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OPTIMAL FILTERS FOR EXTRACTION AND SEPARATION OF PERIODIC SOURCES

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ABSTRACT

In this paper, the problem of extracting periodic signals, like voiced speech or tones in music, from noisy observations or mixtures of periodic signals is considered, and, in particular, the problem of designing filters for such a task. We propose a novel filter design that 1) is specifically aimed at extracting periodic signals, 2) is optimal given the observed signal and thus signal-adaptive, and 3) offers a full parametrization of the periodic signal. The found filters can be used for a multitude of applications including signal compression, parameter estimation, enhancement, and separation. Some illustrative signal examples demonstrate its superior properties as compared to other similar filters.

1. INTRODUCTION

Many natural signals that are of interest to mankind are periodic by nature. This is particularly the case for speech and audio signals where such signals constitute the very atoms of music, namely tones, or, in the case of voiced speech, individual speakers. Perhaps the most fundamental problem of all in signal processing is the source separation problem, as many other problems are trivially, or at least more easily, solved once a complicated mixture has been broken into its constituent parts. There exists a number of different methods for extracting periodic sources like, for example, the algebraic separation method [1] or comb filtering [2], and these can generally also be used for finding the fundamental frequency itself. Recently, some optimal filter designs have been proposed for fundamental frequency estimation [3, 4, 5]. These filter designs are generalizations of Capon's classical optimal beam-former [6]. While these filters have proven to have excellent performance even under adverse conditions, like in the presence of multiple interfering sources [4], they do, however, suffer from several problems when applied to the problem of extracting sources, such as poor performance for high signal-to-noise ratios and high sensitivity to model mismatch or uncertainties (see, e.g., [7]).

In this paper, we propose a novel filter design method which is aimed at extracting periodic signals from noisy observations or from mixtures of periodic signals. The obtained filters are optimal given the observed signal and can thus be said to be signal-adaptive. The proposed filter design is reminiscent of the principle used in the Amplitude and Phase EStimation (APES) method [8, 9], which is well-known to have several advantages over the Capon-based estimators. It can be used for not only extracting or separating periodic signals but also for estimating the fundamental frequency and the number of harmonics of such signals along with the amplitudes of the individual harmonics. In other words, the filtering approach proposed herein provides a full parametrization of periodic signals through the use of the same filter.

The paper is organized as follows. In Section 2, we introduce the fundamentals and proceed to derive the filter designs showing their optimality given the observed signal in Section 3. In the following section, namely Section 4, we illustrate the properties of the proposed design and compare the resulting filters to those obtained using previously published methods. Moreover, we demonstrate its application for the extraction of a periodic signal from a mixture of interfering periodic signals and noise, i.e., for separation and enhancement. Finally, we conclude on the work in Section 5.

2. PROBLEM STATEMENT

We are interested in obtaining the output $y(n)$ of an FIR filter having coefficients $h(m)$ from the input $x(n)$, defined for $n = 0, \ldots, N-1$, as

$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m). \tag{1}$$

We seek to find an optimal set of coefficients $\{h(m)\}$ such that the mean square error between the filter output and a desired output, a signal model if you will, $\hat{y}(n)$, is minimized in the following sense:

$$P = \frac{1}{K} \sum_{n=M-1}^{N-1} |y(n) - \hat{y}(n)|^2 , \tag{2}$$

where $K = N - M + 1$ is the number of samples over which we average. Since we are here concerned with periodic signals, this should be reflected in the choice of the signal model $\hat{y}(n)$. In fact, this should be chosen as the sum of sinusoids having frequencies that are integer multiples of a fundamental frequency ω_0 weighted by their respective complex amplitudes a_l, i.e., $\hat{y}(n) = \sum_{l=1}^{L} a_l e^{j\omega_0 ln}$. This leaves us with the following expression for the mean square error:

$$P = \frac{1}{K} \sum_{n=M-1}^{N-1} \left| \sum_{m=0}^{M-1} h(m)x(n-m) - \sum_{l=1}^{L} a_l e^{j\omega_0 ln} \right|^2 . \tag{3}$$

