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# Reliability-based calibration of load duration factors for timber structures

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**Abstract**

The load bearing capacity of timber structures decrease with time depending on the type of load and timber. Based on representative limit states and stochastic models for timber structures, load duration factors are calibrated using probabilistic methods. Load duration effects are estimated on basis of simulation of realizations of wind, snow and imposed loads in accordance with the load models in the Danish structural codes. Three damage accumulation models are considered, namely Gerhards model, Barrett and Foschi's model and Foschi and Yao's model. The parameters in these models are fitted by the Maximum Likelihood Method using data relevant for Danish structural timber and the statistical uncertainty is quantified. The reliability is evaluated using representative short- and long-term limit states, and the load duration factor $k_{\mathrm{mod}}$ is estimated using the probabilistic model such that equivalent reliability levels are obtained using short- and long-term design equations. Time variant reliability aspect is considered using a simple, representative limit state with time variant strength and simulation of the whole life time load processes.


*Keywords:* Timber; Reliability analysis; Load duration factors; Load models

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## 1. Introduction

Most building codes, national and international, are based on a probabilistic safety philosophy. The code formats are, however, deterministic models with connection to reliability design achieved through failure probability, partial safety factors and characteristic values. Partial safety factors are calibrated for standard cases against probabilistic analyses for similar cases. The condition for calibration is that the probabilistic analysis and the deterministic code fulfil the same requirements of safety. Hence, the required safety is usually not accomplished by using probabilistic theories in everyday design.

Strength reducing effects, often referred to as creep-rupture effects, due to long term loading at high stress ratio levels are known for many materials. Timber materials are highly affected by this reduction in strength with duration of load. Therefore, design of timber structures utilizes a strength-reducing factor to reduce the characteristic short-term strength. The Eurocode for timber structures EC5 [8] refers to this strength modification with a load duration factor, $k_{\mathrm{mod}}$. Traditionally, the load duration factor is determined empirically from experience on timber structures and knowledge of material properties of wood. In North-America probabilistic methods have been used to determine load duration factors, $k_{\mathrm{mod}}$ using reliability methods and stochastic models relevant for North-American conditions, see e.g. Foschi [11], Ellingwood and Rosowsky [9], Rosowsky and Fridley [19], Ellingwood [10] and Rosowsky and Bulleit [20,21]. The results of these studies have been implemented in ASCE Standard 16-95 [1]. This paper presents a similar method to calibrate $k_{\mathrm{mod}}$ using a full probabilistic method based on specific stochastic models for Danish timber strengths and load processes. Requirements are that the lifetime reliability for representative limit states are the same for situations where the load duration is taken into account and situations where short-term strength models are used. The method is used in a Danish study to evaluate the code parameters in the Danish Code DS 413 [5].

The load duration factor $k_{\mathrm{mod}}$ is defined in the Danish code DS 413 [5] and in EC5 [8] as a factor taking into account the effects of load duration and ambient climate on the strength parameters of structural timber elements. In this paper, snow load, wind load, and imposed load are considered. The stochastic models are formulated in accordance with the load models in the Danish structural codes, DS 409 [6] and DS 410 [7]. Three damage accumulation models are considered, namely the models by Gerhards [14], Barrett and Foschi [2] and Foschi and Yao [12]. The parameters in these models are fitted using data relevant for Danish structural timber determined by Hoffmeyer [15].

## 2. Short and long-term strength

The initial (short term) bending strength is assumed to be Lognormal distributed with coefficient of variation equal to 15% and 20%, which are the basic COV's used in Denmark for laminated and structural timber, respectively, see Sørensen and Hoffmeyer [24] and Sørensen et al. [23]. In developing the Danish code it was desired to use a common model for all structural materials, and in this context the Lognormal appeared best overall. The Lognormal distribution is generally also used for all strength parameters in the Eurocode structural codes in order to obtain comparable reliability levels for different materials. It is noted that a Weibull distribution is also a relevant distribution type for timber strengths, but generally results in lower reliability levels.

### 2.1. Damage models

Damage models are used to mathematically describe the long term strength reduction as a function of stress level and duration of loading. In this study three damage models are fitted against data obtained on Nordic structural timber subjected to constant loading. The three damage models have also been used in the North-American studies of load duration effects, see e.g. Foschi [11], Ellingwood and Rosowsky [9]. The characteristics of the three damage models are that $\alpha$ is defined as the degree of damage, i.e. $\alpha = 0$ stands for no damage and $\alpha = 1$ stands for total damage or failure.

### 2.2. Gerhards model

The damage accumulation model presented by Gerhards [14] is written:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \exp\left(-A + B\frac{\sigma}{f_0}\right), \tag{1}$$

where $A$ and $B$ are constants, $\sigma$ is the stress and $f_0$ is the short term strength of the studied member.

Solution of the differential equation in (1):

$$\frac{\sigma}{f_0} = \frac{A}{B} - \frac{\ln 10}{B}\log t = a - b\log t, \tag{2}$$

where

$$a = \frac{A}{B} + \varepsilon; \quad b = \frac{\ln 10}{B}, \tag{3}$$

and $\varepsilon$ models the model uncertainty related to the model in (1). $\varepsilon$ is assumed to be Normal distributed with expected value equal to 0 and standard deviation $\sigma_\varepsilon$.

