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Time-Delay System Identification Using Genetic Algorithm - Part One: Precise FOPDT Model Estimation

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Abstract: Due to the unknown dead-time coefficient, the time-delay system identification turns to be a non-convex optimization problem. This paper investigates the identification of a simple time-delay system, named First-Order-Plus-Dead-Time (FOPDT), by using the Genetic Algorithm (GA) technique. The quality and performance of the GA-based identification are compared with those based on extended Least-Mean-Square (LMS) methods, subject to the consideration of different types of time-delay systems, excitation signals, Signal-to-Noise Ratios, and different evaluation criteria. The obtained results exhibit that the GA technique has a very promising capability in handling this type of non-convex system identification problem.

Keywords: Time-delay system, parameter identification, genetic algorithms, FOPDT

1. INTRODUCTION

The identification of time-delay system is always a challenging task, even for the simplest time-delay system, named First-Order-Plus-Dead-Time (FOPDT) system. Due to the unknown dead-time coefficient, this type of identification problem often turns to be a non-convex optimization problem (Bjorklund and Ljung (2003); Orlov *et al* (2003)).

Illustrative Example: We consider a continuous-time FOPDT system which is described by its transfer function $G(s) = \frac{3e^{-2s}}{s+1}$. In order to simplify the illustration, hereby we assume the system's time constant is precisely known beforehand, but the system's dead-time (denoted as T_d) and DC-gain (denoted as K) are unknown, subject to a condition that we have some pre-knowledge about the boundaries of these unknown parameters, e.g., $K \in [1, 4]$ and $T_d \in [0, 15]$. The considered system is excited by a pseudo white noise signal and both the system's input and response are measured with a reasonable sampling frequency and afterwards filtered by a common low-pass filter. Define a cost function for the parameter identification as a standard quadratic form as

$$C(T_d, K) \hat{=} \sum_{k=l_{max}}^{N} (y(k) - \hat{y}_{(T_d,K)}(k))^2,$$

where N indicates the number of samples and there is $N \gg l_{max}$, and $l_{max} \hat{=} \lfloor T_d/T_s \rfloor$ is an integer representing the largest potential delay steps w.r.t. the sampling period T_s. $y(k)$ is the kth sampled (filtered) response and $\hat{y}_{(T_d,K)}(k)$ is the estimated kth sampled response based on the filtered input signal. The cost function $C(T_d, K)$ subject to the considered parameter boundaries is plotted in Figure 1.

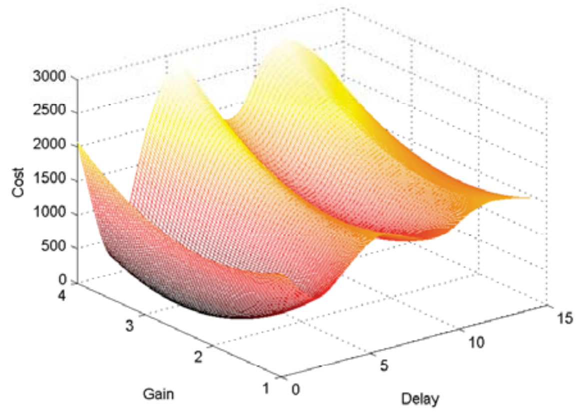

Fig. 1. Cost function surface w.r.t. different T_d and K

The non-convex problem due to the unknown dead-time coefficient can be clearly observed.

The most common and easiest way to estimate the signals' delay is to use the cross-correlation analysis (Bjorklund and Ljung (2003)). In order to estimated a time-delay system, some experimental approaches, such as using system's specific response curve, have been proposed and extensively used over decades (Åström and Hägglund (1995); Richard (2003)). In general, the identification quality and performance of these signal or experimental -based approaches heavily depend on the excitation feature, measured signals' quality and the Signal-to-Noise-Ratio (SNR) level (Ljung (1999)). From a model-based point of view, because the time-delay feature exhibits itself inside the independent time index of state/input/output variables, some specific mathematical operator (or excitation signal) is often needed so as to be able to bring this parameter explicitly out of the time index before any identification algorithm can proceed further. This specific mathematical operator can be realized through an integrator (Wang and Zhang (2001)) or a derivative filter (Ahmed *et al* (2006)) applied on both sides of the system's differential equation model. Correspondingly, some recursive LMS-based procedures to simultaneously estimated all unknown system coefficients (incl. the dead-time) have been proposed in (Ahmed *et al* (2006); Wang and Zhang (2001)). From a theoretical point of view, Orlov *et al* (2003) proposed some conditions to check the identifiability of linear time-delay systems. Moreover, an adaptive identifier is proposed for online identification purpose. However, none of above mentioned methods can get rid of the potential non-convex problem. Yang *et al* (1997) proposed a combined GA and RLS approach for online identification of linear time-delay systems. Each iteration of this proposed method consists of two sequential steps: The binary coded GA is used to estimated the system delay and afterwards the RLS method is employed to estimated the other system parameters. However, there is still no guarantee the solution can converge to the global optimum, even though we could claim that the emphasis of this work is mainly for online purpose. By employing a modified crossover operator within a real coded GA, Shin *et al* (2007) discussed the FOPDT and SOPDT model estimation based on system's step response using the GA technique. However, the proposed approach can not extend to handle other type of excitations, as well as there is no discussion about the algorithm's robustness.

