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UNCERTAINTY OF ENERGY CONSUMPTION ASSESSMENT OF DOMESTIC BUILDINGS

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ABSTRACT

In order to assess the influence of energy reduction initiatives, to determine the expected annual cost, to calculate life cycle cost, emission impact, etc. it is crucial to be able to assess the energy consumption reasonably accurate.

The present work undertakes a theoretical and empirical study of the uncertainty of energy consumption assessment of domestic buildings. The calculated energy consumption of a number of almost identical domestic buildings in Denmark is compared with the measured energy consumption. Furthermore, the uncertainty is determined by means of stochastic modelling based on input distributions found by literature study, industry guidelines, measurements and – when necessary – simple assumptions.

A number of parameters are investigated and ranked in terms of importance to determine which ones contribute the most to the overall level of uncertainty. Measurements and simulations are found to correspond reasonably well; however, it is also found that significant differences may occur between calculated and measured energy consumption due to the spread and due to the fact that the result can only be determined with a certain probability. It is found that occupants' behaviour is the major contributor to the variance of the energy consumption.

INTRODUCTION

In order to assess the influence of energy reduction initiatives, to determine the expected annual cost, to calculate life cycle cost, emission impact, etc. it is crucial to assess the energy consumption reasonably accurate. As buildings account for a substantial part of the overall energy consumption, as the energy prices increase due to lack of resources, and climate change consideration forces governments to control energy consumption more tightly it is increasingly important to be able to determine the actual energy consumption.

However, several investigations reveal significant uncertainties in the estimation of energy consumption in buildings. Deviation between the calculated energy consumption and the actual energy consumption may exceed 100 % in extreme cases. This raises an important question as to the expected level of uncertainty in energy consumption assessment of buildings.

The paper presents an empirical and theoretical investigation of the uncertainty of energy consumption assessment of domestic buildings. The energy consumption of a number of almost identical domestic buildings in Denmark is calculated and compared with the corresponding measured energy consumption. The uncertainty is determined by stochastic modelling using input distributions found by literature study, industry guidelines, measurements and a number of simple assumptions.

The purpose of the work is to improve the understanding of uncertainty in building energy consumption calculations. As part of it the most important input parameters are identified and the total output variance is apportioned to the input parameters. Furthermore, the expected level of uncertainty is examined and expressed in terms of the coefficient of variation. The aim is to be able to improve the estimation of energy consumption and if possible reduce the uncertainty and for the remaining part, at the least, be able to quantify the uncertainty and, thus, to provide building owners and society at large with more detailed and accurate information.

This could also be seen as a step towards a practical application where a reduced set of stochastic input parameters facilitates wider application of Monte Carlo simulation.

In the following the building case is presented after which the measurements and the corresponding simulations (building model and Monte Carlo simulations) are presented. Finally, comparisons and conclusions are made.

METHOD AND RESULTS

Building description

The building case applied for the measurements and simulations comprises eight almost similar redbricked semi-detached houses located in the western part of Denmark, see Figures 1 - 4. The conditioned area is 149.2 m^2 with 0.35 m cavity walls (0.11 m bricks - 0.125 m thermal insulation – 0.11 m bricks). Room height is 2.37 m. The *U*-values for ceiling, floor and walls are 0.19, 0.36 and 0.32 W/(m^2K), respectively. The buildings are naturally ventilated and heated by means of district heating.

Figure 1 Air photograph of the area around the eight Danish residential buildings applied in the present work. The buildings are part of the built-up area shown inside the circle.

Figure 2 Photo of one of the eight residential buildings.

