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Abstract. In analyzing stochastic dynamic systems, analysis of the system uncertainty due to randomness in the loads plays a crucial role. Typically time series of the stochastic loads are simulated using traditional random phase method. This approach combined with fast Fourier transform algorithm makes an efficient way of simulating realizations of the stochastic load processes. However it requires many random variables, i.e. in the order of magnitude of 1000, to be included in the load model. Unfortunately having too many random variables in the problem makes considerable difficulties in analyzing system reliability or its uncertainty. Moreover applicability of the probability density evolution method on engineering problems faces critical difficulties when the system embeds too many random variables. Hence it is useful to devise a method which can make realization of the stochastic load processes with low, say less than 20, number of random variables. In this article we introduce an approach, so-called ”physical modeling of stochastic processes”, and show its applicability for simulation of the wave surface elevation.

Introduction

The application of PDEM requires that the stochastic load process and possible uncertain parameter fields related to the structural model are discretized in terms of a small number of basic random variables. With this specific problem in mind Li et al. [1] devised a turbulence model based on so-called evolutionary phases, which are driven merely a single basic random variable. The method generates time varying random phases for the harmonic components of a spectral decomposition of the turbulence process, which are shown to be mutual independent and uniformly distributed in the interval $[0, 2\pi]$. Hence, the method is stochastic equivalent to the well-known random phase representation of a turbulence field. In the present paper the same approach has been applied as a stochastic model of the irregular sea-state impinging on the point absorber.

Next we perform some tests that checks similarity of the outputs of this method to that of the standard Monte Carlo simulation. In the first step the statistical properties of the time series generated by this method will be represented which reveals the similarity of this method to the random phase method. Outcomes of the time series simulated by this method will be checked to fulfill the requirements for the Monte Carlo simulation. A truncated double exponential distribution is proposed to be used for the initial evolution time of the phases. Finally, it is shown that the simulated time series generated by this method fulfill the statistical requirements of the process.

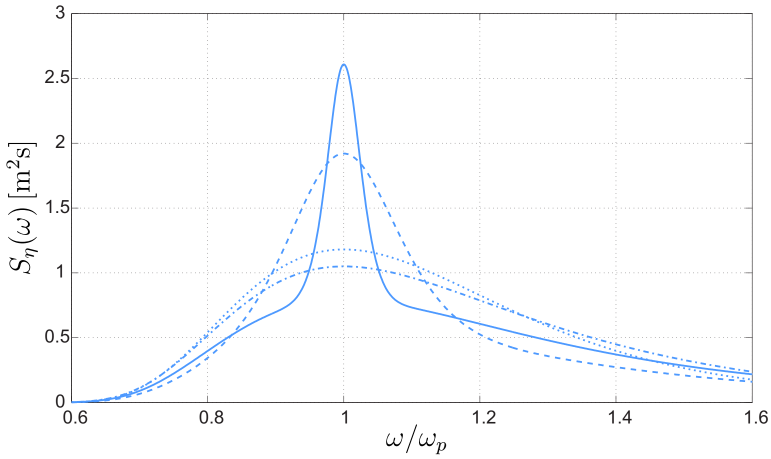

Fig. 1: One-sided modified JONSWAP auto-spectral density function as a function of the bandwidth parameter σ_f, H_s= 3m. —: σ_f=0.03. - - -: σ_f=0.1. · · ·: σ_f=0.5. — · — · —: σ_f=1.0.