This work commits an extensive investigation of the precise FOPDT system identification by using a real coded GA enhanced with a niching technique. The quality and performance of this GA estimation method are evaluated with respect to two different types of time-delay systems, five different excitation signals, three different SNRs, time-domain and frequency-domain fitness criteria. These results are also compared with exhaustive LMS-based methods. The observations exhibit that the GA technique has a very promising capability in handling this type of non-convex system identification problem. The rest of the paper is organized in the following: Section 2 formulates the considered system identification problem; Section 3 introduces the applied GA and its parameters; Section 4 illustrates and discusses different testing scenarios and results; and we conclude the paper in Section 5.

2. PROBLEM FORMULATION

Consider a FOPDT system, which transfer function model is expressed as

$$G_f(s) \hat{=} \frac{K}{\tau s+1} e^{-T_d s}, \tag{1}$$

where K is the system's DC-gain, τ is the system's time constant and T_d is the dead-time coefficient. The FOPDT system identification problem is defined as to precisely determine the system parameters K, τ, T_d of the model (1) based on the sampled system's input and output sequences, denoted as $\{y(k)\}_{k=0}^{N}$ and $\{u(k)\}_{k=0}^{N}$, respectively.

2.1 Discretization

The considered FOPDT model (1) can be converted into its equivalent discrete-time version through the zero-order-hold principle after a proper sampling period T_s is selected, i.e.,

$$H_f(z) \hat{=} \frac{\beta}{z-\alpha} z^{-l}, \tag{2}$$

where $\beta \hat{=} K(1-\alpha)$ and $\alpha \hat{=} e^{-\frac{T_s}{\tau}}$. Integer l is the best approximation of T_d subject to the predefined T_s, i.e., there is $(l-1)T_s \le T_d < lT_s$. From (2), a discrete prediction model can be naturally obtained as:

$$\hat{y}(k) = \alpha \hat{y}(k-1) + \beta u(k-l-1). \tag{3}$$

2.2 Constraint Optimization Problem

The equation (3) can be used to estimated the system output sequence based on the measured system input and previous output signals. We define a quadratic-formed cost function [1] as:

$$C(\alpha,\beta,l) \hat{=} \frac{1}{N} \sum_{k=l_{max}+1}^{N} (y(k) - \hat{y}_{\alpha,\beta,l}(y(k-1), u(k-l-1)))^2 \tag{4}$$

where $\hat{y}_{\alpha,\beta,l}(y(k-1), u(k-l-1))$ is the predicted system output at kth step, based on the measurements $y(k-1)$ and $u(k-l-1)$ for $l_{max}+1 \le k \le N$ according to (3).

The considered (discrete) system identification problem can be formulated into a constraint optimization problem, i.e.,

$$\min_{(\alpha,\beta,l)\in\Theta} E\{C(\alpha,\beta,l)\}, \tag{5}$$

where $E\{.\}$ represents the expectation operator, and Θ represents the admissible set of the unknown parameters. Once the problem (5) is solved, the system parameters of the original system (1) can be derived from the solution of (5), where the precision of the dead-time estimation is pre-determined by the selected sampling frequency.

If the coefficient l is known, the problem (5) reduces to be a standard Prediction Error (PE) formulation, and there are a lot of methods available to solve this kind of problem (Ljung (1999)), e.g., the Recursive Least-Mean-Square (RLMS) based methods can provide an efficient solution to that. However, it has been observed that this type of optimization problem (5) is non-convex subject to the unknown parameter l. Thereby, the precise identification needs to be carefully handled. In the following, the GA-based method is investigated to cope with this kind of non-convex problem.