In the following derivations, we assume the fundamental frequency ω_0 and the number of harmonics L to be known (with $L < M$), although the so-obtained filters can later be used for finding these quantities. Next, we proceed to find not only the filter coefficients but also the complex amplitudes a_l. In doing so, we first introduce some useful notation. First, we introduce a vector containing the filter coefficients as

$$\mathbf{h} = [\, h(0) \ \cdots \ h(M-1) \,]^H \tag{4}$$

(with $(\cdot)^H$ denoting the Hermitian transpose) and a sub-vector containing M samples of the observed signal, i.e.,

$$\mathbf{x}(n) = [\, x(n) \ \cdots \ x(n-M+1) \,]^T . \tag{5}$$

This allows us to write the output of the filter at time n as $y(n) = \mathbf{h}^H \mathbf{x}(n)$. Similarly, we introduce a vector containing the complex amplitudes as

$$\mathbf{a} = [\, a_1 \ \cdots \ a_L \,]^H , \tag{6}$$

and one containing the complex sinusoids at time n, i.e.,

$$\mathbf{w}(n) = \left[\, e^{j\omega_0 1 n} \ \cdots \ e^{j\omega_0 L n} \,\right]^T . \tag{7}$$

Finally, this allows us to write (2) as

$$P = \frac{1}{K} \sum_{n=M-1}^{N-1} |\mathbf{h}^H \mathbf{x}(n) - \mathbf{a}^H \mathbf{w}(n)|^2, \tag{8}$$

which in turn can be expanded into

$$P = \mathbf{h}^H \widehat{\mathbf{R}} \mathbf{h} - \mathbf{a}^H \mathbf{G} \mathbf{h} - \mathbf{h}^H \mathbf{G}^H \mathbf{a} + \mathbf{a}^H \mathbf{W} \mathbf{a}, \tag{9}$$

with

$$\widehat{\mathbf{R}} = \frac{1}{K} \sum_{n=M-1}^{N-1} \mathbf{x}(n) \mathbf{x}^H(n), \tag{10}$$

which can be identified as the sample covariance matrix, and the remaining quantities being defined as

$$\mathbf{G} = \frac{1}{K} \sum_{n=M-1}^{N-1} \mathbf{w}(n) \mathbf{x}(n)^H \tag{11}$$

and

$$\mathbf{W} = \frac{1}{K} \sum_{n=M-1}^{N-1} \mathbf{w}(n) \mathbf{w}^H(n). \tag{12}$$

3. SOLUTION

Solving for the complex amplitudes in (9) yields the following expression

$$\hat{\mathbf{a}} = \mathbf{W}^{-1} \mathbf{G} \mathbf{h}, \tag{13}$$

which depends on the yet unknown filter $\mathbf{h}$. For $\mathbf{W}$ to be invertible, we require that $K \geq L$, but to ensure that also the covariance matrix is invertible (which we will need later), we will also assume that $K \geq M$. By substituting the expression above back into (9), we get

$$P = \mathbf{h}^H \widehat{\mathbf{R}} \mathbf{h} - \mathbf{h}^H \mathbf{G}^H \mathbf{W}^{-1} \mathbf{G} \mathbf{h}. \tag{14}$$

By some simple manipulation, we see that this can be simplified somewhat as

$$P = \mathbf{h}^H \left(\widehat{\mathbf{R}} - \mathbf{G}^H \mathbf{W}^{-1} \mathbf{G} \right) \mathbf{h} \triangleq \mathbf{h}^H \widehat{\mathbf{Q}} \mathbf{h} \tag{15}$$

where

$$\widehat{\mathbf{Q}} = \widehat{\mathbf{R}} - \mathbf{G}^H \mathbf{W}^{-1} \mathbf{G} \tag{16}$$

can be thought of as a *modified* covariance matrix estimate that is formed by subtracting the contribution of the harmonics from the covariance matrix given the fundamental frequency. It can be shown that $\mathbf{W}$ is asymptotically identical to the identity matrix. By replacing $\mathbf{W}$ by $\mathbf{I}$ in (15) one obtains the usual noise covariance matrix estimate, used, for example, in [10]. For finite N, though, this is only an approximation that, nonetheless, may still be useful for practical reasons as it is much simpler.