Assuming constant load and considering $f$ as the residual strength the solution to (2) is:

$$\frac{f}{f_0} = \frac{1}{B}\ln\left(1 + (1-\alpha)(\exp B - 1)\right). \tag{4}$$

This expression is used when simulating the damage due to load duration.

### 2.3. Barrett and Foschi's model

The damage accumulation model by Barrett and Foschi [2] has the following mathematical expression

$$\begin{aligned} &\frac{\mathrm{d}\alpha}{\mathrm{d}t} = A\left(\frac{\sigma}{f_0} - \eta\right)^B + C\alpha; \quad \frac{\sigma}{f_0} > \eta \\ &\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 0; \quad \frac{\sigma}{f_0} \leqslant \eta \end{aligned} \tag{5}$$

where $A$, $B$, $C$ are constants, $\eta$ is a threshold ratio, $\sigma$ is the load in time and $f_0$ is the initial load carrying capacity of the studied member.

Solution of the differential equation in (5) for $\alpha = 1$ and assuming constant load gives:

$$\frac{\sigma}{f_0} = \left(\frac{A}{C}(\exp(C \cdot t) - 1)\right)^{-\frac{1}{B}} + \eta = a(\exp(\exp(b) \cdot t) - 1)^c + \eta, \tag{6}$$

where

$$a = \exp\left(\ln\left(\left(\frac{A}{C}\right)^{-\frac{1}{B}}\right) + \varepsilon\right); \quad b = \ln C; \quad c = -\frac{1}{B}, \tag{7}$$

and $\varepsilon$ models the model uncertainty related to the model in (5).

Considering $f$ as the residual strength:

$$\frac{f}{f_0} = \eta + \left[(1-\alpha)(1-\eta)^B\right]^{\frac{1}{B}}. \tag{8}$$

### 2.4. Foschi and Yao's model

The third damage model by Foschi and Yao [12], also named the Canadian model, is an extended version of (5):

$$\begin{aligned} \frac{d\alpha}{dt} &= A\left(\frac{\sigma}{f_0} - \eta\right)^B + C\left(\frac{\sigma}{f_0} - \eta\right)^D \alpha; \quad \frac{\sigma}{f_0} > \eta \\ \frac{d\alpha}{dt} &= 0; \quad \frac{\sigma}{f_0} \leqslant \eta \end{aligned} \tag{9}$$

where $A$, $B$, $C$, and $D$ are constants, $\eta$ is a threshold ratio, $\sigma$ is the stress and $f_0$ is the initial short term strength.

Solution of the differential equation with short term ramp load ($\sigma = kt$) until failure with initial strength $f_0$ gives (assuming rate of loading is large and $C$ small), see Köhler and Svensson [17]

$$A = \frac{k(B+1)}{f_0(1-\eta)^{(B+1)}}, \tag{10}$$

and time until failure $t_f$:

$$t_f = \ln\left(\frac{\sigma}{k} + \frac{1}{C\left(\frac{\sigma}{k} - \eta\right)^D} \ln\left(\frac{1+\lambda}{\alpha_0 + \lambda}\right)\right) + \varepsilon, \tag{11}$$

where

$$a_0 = \left(\frac{\frac{\sigma}{f_0} - \eta}{1-\eta}\right)^{B+1}; \quad \lambda = \frac{k(B+1)}{Cf_0(1-\eta)^D}\left(\frac{\sigma}{f_0} - \eta\right)^{B-D}, \tag{12}$$

and $\varepsilon$ models the model uncertainty related to the model in (1).

Considering $f$ as the residual strength corresponding to the damage $\alpha$:

$$\frac{f}{f_0} = \eta + (1 - \eta)(1 - \alpha)^{1/(1+B)}. \tag{13}$$

### 2.5. Fit to test results

The constants describing the influence of time on long term strength in the damage models are determined by curve fitting on test results. Often, reference to data is made to the well-known Madison curve, see Wood [26] which is based on results from tests on clear wood specimens. The curve fitting in this investigation is carried out by Maximum Likelihood Method on test results on structural timber (Norway Spruce) in 4-point bending, Hoffmeyer [15]. The optimal estimates of the parameters are shown in Table 1–3. The parameters can asymptotically be assumed Normal distributed when the number of data is larger than 25–30, see Lindley [18]. Based on the Maximum Likelihood estimates also standard deviations and correlation coefficients modelling

Table 1
Maximum Likelihood estimates and standard deviation of parameters in Gerhards model fit against test results on Norway Spruce

| | $a$ | $b$ | $\sigma_s$ |
|---|---|---|---|
| Estimate | 0.90 | 0.0495 | 0.0206 |
| Standard deviation | 0.0055 | 0.0016 | 0.0021 |

The correlation coefficients are: $\rho(a,b) = 0.90$, $\rho(a,\sigma_s) = 0.08$, and $\rho(b,\sigma_s) = 0.02$.

Table 2
Maximum Likelihood estimates and standard deviations of parameters in Barrett and Foschi's model fit against test results on Norway Spruce

| | $a$ | $b$ | $c$ | $\sigma_s$ |
|---|---|---|---|---|
| Estimate | 0.221 | −9.14 | −0.063 | 0.075 |
| Standard deviation | 0.0033 | 0.079 | 0.002 | 0.006 |

The correlation coefficients are: $\rho(a,b) = 0.82$, $\rho(a,c) = 0.65$, $\rho(b,c) = 0.64$, $\rho(a,\sigma_s) = -0.09$, $\rho(b,\sigma_s) = -0.16$, and $\rho(c,\sigma_s) = -0.11$.