3. CONSIDERED GA AND RELEVANT FORMULATIONS

Due to the purpose of this work is to check the GA's application in process system identification, instead of the investigation of any new/improved GA methods/algorithms,

[1] It should be noticed that the cost function is not necessary to be defined as a typical quadratic form if the GA is going to be applied. Here the quadratic form is used mainly due to the fact that we will conduct compatible comparisons between GA-based and LMS-based identification methods.

the standard real coded GA is selected and the quality of GA-based identification is compared with some LMS-based methods.

3.1 Real-coded GA and its parameters

The real coded GA is adopted here regarding to the fact that it is natural and efficient to deal with a continuous searching space (Deb and Agrawal (1995)). The unknown system parameters, except the dead-time coefficient, are real-valued encoded, and the dead-time coefficient is integer-valued encoded. The binary tournament selection is employed to choose which chromosomes to survive and mate, according to the pre-defined cost function (4). The selected chromosomes generate the offsprings according to the Simulated Binary Crossover (SBX) operator and the polynomial mutation (Deb (2000)) with relevant parameters listed in Table 1.

Table 1. Parameters in the used GA

Parameter	Value
Maximal generations	450
Population size	45
Tournament size	2
Crossover probability	0.95
Mutation probability	0.1
SBX distribution index	1
Mutation distribution index	1
Niching shape parameter	1
Number of Elitism	2
Number of real coded parameters	2
Number of integer coded parameters	2
RVCP lower boundary	0
RVCP upper boundary	10
IVCP (Dead time) lower boundary	0
IVCP (Dead time) upper boundary	120

According to the SBX approach (Deb (2000)), two offsprings o_1 and o_2 can be generated from parents p_1 and p_2 through

$$\begin{aligned} o_1(i) &= 0.5[(p_1(i) + p_2(i)) - \beta_{qi}|p_1(i) - p_2(i)|], \\ o_2(i) &= 0.5[(p_1(i) + p_2(i)) + \beta_{qi}|p_1(i) - p_2(i)|], \end{aligned} \quad (6)$$

where $o(i)/p(i)$ indicates the ith gene of offspring/parent, and β_{qi} is determined according to

$$\beta_{qi} = \begin{cases} (\alpha_g u_i)^{\frac{1}{\eta+1}}, & \text{if } u_i \leq \frac{1}{\alpha_g}, \\ (\frac{1}{2 - \alpha_g u_i})^{\frac{1}{\eta+1}}, & \text{otherwise} \end{cases} \quad (7)$$

where u_i is a uniformly distributed random number from the interval $[0, 1]$, η is the non-negative SBX distribution index. $\alpha_g = 2 - \beta_b^{-(\eta+1)}$ and β_b is obtained through $\beta_b = 1 + \frac{2}{(p_2 - p_1)}\lambda$, with $\lambda = min\{(p_1 - p_L), (p_U - p_2)\}$ subject to the assumption that $p_1 < p_2$, where p_L/p_U is the lower/upper boundary of the coded variable.

According to Deb (2000), the polynomial mutation generates an offspring from a parent through

$$o(i) = p(i) + \delta\Delta_m, \quad (8)$$

where Δ_m is the maximal perturbation allowed in the solutions, and δ is determined as

$$\delta = \begin{cases} (2u + (1 - 2u)(1 - \bar{\delta})^{\eta_m + 1})^{\frac{1}{\eta_m + 1}} - 1 & \text{if } u \leq 0.5 \\ 1 - (2(1 - u) + 2(u - 0.5)(1 - \bar{\delta})^{\eta_m + 1})^{\frac{1}{\eta_m + 1}} & \text{otherwise} \end{cases}$$

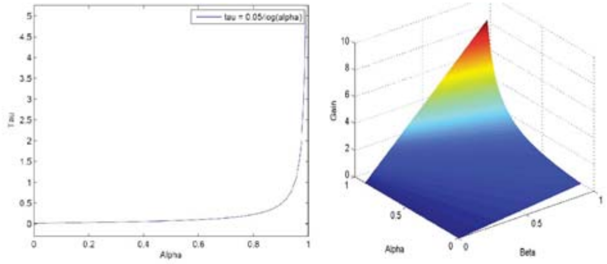

Fig. 2. Exponential relationships between τ and α, K and α, β with $T_s = 0.05sec$

where η_m is the non-negative mutation distribution index, and $\bar{\delta} = \frac{min\{(p - p_L), (p_U - p)\}}{p_U - p_L}$. u is a uniformly distributed random number from the interval $[0, 1]$.

