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Abstract— Influenza is a viral disease that usually affects the nose, throat, bronchi, and seldom lungs. This disease spreads as seasonal epidemics around the world, with an annual attack rate of estimated at 5%–10% in adults and 20%–30% in children. Thus, influenza is regarded as one of the critical health hazards of the world. Early diagnosis (consisting of determination of signs and symptoms) of this disease can lessen its severity significantly. Examples of signs and symptoms of this disease consist of cough, fever, headache, bireme, nasal congestion, nasal polyps and sinusitis. These signs and symptoms cannot be measured with near-100% certainty due to varying degrees of uncertainties such as vagueness, imprecision, randomness, ignorance, and incompleteness. Consequently, traditional diagnosis, carried out by a physician, is unable to deliver desired accuracy. Hence, this paper presents the design, development and application of an expert system to diagnose influenza under uncertainty. The recently developed generic belief rule-based inference methodology by using the evidential reasoning (RIMER) approach is employed to develop this expert system, termed as Belief Rule Based Expert System (BRBES). The RIMER approach can handle different types of uncertainties, both in knowledge representation, and in inference procedures. The knowledge-base of this system was constructed by using records of the real patient data along with in consultation with the Influenza specialists of Bangladesh. Practical case studies were used to validate the BRBES. The system generated results are effective and reliable than from manual system in terms of accuracy.

Keywords— Belief Rule Base (BRB); uncertainty; RIMER; Influenza, Expert System; Inference;

I. INTRODUCTION

Influenza's effects are much more severe and last longer than those of the common cold. Most people recover completely in about one to two weeks, but others may suffer from life-threatening complications (such as pneumonia). Thus, influenza can be deadly, especially for the weak, young and old, or chronically ill. Pregnant women and young children are also at high risk for complications. We know the worldwide death toll exceeds a few hundred thousand a year, but even in developed countries the numbers are uncertain, because medical authorities don't usually verify who actually died of influenza and who died of a flu-like illness. People think of the flu as a minor nuisance [1]. Even healthy people can be affected, and serious problems from influenza can happen at any age [2]. Medical diagnosis is the process of determining any disease or disorder in the human body from the signs and symptoms. Sign is directly discovered by the physician and symptom is obtained from patient experience and feelings. Sometimes,

patients cannot describe exact conditions. Consequently, symptoms become uncertain factors in the diagnosis process. Influenza symptoms may be evaluated as high, medium or low. For example, if nasal congestion is consistent with low Viremia symptom then the disease will be normal influenza but if nasal congestion is consistent with high Viremia symptom then the disease will be a different type of Flu. Usually, there is no definite direction for influenza assessment. Uncertainty prevails in almost every stage of medical decision making process, involving both medical domain knowledge and clinical symptoms [3].Rationally, reliably, and correctly handling uncertainties in medical diagnosis and treatment decisions are major challenges that have been researched for more than four decades [4]. Recently, much attention is given on the development of various expert systems based on different methodologies to support medical diagnosis [4].

This paper presents the development of an expert system (ES), based on a methodology, known as the belief rule-base inference methodology using the evidential reasoning (RIMER)[8–10]. The remaining of the paper is structured as follows. Section two presents the related works. Section three provides an overview of RIMER methodology. Section four presents the architecture, design and implementation of the proposed BRBES. Experimental results and discussion are then presented. A conclusion is included to summaries the contribution of the research.

II. LITERATURE REVIEW

An expert system (ES) in the clinical domain can be defined as software that is designed to be a direct aid to clinical decision-making, in which the characteristics of an individual patient are matched to a computerized clinical knowledge base and patient-specific assessments or recommendations, is then presented to the clinician or the patient for a decision. [4]. An ES consists of two components: knowledge base and inference mechanisms. The knowledge acquisition procedure begins with the selection of target clinical area (for instance, influenza) and selects expert clinicians to gain domain specific knowledge. Then the knowledge is transformed into computer-interpretable knowledge conforming to the design of the knowledge representation method. The knowledge acquisition tools have varying abilities to handle uncertainties involved with diagnostic and therapeutic decisions, by using the argumentation mechanism, a threshold value to compare diagnosis' scores, time annotations, and certainty factor or belief degree to model uncertainty. The inference mechanisms of ES are rule based, Bayesian system, Bayesian belief networks, heuristic, semantic network, neural networks, and genetic algorithms and case-based.

