ZZT-domain Immiscibility of the Opening and Closing Phases of the LF GFM under Frame Length Variations

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ZZT-domain Immiscibility of the Opening and Closing Phases of the LF GFM under Frame Length Variations

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Motivation and contribution

Current research has proposed a non-parametric speech waveform representation (rep) based on zeros of the z-transform (ZZT) [1]. Empirically, the ZZT rep has successfully been applied in discriminating the glottal and vocal tract components in pitch-synchronously windowed speech by using the unit circle (UC) as discriminant [1]. Further, similarity between ZZT reps of windowed speech, glottal flow waveforms, and waveforms of glottal flow opening and closing phases has been demonstrated [1]. Therefore, the underlying cause of the separation on either side of the UC can be analyzed via the individual ZZT reps of the opening and closing phase waveforms; the waveforms are generated by the LF glottal flow model (GFM) [1]. The present study demonstrates this cause and effect analytically and thereby supplements the previous empirical works; moreover, it demonstrates that immiscibility is periodically variant under changes in frame lengths; lengths that maximize or minimize immiscibility are presented.

LF glottal flow model (GFM)

Definition 1 *LF glottal flow (derivative) model [2]*

$$\begin{aligned} e_o(t) &= E_0 e^{\alpha t} sin(\omega_g t), & t_0 \le t \le t_e \\ e_c(t) &= -\frac{E_e}{\epsilon t_a}\left(e^{-\epsilon(t-t_e)} - e^{-\epsilon(t_c-t_e)}\right), & t_e < t \le t_c \\ e_s(t) &= 0, & t_c < t \le T \end{aligned}$$

Let $e_o(t)$, $e_c(t)$ and $e_s(t)$ denote the opening, closing and shut phase respectively. The discretized equivalents of $e_o(t)$ and $e_c(t)$ are $eo = (eo_n)_{n=0}^{N-1}$ and $ec = (ec_n)_{n=0}^{N-1}$ respectively.

Zeros of the z-transform (ZZT)

Definition 2 *Zeros of the z-transform*
The zeros of the z-transform of a sequence $(x_n)_{n=0}^{N-1} \subset \mathbb{R}$ are defined as $z_1, z_2, ..., z_m \in \mathbb{C} \setminus \{0\}$ such that $X(z_i) = \sum_{n=0}^{N-1} x_n z_i^{-n} = 0$ for $1 \le i \le m$.

The ZZT-transformation is denoted $\rho : \mathbb{R} \mapsto \mathbb{C}$, $\rho((x_n)_{n=1}^{N}) = (z_m)_{m=1}^{N-1-k}$, where x is a polynomial coefficient sequence ordered in descending powers, z is a sequence of non-zero zeros, and k is the multiplicity of a zero at zero.

References

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Cauchy bound (CB)

Let $p(a, z)$ denote a univariate polynomial with variable $z \in \mathbb{C}$ and coefficients $(a_n)_{i=0}^{N-1} \subset \mathbb{R}$.

Theorem 1 *Cauchy bound [3]*
All zeros of a complex polynomial,

$$p(a, z) = z^n + \sum_{k=0}^{n-1} a_k z^k$$

lie in the disk $|z| < \lambda$ where $\lambda = 1 + \max_{0 \le k \le n-1} \{|a_k|\}$

Theorem 2 *Cauchy bounded annulus [4]*
Let $p(a, z)$ be a polynomial with zeros $z_1, ..., z_m$ ordered as $0 < |z_1| \le ... \le |z_m|$. Let λ^* denote the CB of $p(a, z)$ and λ_* the CB of $z^m p(a, 1/z)$. Then the following inequalities hold,

$$\frac{1}{\lambda_*} \le |z_1| \le \frac{1}{(2^{1/m} - 1)\lambda_*} \quad and$$
$$(2^{1/m} - 1)\lambda^* \le |z_m| \le \lambda^*$$

Thm. 3 and 1 are equivalent, but thm. 3 yield a tighter bound in the present analysis.

