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PAPER NO. 183

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Assessment of the reliability profiles for concrete bridges

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In this paper calculation of reliability profiles is discussed. ULS as well as SLS limit states are formulated. Corrosion due to chloride penetration is the considered deterioration mechanism. Three models for corrosion are formulated. A definition of service lifetime for concrete bridges is presented and discussed. The proposed method of calculating reliability profiles is illustrated on an existing U.K. bridge. © 1998 Elsevier Science Ltd. All rights reserved.

Keywords: concrete bridges, reliability assessment, corrosion modelling

1. Introduction

This paper is based on research performed for the Highways Agency, London, U.K. under the project DPU/9/44 "Revision of Bridge Assessment Rules Based on Whole Life Performance: Concrete Bridges". It contains details of a methodology which can be used to generate Whole Life (WL) reliability profiles. These WL reliability profiles may be used to establish revised rules for Concrete Bridges. The paper is based on Thoft-Christensen *et al.*[1,2] and Thoft-Christensen[3–5].

2. Limit states

Four limit states are selected for the reliability analysis:

- two ultimate limit state (ULS): collapse limit states (using yield line analysis) and shear failure limit state,
- two serviceability limit states (SLS): crack width limit state and deflection limit state.

2.1. Collapse (yield line) limit state

The following safety margin is used

$$M_1 : g_1(\cdot) = Z_1 E_D - W_D \tag{1}$$

where Z_1 is a model uncertainty variable, E_D is the energy dissipated in yield lines, and W_D is the work done by the applied loads.

The plastic collapse analysis and estimation of the load are performed using the COBRAS program[6]. The reliability analysis (element and system) is done using RELIAB01[7] and RELIAB02[8]. The RELIAB and COBRAS programs have been interfaced and an optimisation algorithm has been included to determine the optimal yield line pattern for each iteration of the reliability analysis, see also Thoft-Christensen[9]. The estimation of the deterioration of the steel reinforcement is based on the program CORROSION[10]. Cobras supports 16 different types of failure mode, 7 are used in this bridge slab analysis (see *Figure 1*). The basic variables used in the yield line ULS are: thickness of slab, cube strength of concrete, density of concrete, depth of reinforcement, yield strength of reinforcement, and two load parameters.

2.2. Shear failure limit state

Shear failure is modelled using a model applicable to reinforced concrete beams[11], which may be written as

$$M_2 : g_2(\cdot) = Z_2 V_{j,ult} - V_j \tag{2}$$

where V_j is the shear force from external loads, Z_2 is a model uncertainty variable, $V_{j,ult}$ is the ultimate shear strength, ν_c is the design shear stress, and ξ_s is the depth factor defined as, where b is the width of the beam and d is the depth of the beam

$$V_u = \xi_s \nu_c bd,\ \nu_c = 0.24\left(\frac{100A_s}{bd}\right)^{1/3} f_c^{1/3},\ \xi_s = \left(\frac{500}{d}\right)^{1/4} \tag{3}$$

The stochastic variables used in the shear limit state are: thickness of slab, cover on reinforcement, concrete cube strength, yield stress of reinforcement, initial area of the reinforcement, density of concrete, static load factor, dynamic load factor, model uncertainty variable, and variables related to the chloride induced corrosion.

2.3. Crack width limit state

Cracking shall be limited to a level that will not impair the proper functioning of the structure or cause its appearance

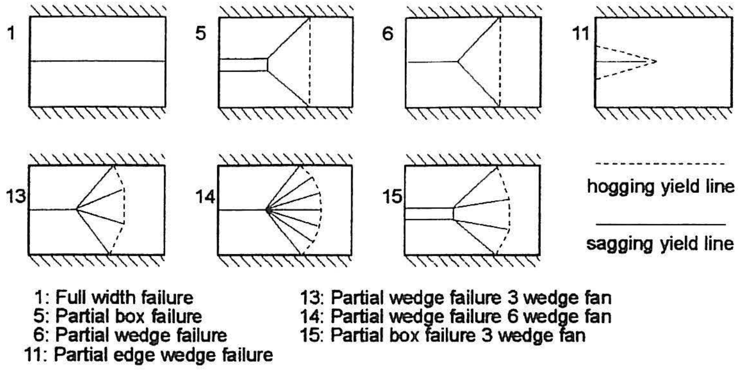

Figure 1 Failure modes for simple supported slab bridges

to be unacceptable. The design crack width may be obtained from[12]