Stochastic Wave Load Model

The irregular plane waves are assumed to propagate in the positive x-direction. The surface elevation at the position x at the time t is described by the homogeneous and stationary zero mean Gaussian process $\{\eta(x,t)\,,\,(x,t)\,\in\,R^2\}$. The double sided auto-spectral density function of the sea-surface elevation process at a given position is defined by the following slightly modified double-sided version of the JONSWAP spectrum, (Hasselmann *et al.* 1973)

$$S_{\eta\eta}(\omega) = \beta\,\frac{H_s^2}{\omega_p}\,\gamma^\alpha\left(\frac{|\omega|}{\omega_p}\right)^{-5}\exp\left(-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right) \tag{1}$$

where

$$\left.\begin{aligned}
&\alpha = \exp\left(-\frac{1}{2}\left(\frac{|\omega|-\omega_p}{\sigma_f\omega_p}\right)^2\right)\\
&T_p = \frac{2\pi}{\omega_p} = \sqrt{\frac{180\,H_s}{g}}\\
&\gamma = 3.3
\end{aligned}\right\} \tag{2}$$

T_p is the peak period, $\omega_p = 2\pi/T_p$ is the related angular peak frequency, H_s is the significant wave height, and σ_f is a band-width parameter. β is a normalization parameter to be determined, so the relation $H_s = 4.0\sigma_\eta$ is fulfilled, corresponding to Rayleigh distributed wave heights. The auto-spectral density function has been shown in Figure 1 for various values of σ_f. Notice the abscissa has been normalized with respect to the angular peak frequency ω_p, which depends on H_s as indicated in the second relation in (2). The surface elevation field admits the following spectral representation

$$\eta(x,t) = \sum_{j=1}^{J}\sqrt{2}\,\eta_j\,\cos\left(\omega_j\,t - k_j\,x - \Phi_j\right)\quad,\quad \omega_j = (j-1)\Delta\omega \tag{3}$$

$$\eta_j = \sqrt{2S_{\eta\eta}(\omega_j)\,\Delta\omega} \tag{4}$$

where J is the number of harmonic components in the spectral decomposition, η_j denotes the standard deviation of harmonic components with angular frequencies in the interval $]\omega_j, \omega_j + \Delta\omega]$, and Φ_j are mutually independent identical distributed random variables, uniformly distributed in $[0, 2\pi]$. k_j is the wave number of the jth harmonic wave component. This is related to the angular frequency ω_j by the well-known dispersion relation of linear gravity waves, (Svendsen and Jonsson 1980)

$$k_j h \tanh(k_j h) = \frac{\omega_j^2 h}{g} \tag{5}$$

The phases Φ_j are referred to as the basic variables of the model. The number J of these variables is typically $10^2 - 10^4$, which renders the model (28) useless for application in the PDEM. Inspired by the so-called evolutionary phase approach by Li *et al.*, (Li *et al.* 2012b) for handling the analog problem of one-dimensional frozen turbulence processes propagating non-dispersive with the mean wind velocity according to Taylor's hypothesis of frozen turbulence, the following alternative spectral representation is suggested

$$\bar{\eta}(x,t) = \sum_{j=1}^{J} \sqrt{2}\,\eta_j\,\cos\left(\omega_j\,t - k_j\,x - \bar{\Phi}_j\right) \tag{6}$$

where

$$\bar{\Phi}_j = \mathrm{mod}\big(k_j \omega_j \eta_j\,T_0\,,\,2\pi\big) \tag{7}$$

$k_j \omega_j \eta_j\,T_0$ is referred to as the evolutionary phases, and $\vartheta_j = k_j \omega_j \eta_j$ denotes a characteristic velocity of the fluid particles in the wave motion. T_0 is an assumed common origin of time for the evolutionary phases, which is considered a random variable. Since the auto-spectral density function is determined by the amplitude of the harmonic components, both representations (3) and (6) represent the exact auto-spectral density $S_{\eta\eta}(\omega)$ for arbitrary values of the phases Φ_j and $\bar{\Phi}_j$ within an error determined by the discretization of the spectrum as given by the parameter $\Delta\omega$. The important point is that the latter representation is generated by merely a single random variable.