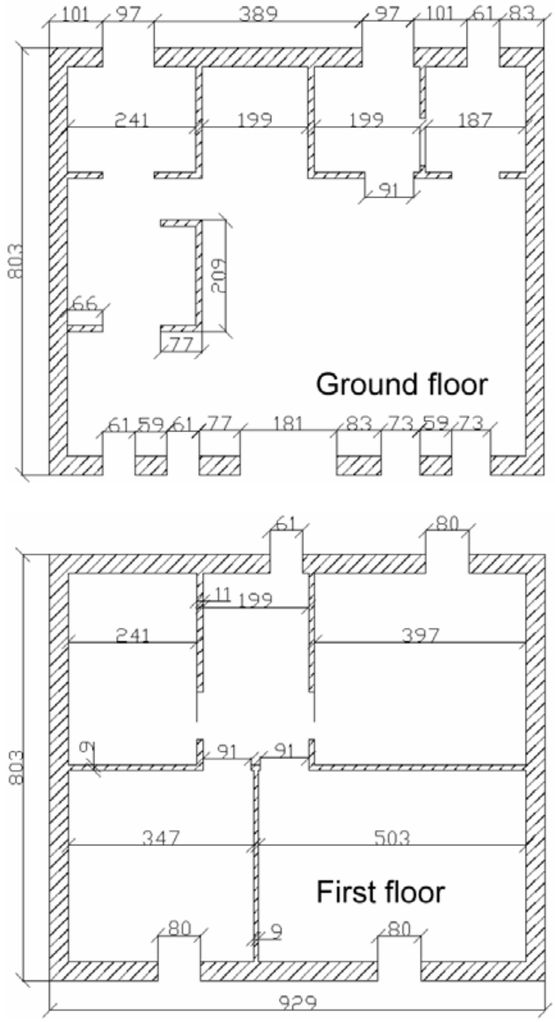

Figure 3 Horizontal plan of one of the building types. Measurements in cm.

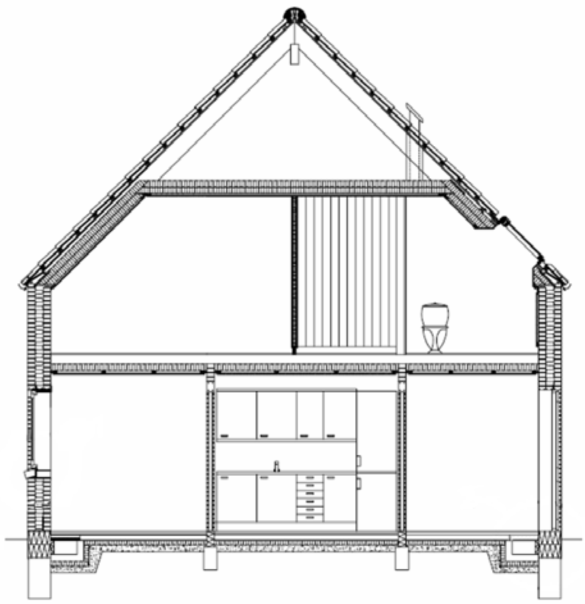

Figure 4 Sectional view of building.

Measurements

A number of detailed measurements are made on the buildings both to obtain detailed knowledge on the building constructions, occupants' behaviour and actual energy consumption, but also to be able to model the buildings properly.

Figure 5 Equipment applied for the blower door pressure test to assess the infiltration and leakage of the buildings.

The building leakage is determined by means of a standard blower door pressure test with pressure differences of 50 Pa, see Figure 5. The blower door test is combined with tracer gas measurements to supplement the knowledge of the overall leakage area with an investigation of the leakage distribution. The leakage information is used to determine the air infiltration and natural ventilation.

Internal temperature is measured to assess the occupants' preferences and determine the heating set-point. The temperature is measured at several locations as a function of time to obtain information on daily variation and general temperature swings. Time-dependent temperature measurement may also reveal information on window and door opening.

Weather data are collected by local external temperature measurements and also by collecting data from the nearest local meteo station: wind speed, wind direction, external air temperature, relative humidity, atmospheric pressure, cloud cover and global solar radiation.

A questionnaire is used to collect information on occupant number and age, occupied period, bathing habits, use of computers, TV, appliances, etc. This is combined with a registration of the use of district heating, water and electricity (readings of the meters in each house). Table 1 provides some statistics on selected main findings.

Table 1
Results from measurements (8 buildings). μ is the mean value and, σ is the standard deviation. Conditioned space 149.2 m^2. 2005 is reported to be a "typical year" according to the occupants.

