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Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):

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**ZTZ-domain Immiscibility of the Opening and Closing Phases of the LF GFM under Frame Length Variations**

**Motivation and contribution**

Current research has proposed a non-parametric speech waveform representation (rep) based on zeros of the z-transform (ZTZ) [1]. Empirically, the ZTZ rep has successfully been applied in discriminating the glottal and vocal tract components in pitch-synchronously windowed speech by using the unit circle (UC) as discriminant [1]. Further, similarity between ZTZ reps of windowed speech, glottal flow waveforms, and waveforms of glottal opening and closing phases has been demonstrated [1]. Therefore, the underlying cause of the separation on either side of the UC can be analyzed via the individual ZTZ reps of the opening and closing phase waveforms; the waveforms are generated by the LF glottal flow model (GFM) [1]. The present study demonstrates this cause and effect analytically and thereby supplements the previous empirical works; moreover, it demonstrates that immiscibility is periodically variant under changes in frame lengths; lengths that maximize or minimize immiscibility are presented.

**LF glottal flow model (GFM)**

**Definition 1** LF glottal flow (derivative) model [2]

\[ e(t) = E_0 e^{-\mu t} \sin(\omega t), \quad 0 \leq t \leq t_e \]

\[ e(t) = E_0 \eta e^{-\mu t} \sin(\omega t - \theta_t), \quad t_e < t \leq t_e + T \]

\[ e(t) = 0, \quad t > t_e + T \]

Let \( e_0(t) \), \( e_1(t) \), and \( e_0(t) \) denote the opening, closing, and shut phase respectively. The discretized equivalents of \( e_0(t) \) and \( e_1(t) \) are \( e_0 = (e_0(n))_{n=0}^{N-1} \) and \( e_1 = (e_1(n))_{n=0}^{N-1} \) respectively.

**Theorem 1 Cauchy bound [3]**

All zeros of a complex polynomial,

\[ p(z) = z^n + \sum_{k=0}^{n-1} a_k z^k \]

Lie in the disk \( |z| < \lambda \) where \( \lambda = 1 + \max_{0 \leq \theta < 2\pi} |a_0| \).

**Theorem 2 Cauchy bounded annulus [4]**

Let \( p(z) \) be a polynomial with zeros \( z_1, ..., z_m \) ordered as \( 0 < |z_1| \leq ... \leq |z_m| \). Let \( \lambda_\alpha \) denote the CB of \( p(z) \) and \( \lambda_\beta \) the CB of \( z^m p(1/z) \). Then the following inequalities hold,

\[ \frac{1}{\lambda_\alpha} \leq |z| \leq \frac{2^{1/m} - 1}{1 - \lambda_\beta} \lambda_\alpha \]

and

\[ (2^{1/m} - 1)\lambda_\alpha \leq |z| \leq \lambda_\beta \]

Thm. 3 and 1 are equivalent, but thm. 3 yield a tighter bound in the present analysis.

**Theorem 3 Alternative Cauchy bound [5]**

All zeros of a \( n \)th degree complex polynomial,

\[ p(z) = z^n + \sum_{k=0}^{n-1} a_k z^k \]

Lie in the disk \( |z| < \lambda_\alpha \) where \( \lambda_\alpha = \max(1, \sum_{k=0}^{n-1} |a_k|) \).

Subscript \( a \) denotes alternative CB.

**ZTZ representation of \( e(t) \) (cf. def. 1)**

\[ z_m = r e^{\mu t}, \quad z_m \neq 0, e^{-\mu t}, \quad m \in \{1; N - 2\} \]

where \( p(x, z) = \sin(k(1 - z)N - \sin(k(N - 1)) \]

\[ k = \omega_0/\pi/\mu \]

**Lower Cauchy bound of the ZTZ rep.**

If \( \lambda_\alpha^{-1}(N) > 1 \) for the ZTZ rep, all zeros lie outside the UC (cf. th. 2). As \( e^{\mu t} \) is just a real scaling of the zeros of \( p(x, z) \), \( \lambda_\alpha^{-1}(N) \) of \( p(x, z) \) can be analysed in isolation heeding

\[ e^{\mu t}(h=1) > 1/(\lambda_\alpha^{-1}(N) \Rightarrow \alpha > \ln(\lambda_\alpha(N)) \]

**Sampling period \( h = 1 \) (cf. ZTZ rep. above).**

The global minima points of \( \lambda_\alpha^{-1}(N) \) are

\[ \lim_{N \to \infty} \left| \frac{\sin(k(1 - z)N)}{\sin(kN)} \right| = 1 \]

\[ \Rightarrow \lim_{N \to \infty} \lambda_\alpha^{-1}(N) = 0 \]

where \( a = \sqrt{k + qf}\) / \( k = 1 + qt, q \in \mathbb{Z} \).

The global maxima points of \( \lambda_\alpha^{-1}(N) \) are

\[ \left| \frac{\sin(k(1 - z)N)}{\sin(kN)} \right| \Rightarrow \lambda_\alpha^{-1}(N) = \frac{\left| \sin(k(1 - z)N) \right|}{\left| \sin(kN) \right|} \]

**Numerical experiment**

The LF GFM params. are set to common values,

\[ b_0 = 0.999086 \]

\[ b_1 = 0.003806 \]

\[ a_0 = 0.000318 \]

\[ a_1 = 0.008080 \]

\[ Ec = 1.00000 Pa \]

The global min point is reached at \( N < t_c \ll 1 \); thus, only the opening phase constraints on \( N \) must be considered when choosing a suitable sequence length.

**References**


