and D_2 are necessary and sufficient to describe the displaments at the time t.

This means, that in the viscoelastic case six parameters are needed to describe the plane state of stress and strain, namely two stress intensity factors, K_1 and K_2, and the four displacment intensity factors, C_1, C_2, D_1 and D_2. In the viscoelastic case the asymptotic solutions for the stress and the displacement fields are then given by

$$\begin{aligned} \sigma_{\alpha\beta}(r,\theta,t) &= \frac{K_\gamma(t)}{\sqrt{2\pi r}} f_{\alpha\beta\gamma}(\theta) \\ u_\alpha(r,\theta,t) &= \sqrt{\frac{r}{2\pi}}\{g_{\alpha\beta}(\theta)C_\beta(t) + h_{\alpha\beta}(\theta)D_\beta(t)\} \end{aligned} \tag{6}$$

Where $f_{\alpha\beta\gamma}(\theta)$, $g_{\alpha\beta}(\theta)$ and $h_{\alpha\beta}(\theta)$ are all known functions of the angle θ only, Rice [17]. It is not difficult to see, that in the 3-dimensional case one extra stress intensity factor and one more deformation factor are needed to descibe the stress and deformation fields.

3. General Solutions for the Crack Tip Parameters

In this paragraph general solutions for the six crack tip parameters defined in the preceeding section are derived.

A general viscoelastic boundary value problem over the region Ω is considered, Christensen [20]. The boundary conditions are given by

$$\begin{aligned} &B^o \ \ on \ \ \partial\Omega^o \\ &B^i(t) \ \ on \ \ \partial\Omega^i(t) \end{aligned} \tag{7}$$

where $\partial\Omega^o$ is the external boundary and $\partial\Omega^i(t)$ is the inner boundary. The inner boundary $\partial\Omega^i(t)$ is assumed to be a monotonicly increasing function of time describing a slow crack growth along an arbitrary crack extension path Π, see figure 1. The boundary conditions B^o on $\partial\Omega^o$ and $B^i(t=0)$ on $\partial\Omega^i(t=0)$ are arbitrary, whereas the boundary $\partial\Omega^i(t) \setminus \partial\Omega^i(t=0)$ is assumed to be traction free.

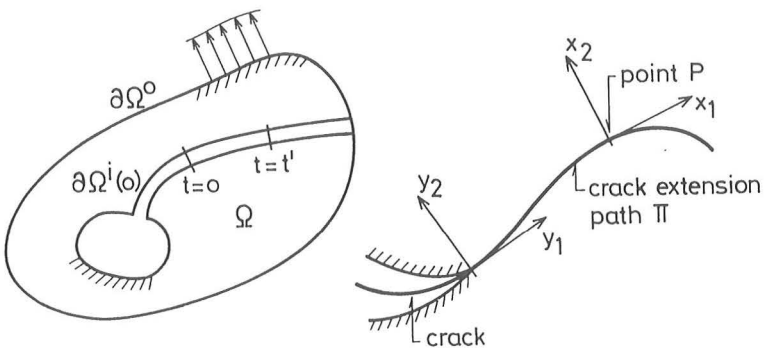

Figure 1. The linear viscoelastic boundary value problem.

Two coordinate systems are used. A local (y_1, y_2) coordinate system moving with the crack tip, and a fixed (x_1, x_2) coordinate system situated at a material fixed point P on the propagation path. The crack is assumed to be at the point P at the time $t = t'$, where the coordinate systems coincide.

Our problem is to find the asymptotic solutions $(y_1^2 + y_2^2 \rightarrow 0)$ for the stress field $\sigma_{\alpha\beta}$ and the displacement field u_α and express the asymptotic fields by the intensity factors defined above. First we will derive the solutions for the stress intensity factors for the moving crack tip at the time $t = t'$, i.e. at the time when the crack tip is at the point P on the crack extension path Π.