Measurement	Unit	μ	σ
Infiltration (leakage)	l/(s·m^2) @ 50Pa	3.00	0.23
Internal air temperature set-point	°C	22.19	1.03
Internal heat load, appliances	W/m^2 (time average)	2.93	1.21
Internal heat load, occupants	W/m^2 (time average)	1.11	0.56
2005 total water consumption	m^3/year	125.3	41.2
2005 electricity consumption	kWh/(m^2·year) (kWh/year)	27.3 (4,079)	11.4 (1,697)
2005 district heating consumption	kWh/(m^2·year) (kWh/year)	73.0 (10,886)	16.6 (2,480)

Simulations

The building simulations including the determination of the energy consumption are made by means of the hygrothermal building simulation programme *BSim* (version 2007) which is developed by the Danish Building Research Institute (Wittchen et al., 2008).

The layout of the building model is shown in Figure 6.

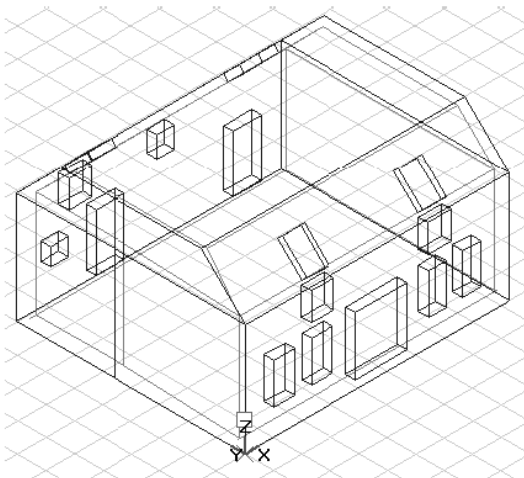

Figure 6 Building model applied for the hygrothermal building simulations using the programme BSim (Wittchen et al., 2008).

The present work on simulations comprises the determination of proper boundary conditions for the building simulation programme including input distributions for the stochastic parameters. To determine what parameters to treat stochastic and what to consider deterministic, an initial screening sensitivity analysis is performed. After the screening analysis a more elaborate quantitative sensitivity analysis is undertaken for the most important parameters to apportion the output variability on the input parameters.

To include a proper description of the natural ventilation and infiltration Computational Fluid Dynamics (CFD) and multizone modelling are applied. The CFD simulations (by means of the programme Flovent 6.1 using the k-ε turbulence model) are used to determine the wind pressure distribution around the buildings for different wind directions to get the C_p-values, see Figure 7. Due to the building location beside some plantation and bushes the corresponding permeability is included in the investigation. The CFD calculations reveal substantial local variation of the wind pressure and, thus, significantly varying driving forces as to air infiltration and natural ventilation.

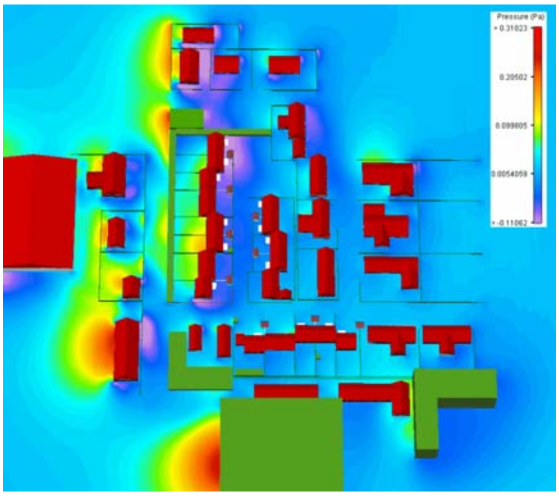

Figure 7 CFD simulation of wind pressure distribution around buildings (Flovent).

Multizone modelling (using the programme COMIS 3.1) applies the leakage information together with the pressure distribution, in shape of C_p-values, to determine the air change rate as a function of the wind speed, wind direction, external and internal densities.

Two overall classes of sensitivity analysis exist, namely local and global analysis. The typical local analysis may usually comprise variation of one variable at a time e.g. by computing partial derivatives or changing a parameter within certain limits all other things being equal. A global sensitivity analysis is characterised by evaluating individual factors varying all other factors as well. Idealistically, the sensitivity analysis should quantify and apportion the total uncertainty related to the model applied for the energy calculation. However, due to the very high number of potential important parameters this procedure is usually not possible in practise. Thus, a screening method is applied in stead to identify the parameter subset the controls most of the output variability including a ranking of the parameters. This could be seen either as a standalone investigation or as part of a more elaborate work where the most important parameters are identified at the initial stage for further investigation.

