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ZTZ-domain Immiscibility of the Opening and Closing Phases of the LF GFM under Frame Length Variations

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Motivation and contribution
Current research has proposed a non-parametric speech waveform representation (rep) based on zeros of the z-transform (ZTZ) [1]. Empirically, the ZTZ rep has successfully been applied in discriminating the glottal and vocal tract components in pitch-synchronously windowed speech by using the unit circle (UC) as discriminant [1]. Further, similarity between ZTZ reps of windowed speech, glottal flow waveforms, and waveforms of glottal opening and closing phases has been demonstrated [1]. Therefore, the underlying cause of the separation on either side of the UC can be analyzed via the individual ZTZ reps of the opening and closing phase waveforms; the waveforms are generated by the LF glottal flow model (GFM) [1]. The present study demonstrates this cause and effect analytically and thereby supplements the previous empirical works; moreover, it demonstrates that immiscibility is periodically variant under changes in frame lengths; lengths that maximize or minimize immiscibility are presented.

LF glottal flow model (GFM)

Definition 1 LF glottal flow (derivative) model [2]
\[ e(t) = E_{e} e^{-\gamma t} \sin(\omega_{m} t), \quad t \leq t \leq t_{e}, \]
\[ e(t) = E_{e} e^{-\gamma t} (e^{-\gamma (t-t_{e})} - e^{-\gamma (t_{e}-t)}), \quad t_{e} < t < t_{c}, \]
\[ e(t) = 0, \quad t_{c} < t < T. \]
Let \( e(t) \), \( e(t) \), and \( e(t) \) denote the opening, closing, and shut phase respectively. The discretized equivalents of \( e(t) \), \( e(t) \), and \( e(t) \) are \( e^{(n)}_{a} = e^{(n)} \) and \( e^{(n)} = e^{(n)} \) respectively.

Cauchy bound (CB)
Let \( p(a, z) \) denote a univariate polynomial with variable \( z \) \in \mathbb{C} \) and coefficients \( a_{n} = a_{n-1} \in \mathbb{R} \).

Theorem 1 Cauchy bound [3]
All zeros of a complex polynomial,
\[ p(a, z) = z^{n} + a_{n-1} z^{n-1} + \ldots + a_{0} \]
lie in the disk \( |z| < \lambda \) where \( \lambda = 1 + \max_{0 \leq k < n} |a_{k}| \).

Theorem 2 Cauchy annular bound [4]
Let \( p(a, z) \) be a polynomial with zeros \( z_{1}, \ldots, z_{n} \) ordered as \( 0 < |z_{1}| \leq \ldots \leq |z_{n}| \). Let \( \lambda_{n} \) denote the CB of \( p(a, z) \) and \( \lambda_{n} \) the CB of \( z^{n} p(a, 1/z) \). Then the following inequalties hold,
\[ \frac{1}{\lambda_{n}} \leq |z| \leq \left( \frac{2^{1/n} - 1}{\lambda_{n}} \right) \] and
\[ (2^{1/n} - 1) \lambda_{n} \leq |z| \leq \lambda_{n} \]
Thm. 3 and 1 are equivalent, but thm. 3 yields a tighter bound in the present analysis.

Theorem 3 Alternative Cauchy bound [5]
All zeros of a \( n \)th degree complex polynomial,
\[ p(a, z) = z^{n} + a_{n-1} z^{n-1} + \ldots + a_{0} \]
lie in the disk \( |z| < \lambda_{n} \) where \( \lambda_{n} = \max_{0 < |a_{k}|} k \).
Subscript \( a \) denotes alternative CB.

Zeros of the z-transform (ZTZ)

Definition 2 Zeros of the z-transform
The zeros of the z-transform of a sequence \( x(n) \) \in \mathbb{C} \) \in \mathbb{R} \) are defined as \( z_{1}, z_{2}, \ldots, z_{n} \in \mathbb{C} \) \in \mathbb{C} \) \in \mathbb{R} \) such that \( x(n) = \sum_{n=0}^{\infty} x_{n} z^{-n} = 0 \) for \( 1 \leq n \leq m \).
The ZTZ-transformation is denoted as \( \rho : \mathbb{C} \rightarrow \mathbb{C} \),
\[ \rho(x(n)) \in \mathbb{C} \] where \( x \) is a polynomial coefficient sequence ordered in descending powers, \( z \) is a sequence of non-zero zeros, and \( k \) is the multiplicity of a zero at zero.