Table 3
Maximum Likelihood estimates and standard deviation of parameters in Foschi and Yao's model fit against test results on Norway Spruce

| | $B$ | $C$ | $D$ | $\sigma_s$ |
|---|---|---|---|---|
| Estimate | 27.3 | 9.78 | 5.44 | 0.35 |
| Standard deviation | 0.92 | 7.57 | 0.46 | 0.03 |

The correlation coefficients are: $\rho(B,C) = 0.38$, $\rho(B,D) = 0.12$, $\rho(C,D) = 0.98$, $\rho(B,\sigma_s) = 0.00$, $\rho(C,\sigma_s) = 0.01$, and $\rho(D,\sigma_s) = 0.00$.

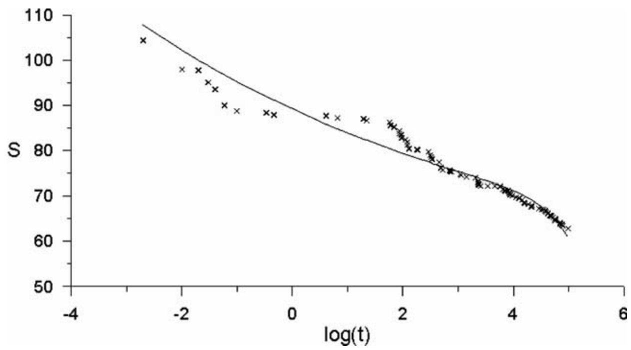


Fig. 1. Fit using model Barrett and Foschi to data. $x$-axis: log($t$) $t$ in hours and $y$-axis: $S = \frac{\sigma}{f_0}$ in %. $x$: data; full line: fit.

the statistical uncertainty are shown, see Sørensen et al. [25] for details. Fig. 1 shows the fit obtained using the Barrett and Foschi's model.

## 3. Load models

### 3.1. Snow load

A stochastic model of snow load in Denmark is established based on meteorological data from five locations in Denmark over a period of 32 years. A model calibrated against direct measurements of snow load is used to transform the meteorological data into snow loads. The load model is illustrated in Fig. 2.

The snow load on terrain, $P_{SG}$, and duration, $T$, of the snow packages (snow pulses) are modeled as follows, where also calibrated model parameters are shown:

- The occurrence of snow packages at times $X_1, X_2, \ldots$ is modelled by a Poisson process. This model has also been used by Ellingwood and Rosowsky [9]. The duration between snow packages is exponential distributed with expected value $1/\lambda$, where $\lambda$ is the expected number of snow packages per year ($\lambda = 1.175$ snow packages per year). Seasonal effects are not taken into account in the model.

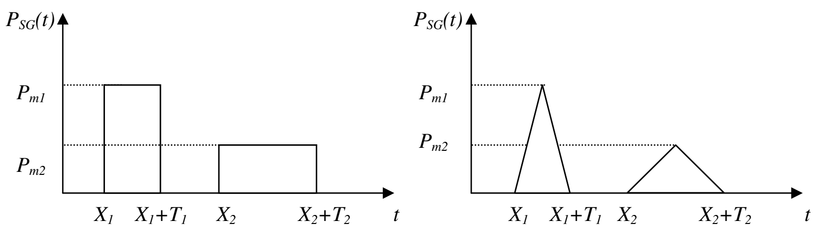


Fig. 2. Snow load model. Left: rectangular snow load. Right: triangular snow load.

- The magnitude of the maximum snow load $P_m$ in one snow package (snow pulse) is assumed to be Gumbel distributed (expected value $\mu_p = 0.33$ kN/m$^2$ and standard deviation $\sigma_p = 0.21$ kN/m$^2$).
- The duration of a snow package $T$ is modeled by $X_T P_m$, i.e. proportional to the maximum snow load of the snow package. $X_T$ is assumed to be Exponential distributed (expected value $\mu_{XT} = 0.35$ days/(kN/m$^2$)).
- The time variation of a snow package is assumed to be rectangular or triangular.

It is noted that in Fridley and Rosowsky [13] a significant effect was found of the time variation of the snow package on the accumulated damage. Therefore and because the observed snow packages are quite different, the triangular and rectangular time variations are included in the present probabilistic calibration of load duration factors.

The annual maximum snow load on a structure is determined from

$$Q_{S,max} = CP_{SG,max}, \tag{14}$$

where $P_{SG,max}$ is the annual maximum snow load on the ground and is Gumbel distributed (expected value $\mu = 0.36$ kN/m$^2$ and standard deviation $\sigma = 0.21$ kN/m$^2$). The 98% quantile in the annual maximum becomes $P_{SG,0.98} = 0.90$ kN/m$^2$. The stochastic model for $Q_{S,max}$ is in accordance with the above stochastic process model for $P_{SG}(t)$. $C$ is a shape factor based on results from Sanpaolesi [22]. The shape factor is assumed Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.35$.

### 3.2. Wind load

The wind velocity is modelled from measurements using sensors typically placed at a height of 10 m above flat farmland or sea in order to reduce disturbance by nearby trees, buildings, hills etc., see CIB W81 [4]. Measurements are stored as 10 min-mean wind velocities in a fixed window and taken over all wind directions. The wind data used in the present investigation are obtained from Sprogø, a small island situated in Great Belt. Measurements have been made in a relatively short period of 16-years. However, the results from Sprogø have been verified against data determined at other locations in Denmark. Only 10 min-mean wind velocities of more than 13 m/s (storms) are included in the data used for the analysis. The generation of time history of the wind pressure $P_W$ is modelled using the data on 10 min periods as well as the wind spectrum and wind action model used in DS 410 [7]. The model is illustrated in Fig. 3.