The fitness sharing in Sareni and Krähenbühl (1998), as a selected Niching technique, is employed to maintain the population diversity. Furthermore, the Elitism (top 2 best) is also used in order to keep the extremal best population. For further more details about the adopted GA, we refer to Seested (2013).

3.2 Searching Spaces

The real coded GA can be employed to handle the optimization problem (2) to identify the discrete system's parameters firstly. However, some of our preliminary investigation discovered that often some poor results in the accuracy of the original (continuous-time) parameter identification are observed, especially with large time-constant systems. The later analysis turned out that this could be due to the exponential relationship between the original parameter τ and the discrete-time parameter α. For instance, the τ and α relationship is illustrated in Figure 2 under the assumption of $T_s = 0.05sec$. It can be observed that α only has the range of $(0, 1)$, and when α moves close to 1, a small deviations of α can have a huge impact on the deviation of τ. The similar relationship exists between K with α and β as well as shown in Figure 2. Thereby, in order to avoid the above mentioned problem, the searching spaces are defined directly based on the original (continuous-time) system parameters.

The evaluation of each selected chromosome consists of two sequential steps: Step one is to convert the original system (1) into its corresponding discrete version (2) subject to the selected parameter values; Step two is to calculate the cost function (4) under these specific parameters. Each chromosome representing the original system parameters is evaluated by this calculated cost. This approach could slightly slow down the computation speed, however, the payback is a much better accuracy in the original parameter identification, as well as faster convergence rate.

3.3 Exhaustive LMS-Based Methods

The optimization problem (5) is a mixed integer nonlinear programming problem with the non-convex feature. If the original dead-time T_d is known beforehand, the problem (5) can be easily solved by some standard linear system methods, e.g., by using the LMS method. Thereby, in order to evaluate the GA identification quality and performance, an Exhaustive LMS-based (ELMS) method is proposed in the following. The basic idea of the ELMS approach (Yang Sun (2011)) is to enumerate all possible dead-time values within its possible range, and for each possible valve, a LMS solution for (5) with a known l is achieved, along with the corresponding specific cost calculated from (4). The specific l together with its corresponding LMS solution which leads to the minimal cost among all possibilities, is claimed to be the best system identification solution.

In order to cope with any potential disturbance caused by colored noises, the Exhaustive Instrumental Variable LMS (E-IV-LMS) solution is also derived. In the following, sometimes we abbreviate the ELMS solution as *LS solution* and IV-ELMS solution as *IV solution*.

4. TESTING RESULTS AND DISCUSSIONS

The quality and performance of GA-based system identification are extensively studied in terms of different types of time-delay systems, different types of excitation signals, different Signal-to-Noise Ratio (SNR) levels, and these are further compared with those of ELMS-based methods.

4.1 Testing Scenarios

Two type of time-delay systems are selected as shown in (9), where $G_1(s)$ is a time-constant dominant system with the Dead-Time to Time-Constant Ratio (DTTCR) of one, while $G_2(s)$ is a time-delay dominant system with the DTTCR of 10.

$$G_1(s) = \frac{5}{s+1}e^{-s},\ G_2(s) = \frac{1}{0.5s+1}e^{-5s}. \tag{9}$$

The sampling frequency is selected as $20Hz$. Five different types of excitation signals are tested respectively, they are all generated from Matlab/Simulink signal blocks:

- (a) White noise;
- (b) A chirp signal, which uniformly sweeps its frequency from $0Hz$ to $5Hz$ during the simulation;
- (c) A Binary Random Sequence (BRS), which shifts the amplitudes between 3 and 6 with a shifting probability of 40%;
- (d) A repeating sequence stair signal generated from the default seed in Matlab/Simulink;
- (e) A pulse sequence generated from default pulse block.

Three SNR levels are also considered, i.e., SNR$=\infty$ (noise-free); SNR$=10$ (reasonable), and SNR$=2$ (poor).

4.2 Identification of $G_1(s)$ System

Different excitation signals under different SNR conditions are used to test the GA identification of $G_1(s)$ system. The evolution of the estimated parameters with SNR=∞ are illustrated in Figure 3, 4 and 5, respectively. It can be noticed that all excitation signals can lead to nearly perfect parameter identification. The BRS excited estimations have some tiny estimation errors w.r.t the τ (T_p) and K, but these errors are within 1.33% and 0.06%, respectively. The convergence speeds along with the GA evolutions for all excitations are also very fast - settling down after around 150-200 generations. From the first two columns in Table 2, it can be observed that the ELMS based methods (LV and IV) also commit perfect performances and accuracies. When the SNR decreases, the GA performance and accuracy start to decrease depending on what type of excitations. These estimation accuracies under different SNR levels are listed in Table 2.