The screening method of Elementary Effects (Morris, 1991; Saltelli et al., 2000) is applied in this work. The method, which can be seen as an extension of a derivative-based screening method, can be characterised as a screening method with global characteristics. The method has been applied in several areas of building sciences e.g. natural night ventilation (Breesch and Janssens, 2004).and thermal building simulation (De Wit, 1997).

The method determines the so-called elementary effect EE of a model $y = y(x_1,\ldots,x_k)$ with input factors x_i. The Elementary Effect for the i^{th} input factor in a point $\mathbf{x}$ is

$$EE(x_1,\ldots,x_k) = \frac{y(x_1, x_2,\ldots,x_{i-1}, x_i + \Delta, x_{i+1},\ldots,x_k) - y(x_1,\ldots,x_k)}{\Delta} \qquad (1)$$

A number of elementary effects EE_i of each factor are calculated within the factor's range of variation. The method comprises a number of individually randomised one-factor-at-a-time simulations where all factors are varied within their input space in a way that spans the entire input space to form an approximate global sensitivity analysis (Morris, 1991; Saltelli et al., 2000).

The model sensitivity to each factor is evaluated by the mean value and the standard deviation of the elementary effects

$$\mu^* = \sum_{i=1}^{r} |EE_i| / r \qquad (2)$$

$$\sigma = \sqrt{\sum_{i=1}^{r} (EE_i - \mu)^2 / r} \qquad (3)$$

where μ^* is the mean value of the absolute values of the elementary effects determining if the factor is important, and σ is the standard deviation of the elementary effects which is a measure of the sum of all interactions of x_i with other factors and of all its nonlinear effects. r is the number of elementary effects investigated for each factor.

One of the most important activities in simulation work is the determination of input distributions. When proper distributions are found they can be reused as long as the underlying data does not change. Some input parameters may be mutually correlated which, in theory, requires that measures of mutual correlation should be established and applied in the simulations. In practise correlation is most often disregarded due to the difficulties of both finding and applying the correlations.

In the present screening analysis input distributions are established for 13 input parameters applied in the building simulation and energy calculations. The distributions are determined using a combination of measurements, questionnaires, literature, theoretical considerations, and also educated guesses depending on the accessibility of material in each case, see Table 4. Due to lack of information the input parameters are assumed to be independent, i.e. uncorrelated. This assumption is discussed later on.

The results are presented in Figure 8 where each input factor is shown as a function of μ^* and σ. The parameters are summarised in Table 4 and ranked according to importance.

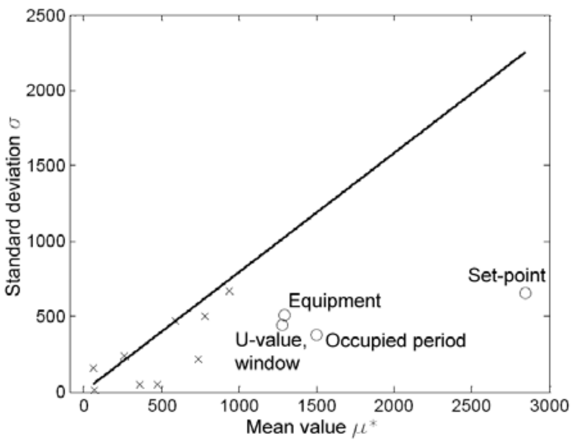

Figure 8 Screening sensitivity analysis performed using the method of elementary analysis based on 140 parameter combinations. The location of the four most important parameters are marked as circles the rest being shown as crosses, see Table 4.

