

## Bayesian Compressed Sensing with Unknown Measurement Noise Level

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# BAYESIAN COMPRESSED SENSING WITH UNKNOWN MEASUREMENT NOISE LEVEL

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## Introduction

- In Bayesian compressed sensing (BCS) we apply approximate Bayesian inference to estimate a sparse vector  $\mathbf{w} \in \mathbb{C}^M$  from noisy measurements  $\mathbf{y} \in \mathbb{C}^N$  taken as

$$\mathbf{y} = \Phi \mathbf{w} + \mathbf{n},$$

with dictionary  $\Phi \in \mathbb{C}^{N \times M}$  and white Gaussian noise  $\mathbf{n} \in \mathbb{C}^N$ .

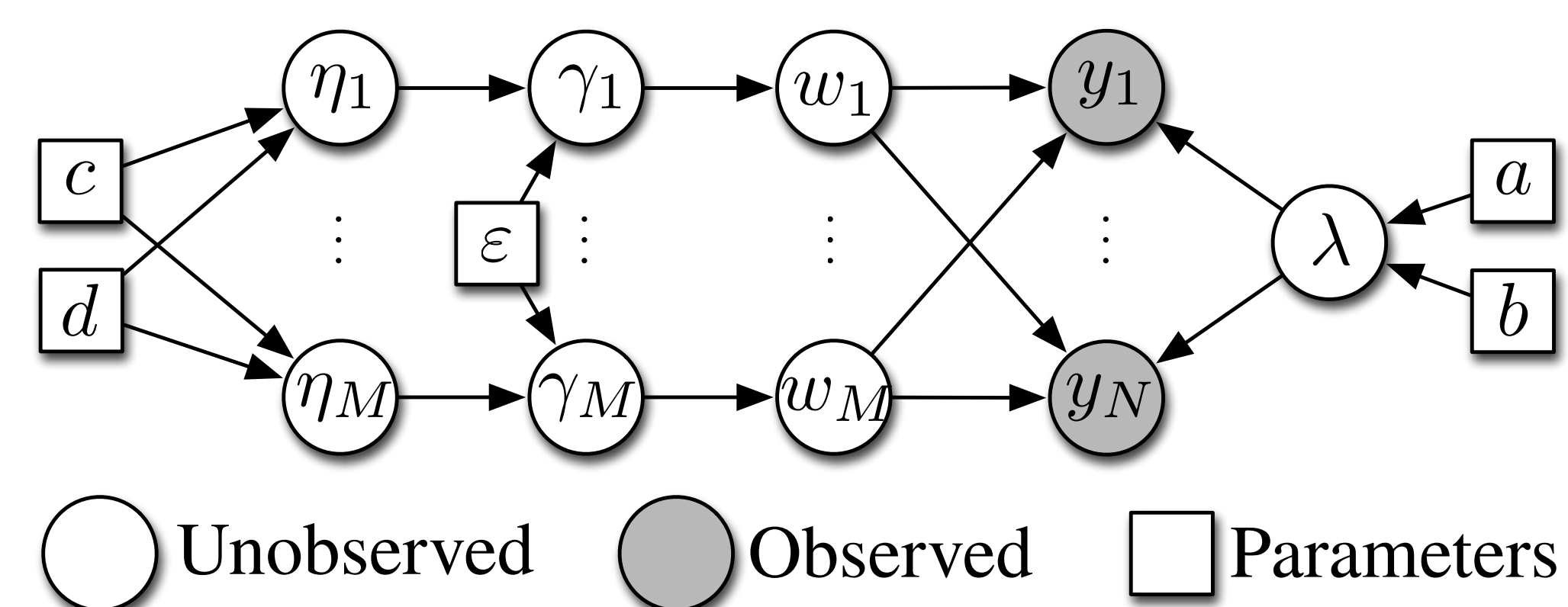
- In most BCS literature it is not tractable to estimate the noise precision  $\lambda$  (inverse variance) as an integral part of the Bayesian inference. Heuristic methods are instead employed resulting in increased computational complexity.
- In this work we propose to modify the three-layer hierarchical prior model in [1] such that the estimation of the noise precision can be included in the inference scheme with no penalty in terms of complexity.