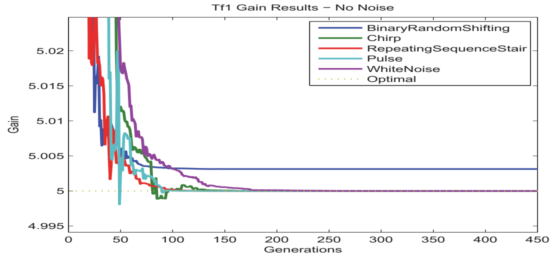

Fig. 3. Estimations of DC-Gain K of $G_1(s)$

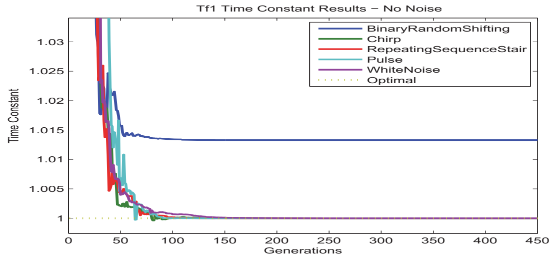

Fig. 4. Estimations of time constant τ of $G_1(s)$

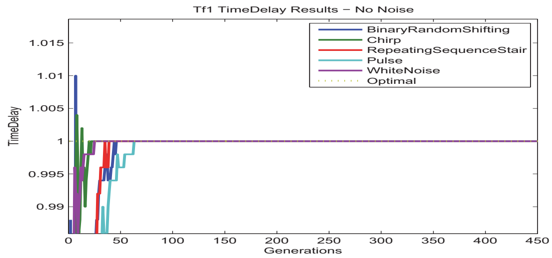

Fig. 5. Estimations of dead-time T_d (τ) of $G_1(s)$

Table 2. Estimation accuracies (%) regarding to $G_1(s)$, where $T_p = \tau$

G1(s) - SNR = ∞

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
b	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01
c	0.07	1.18	0.00	0.03	0.05	1.00	0.00	0.03	0.06	1.33	0.00	0.03
d	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
e	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

G1(s) - SNR = 10

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	0.69	0.71	0.00	5.98	1.49	2.44	0.00	5.97	3.05	4.44	0.00	5.97
b	6.15	3.59	0.00	0.10	6.23	3.83	0.00	0.10	5.76	3.34	0.00	0.10
c	0.00	5.00	5.00	11.52	0.08	4.21	5.00	11.52	0.55	12.31	0.00	11.50
d	0.13	3.51	5.00	2.84	0.20	2.81	5.00	2.84	0.43	10.55	0.00	2.73
e	1.25	6.64	5.00	0.39	0.39	1.38	5.00	0.40	1.74	13.21	5.00	0.39

G1(s) - SNR = 2

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	0.86	4.18	5.00	30.64	3.78	6.25	0.00	29.86	6.74	9.84	0.00	29.86
b	13.34	7.35	0.00	0.47	13.86	9.07	0.00	0.47	12.72	7.70	0.00	0.47
c	0.11	6.24	15.00	57.97	0.16	9.45	10.00	57.69	1.03	20.32	5.00	57.32
d	0.07	10.31	10.00	13.86	0.38	7.51	10.00	13.87	0.96	21.53	2.00	13.72
e	2.40	10.55	10.00	1.94	0.96	2.15	10.00	1.96	4.32	29.32	10.00	1.91

Table 3. Estimation accuracies (%) regarding to $G_2(s)$, where $T_p = \tau$

G2(s) - SNR = ∞

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c	0.02	1.65	0.00	0.00	0.03	1.76	0.00	0.00	0.06	3.65	0.00	0.00
d	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
e	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

G2(s) - SNR = 10

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	5.56	3.08	0.00	0.65	5.72	3.58	0.00	0.66	5.07	1.20	0.00	0.39
b	0.65	4.82	0.00	0.01	1.40	0.97	0.00	0.01	1.19	0.46	0.00	0.01
c	1.80	28.92	1.00	0.74	1.57	21.01	1.00	0.74	1.36	4.93	0.00	0.73
d	1.75	27.84	1.00	0.18	1.53	19.80	1.00	0.18	1.49	12.34	0.00	0.17
e	1.91	8.19	0.00	0.02	1.91	8.27	0.00	0.02	1.65	3.68	0.00	0.02