Probability distribution of T_0

The conventional way to obtain the Probability Density Function (PDF) of the T_0 is to measure time series of the wave surface elevation and perform a backward analysis on each one of them them to obtain its T_0. Finally an empirical PDF can be fitted to these calculated values. This is performed on the data obtained from for the buoys installed in Hanstholm, Denmark with the procedure described in the following. The basic procedure is described in [2]. The basic phase of harmonic wave in the peak frequency is defined as , and its phase evolutionary velocity of the fluid particle is defined as $\Phi(\omega_p, 0)$, and its phase evolutionary velocity of the fluid particle is defined as $\vartheta_p = k_p \omega_p \eta_p$. Propagating the wave inversely from the commencement of time history, a series of values of time quantity T_0, whereby the phase value $\Phi(\omega_p, 0) = 2\pi(n-1)$, can be derived from the following equation

$$T_0 = \frac{\Phi(\omega_p, 0)}{k_p \omega_p \eta_p} + \frac{2\pi(n-1)}{k_p \omega_p \eta_p} \quad , \quad n = 1, \cdots, N_u \tag{8}$$

Where T_0 is a time quantity with positive values; n denotes the number domain of starting-time that belongs to natural numbers; N_u is limited to be 10^9 due to computational costs. The phase value at a typical T_0 is solved by

$$\Phi(\omega_p, T_0) = \Phi(\omega_p, 0) - k_p \omega_p \eta_p T_0 \quad , \quad j = 1, \cdots, m \tag{9}$$

If T_0 satisfies the following constraint condition of maximum deviation

$$\mid \mathrm{mod}\ \{\Phi(\omega_p, T_0), 2\pi\} \mid \leq \epsilon \tag{10}$$

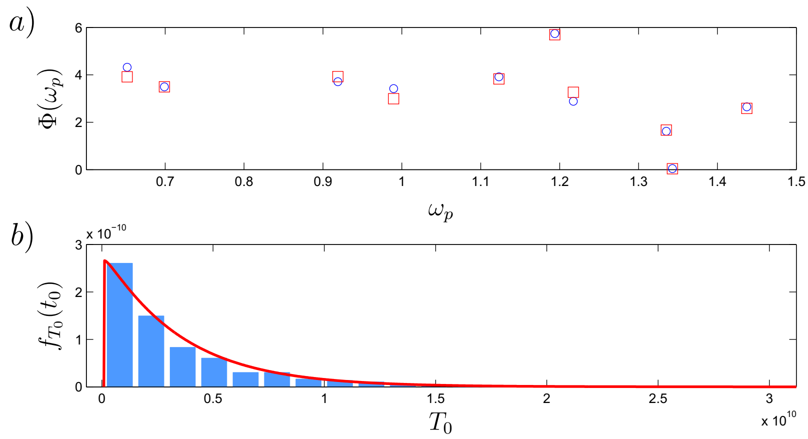

Fig. 2: Analysis of the phases of the wave elevations; a) Measured (○) versus fitted (□) phases of the time series, b) Histogram and fitted PDF of the measured T_0s.

where ϵ is the error tolerance. The corresponding T_0 is then viewed as the desirable starting-time of phase evolution. Since the limited lower frequency harmonic waves contain the primary information and most energy of turbulence, identification of the starting-time of phase evolution is only focused on the lower frequency domain. In general it is extremely difficult to derive the starting-time in every frequency point. Hence the phases related to only a limited number of harmonics within the low frequency range, where most of the energy of the process is concentrated, are used to obtain T_0. Here only 10 phases at high energy part of the surface wave spectrum are used for this purpose. T_0 of totally 2500 measured time series of 800-sec wave elevation (from Hanstholm), whose significant wave height is about 3m, were identified first. The proposed truncated double exponential distribution is then fitted to the identified T_0s. The quality of the fit is shown in figure

Statistical properties of the simulated surface elevations

Three values for T_0 are then chosen from the three regions obtained by the previous analysis and time series of the surface elevation associated to them are simulated. The resulted realizations are shown in figure 3. It is clear that both for very small and very large values of t_0 the realizations of the wave surface elevation deviates from the way they should look like. Only when t_0 is chosen in the calculated range the realization seems to be correct. The histogram of the generated phases for the same t_0 values used in the previous analysis are shown in figure 4. Again it can be seen that distribution of the phases shown in figures 4.a and 4.c deviate to some extent from the uniform distribution while figure 4.b fits better than the former two to the uniform distribution. Here, a truncated exponential distribution (11) is suggested. The scale parameter t_3 specifies the mean value (and variance) of T_0. In appendix A it is shown that the distribution of the phases which their T_0's are drawn from this family of distributions asymptotically approaches uniform distribution.