$$w_k = \beta s_{rm} \epsilon_{sm} \tag{4}$$

where w_k is the design crack width, s_{rm} is the average final spacing, ϵ_{sm} is the mean strain allowing, under the relevant combination of loads, for the effects of tension stiffening, shrinkage, etc., and β is a coefficient relating the average crack width to the design value. The value of ϵ_{sm} may be calculated from

$$\epsilon_{sm} = \frac{\sigma_s}{E_s}\left(1 - \beta_1\left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right) \tag{5}$$

where σ_s is the stress in the reinforcement calculated on the basis of a cracked section. σ_{sr} is the stress in the reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking. β_1 is a coefficient which takes account of the bond properties of the bars.

The average final crack spacing (in mm) for members subjected dominantly to flexure or tension can be calculated from the equation

$$s_{rm} = 50 + 0.25 k_1 k_2 \phi / \rho_r \tag{6}$$

where ϕ is the bar size in use (or the average bar size). ρ_r is the effective reinforcement ratio, $A_s/A_{c.eff}$, where A_s is the area of reinforcement contained within the effective tension area, $A_{c.eff}$. k_1 is a coefficient which takes account of the bond properties of the bar. k_2 is a coefficient which takes account of the strain distribution.

The crack width limit state can then be formulated by

$$M_3{:}g(\cdot) = w_{max} - Z_c w_k \tag{7}$$

where Z_c is a model uncertainty stochastic variable. The stochastic variables used in the crack SLS are: concrete cover, distance between reinforcement bars, diameter of reinforcement bars, thickness of slab, elastic modulus of reinforcement bars, tensile strength of concrete, external bending moment, and one model uncertainty variable.

2.4. *Deflection limit state*

The following deflection limit state is used

$$M_4{:}g(\cdot) = d_{max} - Z_d d_k \tag{8}$$

where d_{max} is the maximum allowable deflection, d_k is the deflection estimated by linear elastic analysis, and Z_d is a model uncertainty variable.

3. Deterioration

Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts to corrode actively. Practical experience of bridges in wetter countries shows that chloride ingress is far bigger a problem that carbonation. The rate of chloride penetration into concrete, as a function of depth from the concrete surface and time, can be represented by Fick's law of diffusion as follows:

$$\frac{\delta C}{\delta t} = D_C \frac{\delta^2 C}{\delta x^2} \tag{9}$$

where C is the chloride ion concentration, as % of the weight of cement, at distance x cm from the concrete surface after t s of exposure to the chloride source. D_C is the chloride diffusion coefficient expressed in cm²/s. The solution of the differential equation (8) is

$$C(x,t) = C_0\left\{1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D_C \cdot t}}\right)\right\} \tag{10}$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % of the weight of cement, x is the distance from the concrete surface in cm, t is the time in s, erf is the error function, D_C is the diffusion coefficient in cm²/s and $C(x,t)$ is the chloride concentration at any position x at time t. In a real structure, if $C_{cr}(x,t)$ is assumed to be the chloride corrosion threshold and x is the thickness of concrete cover, then the corrosion initiation period, T_I, can be calculated based on a knowledge of the parameters C_0 and D_C. The time T_I to initiation of reinforcement corrosion is

$$T_I = \frac{(d_1 - D_1/2)^2}{4D_C}\left(\operatorname{erf}^{-1}\left(\frac{C_{cr} - C_0}{C_i - C_0}\right)\right)^{-2} \tag{11}$$

where C_i is the initial chloride concentration, C_{cr} is the critical chloride concentration at which corrosion starts, and $d_1 - D_1/2$ is the concrete cover. When corrosion has started then the diameter $D_1(t)$ of the reinforcement bars at time t is modelled by

$$D_1(t) = D_i - C_{corr} i_{corr} t \qquad (12)$$

where D_i is the initial diameter, C_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion.