A belief rule-base is an extension of traditional IF- THEN rule base and is capable of handling different types of information with uncertainties[5]. In a traditional IF-THEN rule the consequent is

Boolean i.e., "true" or "false", indicating 100% certainty. For example, IF Viremia consistent with nasal congestion THEN (high clinical probability of) Influenza disease. However, in real-life diagnosis, a doctor cannot judge one patient's vermin to be 100% consistent with nasal congestion and he/she might describe expert opinion or judgment with belief levels, as one of the following: Viremia is *strongly consistent* with nasal congestion; Viremia is *little like* nasal congestion; Viremia *look like* nasal congestion with 50% probability and so on. Hence, the probability of Viremia for the patient can be high, medium or low with different degrees of belief. However, traditional IF-THEN rule is not capable of handling this kind of relationship between antecedents and consequents. If the above rule is extended with a belief structure, a belief rule can be extended in natural language. For example, IF Viremia is strongly consistent with nasal congestion THEN high clinical risk of influenza with a probability of 80%. Therefore, from the above it can be inferred that the uncertain knowledge that exist with the diagnosis of influenza should need to be processed by using refined knowledge representation schema and inference mechanism. This has been achieved by using RIMER methodology as will be discussed in the next section.

III. OVERVIEW OF RIMER METHODOLOGY

RIMER consists of mainly two parts [5]: the first part is the BRB, which is a domain knowledge representation schema with uncertain information, and the second part is Evidential Reasoning (ER) algorithm[6] that is used as an inference mechanism or to deduce inference. BRB is the extended form of traditional IF-THEN rule-base contains appropriate schema to capture different types of uncertainties and allows handling of non-linear causal relationships. Evidential Reasoning (ER) approach deals with multiple attribute decision analysis (MADA) problem having both qualitative and quantitative attributes under uncertainties and hence, facilitates handling of uncertainty in the inference process.

A. Domain Knowledge Representation Using BRB

Belief Rules are the key constituents of a BRB, which include belief degree and are the extended form of traditional IF-THEN rules. In a belief rule, each antecedent attribute takes referential values and each possible consequent is associated with belief degrees [7]. Rule weights, antecedent attribute weights and belief degrees with consequent are considered as the knowledge representation parameters. A belief rule can be defined in the following way.

$$R_k: \begin{cases} \text{IF } (P_1 \text{ is } A_1^k) \wedge (P_2 \text{ is } A_2^k) \wedge \dots \dots \wedge P_{T_k} \text{ is } A_{T_k}^k \\ \text{THEN } \{(C_1, \beta_{1k}), (C_2, \beta_{2k}), \dots \dots, (C_N, \beta_{Nk})\} \end{cases} \quad (1)$$

$$R_k: \left(\beta_{jk} \geq 0, \sum_{j=1}^{N} \beta_{jk} \leq 1 \right) \text{ with a rule weight } \theta_k \text{, attribute}$$

weights $\delta_{k1}, \delta_{k2}, \delta_{k3}, \dots \dots, \delta_{kT_k}$ $k \in \{1, \dots \dots, L\}$ Where P_1, P_2, P_3 ...P_{Tk} represent the antecedent attributes in the kth rule. A_i^k ($i = 1, \dots \dots, Tk, k=1, \dots \dots, L$ represents one of the referential values of the ith antecedent attribute P_i in the kth rule. C_j is one of the consequent reference values of the belief rule. β_{jk} ($j = 1, \dots \dots, N, k = 1, \dots \dots, L$) is the degree of belief to which the consequent reference value C_j is believed to be true. If $\sum_{j=1}^{N} \beta_{jk} = 1$ the kth rule said to be complete; otherwise, it is incomplete. T_k is the total number of antecedent attributes used in kth rule. L is the number of all belief rules in the rule base. N is the number of all possible consequent's referential values in a rule.