Based on recorded Danish wind data over 16 years (Sprogø) the following model is established:

- The occurrence of storms at times $X_1, X_2, \ldots$ is modelled by a Poisson process. The duration between storms thus becomes exponential distributed with expected value $1/\lambda$ ($\lambda = 9.4$ storms per year).
- The magnitude of the maximum wind pressure $P_m$ in one storm is assumed Gumbel distributed (coefficient of variation = 0.44).
- The duration of one storm $T$ (in sequences of 10 min periods) is equal to $X_T P_m$. $X_T$ is assumed to be Exponential distributed (expected value $\mu_{XT} = 1.76$ [10 min/MPa]).

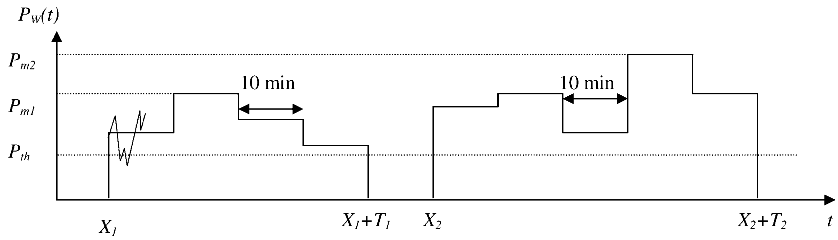


Fig. 3. Wind load model. Time variations in each 10-min period are indicated in the first 10-min period.

- The magnitude of the average wind pressures $P_{mw}$ in each 10-min period in a given storm is modelled as $P_{mw} = P_m - X_{P\ Pm} - P_{th}$ where $P_{th}$ is a lower threshold on wind pressures measured (e.g. proportional to (13 m/s)$^2$). $X_P$ is assumed to be Beta distributed (expected value $\mu_{XP} = 0.58$ and standard deviation $\sigma_{XP} = 0.28$). The wind pressures in one storm are limited to be between a lower threshold measured (e.g. proportional to (average wind velocity = 13 m/s)$^2$) and the maximum value for the storm, implying that $0 \leqslant X_P \leqslant 1$. It is assumed that the sequence of the 10-min periods is unimportant.
- The time history of the wind pressure $P_W(t)$ during each 10-min period is modelled using the wind spectrum and wind action model in DS410 [7].

The annual maximum wind load on a structure is determined from

$$Q_{W,\max} = CP_{W,\max}, \tag{15}$$

where $P_{W,\max}$ is the annual maximum wind pressure (Gumbel distributed with coefficient of variation = 0.25), see Sørensen et al. [23]. The stochastic model for $Q_{W,\max}$ is in accordance with the above stochastic process model for $P_W(t)$. $C$ is a shape factor (modeled as Gumbel distributed with expected value $\mu_C = 1$ and standard deviation $\sigma_C = 0.215$), see Sørensen et al. [23].

The reliability indices (short and long term) are estimated by simulation assuming that the structure considered has a height of 10 m and placed in terrain class II. Further, it is assumed that no dynamic effects influence the wind load. It is noted that the damage accumulation models and the fitted model parameters are not derived for highly variable loads as wind load.

### 3.3. Imposed (live) load

Imposed (live) load is modelled in accordance with the JCSS load model, JCSS [16] and CIB W81 [3]. The model is based on sustained loads and intermittent loads, see Tables 4 and 5. The sustained loads cover ordinary imposed load such as furniture, average utilization by persons, etc. The intermittent loads describe the exceptional load peaks, e.g. furniture assembly while remodeling, people gathering for special occasions, etc.

The following assumptions are made:

- The sustained load changes at times $X_1, X_2, \ldots$ are modelled by a Poisson process. The duration between changes is exponential distributed with expected value $\lambda_{sus}$.

Table 4
Parameters for imposed (live) load, see [16]

| | Sustained load | | | | |
|---|---|---|---|---|---|
| | $A_0$ [m$^2$] | $\mu_{sus}$ [kN/m$^2$] | $\sigma_v$ [kN/m$^2$] | $\sigma_{u,sus}$ [kN/m$^2$] | $\lambda_{sus}$ [year] |
| Office | 2 | 0.5 | 0.3 | 0.6 | 5 |
| Residence | 2 | 0.3 | 0.15 | 0.3 | 7 |
| | Intermittent load | | | | |
| | $\mu_{int}$ [kN/m$^2$] | $\sigma_{u,int}$ [kN/m$^2$] | $\lambda_{int}$ [year] | $t_{int}$ [days] | |
| Office | 0.2 | 0.4 | 0.3 | 1–3 | |
| Residence | 0.3 | 0.4 | 1.0 | 1–3 | |

$A = 5$ m$^2$ and $\kappa = 1.778$.