The solutions will be derived using the correspondence principle. In this case, however, when the boundaries are changing, the classical proof of the correspondence principle is no longer valid. Graham [12,14] has given a proof in the case of changing boundaries which can be used directly on symmetrical crack extension problems. However, following the basic ideas in Grahams proof it is not difficult to extend the proof to the case of non-symmetrical crack extension problems. A general proof is given in Brincker [21]. Also the use of the principle becomes more complicated when the the boundaries are changing. Escpecially in this case it becomes complicated, because an elastic solution in the local (y_1, y_2) coordinate system cannot be used since the correspondence principle is only valid when a solution for a fixed material point is used. This implies that the elastic solution used in the correspondence principle must be expressed in the (x_1, x_2) coordinate system. This complicates the problem.

It proves convenient to consider a modified boundary value problem. The solution to the auxiliary problem is denoted $\sigma'_{\alpha\beta}$, u'_α and the boundary conditions are taken as

$$B^o \quad on \quad \partial\Omega^o \tag{8.a}$$

$$B^i(t) \quad on \quad \partial\Omega'^i(t) \tag{8.b}$$

$$\sigma'_{\alpha\beta} n_\beta = \sigma_{\alpha\beta} n_\beta \quad on \quad \partial\Omega^i(t') \setminus \partial\Omega'^i(t) \tag{8.c}$$

where the the boundary region $\partial\Omega'^i(t)$ is equal to $\partial\Omega^i(t)$ for $t < t'$ and equal to $\partial\Omega^i(t')$ for $t \geq t'$. From the boundary conditions (8) it appears that the solution $\sigma_{\alpha\beta}$, u_α to the original problem and the solution $\sigma'_{\alpha\beta}$, u'_α to the auxiliary problem are identical for $t \leq t'$. The auxiliary problem can be considered as a crack growth problem, where the crack growth history is equal to the crack growth history for the original problem for $t \leq t'$, but where the crack is arrested at the point P and becomes stationary for all times $t > t$.

The six crack tip parameters will therefore be identical at all times $t \leq t'$, i.e. also at $t = t'$. Besides, the solution to the auxiliary problem illustrates how the crack tip parameters will develop with time if the crack tip is arrested at the point P.

The advantage of using the auxiliary problem, is that everything becomes much simpler because the auxiliary problem is formulated as a stationary crack problem with the crack tip situated at the point P.

First let us determine a stress intensity factor K for the stationary crack tip at the point P. The intensity factor can be written as

$$K(t) = K^o(t) + K'^i(t) \tag{9}$$

Here $K^o(t)$ is the contribution from the boundary conditions on the exterior boundary alone, i.e. application of boundary condition (8.a) and the other boundaries being traction free. The other term $K'^i(t)$ is the contribution from the boundary conditions on the interior boundary alone (conditions (8.b) and (8.c)), the exterior boundary being traction free. It should be noticed that the two terms are not independent if the boundary conditions involve any explicit conditions in displacements.

The two contributions $K^o(t)$ and $K'^i(t)$ are now found applying the correspondence principle. The elastic solution for the contribution $K^o(t)$ is assumed to be of the form

$$K^{oe} = a^{ko}(2\mu, 3\kappa)\, b^{ko}(t) \tag{10}$$

i.e. the dependence on the elastic constants 2μ and 3κ and the dependence on the time t introduced by the boundary conditions are assuned to be described by two independent factors. In the general case the contribution must be expressed in terms of a series or an integral where each term in the series or the integrant can be written as an expression like eq. (10) where the first factor is a Green's function for the boundary loads depending on 2μ and 3κ, and where the second factor is the load intensity depending on the time t. For reasons of simplicity, however, only the simple form (10), is used which covers almost all practical cases.