An example of a belief rule by taking account of influenza can be written in the following way.

$$R_k: \begin{cases} \text{IF } (Nasal \ congestion \ is \ High) \wedge (Viremia \ is \ Medium) \\ \text{THEN } Influenza \\ \quad \{(High, (0.80)), (Medium, (0.20)), (Low, (0.00))\} \end{cases} \quad (2)$$

Where $\{(High, (0.80)), (Medium, (0.20)), (Low, (0.00))\}$ is a belief distribution associated with influenza consequent of the belief rule as represented in (2). The belief distribution states that the degree of belief associated with 'High' Influenza is 80%, 20% degree of belief associated with 'Medium' Influenza, 0% degree of belief associated with 'Low' Influenza. Here, 'High', 'Medium', and 'Low' can be

considered as the referential values of the consequent attribute "Influenza" of the belief rule. In this belief rule, the total degree of belief is (0.80+0.40+0.00) =1 and hence, the assessment is complete.

B. Inference System with BRB

The inference procedures in BRB consists of various components such as input transformation, rule activation weight calculation, rule update mechanism, followed by the aggregation of the rules of a BRB by using ER.

The input transformation of the value of an antecedent attribute P_i consists of distributing the value into belief degrees of different referential values of that antecedent. The i*th* value of an antecedent attribute at instant point in time can equivalently be transformed into a distribution over the referential values, defined for the attribute by using their belief degrees[7]. The input value of**,** which is the **i**th antecedent attribute of a rule, along with its belief degree is shown below by (3).

$$H(P_i, \varepsilon_i) = \{(A_{ij}, \alpha_{ij}), j = 1, \cdots, j_i\}, \ i = 1, \dots \dots, T_k \quad (3)$$

Here H is used to show the assessment of the belief degree assigned to the input value of the antecedent attribute. In the above equation A_{ij} (ith value) is the jth referential value of the input P_i. α_{ij} is the belief degree to the referential value A_{ij} with $\alpha_{ij} \geq 0$. $\sum_{j=1}^{j_i} \alpha_{ij} \leq$ $1 i=1, ...,Tk$, and jl is the number of the referential values. In this research, the input value of an antecedent attribute is collected from the patient or from the physician in terms of linguistic values. This linguistic value is assigned degree of belief ε_i by taking account of expert judgment. This assigned degree of belief is then distributed in terms of belief degree α_{ij} of the different referential values A_{ij} [High (H), Medium (M), Low (L)] of the antecedent attribute.

When the kth rule is activated, the weight of activation of the kth rule, w_k is calculated by using the flowing formula [8].

$$\omega_k = \frac{\theta_k \alpha_k}{\sum_{j=1}^{L} \theta_j \alpha_j} = \frac{\theta_k \prod_{i=1}^{T_k} (\alpha_i^k)^{\overline{\delta_{ki}}}}{\sum_{j=1}^{L} \theta_j \left[\prod_{l=1}^{T_k} (\alpha_l^j)^{\overline{\delta_{jl}}} \right]} \text{ and } \overline{\delta_{ki}} = \frac{\delta_{ki}}{max_{i=1,\dots,T_k} \{\delta_{ki}\}} \quad (4)$$

Where $\overline{\delta_{ki}}$ is the relative weight of P_i used in the kth rule, which is calculated by dividing weight of P_i with maximum weight of all the antecedent attributes of the kth rule. By doing so, the value of $\overline{\delta_{ki}}$ becomes normalize, meaning that the range of its value should be between 0 and 1. $\alpha_k = \prod_{i=1}^{T_k} (\alpha_i^k)^{\overline{\delta_{ki}}}$ is the combined matching degree, which is calculated by using multiplicative aggregation function.

