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STRUCTURAL RELIABILITY THEORY
PAPER NO. 146

To be presented at the EURODYN '96, Florence, Italy

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ABSTRACT

This paper considers estimation of the Maximum Softening Damage Indicator (MSDI) by using time-frequency system identification techniques for an RC-structure subjected to earthquake excitation. The MSDI relates the global damage state of the RC-structure to the relative decrease of the fundamental eigenfrequency in an equivalent linear model with slowly varying stiffness properties, displaying the combined damage effects of the maximum displacement ductility of the structure during extreme plastic deformations and the stiffness deterioration in the elastic regime, also called final softening. In order to use the MSDI, robust estimates of the fundamental eigenfrequency of the equivalent linear structure are required. Among many techniques available for system identification application of non-stationary signals, the so-called time-frequency techniques such as the Short-Time Fourier Transform (STFT), the Wavelet Transform (WT) and the Wigner-Ville distribution (WVD) are investigated in this paper. These identification techniques yield an optimum resolution in both time and frequency domain simultaneously. The techniques are evaluated using simulated data generated by the non-linear finite element program SARCOF modelling a 10-storey 3-bay concrete structure subjected to amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The evaluation of the three techniques shows that the WVD compared with the STFT and the WT is the most promising to estimate the softening.

Keywords: Time-Frequency Identification, Short-Time Fourier Transform, Wavelet Transform, Wigner-Ville Distribution, Maximum Softening Damage Indicator.

1. INTRODUCTION

During severe dynamic excitation such as major earthquakes the response of reinforced concrete structures is influenced by non-elastic stress-strain behaviour (hysteresis). Any plastic cyclic deformation implies that the structure suffers local or global damages, ranging from harmless cracking of hitherto uncracked cross-sections to bond deterioration at the interface between reinforcement bars and concrete, crushing of the concrete in compression zones, rupture of reinforcement bars and stirrups, etc. As a consequence of these damages, the damping and stiffness properties of the structure will vary with time. It has become common practice to instrument important buildings which may be exposed to excessive dynamic excitations in order to control the damage accumulation as measured by the sequential stiffness and strength deterioration.These damages are displayed in the dynamic response of the structure in terms of increased eigenperiods. The so-called Maximum Softening Damage Indicator (MSDI) is based on this principle, relating the global damage state of the structure to the relative decrease of the fundamental eigenfrequency, Di Pasquale et al. [1]. The development of any reliability measure such as the MSDI in future excitations is of paramount interest at the estimation of the residual reliability of the partially damaged structures. Furthermore, prediction of damage is also interesting in order to decide whether or not it is necessary to repair or demolish a structure. However, in order to use the MSDI, robust estimates of the fundamental eigenfrequency of the equivalent linear structure are required.

The estimation of the fundamental eigenfrequency may be carried out in the frequency domain or in the time domain. In Ljung [2] and Söderström et al. [3] the general basic features of system identification based on time and frequency domain approaches are highlighted. Much of the system identification research considering civil engineering structures deals with time-invariant systems. For such systems, traditional spectral analysis techniques based on Fourier Transform or digital filtering relying on the assumption of signal stationarity can provide a good description. Unfortunately, these techniques face some limitation when the signal to be analysed is from a non-stationary time-variant system such as an RC-structure subjected to an earthquake.

However, recently different techniques have been investigated for the problem of identifying such systems from signals with the time-varying characteristics. From a system identification the point of view, it is a problem of non-linear structure identification in which the form of the non-linearity as well as its associated parameters are to be identified. Numerous techniques for identification of non-linear structural systems have been developed, see e.g. Ling et al. [4] and Masri et al. [5]. Alternatively, to identify a non-linear model many researchers have tried to identify an equivalent linear model, such as a linear prediction error model, see e.g. DiPasquale et al. [1], Ghanem et al. [6], Shinozuka et al. [7], and Kirkegaard [8]. Others have used the Extended Kalman Filter to linearise the non-linear model and consider an extended state vector which, in addition to the response and its derivative, includes all the parameters to be identified, see e.g. Loh et al. [9] and Koh et al. [10]. All of the above system identification techniques require more or less user expertise in order to obtain satisfactory identification results. The techniques are based on an a priori choice of dynamic model and some rely upon qualified initial parameter estimates. Recently, a system identification technique has been proposed which requires less user expertise, see Mullen et al. [11]. This technique involves computation of an empirical transfer function from input/output time series pairs by using a moving window technique. In a simple way, this technique introduces time dependency into the Fourier analysis of the nonstationary signals. If only the output time series is used the proposed technique corresponds to estimation of the frequency content by using a Short Time Fourier Transform (STFT), see e.g. Oppenheim et al. [12], which maps a signal into a two-dimensional function in a time-frequency plane. The aim of this paper is to compare the STFT with the Wavelet Transform (WT) and the Wigner-Ville distribution (WVD) when only output time series are considered. The WT and the WVD are two other time-frequency signal processing techniques available for analysing nonstationary signals. Recently, these techniques have been considered with increasing interest with respect to the analysis of non-stationary signals from mechanical systems, see e.g. Micaletti et al. [13], Staszewski et al. [14], Newland [15], Surace et al. [16] and Auweraer et al. [17]. The investigations of the accuracy and reliability of the time-frequency techniques will be based on times series simulated by a non-linear finite element program SARCOF, Mørk [18]. This program has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffness of cracked and uncracked concretd of all beam elements must be specified) when the structure is exposed to different levels of peak ground excitation.The program estimates the fundamental eigenfreqüency of the equivalent linear structure at each time step. The example considered in this paper is a 10-storey 3-bay concrete test model of an RC-structure subjected to earthquakes at different levels of peak ground excitation given as amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter.

2. TIME-FREQEUNCY SYSTEM IDENTIFICATION TECHNIQUES

In this section, the three different time-frequency system identification techniques adopted in this paper for time- frequency identification of a deteriorating RC-Structure exposed to stochastic dynamic load are described.