Applying the correspondence principle to eq. (10) yields

$$K^o(t) = \mathcal{L}^{-1}_{s\to t}\{a^{ko}(sR_1^*(s), sR_2^*(s))\, b^{ko*}(s)\} \tag{11}$$

If we introduce the function

$$A^{ko}(t) = \mathcal{L}^{-1}_{s\to t}\{\frac{1}{s}\, a^{ko}(sR_1^*(s), sR_2^*(s))\} \tag{12}$$

then by the differentiation theorem and the convolution theorem of the Laplace theory, Doetsch [25]

$$\begin{aligned} K^o(t) &= \int_{-\infty}^{t} A^{ko}(t-\tau)\, \frac{db^{ko}(\tau)}{d\tau}\, d\tau \\ &= \mathcal{A}^{ko}\{b^{ko}(t)\} \end{aligned} \tag{13}$$

where $\mathcal{A}^{ko}\{.\}$ is a Stieltjes convolution with the kernel $A^{ko}(.)$.

This contribution was easy to obtain. The other contribution $K'^{i}(t)$, however, is somewhat more difficult to determine. We will write the contribution as

$$K'^{i}(t) = K^{i}(t) + K''^{i}(t) \tag{14}$$

where $K''^{i}(t)$ is the contribution from the boundary condition (8.c) alone, i.e. the rest of the boundary being traction free. Again the two terms may be dependent or independent according to the nature of the boundary conditions.

The corresponding elastic solution satisfies the condition

$$K''^{ie}(t) = 0\,; \quad t' \leq t < \infty \tag{15}$$

The corresponding viscoelastic quantity $K''^{i}(t)$ is simple to determine if the following theorem from the theory of elasticity is used:

Theorem: Consider a boundary value problem over a region with a finite number of holes. If the boundary conditions are given as prescibed stresses on all boundaries, and if the resultant force vanishes on each of the boundaries, then the stress solution will be independent of the elastic constans.

A special proof based on complex function theory is given in Muskhelishvili [22], but a simpler proof based on integral equations is given in Brincker [21].

It is seen that the theorem can be applied to the considered boundary value problem. Since the elastic solution $K''^{ie}(t)$ is equal to zero for $t' \leq t < \infty$ as stated by eq. (15) and since the solution does not depend on the elastic constants 2μ and 3κ according to the above theorem, then, by application of the inverse Laplace transform,

$$K''^{i}(t) = 0\,; \quad t' \leq t < \infty \tag{16}$$

which is an important result. Now, the contribution $K''^{i}(t)$ for $0 \leq t < t'$ will be determined. It is known that

$$K(t) = 0\,; \quad 0 \leq t < t' \tag{17}$$

and from eq. (9) and eq. (14)

$$K''^{i}(t) = -(K^{o}(t) + K^{i}(t))\,; \quad 0 \leq t < t' \tag{18}$$

where the contribution $K^{o}(t)$ is given by eq. (13). Following the same ideas as when deriving the expression for $K^{o}(t)$

$$K^{i}(t) = \mathcal{A}^{ki}\{b^{ki}(t)\} \tag{19}$$

Using eq. (16) and (17) an expression for the contribution $K'''^{i}(t)$ valid for all times is now obtained

$$K'''^{i}(t) = -(1-\Delta(t-t'))\,(K^{o}(t)+K^{i}(t)) \tag{20}$$

where $\Delta(.)$ is the Heaviside unit function. Now the final result is obtained by applying eq. (9) and (14), together with eq. (20),

$$\begin{aligned} K(t) &= K^{o}(t)+K^{i}(t)+K'''^{i}(t) \\ &= \Delta(t-t')\,(K^{o}(t)+K^{i}(t)) \end{aligned} \tag{21}$$

and then by using eq. (13) and (19)

$$K(t) = \Delta(t-t')\,(\mathcal{A}^{ko}\{b^{ko}(t)\}+\mathcal{A}^{ki}\{b^{ki}(t)\}) \tag{22}$$