When the kth rule as given in.(1) is activated, the incompleteness of the consequent of a rule can also result from its antecedents due to lack of data. An incomplete input for an attribute will lead to an incomplete output in each of the rules in which the attribute is used. The original belief degree $\overline{\beta_{ik}}$ in the ith consequent C_i of the kth rule is updated based on the actual input information [8,9]. ER approach is used to aggregate all the packet antecedents of the L rules to obtain the degree of belief of each referential values of the consequent attribute by taking account of given input values P_i of antecedent attributes. This aggregation can be carried out either using recursive or analytical approach. In this research analytical approach has been considered since it is computationally efficient than recursive approach [7][8]. Using the analytical ER algorithm [8], the conclusion *O(Y),* consisting of referential values of the consequent attribute, is generated. Equation (5) as given below illustrates the above phenomenon.

$$O(Y) = S(P_i) = \{(C_j, \beta_j), \ j = 1, \dots, N\} \quad (5)$$

Where β_j denotes the belief degree associated with one of the consequent reference values such as C_j. The β_j is calculating by analytical format of the ER algorithm [5][6] as illustrated in (6).

$$\beta_j = \frac{\mu \times \left[\prod_{k=1}^{L} \left((\omega_k \beta_{jk} + 1 - \omega_k \sum_{j=1}^{N} \beta_{jk}) \right) - \prod_{k=1}^{L} (1 - \omega_k \sum_{j=1}^{N} \beta_{jk}) \right]}{1 - \mu \times \left[\prod_{k=1}^{L} 1 - \omega_k \right]} \quad (6) \text{ With}$$

$$\mu = \left[\sum_{j=1}^{N} \prod_{k=1}^{L} \left(\omega_k \beta_{jk} + 1 - \omega_k \sum_{j=1}^{N} \beta_{jk} \right) - (N-1) \right.$$
$$\left. \times \prod_{k=1}^{n} \left(1 - \omega_k \sum_{j=1}^{N} \beta_{jk} \right) \right]^{-1}$$

The final combined result or output generated by ER is represented by $\{(C_1, \beta_1), (C_2, \beta_1), (C_3, \beta_1), \ldots \ldots, (C_N, \beta_N)\}$, where $\boldsymbol{\beta_j}$ is the final belief degree attached to the jth referential value $\boldsymbol{C_j}$ of the consequent attribute, obtained after combining all activated rules in the BRB by using ER. This output can be converted into crisp/numerical value by assigning utility score to each referential value of the consequent attribute [6][8].-

$$H(A^*) = \sum_{j=1}^{N} u(C_j) B_j \qquad (7)$$

Where $H(A^*)$ is the expected score expressed as numerical value and $u(C_j)$ is the utility score of each referential value.

IV BRB EXPERT SYSTEM (BRBES)

The BRBES's architecture along with its components is presented in this section.

A. System Architecture and its Implementation Strategy

The design of the system consists of data structure and program components that are essential to build a computer based system. It also considers the system organization pattern, which is known as architectural style. The architecture of the BRBES consists of user interface, a knowledge engineer, knowledge base, inference engine, documentation and knowledge refinement as shown in fig 1. User interface interacts to a system user to get input data and to receive system generated output. Visual Basic 6.0 has been employed to develop the system interface. Knowledge engineer accommodates data from domain knowledge and expert constructs knowledge-base by using belief rule base. MS SQL-Server is a relational database used at the back-end to store and manipulate initial BRB, which is flexible and user friendly. The inference engine carries the tasks of input transformation, rule activation, rule update and rule aggregation by using ER.
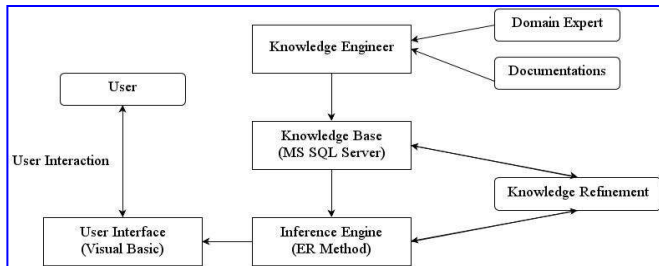

Fig. 1. System architecture of the Influenza suspicion system.

