Bayesian Compressed Sensing with Unknown Measurement Noise Level

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Published in:
Proc. 47th Asilomar Conference on Signals, Systems and Computers

DOI (link to publication from Publisher):
10.1109/ACSSC.2013.6810248

Publication date:
2013

Document Version
Early version, also known as pre-print

Link to publication from Aalborg University

Citation for published version (APA):

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Computational Complexity

- The fast implementation of the RVM [3] can only use its computationally efficient matrix-vector updates when the noise precision estimate is held fixed.
- Using our proposed model, these updates become independent of the noise precision estimate.
- We assume $S \leq N \leq M$, with $N$ compressed measurements, $M$ basis vectors in the dictionary and $S$ nonzero entries in $\mathbf{w}$.
- Computational complexity per iteration:

<table>
<thead>
<tr>
<th>Model</th>
<th>Proposed (3L)</th>
<th>$O(MN)$</th>
</tr>
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<tbody>
<tr>
<td>Fixed $\lambda$</td>
<td>$O(MN)$</td>
<td>$O(MN)$</td>
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</tbody>
</table>

Numerical Results

- Mean number of errors in estimated support
- Noise precision estimate

Graphical Models

Introduction

- In Bayesian compressed sensing (BCS) we apply approximate Bayesian inference to estimate a sparse vector $\mathbf{w} \in \mathbb{C}^M$ from noisy measurements $\mathbf{y} \in \mathbb{C}^N$ taken as

$$\mathbf{y} = \Phi \mathbf{w} + \mathbf{n},$$

with dictionary $\Phi \in \mathbb{C}^{N\times M}$ and white Gaussian noise $\mathbf{n} \in \mathbb{C}^N$.
- In most BCS literature it is not tractable to estimate the noise precision $\lambda$ (inverse variance) as an integral part of the Bayesian inference. Heuristic methods are instead employed resulting in increased computational complexity.
- In this work we propose to modify the three-layer hierarchical prior model in [1] such that the estimation of the noise precision can be included in the inference scheme without penalty in terms of complexity.

Hierarchical Prior Modelling

With $\Gamma = \text{diag}(\gamma)$, the probability distributions in the prior models are:

$$p(\mathbf{y}|\mathbf{w}, \lambda) \quad \text{Model in [1]} \quad \text{Proposed changes}$$

<table>
<thead>
<tr>
<th>Density</th>
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<tbody>
<tr>
<td>Observation model $p(\mathbf{y}</td>
<td>\mathbf{w}, \lambda)$</td>
<td>$N(\mathbf{y}</td>
</tr>
<tr>
<td>Prior on $\lambda$, $p(\lambda)$</td>
<td>$\text{Ga}(\lambda</td>
<td>a, b)$</td>
</tr>
<tr>
<td>Layer 1 on weights, $p(\mathbf{w}</td>
<td>\gamma)$</td>
<td>$\text{Ga}(\gamma</td>
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<tr>
<td>Layer 2 on weights, $p(\gamma</td>
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<tr>
<td>Layer 3 on weights, $p(\eta)$</td>
<td>$\prod_{i=1}^M \text{Ga}(\eta_i</td>
<td>\zeta, \eta_i)$</td>
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References