Solving for the unknown filter in (15) directly results in a trivial and useless results, namely the zero vector. To fix this, we will introduce some additional constraints. Not only should the output of the filter be periodic, i.e., resemble a sum of harmonically related sinusoids, the filter should also have unit gain for all the harmonics frequencies, i.e., $\sum_{m=0}^{M-1} h(m) e^{-j\omega_0 l m} = 1$ for $l = 1, \ldots, L$. Introducing the vector

$$\mathbf{z}(\omega) = \left[\, e^{-j\omega 0} \ \cdots \ e^{-j\omega(M-1)} \,\right]^T , \tag{17}$$

we may also express this as $\mathbf{h}^H \mathbf{z}(\omega_0 l) = 1$. We can now state the filter design problem as the following constrained optimization problem:

$$\min_{\mathbf{h}} \mathbf{h}^H \widehat{\mathbf{Q}} \mathbf{h} \quad \text{s.t.} \quad \mathbf{h}^H \mathbf{z}(\omega_0 l) = 1, \quad \text{for} \quad l = 1, \ldots, L. \tag{18}$$

The constraints for the L harmonics can also be expressed as $\mathbf{h}^H \mathbf{Z} = \mathbf{1}$, where $\mathbf{1} = [\, 1 \ \cdots \ 1 \,]^T$, and

$$\mathbf{Z} = [\, z(\omega_0) \ \cdots \ z(\omega_0 L) \,] \tag{19}$$

The problem in (18) is a quadratic optimization problem with equality constraints that can be solved using the Lagrange multiplier method. Introducing the Lagrange multiplier vector

$$\boldsymbol{\lambda} = [\, \lambda_1 \ \cdots \lambda_L \,]^T , \tag{20}$$

the Lagrangian dual function of the problem stated above can be expressed as

$$\mathcal{L}(\mathbf{h}, \boldsymbol{\lambda}) = \mathbf{h}^H \widehat{\mathbf{Q}} \mathbf{h} - \left(\mathbf{h}^H \mathbf{Z} - \mathbf{1}^T \right) \boldsymbol{\lambda}. \tag{21}$$

By taking the derivative with respect to the unknown filter vector and the Lagrange multiplier vector and setting this to zero, i.e., $\nabla \mathcal{L}(\mathbf{h}, \boldsymbol{\lambda}) = \mathbf{0}$, we obtain

$$\hat{\mathbf{h}} = \widehat{\mathbf{Q}}^{-1} \mathbf{Z} \left(\mathbf{Z}^H \widehat{\mathbf{Q}}^{-1} \mathbf{Z} \right)^{-1} \mathbf{1}. \tag{22}$$

This filter is optimal in the sense that it has unit gain at the harmonic frequencies and an output that resembles a sum of harmonically related sinusoids while everything else is suppressed maximally. It can readily be used for determining the amplitudes of those sinusoids by inserting (22) into (13), which yields the following estimate:

$$\hat{\mathbf{a}} = \mathbf{W}^{-1} \mathbf{G} \widehat{\mathbf{Q}}^{-1} \mathbf{Z} \left(\mathbf{Z}^H \widehat{\mathbf{Q}}^{-1} \mathbf{Z} \right)^{-1} \mathbf{1}. \tag{23}$$

The output power of the filter when this is applied to the original signal can be expressed as $\hat{\mathbf{h}}^H \widehat{\mathbf{R}} \hat{\mathbf{h}}$ which may be used for determining the fundamental frequency by treating ω_0 in $\mathbf{Z}$, $\mathbf{G}$, $\mathbf{W}$ as an unknown parameter and then pick as an estimate the value for which the output power is maximized, i.e.,

$$\hat{\omega}_0 = \arg\max_{\omega_0} \hat{\mathbf{h}}^H \widehat{\mathbf{R}} \hat{\mathbf{h}}. \tag{24}$$

One can also obtain an estimate of the number of harmonics L by estimating the noise variance by filtering out the harmonics and applying one of the many statistical model order estimation tools, like, e.g., the MAP-rule of [11]. From the optimal filter, it is thus possible to obtain a full parametrization of periodic signals as was claimed in the introduction.