$$f_{T_0}(t) = \begin{cases} \dfrac{\exp\left(-\frac{t-t_1}{t_3}\right)}{t_3\left(1-\exp\left(-\frac{t_2-t_1}{t_3}\right)\right)} & , \quad t \in [t_1, t_2] \\ 0 & , \quad t \notin [t_1, t_2] \end{cases} \tag{11}$$

Next we try to check the distribution of the samples of the generates surface elevations using Monte Carlo simulation. Here we have used 5000 samples with different randomly chosen t_0's in the defined range. It is expected that the generated realizations of the wave surface elevation are Gaussian distributed in every time instant. It is seen that the estimated histogram fits acceptably to the normal distribution with the analytical variance of the wave elevation $\sigma_\eta^2 = H_s^2/4$. Moreover since the process

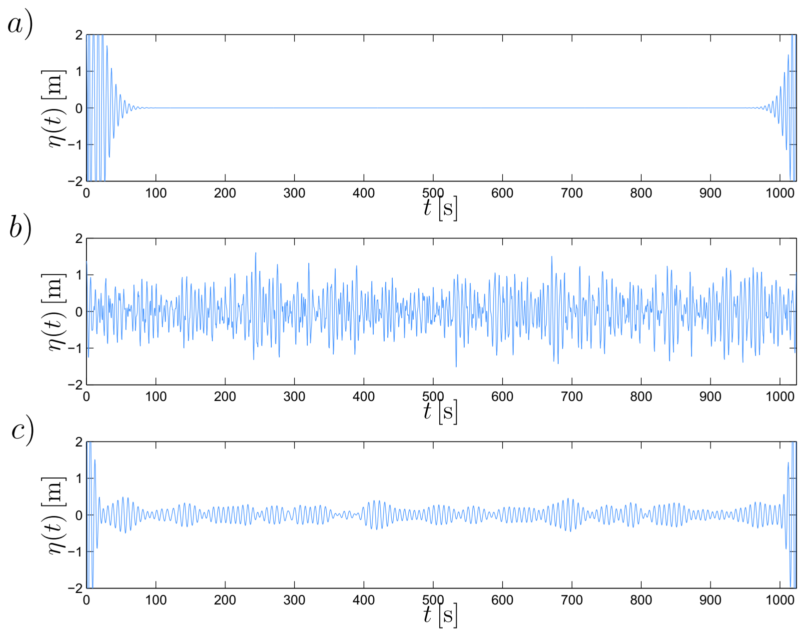

Fig. 3: Samples of the wave elevation time series for different values of t_0; a) $t_0 = 10^3$ b) $t_0 = 10^{10}$ c) $t_0 = 5 \times 10^{18}$.