G2(s) - SNR = 2

	LS - Accuracy				IV - Accuracy				GA - Accuracy			
	K	Tp	Td	MSE	K	Tp	Td	MSE	K	Tp	Td	MSE
a	19.40	27.19	1.00	2.48	16.48	19.11	1.00	2.50	11.78	3.20	0.00	1.95
b	4.12	18.58	0.00	0.05	3.40	2.98	0.00	0.05	2.49	1.51	0.00	0.05
c	4.94	98.78	4.00	3.71	3.64	45.88	2.00	3.70	2.96	7.89	0.00	3.66
d	4.84	97.34	4.00	0.88	3.59	45.26	2.00	0.87	3.23	18.64	0.20	0.87
e	4.86	27.75	0.00	0.08	4.39	19.89	0.00	0.08	3.87	10.03	0.00	0.08

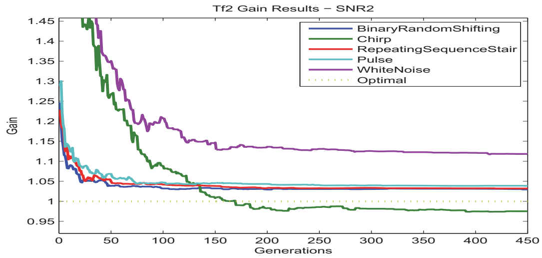

Fig. 6. Estimations of DC-Gain K of $G_2(s)$

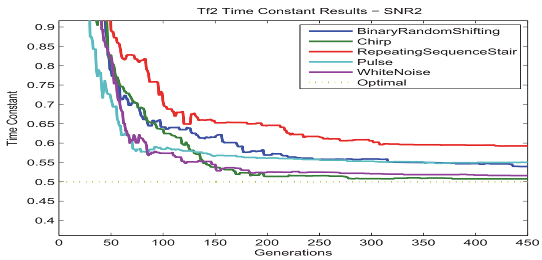

Fig. 7. Estimations of time constant τ of $G_2(s)$

4.3 Identification of $G_2(s)$ System

The accuracies of GA estimation of $G_2(s)$ system under different SNRs are listed as the third column in Table 3. Again, all excitation signals lead to perfect estimations under the noise free condition. When the SNR decreases, the GA estimation performance and accuracy start to decrease. In general, the GA estimations of $G_2(s)$ have better accuracies than those corresponding to $G_1(s)$ estimations when the SNR becomes poor (e.g., SNR$=2$). The GA performances under SNR$=2$ are illustrated in Figure 6, 7 and 8, respectively. It can be noticed that the chirp signal leads the best performance and accuracy. The repeating sequence stair and BRS lead to slowest convergence rates.

4.4 Comparison with ELMS Methods

The accuracies of the GA identification are compared with those generated from the ELMS method and E-IV-LMS method, as shown in Table 2 and 3 for different systems.

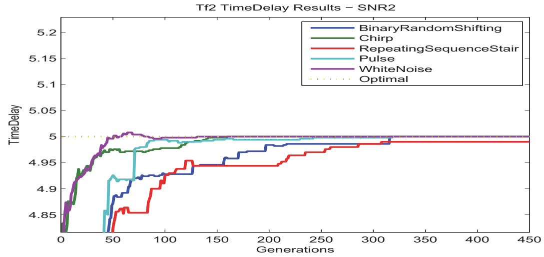

Fig. 8. Estimations of dead-time T_d of $G_2(s)$

Noise Free Case When the system measurements are noise free, all algorithms show perfect accuracies. The largest estimation error is the time constant estimation based on BRS excitation using GA method, which is 1.33% for $G_1(s)$ system and 3.65% for $G_2(s)$ system, respectively.

SNR$=10$ Case Some estimation errors appear with all algorithms in spite of which excitation signal.

For the estimation of $G_1(s)$, in general, the accuracies derived from the E-IV-LMS method is marginally better than those from the ELMS method, while both are marginally better than those from the GA method. The largest error - 13.21% from the GA method is the estimation error of time constant subject to pulse sequence excitation. However, GA method provides almost perfect dead-time T_d estimations except that there is 5% error based on pulse sequence excitation.