The main difference between the design proposed here and the Capon-like designs previously proposed is that the modified covariance matrix $\widehat{\mathbf{Q}}$ is used in (18) in place of $\widehat{\mathbf{R}}$, i.e., the difference is essentially in terms of the output of the filter being periodic. Interestingly, despite this difference, the Capon-like filters of [3, 4] can be obtained as a special case of the solution presented here by setting the modified covariance matrix equal to the sample covariance matrix of the observed signal, i.e., $\widehat{\mathbf{Q}} = \widehat{\mathbf{R}}$. The proposed filter design leads to filters that are generally also much more well-behaved for high SNRs, where Capon-like filters are well-known to perform poorly and require that diagonal loading or similar techniques be applied [7]. The proposed filter also holds several advantages over traditional methods, like the comb filtering approach or sinusoidal filters (also known as FFT filters), namely that it is 1) optimal given the observed signal, and 2) optimized for periodic filter output.

4. EXPERIMENTAL RESULTS

We will start out the experimental part of this paper by showing an example of the optimal filters obtained using the proposed method and the Capon-like filters of [3, 4]. In Figure 1, the magnitude frequency response of the filters are shown for a synthetic signal having $\omega_0 = 0.6283$, $L = 5$, unit amplitudes and random phases with white Gaussian noise added at -20 dB. Both the proposed and the Capon-like filters can be seen to exhibit the expected response following the harmonic structure of the signal, and they are also quite similar. In Figure 2, the same is shown only the SNR is now 20 dB. It can clearly be seen that the proposed filters still exhibit the desired response emphasizing the harmonics of the signal. The Capon-like design, however, behaves erratically. In the next example, we will use the proposed signal-adaptive filter to extract a real trumpet signal, a single tone sampled at approximately 8 kHz using 60 ms segments and a filter length of 100. For each segment the fundamental frequency was found using the subspace method of [12]. The single tone has been buried in noise at 10 dB and interfering tones (also trumpet tones) have been added also at 10 dB. The spectrogram of the original signal is shown in Figure 3 and the same signal with noise and interference added is shown in Figure 4. The spectrogram of the extracted signal is shown in Figure 5. A slice of the time-domain signals are depicted in Figure 6. These figures clearly demonstrate the ability of the proposed filters to extract the signal while rejecting not only noise, but also strong periodic interference even when these are fairly close to the harmonics of the desired signal.

5. CONCLUSION

In this paper, we have proposed novel filter designs for extracting periodic signals from noisy mixtures. The filters are optimal given the observed signal and are designed such that their output is periodic as possible, meaning that it should resemble a sum of harmonically related sinusoids. This idea can be seen as a generalization of the principle employed in the well-known APES filters. Moreover, the filters are obtained as solutions to a linearly constrained quadratic optimization problem that has a closed-form solution. We have demonstrated that the new filters lead to a number of advantages over previous methods, including Capon-like designs. The filters can be used for a number of application, including separation, enhancement, and parameter estimation.

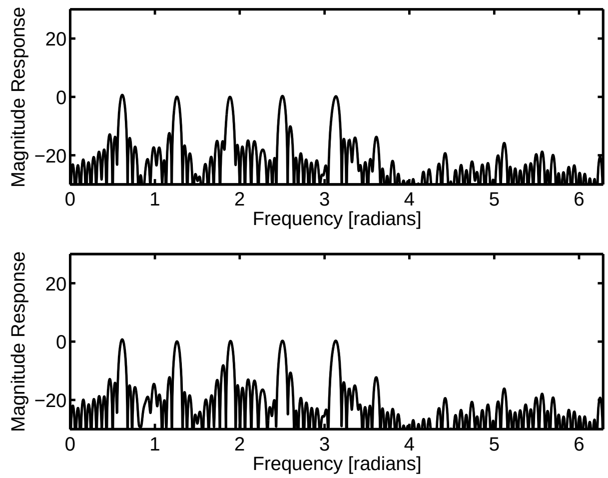

Fig. 1. Frequency response of the proposed (top) and Capon-like filters (bottom) for an SNR of -20 dB.