Based on a survey, three models for chloride penetration are proposed (the initial chloride concentration is assumed to be zero): low deterioration, medium deterioration and high deterioration. The deterioration parameters for these three levels are ($N(a,b)$ is a normal distribution with the mean a and the standard deviation b, Uniform$[a,b]$ is a uniform distribution in the interval $[a;b]$):

3.1. Low:

Diffusion coefficient D_C: $N(25.0, 2.5)$ [mm²/year]
Chloride concentration, surface C_0: $N(0.575, 0.038)$ [%]
Corrosion density i_{corr}: Uniform[1.0, 2.0] [mA/cm²].

3.2. Medium:

Diffusion coefficient D_C: $N(30.0, 2.5)$ [mm²/year]
Chloride concentration, surface C_0: $N(0.650, 0.038)$ [%]
Corrosion density i_{corr}: Uniform[1.5, 2.5] [mA/cm²].

3.3. High:

Diffusion coefficient D_C: $N(35.0, 2.5)$ [mm²/year]
Chloride concentration, surface C_0: $N(0.725, 0.038)$ [%]
Corrosion density i_{corr}: Uniform[2.0, 3.0] [mA/cm²].

Figure 2 shows sample realisations of the history of the reinforcement area for all tree deterioration models.

4. Service life time

In Thoft-Christensen[5] the service life time is defined as the initiation time T_I, see equation (11) for corrosion of the reinforcement. This is a rational definition from a life-cycle cost of view since repair of corroded reinforced elements is a major contributor to the life-cycle cost. It is relatively inexpensive to repair a structural element by replacing some part of the concrete instead of waiting until corrosion has taken place.

On basis of equation (11) outcomes of the corrosion initiation time T_I has been performed on basis of the following data by simple Monte Carlo simulation (1000 simulations) using the software program Corrosion[10]:

Initial chloride concentration: 0%
Surface chloride concentration: Normal(0.650;0.038)
Diffusion coefficient: Normal(30;5)
Critical concentration: Normal(0.3;0.05)
Cover: Normal(40;8)
Number of samples: 1000.

The simulated values are plotted on Weibull probability paper in *Figure 3*. In the same figure is shown that a Weibull distribution can be used to approximate the distribution of the simulated data.

The straight line in *Figure 3* corresponds to a Weibull distribution $W(x;\mu,k,\epsilon)$, where $\mu = 63.67$, $k = 1.81$ and $\epsilon = 4.79$. The corresponding histogram and the density function is shown in *Figure 4*.

5. Reliability profiles

This example is used to illustrate the proposed methodology. The example is based on an existing U.K. bridge, but some limitations and simplifications are made. The bridge was built in 1975 and was designed for 45 units HB load. The bridge has a span of 9.75 m, the width is 2 times 13.71 m, and the slab thickness is 550 mm (see *Figure 5*). Based on the corrosion data shown in *Table 1* the expected

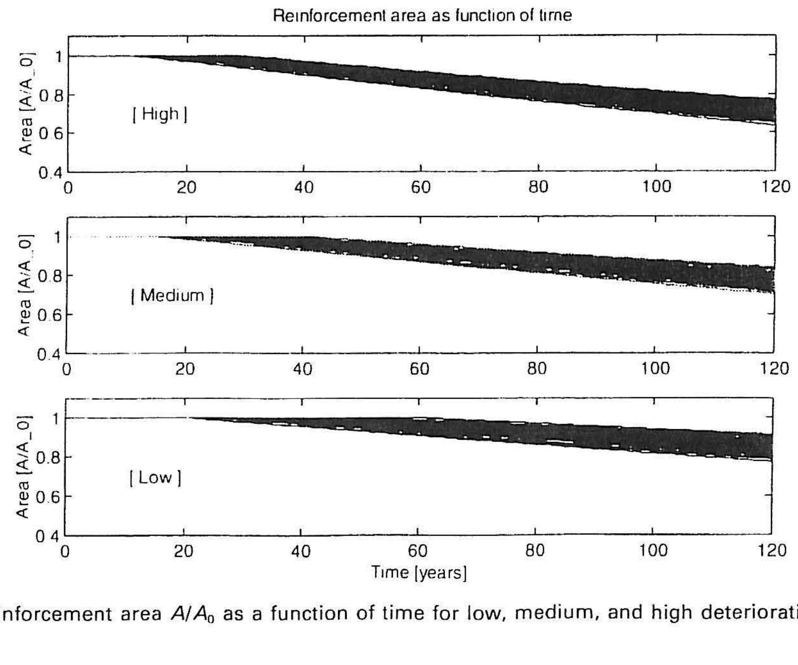

Figure 2 Normalised reinforcement area A/A_0 as a function of time for low, medium, and high deterioration