B. Knowledge Base Construction in BRB

In order to construct the knowledge base for this expert system by using belief rule base, a BRB framework (by taking account of signs and symptoms associated with influenza) have been developed as illustrated in fig 2.
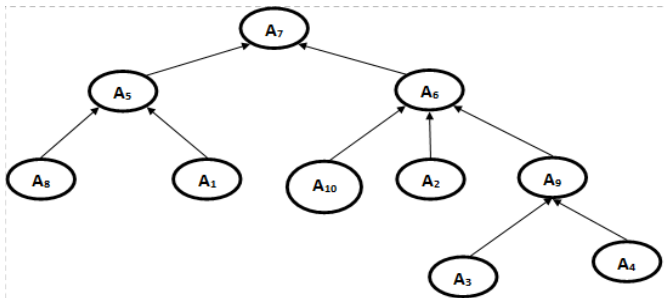

Fig. 2. BRB Framework to Diagnose the Influenza.

This BRB consists of four sub-rule-bases. For example, A_5 sub rule-base has two antecedent attributes. Each antecedent attribute consists of three referential values. Hence, this sub-rule-base consists of

9 rules. The entire BRB (which consists of 4 sub-rule bases) consists of $(9+27+9+9) = 54$ initial belief rules. It is assumed that all belief rules have equal rule weight; all antecedents have equal weight, and the initial belief degree assigned to each consequent of a rule by an expert by accumulating patient's data.

A BRB can be established in the following four ways[20]- (1) Extracting belief rules from expert knowledge (2) Extracting belief rules by examining historical data; (3) Using the previous rule bases if available, and (4) Random rules without any pre-knowledge. In this paper, we constructed initial BRB by the domain expert knowledge. The initial rule-base for the sub rule base 1 where antecedent A3 is for Headache and A4 is for Cough and consequent A9 is for achness is shown in Table 1.

TABLE I. THE INITIAL RULE BASE FOR A SUB RULE BASE

No.	W	Antecedent	Consequent
1.	1	A3 is H ^ A4 is H	A9 is {(H,1),(M,0),(L,0)}
2.	1	A3 is H ^A4 is M	A9is{(H,0.6),(M,0.4),(L,0)}
3.	1	A3 is H ^A4 is L	A9 is{(H,0.8),(M,0),(L,0.2)}
4.	1	A3 is M ^A4 is H	A9 is {(H,0.6),(M,0.4),(L,0)}
5.	1	A3 is M ^ A4 is M	A9 is {(H,0),(M,1),(L,0)}
6.	1	A3 is M ^ A4 is L	A9 is {(H,0),(M,0.8),(L,0.2)}
7.	1	A3 is L ^ A4 is H	A9 is{(H,0.8),(M,0),(L,0.2)}
8.	1	A3 is L ^ A4 is M	A9 is {(H,0),(M,0.6),(L,0.4)}
9.	1	A3 is L ^ A4 is L	A9 is {(H,0),(M,0)(L,1)}

An example of a belief rule taken from Table 1 is illustrated below:

R1: IF Headache (A3) is high AND Cough (A4) is high THEN the **Acheness (A9)** can be considered as high. When **Headache** is low but **Cough** is high then the belief degree of the **Acheness** be distributed among High (as 0.8) and Low (as 0.2) as can be seen from rule (7). The weight of each rule has been considered as 1.

C. BRBES Interface

A system interface can be defined as the media, enabling the interaction between the users and the system. Fig. 3 illustrates a simple interface of the BRBES. This interface facilitates the acquiring of the leaf nodes' (antecedent attributes) data of the BRB framework (fig 2), which is collected from patients and experts. The system interface enables the displaying of the assessment results (the top node) and sub-results. For example, fig 3 illustrates the result for the data of leaf nodes (A3 = 12, A4 = 18) associated with A9 sub-rule-base.
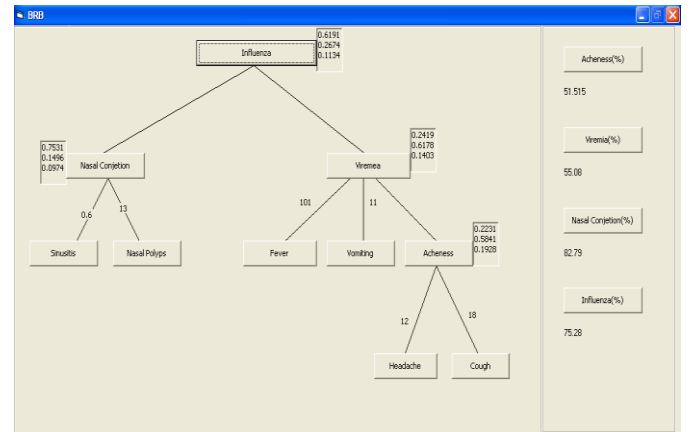