2.1 The Short-Time Fourier Transform (STFT)

Traditional spectral analysis techniques based on Fourier Transform or digital filtering, provides a good description of stationary and pseudo-stationary signals. Unfortunately, these techniques face some limitation when the signals to be analysed are highly non-stationary, i.e. signals with time-varying spectral contents. Therefore, an analysis adapted to nonstationary signals requires more than the Fourier Transform, e.g. a Short-Time Fourier Transform (STFT), see e.g. Oppenheim et al. [12]. The idea of the STFT is to split a nonstationary signal into fractions within which stationary assumptions apply and to carry out a Fourier Transform on each of these fractions. The STFT defined as the Fourier Transform of a Gaussian windowed time signal for various position, *b*, of the window. The two-dimensional time-frequency representation S(b,f) can be stated in terms of the inner products between the signal and a window

$$S(b,f) = \int_{-\infty}^{\infty} x(t) g^{*}(t-b) e^{-2j\pi f t} dt \tag{1}$$

where $x(t)$ is the signal, b is the time parameter, f the frequency parameter and $g(\ )$ is a complex window function. The series of spectra estimated by (1), each of them related to a time index, form a time-frequency representation of the signal. When $x(t)$ is windowed at the time b by multiplication by the window function g whose magnitude is approximately zero outside some effective time interval around b, and the Fourier Transform of the windowed signal is taken, the resulting spectrum indicates the frequency content of the signal at or around time b. As the window is lengthened, the frequency resolution improves since more samples are used in the Fourier Transform. However, longer windows reduce the time localization of the STFT since more of the signal is included in the Fourier Transform. This is an example of the uncertainty principle, and it can be shown that the product of frequency resolution Δf and time resolution Δt is limited to, see e.g. Rioul et al. [19]

$$\Delta t \Delta f \geq \frac{1}{4\pi} \tag{2}$$

Various choices of the window functions are possible, but the resolution relationship is only an equality where the windowing function is a Gaussian, and the resulting STFT is known as the Gabor Transform due to Gabor (1946) who first adapted the STFT for time-frequency analysis. STFT provides constant absolute bandwidth analysis and constant resolution in time as well as in the frequency domain irrespective of the actual frequency.

2.2 The Wavelet Transform (WT)

In 1982 the Wavelet Transform, WT, was introduced in signal analysis by the geophysicist J. Morlet and since then it has received much attention, especially in mathematics. In the 90'es there has also been an increasing interest in the field of sound and vibration, see. e.g. Staszewski et al. [14], Newland [15], Surace et al. [16], Auweraer et al. [17], Rioul et al. [19], Daubechies [20], Chui [21] and Meyers [22].

The WT is an example of a time-scale decomposition which is obtained by dilating a chosen function named a basic mother wavelet. The continuous WT is defined as follows, see e.g. Rioul [20]

$$W(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) g^{*}\left(\frac{t-b}{a}\right) dt \tag{3}$$

where b is a translation coefficient localising the window of data to be transformed, a is a dilatation parameter indicating a sort of the amount of window stretching and $g^{*}(\ )$ is the complex conjugated of a so-called basic mother wavelet function. So in short, the wavelet function, $g(\ )$ extracts time scaled information from the signal around time, b, by means of inner products between the signal and scaled versions of the wavelet. Another interpretation of the wavelet is that the WT for different a and b is simply the projection of $x(t)$ onto the space with the bases consisting of the family of mother basic wavelet functions derived from dilatations, by changing a, and translations, by changing b of the mother wavelet. This implies that a given time series can be written by a decomposition of the signal in terms of the wavelet functions. Compared with the Fourier Transform this corresponds to a signal $x(t)$ can be described by means of a set of functions $e^{i\omega t}$, forming a "basis" for signal expansion.The difference between STFT and WT can be explained by the way the weighting function works. The STFT uses a weighting function which consists of a number of modulation frequencies, as many as frequencies lines produced by the chosen FFT-transform. The time signal to be analysed is projected (compared with) onto these modulation frequencies in order to obtain the frequency contents of the time signal. The basic wavelet contains only one modulation frequency, thus the wavelet must be rescaled (i.e. compressed or expanded) in order to extract the frequency content of the signal at other frequencies than the frequency of the basic wavelet. This means that the WT is not a true time-frequency technique as the STFT, but rather a time scale-tool. So in order to interpret the WT as a time-frequency technique, a connection between scale a and frequency f has to be established.

If a wavelet is considered in the frequency domain, the spectrum corresponds to a bandpass filter centered around the frequency f_o where this centre frequency is the reciprocal of the time period of the wavelet and the bandwitdh depends on how many time periods (oscillations) that are included in the wavelet, i.e. the length of the wavelet. Scaling in time domain corresponds to a translation in the frequency domain: The spectrum of the expanded wavelet is translated towards low frequencies, while the compressed wavelet is translated towards high frequencies. The relation between "scale" and "frequency" becomes evident here and the frequency of the signal f is given by the following relationship, Staszewski et al. [14].

$$f = \frac{f_0}{a} \tag{4}$$

Basically, by expanding the wavelet, low frequency information is extracted, and by compressing the wavelet, high frequency information is extracted. So WT favours the time resolution, when analysing high frequency components and privileges the frequency resolution when dealing with low frequencies, in contranst to STFT, which offers constant resolution in both time and frequency. In addition, the WT leads to an analysis with constant percentage (or relative) band width, while STFT provides constant band width analysis, as mentioned above. Since the duration, Δt, of the wavelet in the time domain is proportional to the scaling factor *a*, while the wavelet filter bandwidth, Δf, in the frequency domain is inversely proportional to the scaling factor, the product of the time duration and wavelet filter bandwidth is a constant. It can be shown, see e.g. Rioul [19] that this constant is always larger than or equal to 1/4π, which means that the uncertainty relationship (2) for the STFT also applies to the WT. In this paper the Morlet wavelet is used because it is closely related to Fourier analysis and therefore easy to understand and implement. The Morlet or "Gaussian" wavelet is defined as, see e.g. Cohen [23]