Table 5
Parameters for imposed (live) load with 50 year max (Unit: [kN/m$^2$])

| | Expected value | Standard deviation | COV | 98% quantile |
|---|---|---|---|---|
| Office | 3.05 | 0.88 | 0.29 | 2.97 |
| Residence | 2.71 | 0.95 | 0.35 | 2.10 |

- The magnitude of the sustained load $P_{sus}$ is assumed Gamma distributed with expected value $\mu_{sus}$ and standard deviation $\sigma_{sus} = \sqrt{\sigma_v^2 + \sigma_{u,sus}^2 \kappa A_0/A}$ with parameters defined in Table 4.
- The intermittent loads occurring at times $X_1, X_2, \ldots$ are also modelled by a Poisson process. The duration between the intermittent loads is thus exponential distributed with expected value $\lambda_{int}$.
- The magnitude of the intermittent loads $P_{int}$ is assumed Gamma distributed with expected value $\mu_{int}$ and standard deviation $\sigma_{int} = \sqrt{\sigma_{u,int}^2 \kappa A_0/A}$ with parameters defined in Table 4. The time length of the intermittent loads is $t_{int}$ (Table 5).

## 4. Calibration of load duration factor

Reliability based calibration of load duration factors are performed using probabilistic models of loads and short- and long-term strength. One realization during the design life time (50 years) is simulated as follows (snow and wind load):

1. Simulate a realization of the model uncertainty $C$ (incl. uncertainty on shape factors).
2. Simulate durations between load occasions.
3. Simulate magnitudes of maximum load $P_m$ in load occasion.

4. Simulate duration $T$ of load occasion.
5. Calculate the time history of the load occasions as $Q(t) = C \cdot P(t)$
6. Apply the load to a damage accumulation law.
7. Calculate: $T_{F,S}$ = the first time when the load is larger than the short term strength; $T_{F,L}$ = the first time when the accumulated damage is larger than 1; $T_{F,R}$ = the first time when the load is larger than the damage reduced short term strength.

In the following a full probabilistic approach is described where all uncertainties related to strength, model and loads are included in a way consistent with the background for the partial safety factors in the Danish structural codes, see Sørensen et al. [23]. The stochastic model is shown in Table 6. The coefficient of variation $V_Z$ for the model uncertainty of the damage accumulation model is subjectively assessed in the present investigation.

The following *short-term limit state equation* is used:

$$g = zf_0X_R - ((1-\kappa)G + \kappa Q), \tag{16}$$

where $z$ is the design parameter; $\kappa$, the coefficient, $0 \leqslant \kappa \leqslant 1$; $f_0$, the short term strength; $X_R$, the model uncertainty for short term strength; $G$, the permanent load; $Q$, the variable load.

The corresponding design equation is:

$$\frac{zf_k}{\gamma_m} - \left((1-\kappa)\gamma_G G_k + \kappa\gamma_Q Q_k\right) = 0, \tag{17}$$

where the characteristic values and partial safety factors in Danish codes DS409 [6] and DS413 [5] are: $f_k$ is the characteristic value for short term strength (5% quantile); $G_k$, the characteristic value for permanent load (mean value); $Q_k$, the characteristic value for variable load (98% quantile in annual maximum distribution); $\gamma_m$, the partial safety factor for material parameter (=1.5/1.64 if coefficient of variation = 0.15/0.20); $\gamma_G$, the partial safety factor for permanent load (=1.0); $\gamma_Q$, the partial safety factor for variable load (=1.5).

The design variable $z$ is determined from (17) and next, the reliability index $\beta$ is calculated on the basis of (16) and the stochastic model in Table 6, see also Sørensen and Hoffmeyer [24] It is noted that in the Danish codes the reference one-year reliability index is $\beta = 4.8$ and $V_Q = 0.2$ and 0.4 for imposed and environmental load, respectively.

Table 6
Stochastic model

| | Distribution | Expected value | Coefficient of variation | Comment |
|---|---|---|---|---|
| $f_0$ | Lognormal | 1 | $V_R = 0.15/V_R = 0.20$ | Short term strength |
| $X_R$ | Lognormal | 1 | 0.05 | Model uncertainty on short term load bearing capacity |
| $G$ | Normal | 1 | 0.1 | Permanent load |
| $Q$ | Gumbel | 1 | $V_Q = 0.2/0.4$ | Variable load |
| $Z_R$ | Lognormal | 1 | $V_Z = 0.0/0.1$ | Model uncertainty on damage accumulation model |
| $A$ | Normal | Table 1,2 or 3 | Table 1,2 or 3 | Regression parameter |
| $B$ | Normal | Table 1,2 or 3 | Table 1,2 or 3 | Regression parameter |
| $C$ | Normal | Table 1,2 or 3 | Table 1,2 or 3 | Regression parameter |
| $\varepsilon$ | Normal | 0 | Table 1,2 or 3 | Lac-of-fit regression parameter |

The following *long-term limit state equation* is used:

$$g = 1 - Z_R \alpha(f_0, G, Q, A, B, C, \varepsilon, SR(z), \eta, \kappa, T_L), \tag{18}$$

where $Z_R$ is the model uncertainty for long term strength with mean 1 and coefficient of variation $V_Z$; $\alpha$, the damage function. Gives the accumulated damage after $T_L = 50$ years with a time varying variable load $Q = Q(t)$; $T_L$, the design life time ( = 50 years); $A$, $B$, $C$, $\varepsilon$, the parameters in damage accumulation model; SR, the stress ratio = $\frac{(1-\kappa)G+\kappa Q}{zR}$; $\eta$, the threshold value; $f_0$, the short term strength; $G$, the permanent load; $Q = Q(t)$, the variable load as function of time.