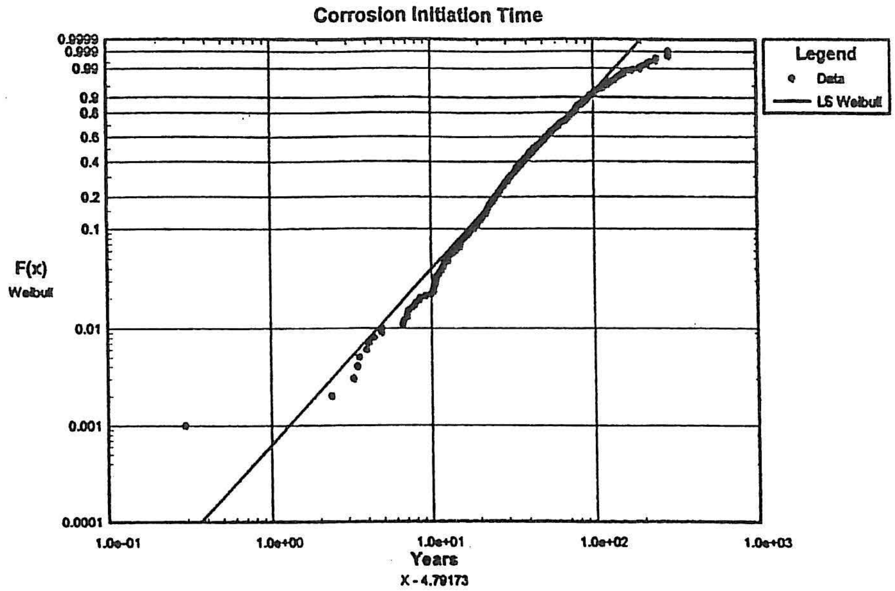

Figure 3 Plotting of simulated data on Weibull probability paper

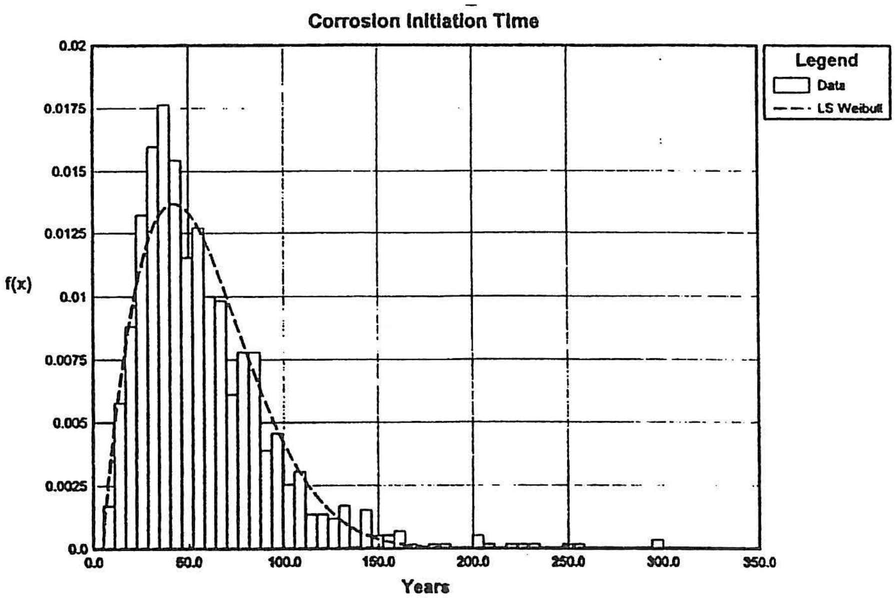

Figure 4 Density function of the corrosion initiation time

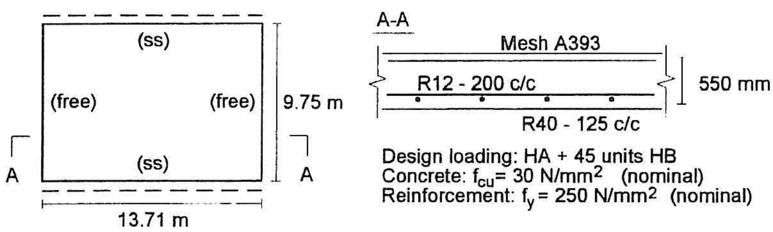

Figure 5 Bridge data

area of the reinforcement as a function of time can be calculated (see *Figure 6*).