$$g(t) = e^{j2\pi f_0 t} e^{-0.5|t|^2} \tag{5}$$

However, the question of which wavelet is best suited for a particular application is very difficult to answer and is the subject of much research. Any wavelet must satisfy certain admissibility criteria, see e.g Daubechies [20], e.g. absence of negative frequencies, linearity, and square integrability (finite power). Although the Morlet wavelet is strictly compact in neither the time nor the frequency domain, admissible wavelets may turn out to be compact in either domain, but not both. The work of Daubechies [20] produces families of wavelets, compact in the time domain, generated from dilatation equations. In Newland [15] algorithms for finding discrete wavelets transform based on Daubechies wavelets which was developed by Mallat [24]. Newland has also developed harmonic wavelets, see Newland [15], whose dilatations are more readily converted to frequencies; these wavelets are compact in the frequency domain, but extend to infinity in the time domain.

2.3 The Wigner-Ville Distribution

The Wigner-Ville Distribution, WVD is a more global transform and is regarded as being the most fundamental of all time-frequency distributions. It is the main particularity of this transform that it does not give any restriction on the simultaneous resolution in time and in frequency. In other words, the WVD is not limited by the uncertainty principle, since it is a more general transform, not using an analysing function.The so-called Wigner distribution is defined by, see e.g. Cohen [23]

$$WV(b,f) = \int_{-\infty}^{\infty} x(b+\tau/2)x^*(b-\tau/2)e^{-j2\pi f\tau}d\tau \qquad (6)$$

where b is the time parameter and f is the frequency. It is seen from (6) that this distribution is a kind of combined Fourier Transform and autocorrelation calculation, i.e. autospectrum estimate as a function of time or autocorrelation estimate as function of frequency. Although the distribution is truely two-dimensional, its interpretation is not straightforward since the link between time-localization of the phenomena and the time parameter of the distribution is not unique. It can be seen from (6) that the time-localised phenomena at the time t_0 or repeated phenomena occurring at t_0-t_1 and t_0+t_1 will both give an output at t_0. This means that this technique can be a highly non-local distribution, since it reflects properties of the signal that it has at other times. Another problem with the Wigner distribution is the signal multiplication. This implies that the sampling frequency must be at least 4 times higher than the Nyquist frequency in order to avoid spectrum aliasing. One can use the usual sampling frequency according to the Nyquist criterion if, instead of the signal $x(t)$, the complex analytical signal of it is used, where the imaginary part is calculated via the Hilbert Transform. When the complex analytical signal is used, the Wigner distribution is very often called the Wigner-Ville distribution, WVD. An advantage of the WVD is that all the signal energy is concentrated around the sought frequency when a single component signal is considered. Far more important, however, the same feature is maintained also in the case of multiple component signals allowing for the isolation of each single vibrating contribution in $x(t)$. However, resolution problems can arise in the case of close frequencies due to cross-terms introduced by the bilinearity of the WVD, see e.g. Cohen [23], which are irrelevant from a physical point of view. Such interference can be removed by using an appropriately smoothed version of the WVD, see e.g Cohen [23]. The smoothed WVD can be defined by, see e.g. Rioul et al. [19]

$$\hat{WV}(b,f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(b-\tau,f-\xi)WV(\tau,\xi)d\tau d\xi \qquad (7)$$

It can be shown, Rioul et al. [19], that it is possible to obtain the STFT or WT from the WVD by a proper choice of smoothing function $G(\; , \;)$. Therefore, the WVD is a more general time-frequency analysing technique than the STFT and the WT. Unfortunately the amount of calculation involved is much bigger than in the case of the STFT and WT.

3. EXAMPLE

In this example the system identification techniques described in section 2 will be investigated in a simulation study. The investigations will be based on times series simulated by a non-linear finite element program SARCOF, Mørk [18], which has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffnesses of cracked and uncracked concrete of all beam elements must be specified). The program estimates the fundamental eigenfrequency of the equivalent linear structure at each time step.

The computer program SARCOF is a non-linear finite element program which is able to handle severe inherent material non-linearities and it is able to handle the following items:

- Unsymmetric cross-sections with different yield capacities at positive and negative bending.
- Interaction of bending moments and axial forces.
- Stiffness and strength degradation during plastic deformation.
- Pinching effect of moment-curvature relation due to shear loading.
- Finite extensions of plastic zones at the end of the beams.

The program is based on a full non-linear description of the internal degrees of freedom, which controls the hysteresis. In order to save computertime the external degrees of freedom, i.e. the global displacements are described by a truncated expansion in the eigenmodes of the undamaged structure, see Mørk [18].

3.1 Test Structure

The computer model, see figure 1, models a test structure, see Cecen [25], which consists of two 10-storey, 3-bay frames working in parallel with storey weights, uniformly distributed, attached in between.
The total height of the structure is 2.29 m and all storey heights are uniformly distributed. The columns and beams in the structure are 0.038 m wide, 0.051 m and 0.038 m deep for the columns and beams, respectively. Furthermore, all columns and beams are symmetrically reinforced. The following values are used for the density $\rho = 2500$ kg/m^3, the stiffness $E = 2.0 \cdot 10^{10}$ N/m^2 and the damping ratio $\zeta = 0.035$. The stiffness and strength deterioration are modelled using a Clough-Johnston hysteretic model. In this deteriorating model the limit value e_0 is taken as 26 and the decay parameter e_1 is taken as 12. The first eigenfrequency of the structure is 3 Hz. The test model is shown in figure 1. The M_i's refer to different material properties, see Cecen [25].