The design equation corresponding to the limit state function (18) is:

$$\frac{zf_k}{\gamma_m} k_{mod} - \left((1-\kappa)\gamma_G G_k + \kappa\gamma_Q Q_k\right) = 0, \tag{19}$$

where $k_{mod}$ is the load duration factor.

In order to take into account (in the short-term model) the decrease with time of the strength due to accumulated damage the following *alternative limit state equation* is used:

$$g = z\frac{f}{f_0} f_0 X_R - ((1-\kappa)G + \kappa Q), \tag{20}$$

where $f$ is the residual strength corresponding to the damage $\alpha$ at time $t$. $f/f_0$ is determined from (4), (8) or (13). A simple, conservative alternative is to use $(1 - \alpha f_0)$ instead of $f_0$ in (16):

$$g = z(1-\alpha) f_0 X_R - ((1-\kappa)G + \kappa Q), \tag{21}$$

where $\alpha$ is the damage function obtained from (1), (5) or (9). In (21) the strength is reduced linearly by the damage.

The limit state functions (20) and (21) are used with the design equation (19). The $k_{mod}$ factor is calibrated by the following steps:

1. Calculate the short term reliability index $\beta_{50}^S$ for a 50 year reference period using the limit state function (16) and the design equation (17). $\beta_{50}^S$ is calculated as function of $\gamma_m$ by simulation ($\gamma_G$ and $\gamma_Q$ are fixed).
2. Calculate the long term reliability index $\beta_{50}^L$ for a 50 year reference period using the limit state function (18) and the design equation (19) and $k_{mod} = 1$. $\beta_{50}^L$ is calculated as function of $\gamma_m$ by simulation ($\gamma_G$ and $\gamma_Q$ are fixed).
3. $k_{mod}$ is estimated from

$$k_{mod} = \frac{\gamma_m^S(\beta)}{\gamma_m^L(\beta)} \tag{22}$$

for a reasonable range of values of the reliability index $\beta$ corresponding to the 50 year reference period. $\gamma_m^S(\beta)$ is the short term partial safety factor as function of $\beta$ and $\gamma_m^L(\beta)$ is the long term partial safety factor as function of $\beta$.

In order to evaluate the time-variant reliability the following two supplementary reliability indices are determined: $\beta_{50}^{R1}$ based on the limit state function (20) and $\beta_{50}^{R2}$ based on the limit state function (21).

## 5. Results

For *snow load* Figs. 4 and 5 show the relative reliability indices $\beta(\gamma_m)/\beta_0$ as function of $\gamma_m$ for rectangular and triangular snow packages for the Gerhard and the Barrett and Foschi damage models. $V_R = 0.15$, $V_Z = 0$ and no statistical uncertainty is included. $\beta_0 = \beta(\gamma_m = 1.5)$ is the reliability index for the short term limit state with $\gamma_m = 1.5$. It is seen that:

- The long term reliability is smaller than the short term reliability.
- $\beta_{50}^{R1} \approx \beta_{50}^{L}$ indicating that damage reduced strength does not change the reliability compared to the long term reliability.
- The conservative model in (21) gives reliability indices $\beta_{50}^{R2}$ equal to $\beta_{50}^{L}$ for rectangular snow packages and slightly less than $\beta_{50}^{L}$ for triangular snow packages. This indicates that the long-term effects can be reasonable well be approximated by the limit state Eqs. (20) and (21).

The results also show that Gerhards and Barrett and Foschi's damage models give almost the same relative reliability levels. $k_{mod}$ factors are calculated using (22). Table 7 shows the calculated $k_{mod}$ factors. It is seen that:

- triangular snow packages give $k_{mod} = 0.80$–$0.87$ and rectangular snow packages give $k_{mod} = 0.75$.

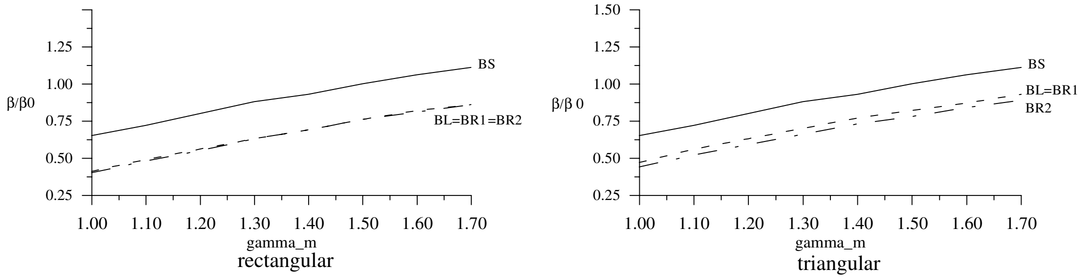


Fig. 4. Relative reliability index $\beta/\beta_0$ for snow load with $\beta_0 = \beta(\gamma_m = 1.5)$ as function of $\gamma_m$. BS $= \beta_{50}^{S}$; BL $= \beta_{50}^{L}$; BR1 $= \beta_{50}^{R1}$; BR2 $= \beta_{50}^{R2}$. Damage model: Gerhard.