Based on the results from the screening analysis a reduced number of stochastic input parameters are chosen for further investigation, see Table 2. In the screening analysis infiltration and natural ventilation are treated rather simplified, thus, even though they are not ranked as the highest they are included in the quantitative analysis using a somewhat more detailed model in *BSim*, see formula (4), that considers the influence of wind and temperature

$$n = n_0 + c_t \cdot (t_i - t_e)^{t_p} + c_v \cdot v \qquad (4)$$

where n is the air change rate [h^{-1}], n_0 is the basic air change rate [h^{-1}], c_t is the temperature factor [1/(h·K)], t_i is the internal temperature [°C], t_e is the external temperature [°C], t_p is the temperature exponent [-], c_v is the wind factor [s/(h·m)], and v is the wind speed [m/s] (Wittchen et al., 2008).

As to weather data the Danish DRY (Design Reference Year) is applied. Multizone modelling using input from the local meteo station from 2005 and 2006, respectively, are compared with models using the Danish DRY. It is found that the DRY provides a reasonable good description of the infiltration and natural ventilation in this case.

The quantitative sensitivity analysis is based on regression analysis. PEAR (Pearson product moment correlation coefficient) and SRC (Standardised Regression Coefficient) and their rank transformations SPEA (Spearman coefficient) and SRRC (Standardised Rank Regression Coefficient) are applied. Whereas PEAR is suited for linear models, SPEA is a good measure of correlation in case of non-linear models. Especially, the SPEA coefficient is assumed to work well in this case and taken as the most reliable quantitative measure of the sensitivity, i.e. how the output uncertainty is apportioned on the input parameters (Saltelli et al., 2000). Table 3 presents the results.

Table 2

Input distributions for the detailed sensitivity and the uncertainty analysis

Type N is a truncated normal distribution and E is a truncated exponential distribution.

Interval defines distribution boundaries of the 99% confidence interval, μ is mean value and σ is standard deviation (Normal distribution) and d is a positive displacement of the distribution (Exponential)

Input parameter		Distribution	
Name	Unit	Type	Interval; μ; σ or d
Set-point space heating	°C	N	20 - 25; 22.27; 0.75
Infiltration / Nat. vent. Basic air change rate, n_0 Temperature exponent, t_p Wind factor, c_v	 1/h - s/(h·m)	 N N N	 1.0 - 1.2; 1.1; 0.033 0.5 – 0.6; 0.55; 0.017 0.034 – 0.064; 0.058; 0.007
Occupied period	h/day	N	12 – 18; 14.9; 0.95
Occupant heat load	W/m^2 (Occupants)	E	0.68 – 6.00; 1.10; 0.68 (1.01 – 8.95; 1.64; 1.01)
Appliances heat load	W/m^2 (kW)	E	1.62 – 6.00; 0.70; 1.62 (0.24 – 0.90; 0.10; 0.24)

Table 3
Results from quantitative sensitivity analyses
SPEA is Spearman coefficient, PEAR is Pearson product moment correlation coefficient, SRC is Standardised Regression Coefficient and SRRC is Standardised Rank Regression Coefficient.

Input parameter	SPEA		PEAR		SRC		SRRC	
	%	Rank	%	Rank	%	Rank	%	Rank
Set-point space heating	36.9	1	38.0	1	34.7	1	41.1	1
Occupant heat load	22.6	2	22.8	2	25.5	2	22.1	2
Appliances heat load	15.7	3	20.7	3	15.6	4	19.3	3
Occupied period	5.9	5	6.8	4	5.6	5	7.7	4
Basic air change rate, n_0	14.6	4	4.7	5	16.0	3	6.2	5
Temperature exponent, t_p	2.4	6	3.0	7	0.3	7	0.8	7
Wind factor, c_v	2.0	7	3.9	6	2.4	6	2.9	6

In Figure 9 results from the uncertainty analysis are presented. The uncertainty is determined via the cumulative distribution function for the yearly district heating energy consumption found by means of Monte Carlo analysis and Latin Hypercube sampling. Latin Hypercube sampling is applied to ensure that the ensemble of random numbers is representative of the real variability (Saltelli et al., 2000).