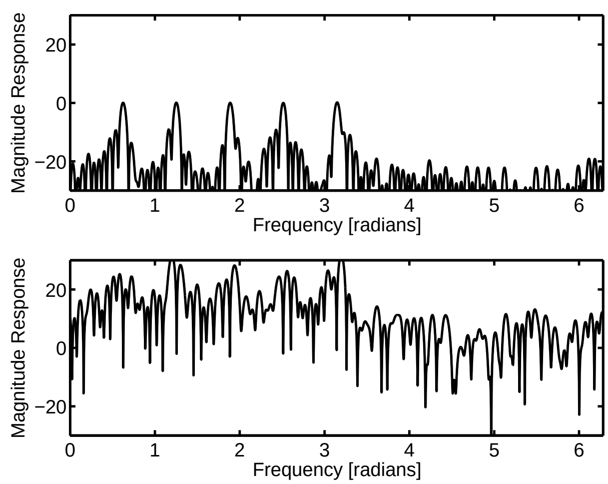

Fig. 2. Frequency response of proposed (top) and Capon-like filters (bottom) for an SNR of 20 dB.

6. REFERENCES

[1] M.-Y. Zou, C. Zhenming, and R. Unbehauen, "Separation of periodic signals by using an algebraic method," in *Proc. IEEE Int. Symp. Circuits and Systems*, 1991, vol. 5, pp. 2427–2430.

[2] A. Nehorai and B. Porat, "Adaptive comb filtering for harmonic signal enhancement," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34(5), pp. 1124–1138, Oct. 1986.

[3] M. G. Christensen, J. H. Jensen, A. Jakobsson, and S. H. Jensen, "On optimal filter designs for fundamental frequency

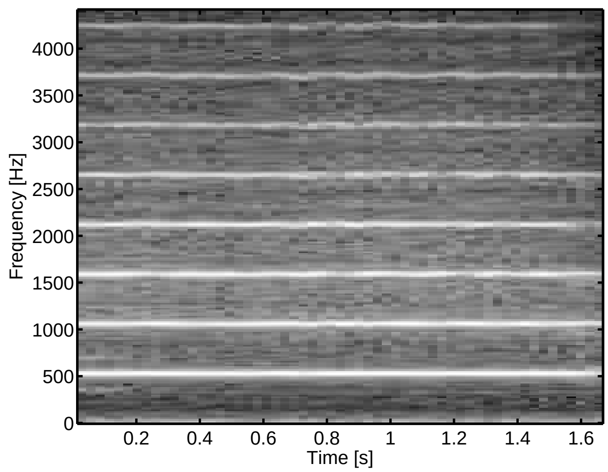

Fig. 3. Spectrogram of trumpet signal.

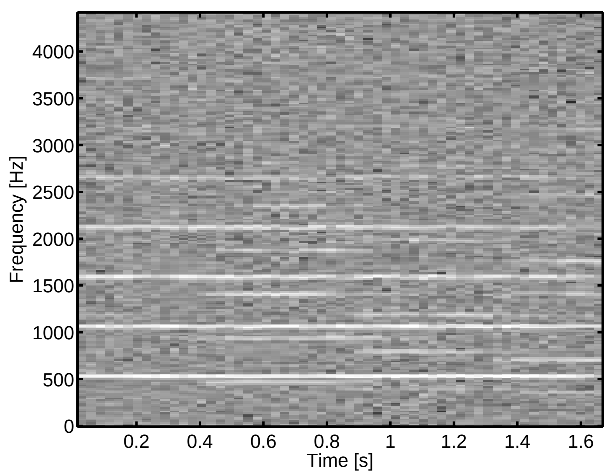

Fig. 4. Spectrogram of trumpet signal in noisy and with periodic interference added at 10 dB SNR.

estimation," *IEEE Signal Process. Lett.*, vol. 15, pp. 745–748, 2008.