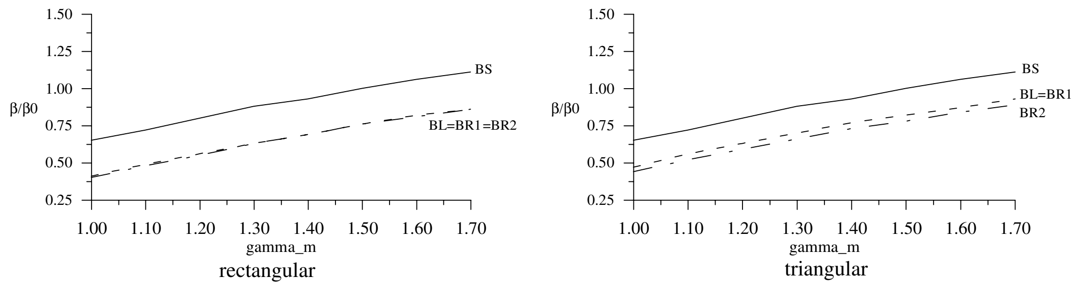


Fig. 5. Relative reliability index $\beta/\beta_0$ for snow load with $\beta_0 = \beta(\gamma_m = 1.5)$ as function of $\gamma_m$. BS $= \beta_{50}^{S}$; BL $= \beta_{50}^{L}$; BR1 $= \beta_{50}^{R1}$; BR2 $= \beta_{50}^{R2}$. Damage model: Barrett and Foschi.

Table 7
$k_{mod}$ factors for Danish snow load ($V_Z = 0.0$)

| | $V_R$ | Without statistical uncertainty | | With statistical uncertainty | |
|---|---|---|---|---|---|
| | | Rectangular | Triangular | Rectangular | Triangular |
| Gerhards | 0.20 | 0.75 | 0.81 | 0.75 | 0.79 |
| Barrett and Foschi | 0.20 | 0.75 | 0.81 | 0.75 | 0.79 |
| Foschi and Yao | 0.20 | 0.75 | 0.87 | 0.75 | 0.86 |

- statistical uncertainty is not important.
- the three damage models give almost the same $k_{mod}$ values.

It is noted that although different stochastic models were used, almost the same range of load reduction factors were found in the US/Canadian studies described in e.g. Ellingwood and Rosowsky [9] and Foschi [11], namely $k_{mod} = 0.74–0.83$.

Next the effect of using a reduced duration of the snow packages is investigated. Table 8 shows the calculated $k_{mod}$ factors for different expected durations of a snow package. Rectangular snow packages, no statistical uncertainty, $V_Z = 0.0$, Gerhards damage model and $V_R = 0.15$ are used. It is seen that the duration should be decreased to less than 25% before a small effect is observed.

For *wind load* Fig. 6 shows the relative reliability indices $\beta(\gamma_m)/\beta_0$ as function of $\gamma_m$ for the Gerhards and the Barrett and Foschi damage models. $V_R = 0.15$, $V_Z = 0$ and no statistical uncertainty

Table 8
$k_{mod}$ factors for different expected durations of snow packages

| $\mu_{X_T}$/75days/(kN/m$^2$) | $k_{mod}$ |
|---|---|
| 1 | 0.75 |
| 0.5 | 0.75 |
| 0.25 | 0.76 |
| 0.20 | 0.77 |
| 0.15 | 0.78 |
| 0.10 | 0.81 |
| 0.05 | 0.82 |

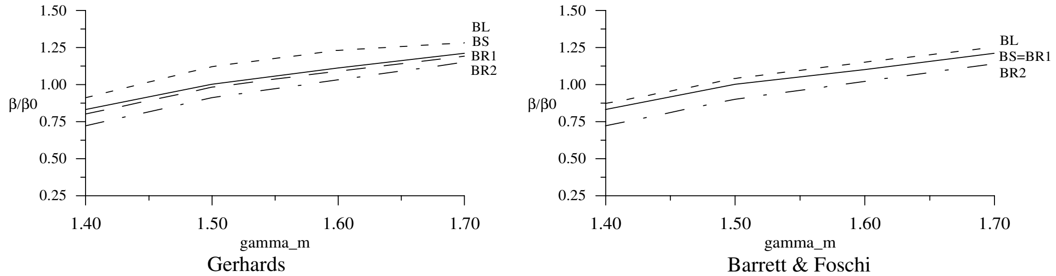


Fig. 6. Relative reliability index $\beta/\beta_0$ for wind load with $\beta_0 = \beta(\gamma_m = 1.5)$ as function of $\gamma_m$. BS $= \beta_{50}^{S}$; BL $= \beta_{50}^{L}$; BR1 $= \beta_{50}^{R1}$; BR2 $= \beta_{50}^{R2}$. Damage models: Gerhards and Barrett and Foschi.

is included. $\beta_0 = \beta(\gamma_m = 1.5)$ is the reliability index for the short term limit state with $\gamma_m = 1.5$. It is seen that:

- The long term reliability is larger than the short term reliability.
- For Gerhards damage model $\beta_{50}^{R1}$ is slightly smaller than $\beta_{50}^{S}$ and for Barrett and Foschi's damage model $\beta_{50}^{R1} \approx \beta_{50}^{S}$.
- The conservative model in (21) gives reliability indices $\beta_{50}^{R2}$ less than the reliability indices $\beta_{50}^{R1}$ based on the damage reduced strength model in (20).
- Gerhards and Barrett and Foschi's damage models give almost the same relative reliability levels.

The calculated $k_{mod}$ factors in Table 9 shows:

- $k_{mod}$ is approximately 1.05 (from 1.01 to 1.04),
- the Barrett and Foschi and Foschi and Yao damage models give slightly larger $k_{mod}$ factors than the Gerhard model,
- statistical uncertainty has some importance, especially for the Barrett and Foschi damage model.