Reliability profiles for the yield line limit state (ULS) are as an illustration calculated on the basis of the stochastic modelling shown in *Table 1*.

The general traffic highway load model in the Eurocode 1, Part 3 (ENV 1991-3:1995) for lane and axle load is applied. The load effects produced by the Eurocode model (lane and axle load) are multiplied by a static load factor (extreme type 1) and a dynamic load factor (normal).

The normalised reliability profile for the yield line ULS (full width failure) and the corresponding probability of failure profile are shown in *Figure 7*. The reliability index at time $t = 0$ is $\beta_0 = 11.5$. Due to the size of the concrete

Table 1 Stochastic modelling used for the ULS

No.	Type	Par. 1	Par. 2	Description
1	Normal	550.0	10.0	Thickness of slab [mm]
2	LogNormal	30.0	6.0	Cube strength of concrete [MPa]
3	Normal	23.6	0.4	Density of concrete [kN/m^3]
4	LogNormal	289.0	25.0	Yield strength: longitudinal reinforcement [MPa]
5	Normal	60.0	8.0	Cover on longitudinal reinforcement [mm]
6	LogNormal	289.0	25.0	Yield strength: transverse reinforcement [MPa]
7	Normal	86.0	8.0	Cover on transverse reinforcement [mm]
8	Fixed	10053.0	—	Longitudinal reinforcement area (initial) [mm^2]
9	Fixed	565.0	—	Transverse reinforcement area (initial) [mm^2]
10	Gumbel	0.352	0.026	Static load factor [—]
11	Normal	1.27	0.20	Dynamic load factor [—]
12	Normal	1.08	0.072	Chloride concentration on surface [%]
13	Fixed	0.0	—	Initial chloride concentration [%]
14	Normal	35.0	2.5	Diffusion coefficient [cm^2/s]
15	Normal	0.4	0.05	Critical chloride concentration [%]
16	Uniform	2.5	0.29	Corrosion parameter [—]
17	Normal	1.0	0.05	Model uncertainty variable [—]

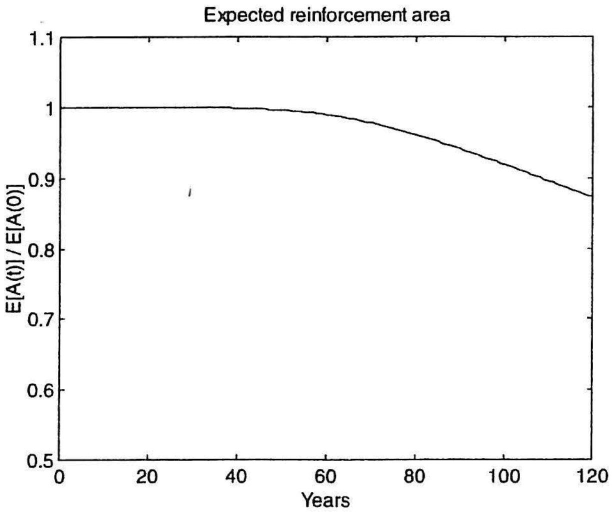

Figure 6 Relative expected reinforcement area $E[A(t)]/E[A(0)]$ as a function of time

cover (mean value 60 mm) the deterioration does not have any effect until year 70.

The results from the sensitivity analysis with regard to the mean values are shown for $t = 0$ years and $t = 120$ years in *Figure 8*. The sensitivity measure shown is the reliability elasticity coefficient. The meaning of the elasticity coefficient e_p is the following. If a parameter p is changed 1% then the reliability index is changed e_p%. The most important variables are, as expected, the thickness of the slab, the yield strength of the reinforcement, and the model uncertainty. Observe that the magnitude of sensitivity with regard to the cover changes from negative at time $t = 0$ to positive at time $t = 120$ due to the corrosion.