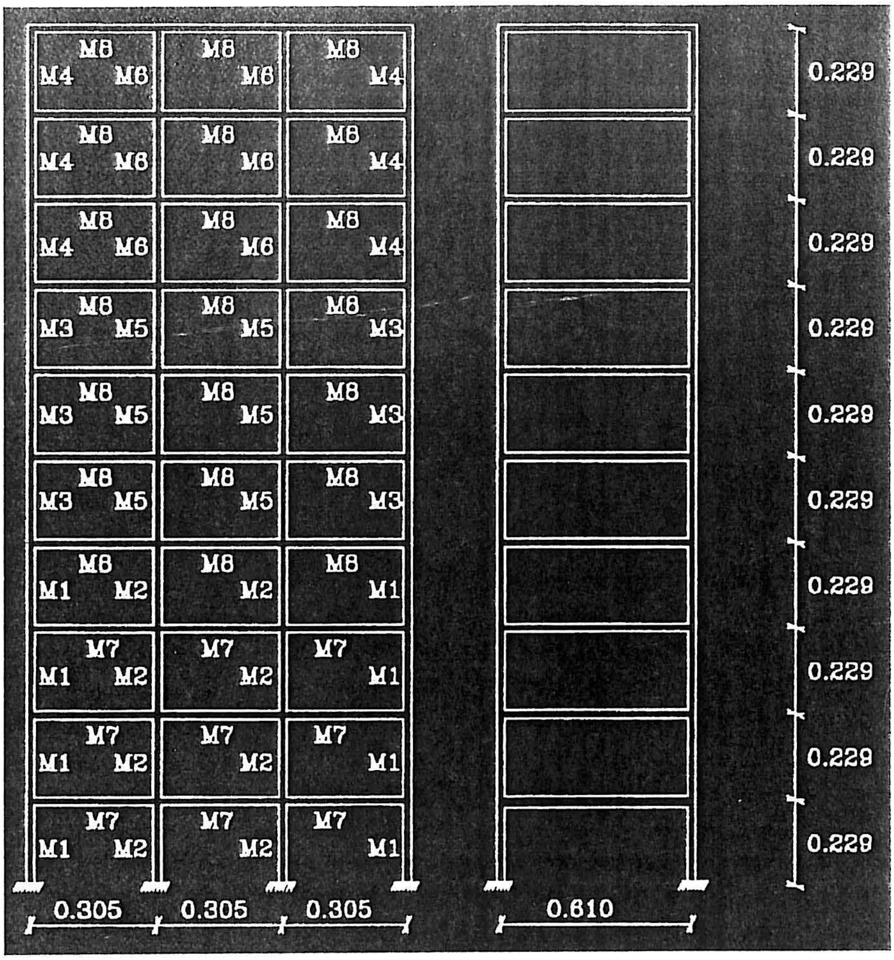

Figure 1: Computer model for the 10-storey 3-bay Reinforced Concrete Frame.

The excitation applied to the test structure was simulated earthquake runs for the basement motions that were patterned after the North-South component of the acceleration history measured at El Centro during the Imperial Valley Earthquake of 1940. The acceleration process at the ground surface is determined as the response process of an intensity modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The excitation process *F(t)* is then obtained from the stochastic differential equations, see Tajimi [26]

$$F(t)=2\zeta_s\omega_s\dot{U}(t)+\omega_s^2U(t) \tag{8}$$

$$\ddot{U}(t)+2\zeta_s\omega_s\dot{U}(t)+\omega_s^2U(t)=-\beta(t)W(t) \tag{9}$$

F(t) can then be interpreted as the negative part of the ground surface acceleration, and ζ_s and ω_s are the damping ratio and the circular eigenfrequency of a single degree-of-freedom shear model of the underlying subsoil. *W(t)* is a unit intensity white noise with the auto-spectral density function 0.5π generated by the broken line process model of Ruiz and Penzien, see Clough et al. [27]. The deterministic modulation function is given as follows see, Jennings et al. [28]

$$\beta(t)=\beta_0 \begin{cases} (\frac{t}{t_1})^2 , & 0 \le t \le t_1 \\ 1 , & t_1 < t < t_0 + t_1 \\ \exp(-c(t-t_0-t_1)) , & t_0 + t_1 \le t \end{cases} \tag{10}$$

The following parameters are used for the Kanai-Tajimi filter and the modulation function. The damping ratio in the Kanai-Tajimi filter is chosen as $\zeta_s = 0.65$ and the circular frequency is chosen as ω_s = 19.8.s^{-1}. In the modulation function the decay parameter c is 0.2. The excitation has maximum acceleration at t_1 = 3 sec. and duration of the strong motion is t_0 = 3 sec.

The integrated dynamic system is in SARCOF solved by a 4th Runge-Kutta Scheme. The time step is selected as 0.004 sec., where it has been proven that no drift occurs in the simulated signal.

3.2 Maximum Softening Damage Indicator (MSDI)

The changes in stiffness are normally very high and the stiffness changes are very fluctuating during an earthquake. An example of how the first eigenperiod *T(t)* of the equivalent linear structure, and therefore also the stiffness of the 10 storey, 3 bay reinforced concrete structure is slowly varying is presented in figures 2-4.The reason for this fluctuating behaviour is simply due to the fact that the structure changes rapidly from being in the elastic to the plastic regime. A measured change in the eigenperiod as illustrated in figure 2 is of course not suitable for prediction of the damage at a given time t_1 during the quake. It is therefore necessary to perform a smoothing of the measured eigenperiod which corresponds to time-averaging the structural degradation. A time-averaging method of the instantaneous period has been proposed by Rodriguez-Gomez [29] and is based on the principle of a moving averaging window in the following way. The smoothed value $<T(t_1)>$ at the time t_1 is evaluated as

$$<T(t_1)> = \frac{1}{T_a}\int_{t-\frac{T_a}{2}}^{t+\frac{T_a}{2}} T(t)dt \tag{11}$$

where T_a is the length of the averaging window, which should be sufficiently large, so that the local peaks are removed. On the other hand, T_a should not be selected so large that intervals of increased plastic deformation are not displayed in $<T(t_1)>$. The value $T_a = 2.4\, T_0$ is recommended as a reasonable compromise, Rodriquez-Gomez [29], where T_0 is the first eigenperiod of the equivalent linear structure.