Theorem 3 *Alternative Cauchy bound [5]*
All zeros of a n'th degree complex polynomial,

$$p(a, z) = z^n + \sum_{k=0}^{n-1} a_k z^k$$

lie in the disk $|z| \le \lambda_a$ where $\lambda_a = max\left\{1, \sum_{i=0}^{n-1} |a_i|\right\}$

Subscript a denotes *alternative* CB.

Analysis of opening phase

ZZT representation of eo (cf. def. 1)

$z_m = e^{\alpha} \rho(x_p)$, $z_m \ne 0, e^{\alpha \pm ik}$, $m \in [1; N-2]$

where

$p(x_p, z) = sin(k)z^N - sin(kN)z + sin(k(N-1))$,
$k = \omega_g = \pi/t_p$

Lower Cauchy bound of the ZZT rep.

If $\lambda_*^{-1}(N) > 1$ for the ZZT rep., all zeros lie outside the UC (cf. th. 2). As e^{α} is just a real scaling of the zeros of $p(x_p, z)$, $\lambda_*^{-1}(N)$ of $p(x_p, z)$ can be analysed in isolation heeding

$$e^{\alpha(h=1)} > (1/\lambda_*^{-1}(N) \Leftrightarrow \alpha > ln(\lambda_*(N))$$

Sampling period $h = 1$ (cf. ZZT rep. above).
The global minima points of $\lambda_*^{-1}(N)$ are

$$\left.\begin{aligned} \lim_{N \to a^{\pm}} \left|\frac{sin(k)}{sin(k(N-1))}\right| = \infty \\ \lim_{N \to a^{\pm}} \left|\frac{sin(kN)}{sin(k(N-1))}\right| = \infty \end{aligned}\right\} \Rightarrow \lim_{N \to a^{\pm}} \lambda_*^{-1}(N) = 0$$

where $a = (k + q\pi)/k = 1 + qt_p$, $q \in \mathbb{Z}$.
The global maxima points of $\lambda_*^{-1}(N)$ are

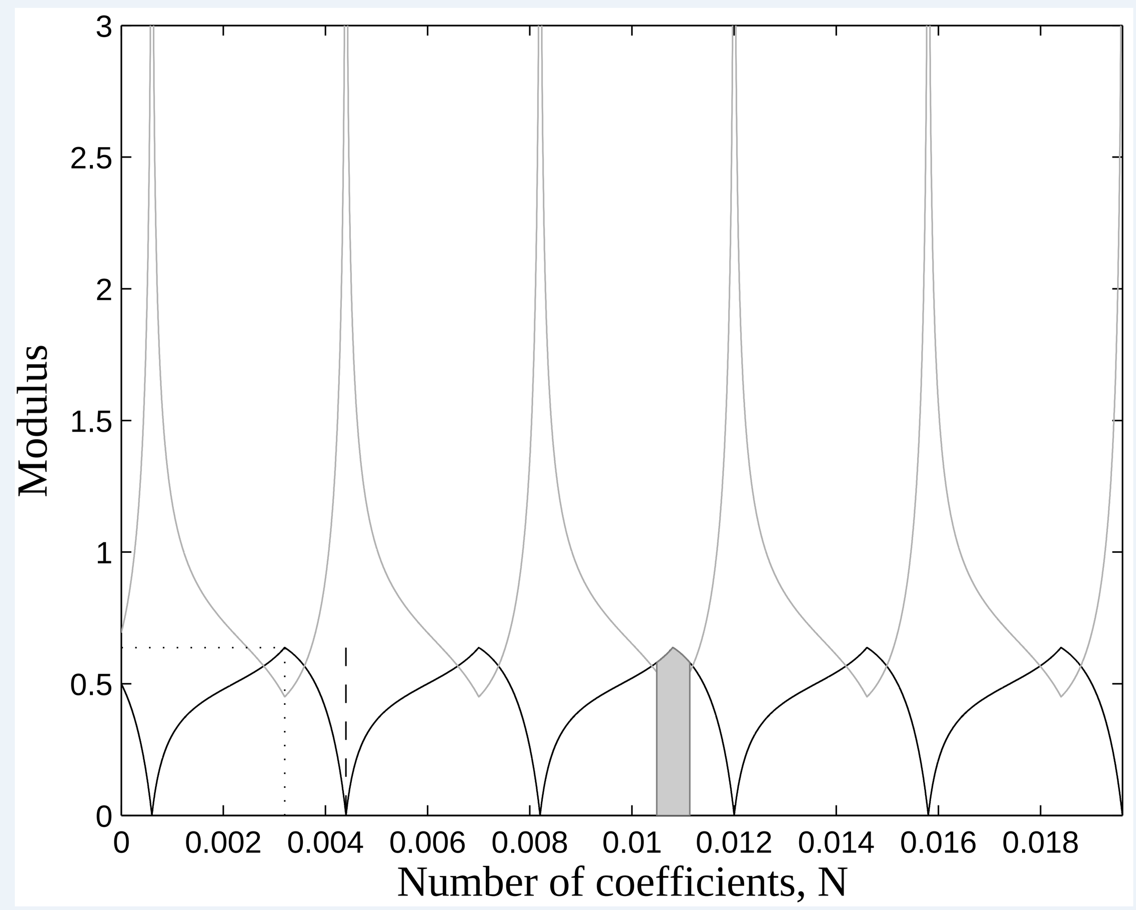
$$\left|\frac{sin(k)}{sin(k(N-1))}\right| = \left|\frac{sin(kN)}{sin(k(N-1))}\right| \Rightarrow \lambda_*^{-1}(N) = \frac{|2cos(\pi/t_p)|}{|2cos(\pi/t_p)|+1}$$