## Hierarchical Prior Modelling

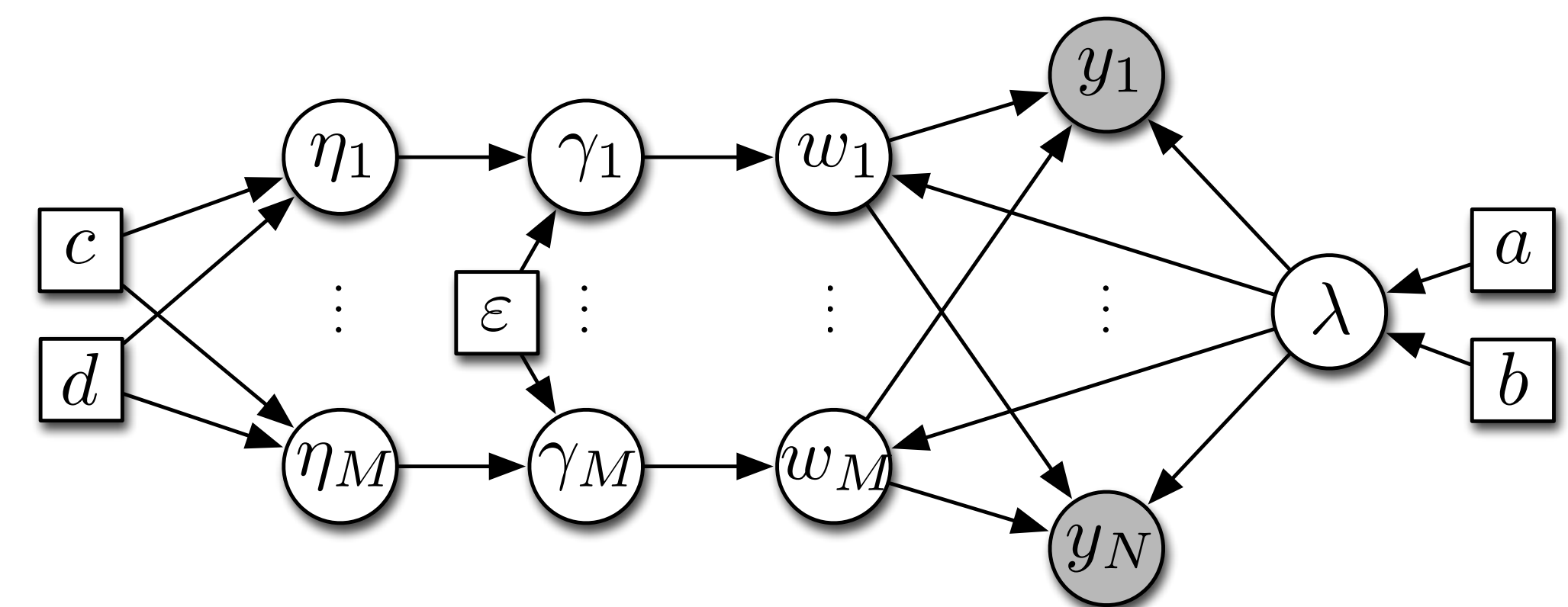
With  $\Gamma = \text{diag}(\gamma)$ , the probability distributions in the prior models are:

Density	Model in [1]	Proposed changes
Observation model $p(\mathbf{y} \mathbf{w}, \lambda)$	$N(\mathbf{y} \Phi \mathbf{w}, \lambda^{-1} \mathbf{I})$	-
Prior on $\lambda$ , $p(\lambda)$	$\text{Ga}(\lambda a, b)$	-
Layer 1 on weights, $p(\mathbf{w} \gamma)$	$N(\mathbf{w} \mathbf{0}, \Gamma)$	$N(\mathbf{w} \mathbf{0}, \Gamma \lambda^{-1})$
Layer 2 on weights, $p(\gamma \eta)$	$\prod_{i=1}^M \text{Ga}(\gamma_i \varepsilon, \eta_i)$	-
Layer 3 on weights, $p(\eta)$	$\prod_{i=1}^M \text{Ga}(\eta_i c, d)$	-