This is the solution for the stress intensity factor $K(t)$ for the auxiliary problem, i.e. for the crack at the time t' when the crack arrives at the point P and for $t > t'$, when the crack is arrested at the point P. The stress intensity factor $K(t)$ for the original problem at the time $t = t'$ is then given by

$$K(t') = \lim_{t\to t'}(\mathcal{A}^{ko}\{b^{ko}(t)\}+\mathcal{A}^{ki}\{b^{ki}(t)\})\;;\;\; t > t' \tag{23}$$

and if the crack tip is arrested at the point P at the time $t = t'$, then the development af K at times greater than t' is given by

$$K(t) = \mathcal{A}^{ko}\{b^{ko}(t)\}+\mathcal{A}^{ki}\{b^{ki}(t)\}\;;\;\; t > t' \tag{24}$$

Now the solution for the stress intensity factor K is obtained for a linear viscoelastic crack problem under general loading conditions both for a steadily growing crack and for a crack that is suddenly arrested at a certain point a the crack extension path.

However, the solution for the corresponding deformation intensity factors C and D remanins to be obtained. The elastic solution for the crack tip at the point P for the auxiliary problem can be written, see eq. (5)

$$\begin{aligned} C^{e}(t) &= a^{c}(2\mu,3\kappa)\,K^{e}(2\mu,3\kappa,t) \\ D^{e}(t) &= a^{d}(2\mu,3\kappa)\,K^{e}(2\mu,3\kappa,t) \end{aligned} \tag{25}$$

where $a^{c}(2\mu,3\kappa) = 1/(2\nu)$ and $a^{d}(2\mu,3\kappa) = \lambda(2\mu,3\kappa)/(2\mu)$. To these expressions the correspondence can be applied directly, and the deformation intensity factors for the auxiliary problem are then given by

$$\begin{aligned} C(t) &= \mathcal{A}^c\{K(t)\} \\ D(t) &= \mathcal{A}^d\{K(t)\} \end{aligned} \tag{26}$$

where $K(t)$ is the viscoelastic stress intensity factor given by eq. (22) and $\mathcal{A}^c$ and $\mathcal{A}^d$ are Stieltjes convolutions with the kernels $A^c(.)$, $A^d(.)$ defined by their Laplace transforms

$$\begin{aligned} A^{c*}(s) &= \frac{1}{s}\, a^c(sR_1^*(s), sR_2^*(s)) \\ A^{d*}(s) &= \frac{1}{s}\, a^d(sR_1^*(s), sR_2^*(s)) \end{aligned} \tag{27}$$

Similarly to eq. (23) and (24), the deformation intensity factors for the original problem at the time $t = t'$ and the development of the intensity factors if the crack tip is arrested at the point P at the time $t = t'$ are given by

$$\begin{aligned} C(t') &= \lim_{t \to t'} \mathcal{A}^c\{K(t)\}\ ;\ t > t' \\ D(t') &= \lim_{t \to t'} \mathcal{A}^d\{K(t)\}\ ;\ t > t' \end{aligned} \tag{28}$$

$$\begin{aligned} C(t) &= \mathcal{A}^c\{K(t)\}\ ;\ t > t' \\ D(t) &= \mathcal{A}^d\{K(t)\}\ ;\ t > t' \end{aligned} \tag{29}$$

and the problem of determining the crack tip parameters for a growing crack under general loading is finally solved.

4. Solutions for Prescibed Stresses

A boundary value problem over $\Omega = \Omega(t)$ with prescribed stresses, where the loads on the exterior boundary $\partial\Omega^o$ and the loads on the interior $\partial\Omega^i$ each form an equilibrium system is now condsidered.

The material is linear viscoelastic, but for simplicity, Poisson's ratio ν will be assumed to be constant, which yields the following constitutive equation

$$\sigma_{ij} = \frac{1}{1+\nu}\, \mathcal{R}\{\epsilon_{ij} + \frac{\nu}{1-2\nu}\delta_{ij}\epsilon_{kk}\} \tag{30}$$

where $\mathcal{R}\{.\}$ is a Stieltjes convolution with the relaxation kernel $R(.)$. The kernel function $R(.)$ corresponds to Young's modulus E in the elastic case, and the corresponding creep function $C(.)$ can be found from the condition, Christensen [20]