Analysis of opening phase

ZTZ representation of \( e(t) \) (cf. def. 1)
\[ z_{m} = e^{\gamma} p(x), \ z_{m} \neq 0, e^{\gamma} \neq 0, \ m \in [1; N - 2] \]
where
\[ p(x_{p}, z) = \sin(k) N \sin(k N) z + \sin(k (N - 1)) \]
\[ k = \omega_{m} \pi / f_{p} \]
Lower Cauchy bound of the ZTZ rep.
If \( \lambda_{n}^{-1} > 1 \) for the ZTZ rep., all zeros lie outside the UC (cf. th. 2). As \( e^{\gamma} \) is just a real scaling of the zeros of \( p(x_{p}, z) \), \( \lambda_{n}^{-1} \) (of \( p(x_{p}, z) \) can be analysed in isolation heeding
\[ e^{\gamma (1/n)} > (1/\lambda_{n}^{-1}) \Rightarrow \alpha > n \ln(\lambda_{n}) \]
Sampling period \( h = 1 \) (cf. ZTZ rep. above). The global minima points of \( \lambda_{n}^{-1} \) (of \( p(x_{p}, z) \) are
\[ \lambda_{n}^{-1} \]

Theorem 2 Alternative Cauchy bound (5)
All zeros of a \( n \)th degree complex polynomial,
\[ p(a, z) = z^{n} + a_{n-1} z^{n-1} + \ldots + a_{0} \]
lie in the disk \( |z| < \lambda_{n} \) where \( \lambda_{n} = \max_{0 < |a_{k}|} k \).
Subscript \( a \) denotes alternative CB.

Analysis of closing phase

ZTZ representation of \( e(t) \) (cf. def. 1)
\[ z_{m} = e^{\gamma} p(x), \ z_{m} \neq 0, e^{\gamma} \neq 0, \ m \in [1; N - 1] \]
where
\[ p(x_{p}, z) = (c_{1} - c_{2}) z^{N+1} + (c_{2} e^{-k} - c_{1}) z^{N} + \]
\[ (c_{2} e^{-k} - c_{1}) z^{N-1} + (c_{1} e^{-k} - c_{2} e^{-k}) \]
\[ c_{1} = e^{-k}, \ c_{2} = e^{-\left(k_{1} - k_{2}\right)}, \ k = \epsilon \]
Upper Cauchy bound of the ZTZ rep.
If \( \lambda_{n}^{-1} < 1 \) for the ZTZ rep., all zeros lie inside the UC (cf. th. 3). The global minimum value of \( \lambda_{n}(N) \) is
\[ \lambda_{n}(N) = 1 \]
which is achieved at
\[ N = -\ln(2) / (\omega_{m} - e^{-\left(k_{1} - k_{2}\right)}) / \epsilon \approx -\ln(\epsilon) / \epsilon \]
The global max point and value of \( \lambda_{n}(N) \) are
\[ \lambda_{n}(0) = e^{-\epsilon} + 1 + \frac{1 - e^{-\left(k_{1} - k_{2}\right)}}{1 - e^{-\epsilon}} \approx 2 \]
Numerical experiment
The LF GFM params. are set as common values,
\[ b_{0} = 0.00000000000000000000000000000000 \]
\[ b_{1} = 0.00000000000000000000000000000000 \]
\[ c_{0} = 0.00000000000000000000000000000000 \]
\[ c_{1} = 0.00000000000000000000000000000000 \]
\[ \epsilon = 1.000000 \]
Back line: Lower Cauchy bound. Vert. dotted line: A global min point. Vert. and hortz. dotted lines: A global max point and the max value respectively. Grey line lower bound of \( \epsilon \). Grey region exemplifies a feasible neighbourhood for \( N \). When \( \lambda_{n}^{-1}(N) = 0 \Rightarrow \alpha = -\infty \) why \( N \) must be chosen outside a neighbourhood of the global min points.
The illustrated feasible neighbourhood for \( N \)
\[ \lambda_{n}^{-1}(N) > \left( \frac{\pi}{\omega_{m} - e^{-\left(k_{1} - k_{2}\right)}} \right) \]

The global min point is reached at \( N < t_{e} << 1 \); thus, only the opening phase constraints on \( N \) must be considered when choosing a suitable sequence length.

References