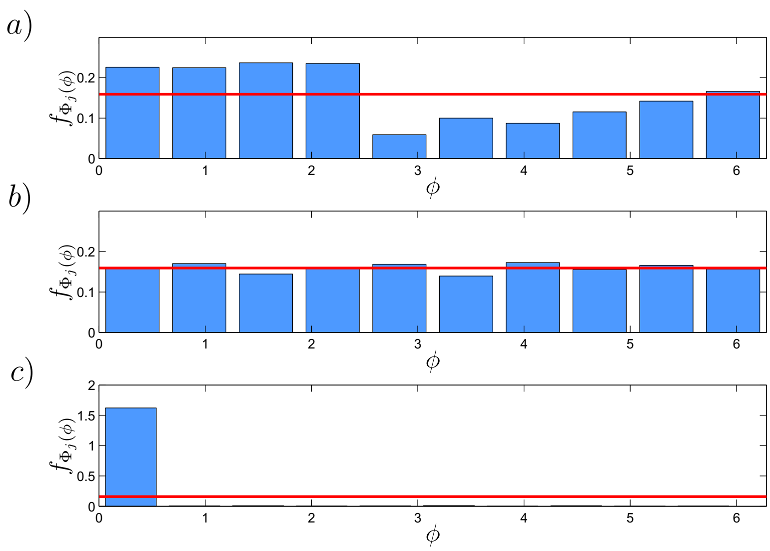

Fig. 4: Histogram of sampled phases $\bar{\Phi}_j$, $H_s = 3$ m, $\sigma_f = 0.1$, J=2048; a) $t_0 = 10^3$ s, b) $t_0 = 10^{10}$ s, c) $t_0 = 5 \times 10^{18}$ s.

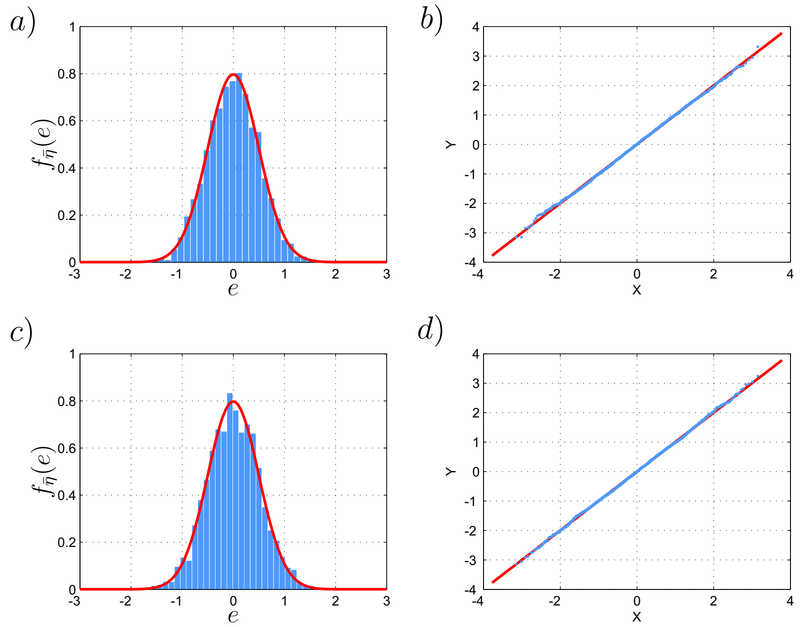

Fig. 5: Statistical properties of the simulated time series by the proposed empirical PDF; a) PDF of the surface elevation $\eta(t)$ obtained as ensemble average of one realization, b) Comparison of histogram of samples of one realization with Gaussian distribution $\mathcal{N} \sim (0, \sigma_\eta^2)$, c) PDF of the surface elevation $\eta(t)$ obtained as ensemble average at the time $t = 732$ s of 5000 time-series, d) Comparison of histogram with Gaussian distribution $\mathcal{N} \sim (0, \sigma_\eta^2)$.

is stationary, the distribution of the samples could be achieved by ergodic sampling. Figures 5.a and 5.b show the estimated PDF and QQ-plots of samples of one realization of wave surface elevation by ergodic sampling respectively. Figures 5.c and 5.d show the estimated PDF and QQ-plots of the samples of 5000 realization of wave surface elevation at a certain time using Monte Carlo simulation respectively. Finally the covariance function of the samples generated by the introduced method are shown in figure 6. The covariance of the samples of the process are estimated as ensemble average of 100 samples of the process simulated by the introduced method. For both too small and too large values of t_0 the covariances in the non zero time lags underestimated the covariance of the process. All figures confirm that distribution of the samples is well matched with that of Gaussian. As verified by these results the $\eta(x, t)$ becomes zero-mean and normal distributed, if the sample values of T_0 is drawn from a finite interval $[t_1, t_2]$. Under this condition (6) becomes stochastically equivalent to (3).