$$s^2 C^*(s) R^*(s) = 1 \tag{31}$$

First the stress intensity factor K for the considered crack tip is to be obtained. Using eq. (10) and (19) and the theorem in the preceding section

$$\begin{aligned} K^{oe}(t) &= a^{ko} b^{ko}(t) \\ K^{ie}(t) &= a^{ki} b^{ki}(t) \end{aligned} \tag{32}$$

where the factors a^{ko} and a^{ki} are independent of the elastic constants. It is assumed that the crack is arrested at the point P at the time $t = t'$. Then from eq. (22)

$$K(t) = \Delta(t - t') K^e(t) \tag{33}$$

where

$$K^e(t) = a^{ko} b^{ko}(t) + a^{ki} b^{ki}(t) \tag{34}$$

On the basis of this result it can be concluded that the stress intensity factor for a growing crack in the case of prescribed stresses does not depend in any way on the crack growth history, but has the same value as in the elastic case, i.e. as for the corresponding elastic problem. Also, this result can easily be achieved by direct application of the correspondence principle, Graham [13].

Now the corresponding values for the deformation intensity factors C and D are to be obtained. From eq. (5)

$$\begin{aligned} C^e &= \frac{\rho^c(\nu)}{E} K^e \\ D^e &= \frac{\rho^d(\nu)}{E} K^e \end{aligned} \tag{35}$$

where $\rho^c(\nu)$ and $\rho^d(\nu)$ are known functions and where E is Young's modulus. Combining this with eq. (26) and (33) yields

$$\begin{aligned} C(t) &= \rho^c(\nu)\, \mathcal{C}\{\Delta(t - t') K^e(t)\} \\ D(t) &= \rho^d(\nu)\, \mathcal{C}\{\Delta(t - t') K^e(t)\} \end{aligned} \tag{36}$$

where $\mathcal{C}\{.\}$ is a Stieltjes convolution with the kernel $C(.)$.

We see that the deformation intensity factors C and D are directly proportional, which means that in this case ($\nu = constant$), the asymptotic fields around the crack tip are completely described for each mode by two quantities only, for example the stress intensity factor K and the deformation intensity factor C, i.e. in the mixed mode case four parameters are needed to describe the crack tip fields.

4. A Specific Solution for Prescibed Displacements

A specific boundary value problem with prescribed displacements will now be considered. In this case the region $\Omega = \Omega(t)$ is an infinite strip at a width of $2h$. A crack is growing at a constant velocity v from the left to the right, se figure 2, and the boundary conditions are given by

$$\begin{aligned} x_2 = h &: \; u_2 = u_0\,\Delta(t)\,; \;\; \sigma_{12} = 0 \\ x_2 = -h &: \; u_2 = -u_0\,\Delta(t)\,; \;\; \sigma_{12} = 0 \end{aligned} \tag{37}$$

The crack surfaces $\partial\Omega^i$ are traction-free. The problem is well known from the literature. The stationary elastic problem has been studied by Rice [24], and the viscoelastic problem has been studied by Mueller [6,15].

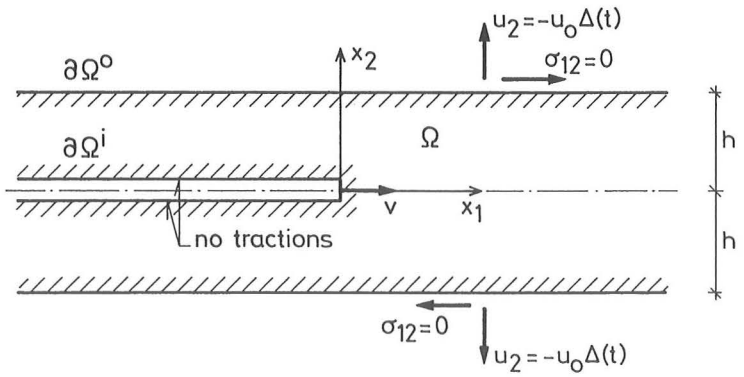

Figure 2. Crack growth in an infinite strip.