Based on $<T(t_j)>$, the instantaneous softening, $\delta(t)$, of a structure is defined as Çakmak et al.[1]

$$\delta(t) = 1 - \frac{T_0}{<T(t)>} \tag{12}$$

The maximum softening damage indicator, δ_M, is the maximum of $\delta(t)$ during the seismic excitation. Obviously, the damage indicator $\delta(t)$ is non-decreasing with time and attains values in the value [0;1], where $\delta(t) = 0$ corresponds to an undamaged structure.

In figure 1 the instantaneous damage $\delta(t)$ is shown as a function of time. The damage is estimated by using different lengths of the running window.

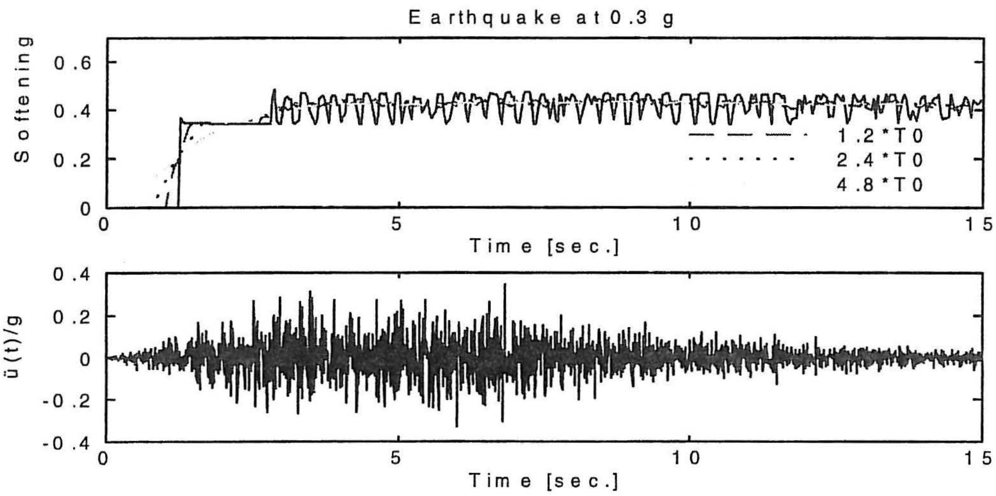

Figure 2: The damage indicator $\delta(t)$ as a function of time for different choices of the length of the running window. Peak ground surface acceleration = 0.3 g.

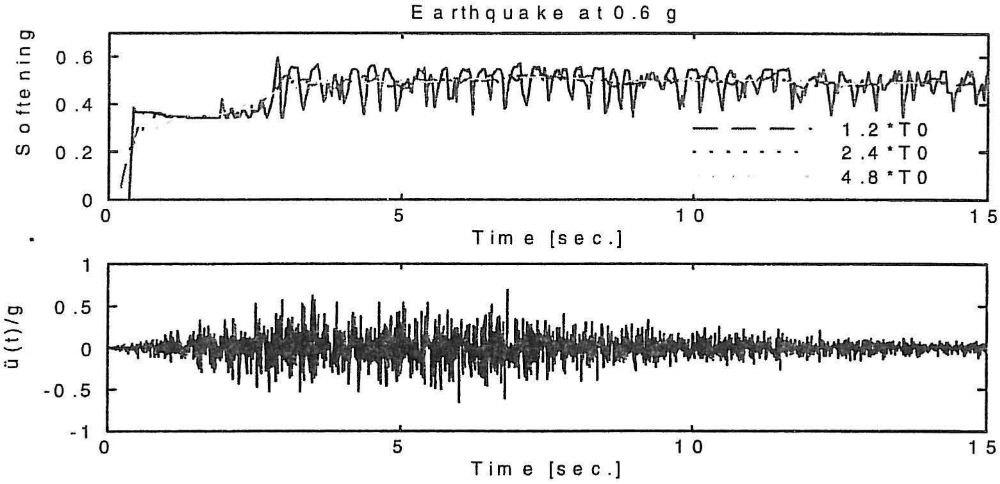

Figure 3:. The damage indicator δ(t) as a function of time for different choices of the length of the running window. Peak ground surface acceleration = 0.6 g.

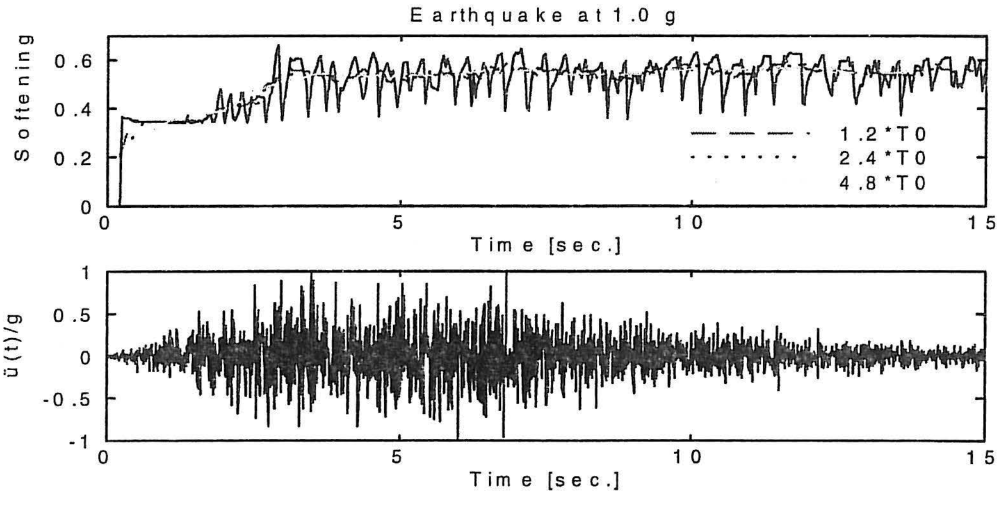

Figure 4: The damage indicator δ(t) as a function of time for different choices of the length of the running window. Peak ground surface acceleration = 1.0 g

From figures 2, 3 and 4 it is seen that the choice of the length of the running window proposed in Rodriques et al. [29] at 2.4 T_0 seems to be a reasonable compromise. This means that in the following, that size of window is used.