The results show as expected that the load reduction factor $k_{mod}$ for wind load is larger than 1. This could be expected since the time scale of wind load effects is much smaller than the typical time for short term load tests (300 ± 120 s). Further, the wind load effects have a time-varying behaviour which is also very different from that used in the load duration tests (with time independent, constant load). This effect should be investigated more in future tests.

Next *combined wind and snow load* is considered, namely the load combination with dominating (extreme) snow load and companion wind load. The companion wind load is modeled on basis of instantaneous (daily) wind load. The following assumptions are made:

1. the magnitude of the instantaneous (daily) average 10 min velocity $V_{10}$ is assumed Weibull distributed.
2. the time history of the wind pressure $P(t)$ during each 10 min period is modeled using the wind spectrum in DS410 [7].

Based on recorded Danish wind data the instantaneous (daily) average 10 min velocity in 10 m height is modeled as Weibull distributed with expected value = 5.9 m/s and standard deviation = 3.3 m/s.

Table 9
$k_{mod}$ factors for Danish wind load ($V_Z = 0.0$ and $V_R = 0.20$)

| | Without statistical uncertainty | With statistical uncertainty |
|---|---|---|
| Gerhards | 1.03 | 1.01 |
| Barrett and Foschi | 1.04 | 1.02 |
| Foschi and Yao | 1.04 | 1.04 |

Table 10
$k_{mod}$ factors for imposed load

| | $V_R$ | Without statistical uncertainty | | With statistical uncertainty | |
|---|---|---|---|---|---|
| | | Office | Residence | Office | Residence |
| Gerhards | 0.15 | 0.79 | 0.81 | 0.79 | 0.81 |
| | 0.20 | 0.80 | 0.79 | 0.81 | 0.80 |
| Barrett and Foschi | 0.15 | 0.77 | 0.80 | 0.78 | 0.79 |
| | 0.20 | 0.76 | 0.80 | 0.77 | 0.80 |

The design equation is written:

$$\frac{zf_k}{\gamma_m} - \left((1-\kappa)\gamma_G G_k + \kappa\left[\gamma_Q Q_{S,k} + \psi Q_{W,k}\right]\right) = 0, \tag{23}$$

where $Q_{S,k}$ is the characteristic value for snow load (98% quantile in annual maximum distribution); $Q_{W,k}$, the characteristic value for wind load (98% quantile in annual maximum distribution); $\psi$, the load combination factor for wind load (=0.5).

Load duration factors $k_{mod}$ are calculated using (22) for Barrett and Foschi's damage model, triangular and rectangular snow packages, $V_R = 0.15$, $V_Z = 0$, $\kappa = 0$ and no statistical uncertainty. The result is

$$k_{mod} = 0.91\text{–}0.94 \text{ for constant snow package,}$$
$$k_{mod} = 0.94\text{–}0.97 \text{ for triangular snow package.}$$

It is seen that the load duration factor is larger than those for extreme snow load alone (0.75 for constant and 0.80 for triangular snow packages), but a little smaller than the load duration factor for wind alone (1.0). It seems to be a reasonable approximation to use the $k_{mod}$ value of the fastest varying load when snow and wind loads are combined.

For *imposed load* $k_{mod}$ factors are shown in Table 10. It is seen that:

- Gerhards and Foschi and Yao's damage model give $k_{mod} \approx 0.80$ whereas the Foschi damage model gives $k_{mod}$ = 0.75–0.80,
- $k_{mod}$ is almost the same for office and residence loads,
- statistical uncertainty is not important,
- the uncertainty of the short term timber strength is not important.

It is noted that although different stochastic models were used, almost the same range of load reduction factors were found in the US studies described in e.g. Ellingwood and Rosowsky [9], namely $k_{mod}$ = 0.76–0.84.

## 6. Conclusions

It is shown how the load duration effect can be determined on basis of simulation of realizations of the time varying load processes. Stochastic models are presented for wind, snow and imposed loads in accordance with the load models in the Danish structural codes.

Three damage accumulation models are considered, namely Gerhards model, Barrett and Foschi's model and Foschi and Yao's model. The parameters in these models are fitted by the Maximum Likelihood Method using data relevant for Danish structural timber and the statistical uncertainty is quantified.

The reliability is evaluated using representative short and long term limit states, and the load duration factor $k_{mod}$ is estimated using the probabilistic model such that equivalent reliability levels are obtained using short and long term design equations. The results are:

- Snow: $k_{mod} = 0.75$–$0.80$
- Wind: $k_{mod} = 1.00$–$1.05$
- Imposed: $k_{mod} = 0.80$

The load reduction factors for snow load and imposed (live) load are almost the same as those found in US/Canadian studies although different stochastic models were used. Further, the results show that inclusion of statistical uncertainty for the damage accumulation parameters has only minor influence on the load reduction factors.

Time variant reliability aspect is considered using a simple, representative limit state with time variant strength and simulation of the whole life time load processes. The results indicate that inclusion of the time-variant aspects is unimportant for snow and imposed load, but has some importance for wind load implying that $k_{mod} = 1.00$. The results are based on tests with constant load. However, the loads from the considered variable loads (snow, wind and imposed) have a time-varying behavior. In order to obtain more realistic load duration factors, tests should be performed with time-varying load corresponding to typical variations of the real loads. Especially for wind loads with fast changes in the load level an influence on the load duration factor can be expected.

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