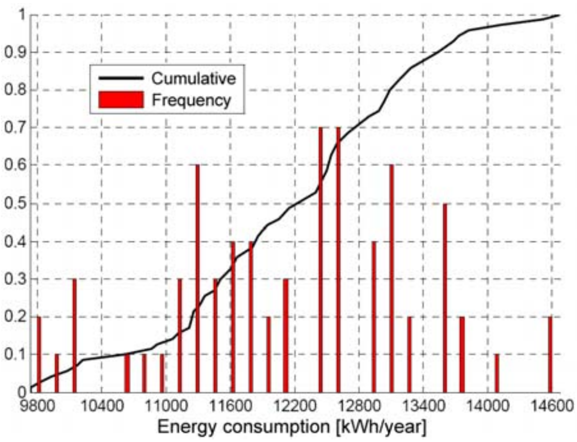

Figure 9 Frequency and cumulative distribution function for the uncertainty analysis of the heating energy consumption (district heating). The analysis is based on 70 realisations using Monte Carlo analysis and Latin Hypercube sampling.

For the 70 realisations the following statistics are found; mean value μ = 12,008 kWh/year, median x_m = 11,925 kWh/year, standard deviation σ = 1,294 kWh/year and, thus, the coefficient of variation δ = 10.8 %. Apart from those statistics it is possible to estimate the probability of a certain yearly heating energy consumption using the cumulative distribution function directly.

Due to the relatively limited number of realisations the cumulative distribution shown in Figure 9 is somewhat serrated. However, except from the lowest values the distribution seems to be normally distributed which is also supported by the low values of the skewness, 0.025, and the kurtosis, 2.269.

DISCUSSION AND RESULT ANALYSIS

The measurements in this work are used partly to collect information for the development of proper input distributions and partly to investigate the actual energy consumption. The input distributions are presented in Tables 2 and 4.

The measurements reveal a substantial variance of several parameters like the internal heat load related to appliances and occupants, see Table 1. The energy consumption coefficient of variance is approximately 0.2. It is found that the internal temperature corresponds well with the thermal comfort requirements in standards like EN 15251 (2007).

The simulations using a hygrothermal building simulation programme apply the building model shown in Figure 6. Several other building models have been tried with more and less detail, however, the present model is found to provide a reasonable estimation of the energy consumption compared with the setup and simulation time. A rather detailed model with internal space divided into rooms was found to be unnecessary difficult unless specific results from separate rooms are requested. In that case the model should be coupled with multizone modelling or CFD to ensure proper determination of interzonal airflow.

Ranking of input parameter is performed using sensitivity analysis. The purpose of the ranking is to be able to select the most important input parameters for further stochastic modelling assuming that the rest are deterministic. In that way a reduced set of stochastic parameters may be identified and facilitate practical application of Monte Carlo simulation and focus of resources when the knowledge of distributions is to be expanded.

The most important parameters are identified by screening and global sensitivity analysis to be

- set-point of heating, i.e. the occupants' preferred internal temperature
- internal heat load due to occupants and appliances
- occupied period
- *U*-values

- natural ventilation and air infiltration

Using regression coefficients, see Table 3, the variability of the energy consumption may be apportioned to the relevant input parameters. For instance, it is found that more than 1/3 of the variability is due to the set-point of space heating.

It is worth to note that the most important parameters are all related to occupants' behaviour. This strong influence of occupants is found in other investigations, too. Even the natural ventilation and infiltration, which is influenced by weather data to a great extent, is also influenced by occupants by window and door opening, airing habits, etc. In general the influence of occupant behaviour is not well understood and more research should be undertaken to investigate this topic to be able to understand and to predict building energy consumption. This point will be even more important when the permitted energy consumption is further reduced in future due to the fact that a relatively higher proportion of the consumed energy is related to user behaviour including hot water consumption.

The screening sensitivity analysis, using the method of Elementary Effects, reveals that none of the input parameters shows significantly non-linear or correlation behaviour in the building energy consumption model. This is concluded by comparing the mean values, μ^*, and the standard deviations, σ, of the analysis results in Figure 8 and Table 4.

Correlation of input parameters is ignored due to lack of information and, as a consequence, the input parameters are assumed statistically independent. The quality of this assumption may be questioned to some extent. Yet, in general it is felt that the assumption does not violate the overall conclusions. More research including measurements is needed to provide sound and detailed evaluation of the correlation issue.