Concluding Remarks

A new method for modeling wave surface elevation is represented in the article. The concept of the method is similar to that of the random phase. The advantage of the method is that it binds all the random phases of the problem into one by some technique. It is useful in view of applying PDEM on stochastic dynamic systems where appreciably high number of random variables are associated in analysis. Normally most of these random numbers come from the stochastic loads of the model i.e. wave loads or seismic loads. Dynamic analysis of structural systems excited by such loads involves thousands of random variables which is out of reach of PDEM. Hence such a method which can decrease the number of the random variables appreciably is necessary to be used in this regards. It is shown in the paper that this method fulfills the statistical requirements of the stochastic process i.e. uniform distribution of random phases and Gaussian distribution of the samples of the process and correct power spectrum.

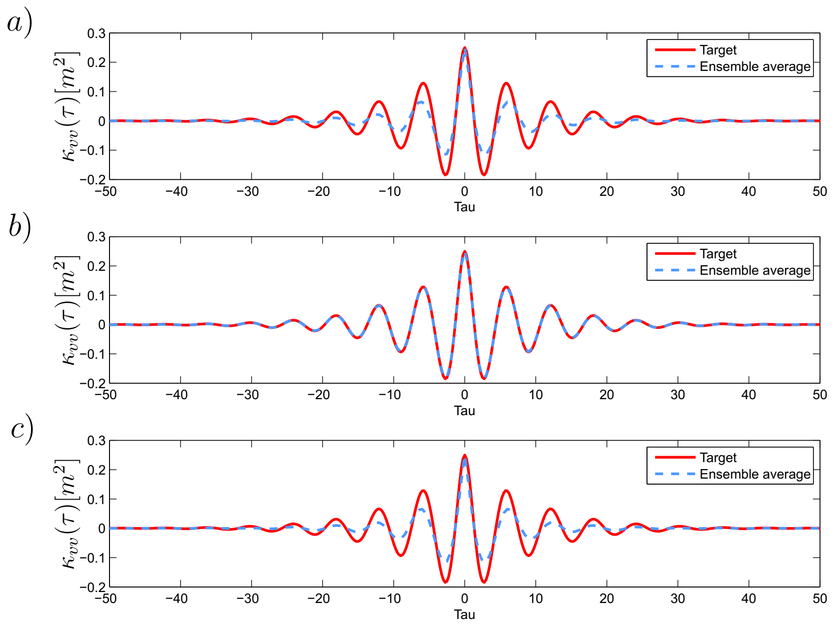

Fig. 6: Auto covariance function of the simulated time series; a) $1 < t_0 < 10^2$ b) $10^4 < t_0 < 10^6$ c) $10^{18} < t_0 < 10^{20}$.

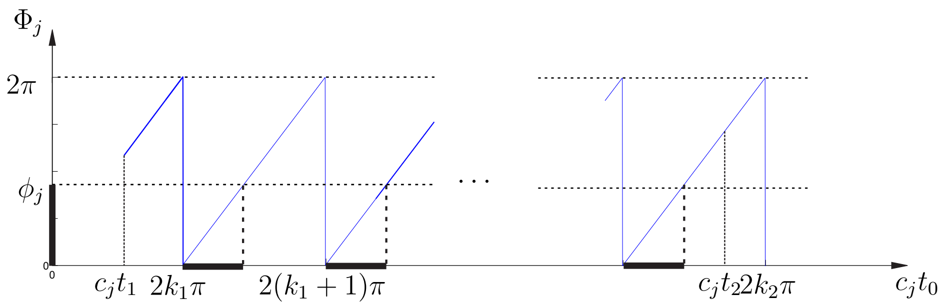

Fig. 7: Schematic representation of the phases generated by the physical modeling method.