3.3 Results

This section presents the softening results estimated by the STFT, the WT and the WVD, respectively, from the response presented in section 3.2.

Figure 5, 6 and 7 show the softening results obtained by the STFT, the WT and the WVD, respectively for the three different earthquake levels. The estimated softening values are shown together with the smoothed softening values determined by SARCOF from the simulations.The instantaneous frequency used for the estimation of the softening values is assumed to correspond to the maximum peak values in the Spectrogram for the STFT, the Scalogram for the WT and the WVD. The Spectrogram defined as the square modulus of the STFT provides a distribution of the energy of the signal in the time-frequency plane. A similar distribution can be defined in the wavelet case. The total energy E_x of the signal can be expressed in terms of the WT as

$$E_x = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |W(b,a)|^2 \frac{da\,db}{a^2} \tag{13}$$

where the Scalogram is defined as the modulus of the WT. Equation (13) allows us to interpret the Scalogram as a density of the energy distribution over the time-scale plane *(b;a)*.

The results in figure 5 are obtained by the STFT where signals were analysed using 512 points windowed by a Hanning window. The STFT was estimated using 500 overlapping window points. A robust identification of the fundamental frequency seemed not to be possible using less than approximately 512 points. The results in figure 5 show that the STFT overestimates the maximum softening for earthquakes at level 0.3 g and 0.6 g, respectively. This means that the fundamental frequency has been underestimated. For the results corresponding to an earthquake at 1.0 g the maximum softening is estimated in the confidence band.

Figure 6 shows the results obtained by the WT. As mentioned above, the instantaneous frequency is determined as the frequency corresponding to the maximum peak of the Scalogram, i.e. the frequency corresponding to the maximum value of $|W(m,n)|^2$, where $W(m,n)$ is a frequency implementation of (1) given as, Staszewski et al. [14]

$$W(m,n) = \sqrt{m\Delta a} \sum_{f_n} X(f_n) G^*(m\Delta a f_n) e^{i2\pi\Delta b f_n} \tag{14}$$

where f_n is the discretised frequency whilst Δa and Δb, respectively, are dicretised increments of dilatation and translation parameters. $X(\ )$ and $G(\ )$ are the Fourier Transform of the $x(t)$ and $g(t)$, respectively. This expression can take advantage of the FFT algorithm by transforming convolutions in products and returning back to time domain. Values of the WT were estimated based on a procedure using 512 points in the FFT. The scale parameter was chosen as Δa=0.01. Figure 6 shows that the WT predicts values of the softening very well for an earthquake at level

1.0 g. However, it is also seen that the values of the softening are underestimated and overestimated for the earthquakes at level 0.3 g and 0.6 g, respectively. It was found that the reliability of the values of the WT were sensitive to the choice of number of points used in the FFT. This problem is perhaps due to the fact that the WT favours the time resolution when analysing high frequency components. Perhaps, the problem could also be a result of a frequency implementation of the WT.

The results obtained by the WVD are presented in figure 7.The practical implementation of the WVD requires that the signal $x(t)$ has a finite duration. To ensure this the Pseudo WVD is introduced, which is defined as the WVD of a windowed time signal

$$W(t,f) = \int_{-\infty}^{\infty} |w(\tau/2)|^2 x(t+\tau/2) x^*(t-\tau/2) e^{-j2\pi f\tau} d\tau \qquad (15)$$

The time resolution of the WVD (15) is the sampling interval, while the chosen frequency resolution is directly related to the length of the chosen window $w(\ )$. This means that larger values of the window length can be chosen in order to obtain a better approximation of the true WVD. However, a larger window involves significantly more calculations. The window length was chosen as 800 points. Therefore the WVD is not calculated for the first and the last 400 points, respectively. The WVD was not smoothed by a double convolution, see (7), in order to get rid of the drawbacks with cross-terms, mentioned in section 2.3. Instead of a time-averaging method of the instantaneous frequncy has been used based on the principle of a moving averaging window. When the results in figure 7 are compared with the results obtained with the STFT and the WT, it seems that a smoothed WVD offers the best possible means of estimating the softening of an RC structure subjected to an earthquake. However, further investigations with respect to the smoothing of the WVD are required to fully establish the technique. It should also be mentioned that the amount of calculation involved is much bigger for the WVD than in the case of the STFT or WT. The implementaion of the three techniques used for the calculations in this paper showed that the ratio of the amount of calculation between the STFT, the WT and the WVD, respectively, was 1:300:400.