Again we will assume Poissons ratio to be constant and use the constitutive equation (30). All shearing mode parameters K_1, C_2, D_2 vanish because of the symmetry of the problem. The stress intensity factor K_1 is then obtained using the results of section 3. While the crack surfaces are traction-free, $K_1^{ie} = 0$ for all t. The contribution K_1^{oe} may be found in Rice [24] or in a handbook, for example in Tada [23],

$$K^{oe}(t) \;=\; \rho^k(\nu)\; E\; \Delta(t) \tag{38}$$

where $\rho^k(\nu) = u_0/\sqrt{h}$ for plane stress and $\rho^k(\nu) = u_0/((1-\nu^2)\sqrt{h})$ for plane strain. We see that the elastic solution is of the form given by eq. (10). If we assume

that the crack tip is arrested at the point P at the time $t = t'$, then, from eq. (22),

$$\begin{aligned} K_1(t) &= \rho^k(\nu)\, \Delta(t-t')\, \mathcal{R}\{\Delta(t)\} \\ &= \rho^k(\nu)\, \Delta(t-t')\, R(t) \end{aligned} \tag{39}$$

If the crack is moving at the arbitrary time $t = t'$, then from eq. (23)

$$\begin{aligned} K_1(t') &= \lim_{t \to t'} \rho^k(\nu)\, \Delta(t-t')\, R(t)\ ;\ \ t > t' \\ &= \rho^k(\nu)\, R(t') \end{aligned} \tag{40}$$

Again it is noted that the stress intensity factor does not depend upon the crack growth history or the velocity of the crack tip, but only on the time that has elapsed since the application of the loads. The result is in agreement with the solution found by Mueller [15].

As in the preceding example, the deformation intensity factors are proportional, since Poissons ratio is assumed to be constant, and only one of them, say C_1, will be obtained. Similar to eq. (36) we get from eq. (26) and (39)

$$C_1(t) = \rho^c(\nu)\, \rho^k(\nu)\, \mathcal{C}\{\Delta(t-t')\, R(t)\} \tag{41}$$

Using the identity (31) and some properties of the Heaviside function $\Delta(.)$ it is not difficult to see that

$$\mathcal{C}\{\Delta(t-t')\, R(t)\} = \Delta(t-t')\, C(t-t')\, R(t') \tag{42}$$

so that

$$C_1(t) = \rho^c(\nu)\, \rho^k(\nu)\, C(t-t')\, R(t') \tag{43}$$

Again, if the crack is moving at the arbitrary time $t = t'$, then, from eq. (28),

$$\begin{aligned} C_1(t') &= \rho^c(\nu)\, \rho^k(\nu)\, C(0+)\, R(t') \\ &= \rho^c(\nu)\, C(0+)\, K_1(t') \end{aligned} \tag{44}$$

Note that the deformation intensity factor only depend on the velocity of the crack tip through on on-off dependency. This result might seem surprising. Mueller, [15] analysed the deformations around a growing crack in the infinite strip, and found a solution for the crack opening that was dependent upon the crack tip velocity. However, he did not express the solution by the properties of the asymptotic displacement field, but derived an equation for the total displacements. The dependency of the velocity found by Knauss might therefore be caused by far field properties of the displacement field. This is indicated by the fact that his solutions for the crack opening do not correspond

to a squareroot dependency on the distance from the crack tip, but is clearly influenced by higher order terms.

Hovever, the result given by eq. (44) should not be surprising as only the asymptotic fields are considered. When the crack is moving, no matter how slow the crack growth is, all points at the crack extension path will experience infinite rates just prior to arrival of the crack tip. This justifies the result that only the material properties corresponding to infinite rates (the intial value of the creep function $C(0+)$) influence the asymptotic deformation fields.

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Department of Building Technology and Structural Engineering
The University of Aalborg, Sohngaardsholmsvej 57, DK 9000 Aalborg
Telephone: 45 98 14 23 33 Telefax: 45 98 14 82 43