For the estimation of $G_2(s)$, the accuracies derived from the GA method are overall slightly better than those from the E-IV-LMS method, and E-IV-LMS method is marginally better than ELMS method. The largest estimation error - 28.92% is from the ELMS method for time constant estimation subject to BRS excitation. Furthermore, the GA method provides perfect dead-time T_d estimations for all excitations. In terms of MSE and dead-time estimations, the estimations of $G_2(s)$ by using all algorithms subject to any excitation show better accuracies than the corresponding situations of $G_1(s)$.

SNR$=2$ Case When the SNR decreases down to 2, the accuracies from all algorithms decrease as well, but with different percentages.

For the estimation of $G_1(s)$, the GA method provides the best dead-time estimation no matter which excitation is used. The ELMS method provides dead-time estimation errors from $5\% - 15\%$ for all excitations except the chirp signal. In terms of MSE, the chirp signal is the best excitation for both cases of SNR$=10$ and 2. Roughly, the estimation errors for K and τ of all algorithms subject to any excitation are more or less doubled comparing with the corresponding situations when SNR$=10$.

For the estimation of $G_2(s)$, the GA method achieves overall best results, and the E-IV-LMS method is marginally better than the ELMS method. The largest error derived by GA is 18.64% (for T_p under signal (d)), the largest error derived by E-IV-LMS is 45.88% (for T_p under signal (c)) while that derived by ELMS method is 98.78% (for T_p under signal (c)). Compared with the situation when SNR$=10$, the estimation errors using GA method are roughly doubled subject to any excitation when SNR$=2$,

Table 4. Estimation accuracies (%) regarding to $G_1(s)$, where T_p is τ

G1(s) - SNR = ∞

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	0.00	0.00	0.00	0.00	5.21	12.03	--	24.44	62.43	--
b	0.00	0.00	0.00	0.01	0.10	0.23	--	0.02	0.10	--
c	0.08	1.04	0.20	0.16	14.80	54.10	--	18.20	100.00	--
d	0.01	0.25	0.20	0.08	14.46	52.04	--	17.89	100.00	--
e	0.00	0.00	0.00	0.00	12.01	7.48	--	21.56	50.49	--

G1(s) - SNR = 10

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	1.52	1.88	0.00	6.00	4.72	11.58	--	25.32	63.70	--
b	6.18	3.88	0.00	0.10	5.76	1.61	--	5.94	2.68	--
c	0.40	2.54	5.20	11.81	15.54	56.52	--	19.23	100.00	--
d	0.53	1.02	5.20	2.89	15.19	54.09	--	18.92	100.00	--
e	1.40	1.80	5.00	0.40	13.27	0.11	--	23.23	51.40	--

G1(s) - SNR = 2

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	6.55	13.85	5.00	30.76	3.46	12.07	--	25.82	64.37	--
b	13.73	9.09	0.00	0.47	12.76	4.18	--	13.21	6.39	--
c	1.15	0.14	15.00	58.07	16.45	61.07	--	20.50	100.00	--
d	1.29	0.52	14.00	13.89	16.11	58.10	--	20.19	100.00	--
e	3.08	2.76	10.00	1.97	14.75	9.71	--	25.28	52.94	--

while the estimation errors using both LMS-based methods are more or less 3-4 times. This indicates the GA method is more robust than the others subject to different noise disturbances.

4.5 Computation Loads

In average, the GA method usually takes about 20 sec to accomplish one estimation procedure, while the ELMS method only takes about one second and E-IV-LMS method takes about 3-4 sec. Of course, the GA method can shorten the computation time by reducing the requested number of generations (450 in our case), as long as the convergence can be obviously observed. Nevertheless, the pre-filtering of the measured data is required for using the LMS based methods, which consumed time is not taken into account here. The GA results reported so far directly applied all measurements without any pre-filtering.

4.6 Data Pre-filtering Impacts

The GA estimation using the data pre-filtered by a low-pass filter (i.e., the data used by LMS-based methods) is also studied so as to investigate whether the pre-filtering could make GA perform better or not. The results are listed as the first column in Table 4 and 5, respectively. It can be observed that: for the $G_1(s)$ estimation, the pre-filtered data result in more precise results of τ (T_p) for signal (c), (d) and (e), but with costs of worse dead-time estimations. There are no much differences about estimations of K. For the $G_2(s)$ case, there is no any positive sign observed by using the pre-filtering.