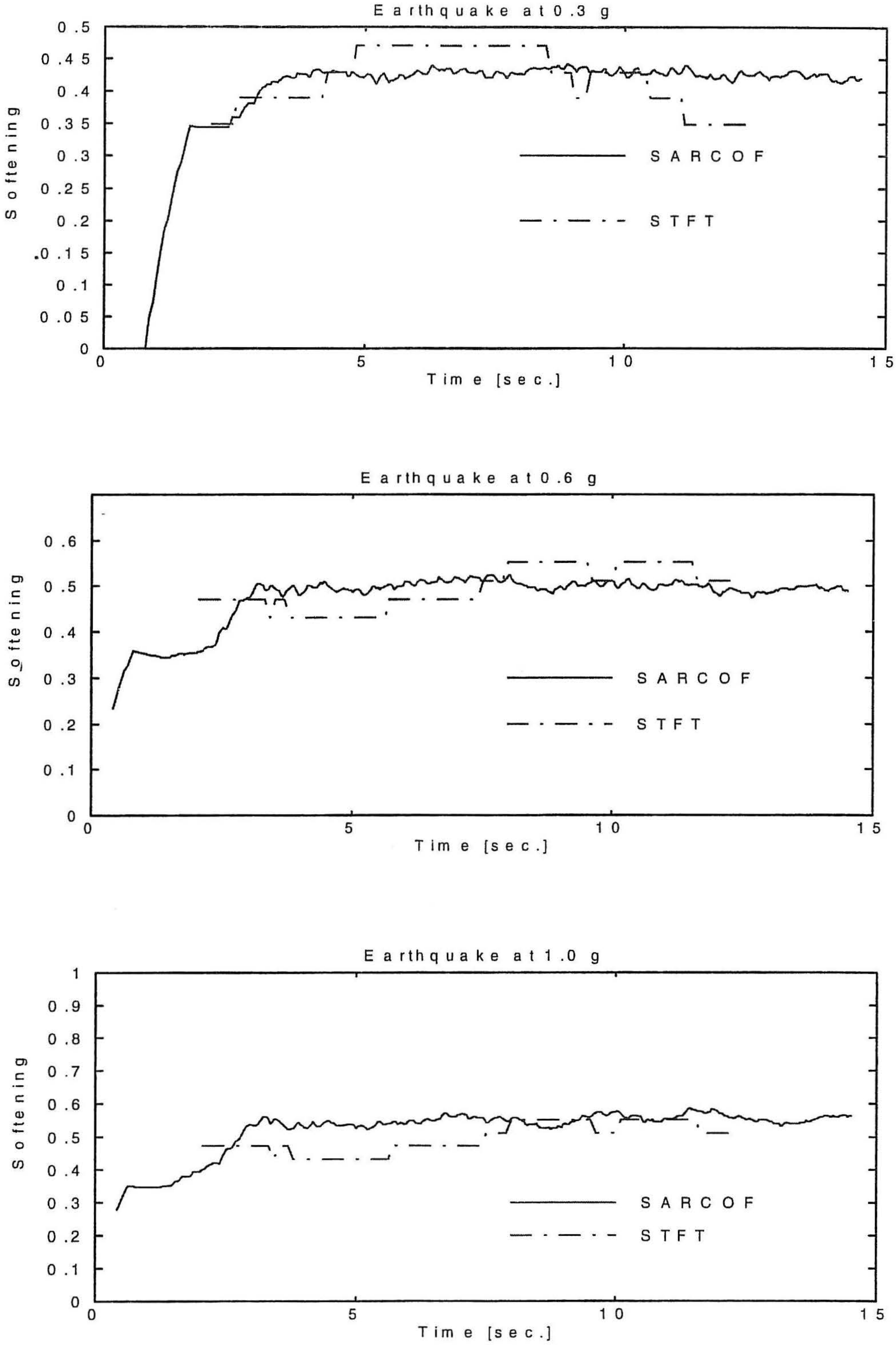

Figure 5: The damage indicator $\delta(t)$ as a function of time estimated by STFT.

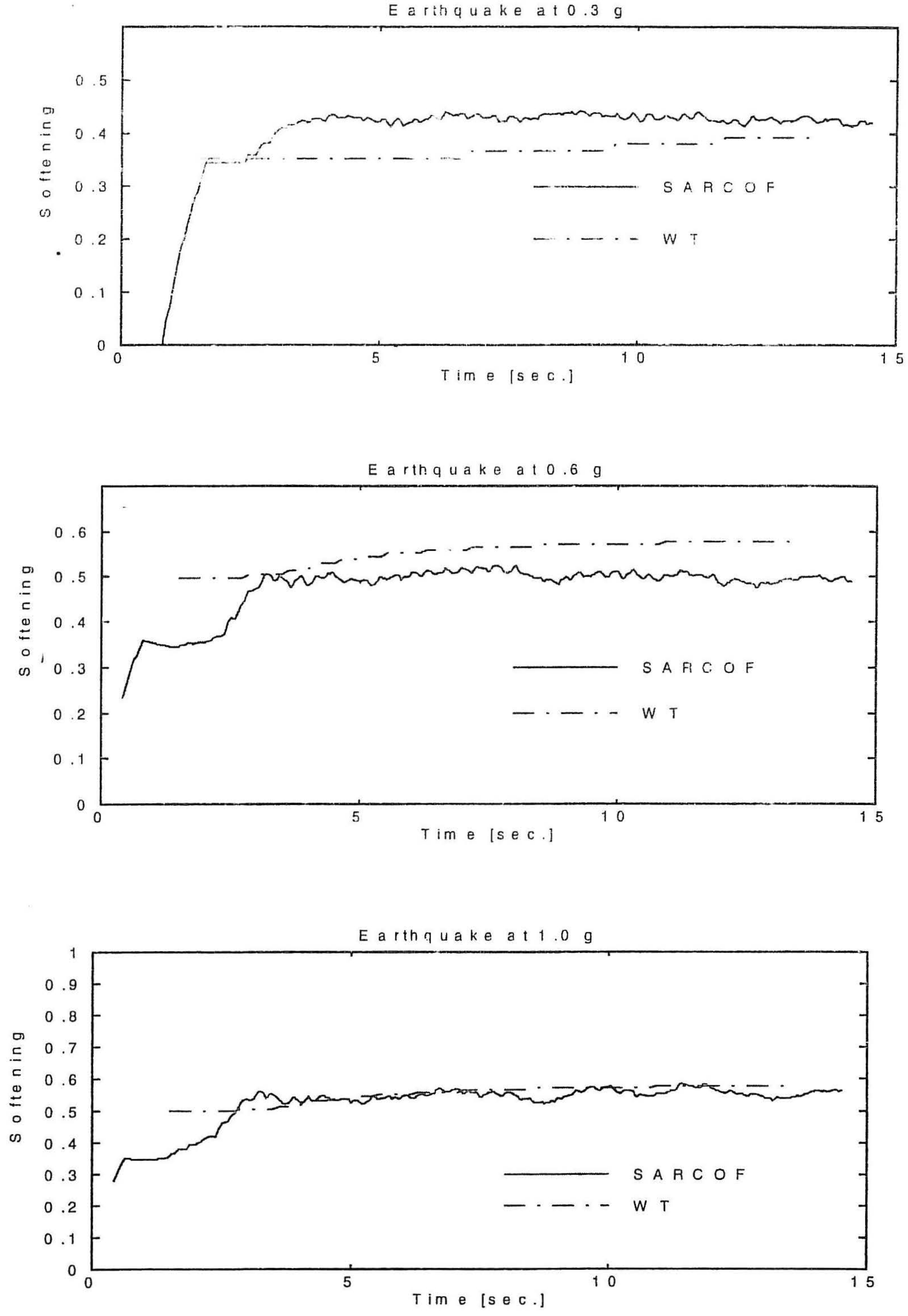

Figure 6: The damage indicator $\delta(t)$ as a function of time estimated by WT .