## Graphical Models



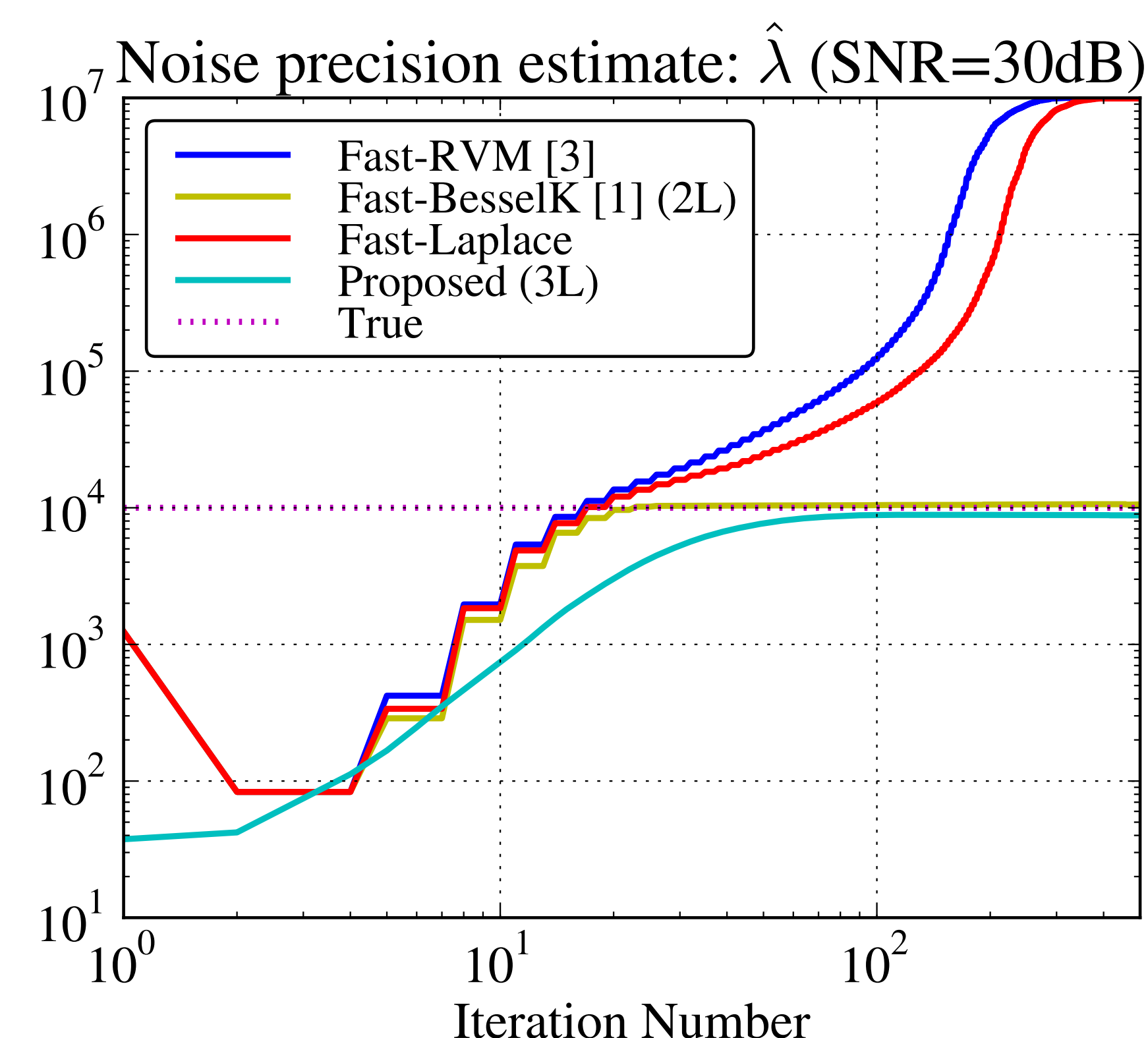
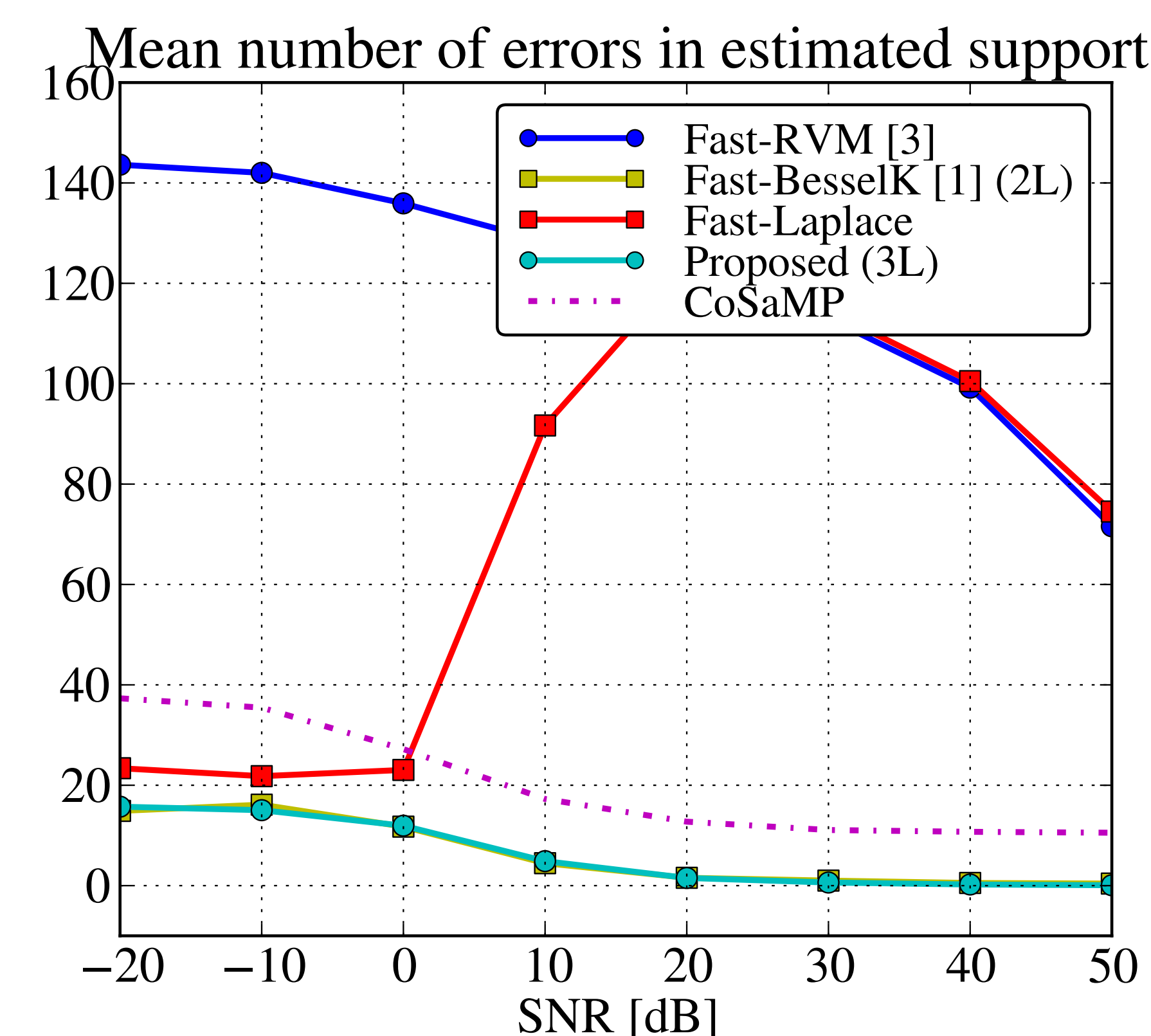
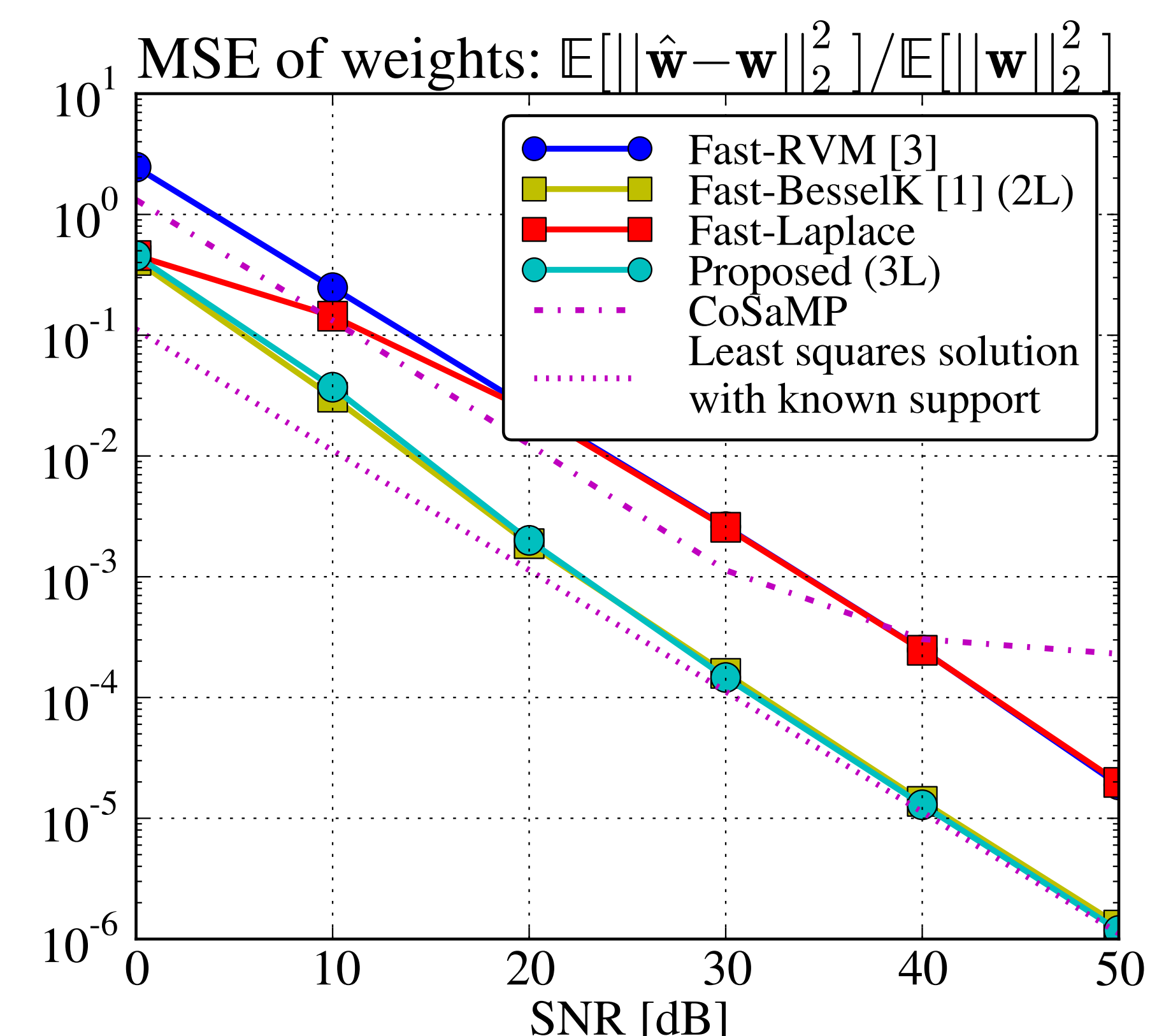
**Fig 1:** Hierarchical prior model as proposed by Lovmand et. al. [1].



**Fig 2:** Proposed model with the modification inspired by [2].

The relevance vector machine (RVM), its fast variant [3] and other algorithms proposed in the literature can be derived from special cases of the model in Fig. 1.

## Numerical Results



## Computational Complexity

- The fast implementation of the RVM [3] can only use its computationally efficient matrix-vector updates when the noise precision estimate is held fixed.
- Using our proposed model, these updates become independent of the noise precision estimate.
- We assume  $S \leq N \leq M$ , with  $N$  compressed measurements,  $M$  basis vectors in the dictionary and  $S$  nonzero entries in  $\mathbf{w}$ .
- Computational complexity per iteration:

	Model in [1] & [3]	Proposed
Fixed $\hat{\lambda}$	$\mathcal{O}(MN)$	$\mathcal{O}(MN)$
Updating $\hat{\lambda}$	$\mathcal{O}(MNS)$	$\mathcal{O}(MN)$

## References

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