4.7 Frequency-Domain Identification

The GA system identification in frequency-domain is also exploited under the assumption that the dead-time is already known/identified. Firstly, the measured data is converted into its DFT format using the FFT algorithm, then a quadratic formed cost function in frequency-domain is constructed as

$$C_f(K,\tau) \hat{=} \frac{1}{N_f}\sum_{k=1}^{N_f} W(k)(|Y(k)| - |\hat{Y}_{K,\tau}(k)|)^2, \qquad (10)$$

Table 5. Estimation accuracies (%) regarding to $G_2(s)$, where T_p is τ

G2(s) - SNR = ∞

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	0.27	1.15	0.08	0.23	23.53	9.95	--	29.04	38.43	--
b	0.00	0.00	0.00	0.00	0.06	0.18	--	0.01	0.11	--
c	0.02	1.87	0.00	0.00	20.22	69.09	--	20.87	100.00	--
d	0.00	0.00	0.00	0.00	19.90	0.89	--	21.64	67.91	--
e	0.00	0.00	0.00	0.00	19.89	0.98	--	21.64	67.96	--

G2(s) - SNR = 10

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	6.05	4.63	0.24	0.66	19.72	11.46	--	24.20	33.03	--
b	0.40	1.17	0.00	0.01	1.61	7.73	--	0.42	0.04	--
c	1.42	21.09	1.00	0.74	19.46	99.99	--	20.15	100.00	--
d	1.39	19.68	1.00	0.18	19.76	100.00	--	20.14	100.00	--
e	1.67	8.03	0.00	0.02	18.84	4.73	--	20.75	76.63	--

G2(s) - SNR = 2

	GA Filter - Accuracy				GA Frequency			GA Filter Frequency		
	K	Tp	Td	MSE	K	Tp	Td	K	Tp	Td
a	15.46	16.72	0.96	1.95	15.54	16.67	--	18.03	27.73	--
b	0.92	2.51	0.00	0.05	3.13	12.51	--	0.42	1.07	--
c	3.23	44.88	2.00	3.70	18.19	100.00	--	19.20	100.00	--
d	3.21	44.37	2.00	0.87	18.49	100.00	--	19.22	100.00	--
e	3.83	20.35	0.00	0.08	17.54	28.83	--	19.54	100.00	--

where $W(k)$ is a frequency weighting sequence, N_f is the length of signal's DTF sequence. the amplitude of the kth estimated output sample $\hat{Y}_{K,\tau}(k)$, which is a function of unknown parameter K and τ, is calculated according to

$$|\hat{Y}_{K,\tau}(k)| = \frac{K}{\sqrt{(2\pi f_s k\tau/N_f)^2 + 1}}|U(k)|. \qquad (11)$$

where f_s is the sampling frequency. The GA algorithm is used to obtain the best estimation of K and τ through minimizing the cost function (10). The accuracies of the obtained results are listed as the second columns in Table 4 and 5, respectively. It can be observed that in general the accuracies by this frequency method is not as good as the time-domain based methods. This is mainly due to different emphases (cost functions). The same GA frequency method using the pre-filtered data is also investigated, and the results are listed as the third columns in Table 4 and 5. It can be noticed that this is the worst case comparing with all methods studied through this work no matter for the $G_1(s)$ or $G_2(s)$ case.

5. CONCLUSION

The real coded GA method for identifying a simple class of time-delay systems, named FOPDT system, is extensively investigated w.r.t. different system features, excitation signals, SNRs and evaluation criteria. The accuracies and performances of the employed GA are further compared with those of exhaustive LMS-based methods. It can be concluded that (a) GA acts almost as good as these ELMS-based methods in general, and even better for time-delay dominant systems in term of estimation accuracy; (b) There is no need for GA method to pre-filter the measured data; (c) GA always plays best in estimating the dead-time coefficient; (d) GA method is more robust to different SNRs; (e) It seems that estimation of T_p turns to be the most challenging task for GA when the SNR becomes poor, especially with excitations of (c), (d) and (e); (f) The T_p estimation could achieve better accuracies by using the frequency-domain evaluation criterion instead of the time-domain one, but only for $G_1(s)$ type of time constant dominant systems; (g) In general it seems that the chirp signal is the best excitation, especially for using the frequency-domain criterion.

The obtained results exhibit that the GA technique has a very promising capability in handling this type of non-convex system identification problem. The proposed GA method can be naturally extended to estimate higher-order linear time-delay systems, even for nonlinear time-delay systems. The same method can also be used to obtain some low-order time-delay system model based on measurements from a sophisticated complex systems, this part is reported in our second serial paper (Yang Seested (2013)).

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