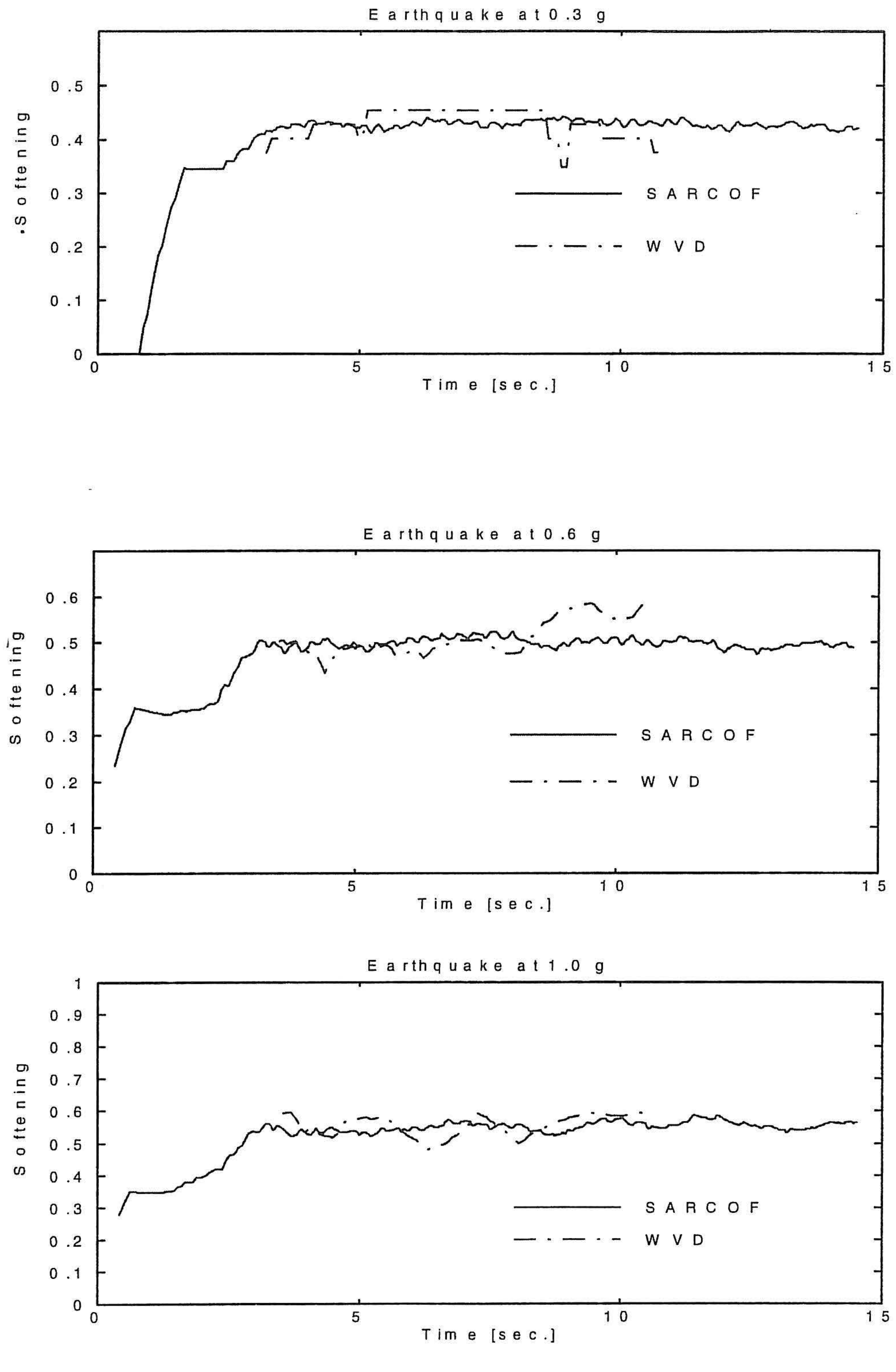

Figure 7: The damage indicator $\delta(t)$ as a function of time estimated by the WVD.

4. CONCLUSIONS

This paper considers estimation of the maximum softening by using the Short-Time Fourier Transform (STFT), the Wavelet Transform (WT) and the Wigner-Ville distribution (WVD) on simulated response from an RC-structure subjected to earthquake excitation. From the results presented in the paper, following conclusions can be drawn with respect to the three different time-frequency identification techniques. The STFT gives a relatively poor overall resolution which is restricted to the uncertainty principle but together with the WT it takes less computation time compared with the WVD. The STFT is easy to implement compared with the WT and the WVD, and therefore it is time-frequency identification techniques which should be used when it salifies the needs of the application. However, in this paper it is shown that the STFT does not satisfies the needs of the application. It was found that the reliability of the values of the WT was sensitive to the choice of number of points used in the implementation using the FFT. This problem is perhaps due to the fact that the WT favours the time resolution when analysing high frequency components. The evaluation of the three techniques shows that the WVD compared with the STFT and the WT are the most promising for estimation of the softening so the WVD seems to offer a possible means of estimating the softening of an RC structure subjected to an earthquake. However, further investigations are required to fully establish the WVD technique. Especially, different smoothing techniques of the WVD in the time domain as well as in the frequency domain should be investigated in order to give less fluctuating estimates of the softening.

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