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Appendix

Distribution of the random phases generated by the proposed method is committed here. Assume t_0 is distributed as

$$f_{T_0}(t_0) = \frac{1}{t_3\beta} e^{-\frac{t_0 - t_1}{t_3}} \quad , \quad \beta = 1 - e^{-\frac{t_2 - t_1}{t_3}} \tag{12}$$

according to figure 7 we have

$$\left. \begin{aligned} F_{\Phi_j}(\phi_j) = p(\Phi_j < \phi_j) = \sum_{k=k_1}^{k_2} p\left(2k\pi < c_j t_0 < 2k\pi + \phi_j\right) \\ = \sum_{k=k_1}^{k_2} \left(F_{T_0}\left(\frac{2k\pi + \phi_j}{c_j}\right) - F_{T_0}\left(\frac{2k\pi}{c_j}\right)\right) \end{aligned} \right\} \tag{13}$$

where $c_j t_1 = 2(k_1-1)\pi + \mathrm{mod}(c_j t_1, 2\pi)$ and $c_j t_2 = 2(k_2-1)\pi + \mathrm{mod}(c_j t_2, 2\pi)$. Then

$$f_{\Phi_j}(\phi_j) = \frac{d}{d\phi_j} F_{\Phi_j}(\phi_j) = \frac{1}{c_j} \sum_{k=k_1}^{k_2} f_{T_0}\left(\frac{2k\pi + \phi_j}{c_j}\right) \tag{14}$$

Next, the characteristic function of Φ_j can be calculated [3]

$$\left.\begin{aligned} S_{\Phi_j}(\omega) &= \int_0^{2\pi} e^{i\omega\phi_j} f_{\Phi_j}(\phi_j) d\phi_j = \int_0^{2\pi} \frac{e^{i\omega\phi_j}}{c_j} \sum_{k=k_1}^{k_2} f_{T_0}\left(\frac{2k\pi + \phi_j}{c_j}\right) d\phi_j \\ &= \sum_{k=k_1}^{k_2} \frac{1}{c_j} \int_0^{2\pi} e^{i\omega\phi_j} f_{T_0}\left(\frac{2k\pi + \phi_j}{c_j}\right) d\phi_j \end{aligned}\right\} \tag{15}$$

substituting (12) into (15) results

$$\left.\begin{aligned} S_{\Phi_j}(\omega) &= \sum_{k=k_1}^{k_2} \frac{1}{c_j t_3 \beta} \int_0^{2\pi} \exp\left(i\omega\phi_j - \frac{2k\pi + \phi_j - c_j t_1}{c_j t_3}\right) d\phi_j \\ &= \left(\frac{e^{t_1/t_3}}{c_j t_3 \beta} \int_0^{2\pi} e^{\gamma\phi_j} d\phi_j\right) \times \sum_{k=k_1}^{k_2} \left(e^{-\frac{2\pi}{c_j t_3}}\right)^k \\ &= \left(\frac{e^{t_1/t_3}}{c_j t_3 \beta\gamma}\left(e^{2\pi\gamma} - 1\right)\right) \times \sum_{k=k_1}^{k_2} \left(e^{-\frac{2\pi}{c_j t_3}}\right)^k = \frac{e^{t_1/t_3}}{c_j t_3 \beta\gamma} \frac{\Omega^{k_1} - \Omega^{k_2+1}}{1-\Omega}\left(e^{2\pi\gamma} - 1\right) \end{aligned}\right\} \tag{16}$$

where $\gamma = i\omega - \frac{1}{c_j t_3}$ and $\Omega = \exp(-\frac{2\pi}{c_j t_3})$. As a special case assume $t_1 \to 0$ and $t_2 \to \infty$ then $k_1 \to 0, k_2 \to \infty$ and $\beta \to 1$. For this case we have

$$f_{T_0}(t_0) = \frac{1}{t_3} e^{-\frac{t_0}{t_3}} \quad \Leftrightarrow \quad S_{\Phi_j}(\omega) = \frac{1}{c_j t_3 \gamma} \frac{e^{2\pi\gamma} - 1}{1-\Omega} \tag{17}$$

As $c_j t_3 \to \infty$

$$\lim_{c_j t_3 \to \infty} S_{\Phi_j}(\omega) = \frac{e^{i2\pi\omega} - 1}{i2\pi\omega} \tag{18}$$

(18) proves that the T_0 will be distributed uniformly between 0 and 2π asymptotically as $c_j t_3 \to \infty$ for the proposed distribution.

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