where $N = -1 + qt_p \ \vee \ N = t_p - 1 + qt_p$, $q \in \mathbb{Z}$

Numerical experiment

The LF GFM params. are set to common values,

$t_0 = 0.00000s$ $\quad t_p = 0.00380s$ $\quad t_e = 0.00480s$
$t_a = 0.00031s$ $\quad t_c = 0.00800s$ $\quad Ee = 1.00000Pa$



Black line: Lower Cauchy bound. Vert. dashed line: A global min point. Vert. and horiz. dotted lines: A global max point and the max value respectively. Grey line: Lower bound of α. Grey region exemplifies a feasible neighbourhood for N. When $\lambda_*^{-1}(N) \to 0 \Rightarrow \alpha \to \infty$ why N must be chosen outside a neighbourhood of the global min points.

The illustrated feasible neighbourhood for N,

$$\lambda_*^{-1}(N) > (e^{-(\alpha=543.6428)\cdot(h=0.001)} \approx 0.58062) \Rightarrow$$
$$N \in [-1 + qt_p - 82.68‰ t_p; -1 + qt_p + 86.80‰ t_p], \ q \in \mathbb{Z}$$

Analysis of closing phase

ZZT representation of ec (cf. def. 1)

$z_m = \rho(x_p)$, $z_m \ne 0, 1, e^{-k}$, $m \in [1; N-1]$

where

$$p(x_p, z) = (c_1 - c_2)z^{N+1} + (c_2 e^{-k} - c_1)z^N + (c_2 - c_1 e^{-kN})z + (c_1 e^{-kN} - c_2 e^{-k}),$$

$c_1 = e^{\epsilon t_e}$, $c_2 = e^{-\epsilon(t_c - t_e)}$, $k = \epsilon$

Upper Cauchy bound of the ZZT rep.

If $\lambda_a(N) < 1$ for the ZZT rep., all zeros lie inside the UC (cf. th. 3).
The global minimum value of $\lambda_a(N)$ is

$$\lambda_a(N) = 1$$

which is achieved at

$$N = -ln\left(\tfrac{1}{2} - \tfrac{1}{2}e^{-\epsilon} + e^{-\epsilon(t_c+1)}\right)/\epsilon \approx -ln(\tfrac{1}{2})/\epsilon$$