Having both measurements and simulations it is possible to perform a comparison even though the number of buildings does not justify simplistic general conclusions. Measurement for 2005 may be taken as a typical year with mean value μ = 10,886 kWh/year, standard deviation σ = 2,480 kWh/year and, coefficient of variation δ = 22.8 % (Table 1). The corresponding simulations provide the mean value μ = 12,008 kWh/year, standard deviation σ = 1,294 kWh/year and, coefficient of variation δ = 10.8 % (Figure 9). Thus, the measurements' mean value is found within the distance of one standard deviation and vice versa for the simulations. It is noted that the standard deviation of the measurements ($\delta \sim 0.2$) is twice as high as for the simulations ($\delta \sim 0.1$). This may partly be due to the low number of samples for the measurements (8 buildings) and additional sources of variance not reflected in the simulation approach like model uncertainties.

CONCLUSIONS

The present work undertakes a theoretical and empirical study of the uncertainty of energy consumption assessment of domestic buildings. The calculated energy consumption of a number of almost identical domestic buildings in Denmark is compared with the measured energy consumption. Furthermore, the uncertainty is determined by means of stochastic modelling based on input distributions found by literature study, industry guidelines, measurements and – when necessary – simple assumptions.

The measurements reveal substantial variance of several parameters like the internal heat load related to appliances and occupants. It is found that the internal temperature corresponds well with usual thermal comfort requirements.

A number of parameters are investigated and ranked in terms of importance to determine which ones contribute the most to the overall level of uncertainty. Ranking of input parameter are performed using sensitivity analysis. The most important parameters are identified by screening and global sensitivity analysis to be

- set-point of heating, i.e. the occupants' preferred internal temperature
- internal heat load due to occupants and appliances
- occupied period
- *U*-values
- natural ventilation and air infiltration

It is worth to note that the most important parameters are all related to occupants' behaviour.

Measurements and simulations are found to correspond reasonably well; however, it is also found that significant differences may occur between calculated and measured energy consumption due to the spread and the fact that the result can only be determined with a certain probability (coefficient of variation of approximately 0.1 – 0.2). It is found that occupants' behaviour is the major contributor to the variance of the energy consumption.

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Table 4

Screening sensitivity analysis with input distributions ranked according to importance (yearly heating energy consumption) based on 140 samplings according to the method of Elementary Effects.

Type N is a truncated normal distribution, L is a truncated lognormal distribution, and D is a discrete distribution. Interval defines distribution boundaries, μ is mean value and σ is standard deviation

Input parameter		Distribution		Elementary Effects [kWh/year]		
Name	Unit	Type	Interval; μ; σ	Rank	μ^*	σ
Set-point space heating	°C	N	21 - 24; 22; 0.71	1	2850	656
Occupied period	h/day	D	10 – 18; see NOTE 1	2	1500	376
Appliances heat load	W	N	215 – 730; 437; 100	3	1290	510
U-value windows	W/m^2K	N	1.1 – 2.9; 2.4; 0.4	4	1280	439
U-value doors	W/m^2K	N	2.2 – 3.3; 2.9; 0.2	5	940	666
Thermal conductivity (re *U*-value of walls)	%	L	0 - 40; 2.5; 0.7	6	782	501
Occupant heat load	Occupants	N	0.522 – 2.861; 1.594; 0.5	7	741	218
Natural ventilation	m^3/s	N	0.0386 – 0.0433; 0.041; 0.001	8	587	473
Solar shading factor	-	N	0.5 – 1.0; 0.8; 0.1	9	472	49
Glass *g*-value	-	N	0.59 – 0.76; 0.67; 0.04	10	362	47
Building orientation	°	D	21 – 291; see NOTE 2	11	261	236
Infiltration	$l/(s\cdot m^2)$	N	0.20 – 0.24; 0.215; 0.008	12	71	9
Building heat capacity	Wh/m^2K	N	120 - 144; 132; 4	13	58	158
NOTE 1, data format (occupied period [h/day], relative frequency): (10,0.0526; 11,0.1053; 12,0.0526; 13,0.1579; 14,0.1053; 15,0.1053; 16,0.2105; 17,0.1579; 18,0.0526)						
NOTE 2, data format (building orientation [°], relative frequency): (21,1/3; 111,1/3; 291,1/3) [0° = North]						