[4] M. G. Christensen, P. Stoica, A. Jakobsson, and S. H. Jensen, "Multi-pitch estimation," *Elsevier Signal Processing*, vol. 88(4), pp. 972–983, Apr. 2008.

[5] M. G. Christensen and A. Jakobsson, *Multi-Pitch Estimation*, vol. 5 of *Synthesis Lectures on Speech & Audio Processing*, Morgan & Claypool Publishers, 2009.

[6] J. Capon, "High-resolution frequency-wavenumber spectrum analysis," *Proc. IEEE*, vol. 57(8), pp. 1408–1418, 1969.

[7] P. Stoica and R. Moses, *Spectral Analysis of Signals*, Pearson Prentice Hall, 2005.

[8] J. Li and P. Stoica, "An adaptive filtering approach to spectral estimation and SAR imaging," *IEEE Trans. Signal Process.*, vol. 44(6), pp. 1469–1484, June 1996.

[9] P. Stoica, H. Li, and J. Li, "A new derivation of the APES filter," *IEEE Signal Process. Lett.*, vol. 6(8), pp. 205–206, Aug. 1999.

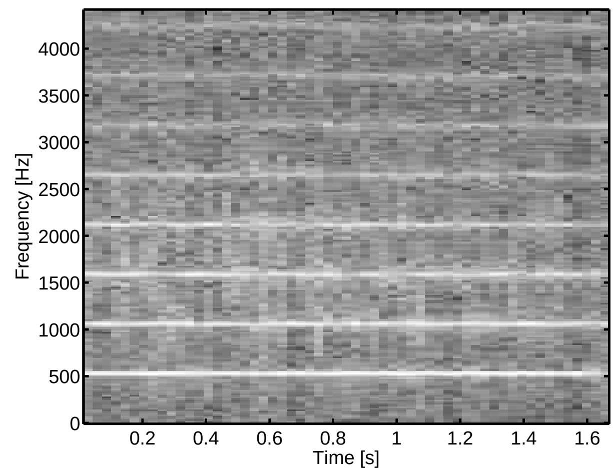

Fig. 5. Spectrogram of signal extracted using the optimal filter.

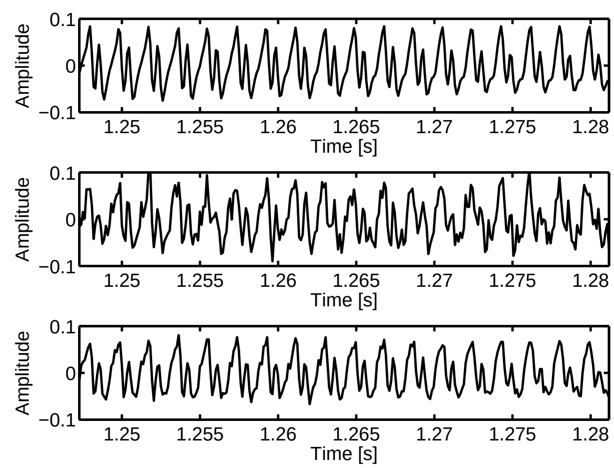

Fig. 6. Original signal (top), observed signal with Gaussian noise and periodic interference added at 10 dB SNR (middle), extracted signal (bottom).

[10] P. Stoica, H. Li, and J. Li, "Amplitude estimation of sinusoidal signals: Survey, new results and an application," *IEEE Trans. Signal Process.*, vol. 48(2), pp. 338–352, Feb. 2000.

[11] P. M. Djuric, "Asymptotic MAP criteria for model selection," *IEEE Trans. Signal Process.*, vol. 46, pp. 2726–2735, Oct. 1998.

[12] M. G. Christensen, A. Jakobsson, and S. H. Jensen, "Joint high-resolution fundamental frequency and order estimation," *IEEE Trans. Audio, Speech, and Language Process.*, vol. 15(5), pp. 1635–1644, July 2007.