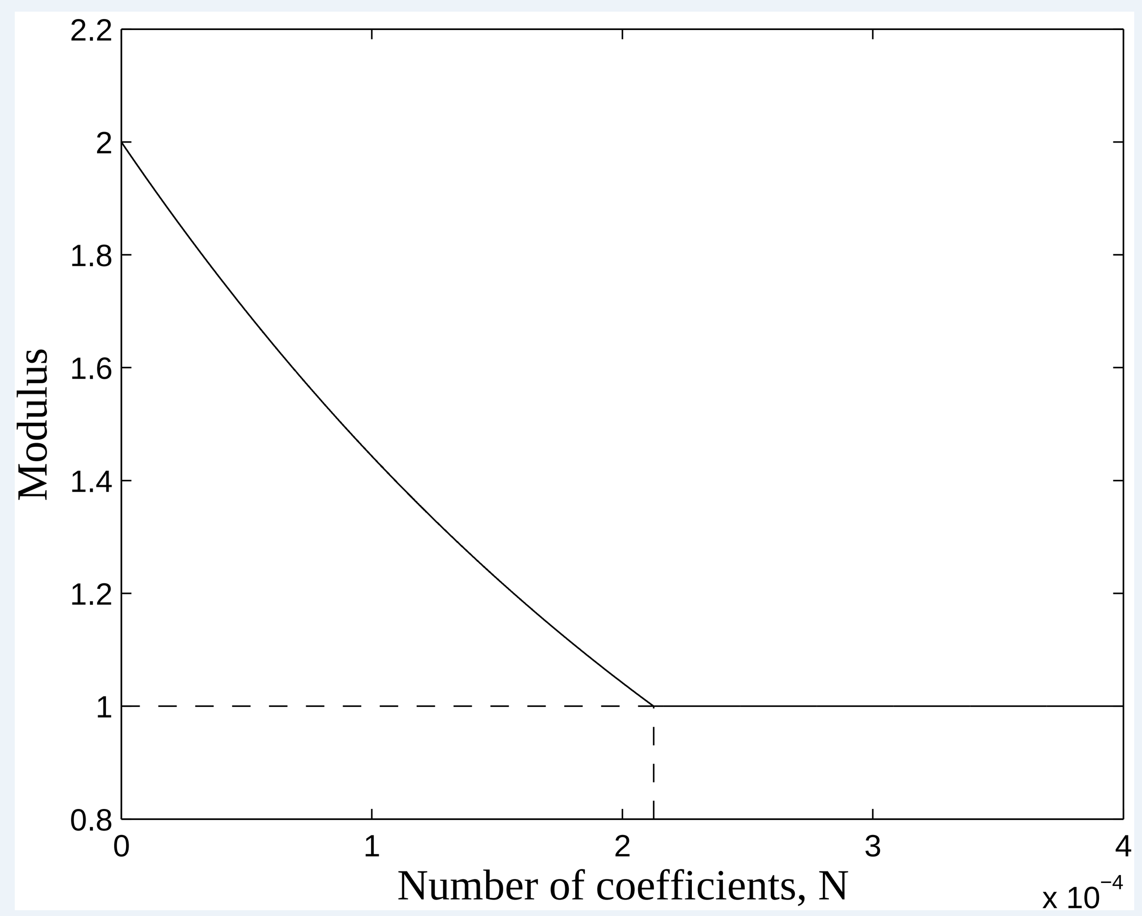
The global max point and value of $\lambda_a(N)$ are

$$\lambda_a(0) = e^{-\epsilon} + 1 + \frac{1 - e^{-\epsilon(t_c+1)}}{1 - e^{-\epsilon t_c}} \approx 2$$

Numerical experiment

The LF GFM params are the same as for the opening phase experiment. Further, ϵ is estimated iteratively by [2],

$$\epsilon t_a = 1 - e^{-\epsilon(t_c - t_e)} \quad \Leftrightarrow \quad \epsilon \approx 3261.44143$$



Solid line: Upper Cauchy bound of LF GFM closing phase. Vert. dashed line: A global min point. Horiz. dashed line: The global min value.

The global min point is reached at $N < t_c << 1$; thus, only the opening phase constraints on N must be considered when choosing a suitable sequence length.