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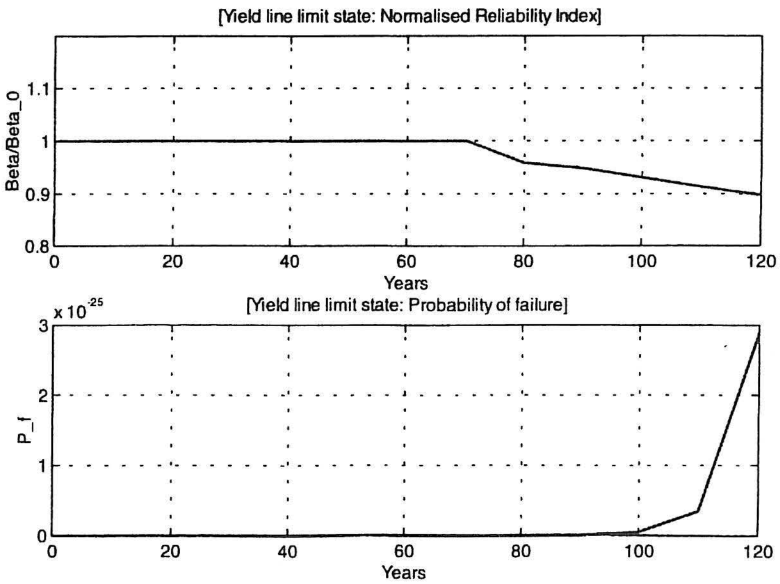

Figure 7 Reliability profiles using a yield line limit state

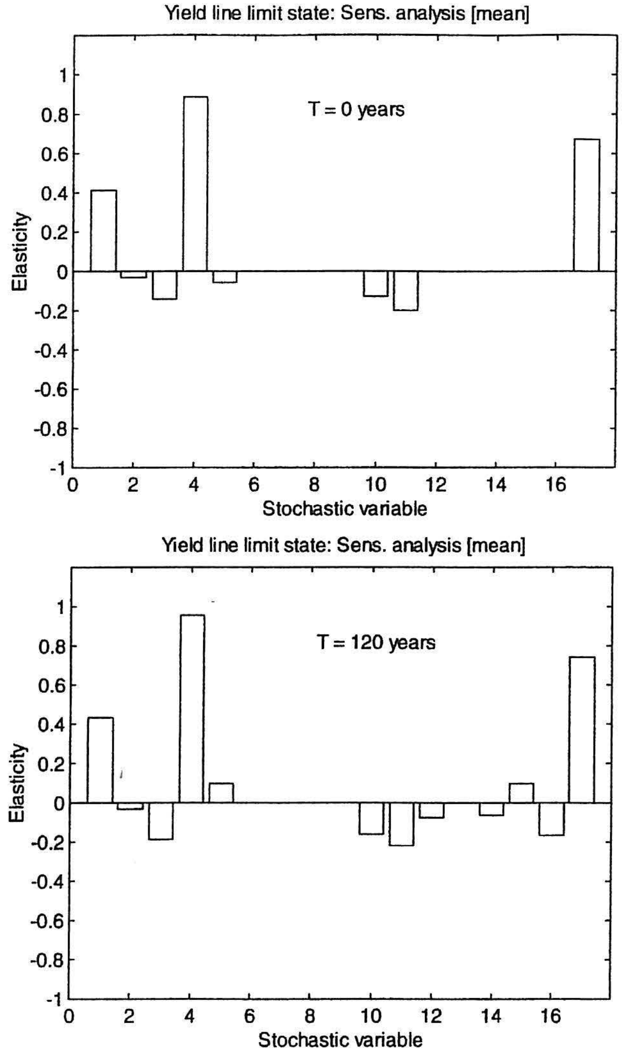

Figure 8 Sensitivity analysis for yield line limit state at $t = 0$ years and at $t = 120$ years

mental philosophy of this work. However, any views expressed in this paper are not necessarily those of the Highways Agency of the U.K. Department of Transport. The author would also like to thank Dr F.M. Jensen, Dr C. Middleton and Dr A. Blackmore for their significant contributions to the research presented here and in Thoft-Christensen *et al.*[2].

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