Fig. 3. BRBES's Interface

From fig 3, it can be observed that the degree of belief obtained for the referential values of the consequent attribute "A9" of this sub-rule-base is {High (0.6191), Medium (0.2674), Low(0.1134)}. Similarly, the degree of belief of the referential values of other consequent attributes can be understood from fig 3. It is interesting to note that the child nodes (A5, A6, A7, A9) are not the leaf nodes and hence, their data can't be acquired externally to feed the system. These child nodes actually the consequent of leaf nodes of the sub-rule-bases

and their referential values have already been calculated by the system as the degree of belief. However, in order to obtain a single data value, each referential value of the consequent has been multiplied by the utility values as mentioned in Section III. The calculated single data value of A5, A6, A7 and A9 have been considered as the antecedent value of the upper level sub-rule-bases. Fig 3 also illustrates the overall Influenza assessment (can be considered at a high level or aggregated), which is {High (0.6191), Medium (0.2674), Low (0.1134)}. This is transformed into a crisp value by using (7), which is 75.28% as shown in fig 3.

V. RESULTS AND DISCUSSION

In this research, the leaf nodes data of the BRB framework (fig 2 and 3) have been collected from the patients. The patient's data have been used in the BRBES to assess the influenza. Expert's opinion on the influenza diagnosis of the patients is also collected as shown in Table 2. If the experts perception on the diagnosis is greater than 40%, then outcome is considered as one otherwise zero and this data have been considered as the baseline as shown in Column 6 of Table 2. The data set consists of 200. For simplicity, only ten patient's data set are presented in Table 2. Colum 5 of Table 2 illustrates BRBES' generated output in percentages, which is calculated by using utility equation (7).For example, the overall system output of influenza diagnosis is 75.28% can be obtained, by using a degree of belief associated with referential values such as {High (0.6191), Medium(0.2674), Low(0.1134)}.

The Receiver Operating Characteristic (ROC) curve can be used to analyze effectively performances of suspicion tests having ordinal or continuous results[21]. It can be used to test the reliability of the BRBES's output in comparison with manual system by taking account of benchmark data as mentioned earlier. The performance of the system can be measured by calculating the Area Under Curve (AUC) [9–10]. A larger AUC refers to a more accurate and reliable result. Fig 4 shows the two ROC curves; one represents the performances of the BRBES and the other for the manual system.

TABLE II. INFLUENZA ASSESSMENT BY BRBES AND HUMAN EXPERT

ID	Sex	Age	Experts' Opinion	BRBES' Results	Benchmark Data
P1	F	52	8.89%	9.0%	0
P2	M	60	21.94%	21.9%	0
P3	M	3	14.92%	15.0%	0
P4	F	4	45.64%	16.12%	1
P5	M	65	45.16%	45.9%	1
P6	M	67	35.16%	36.0%	1
P7	F	45	4.75%	4.89%	0
P8	M	4	56.08%	56.89%	1
P9	M	68	23.18%	24.0%	0
P10	F	78	76.34%	76.0%	1

The ROC curve with a blue line in fig 4 illustrates the BRBES influenza diagnosis result while the curve with green line illustrates the manual system influenza diagnosis result. The AUC for BRBES is 0.989 (95% confidence intervals 0.960 – 1.012), and the AUC of manual system is 0.977 (95% confidence intervals 0.939 – 1.014). From the AUC of the two systems result, it can be observed that AUC of BRBES is greater than the AUC of the manual system. This implies that results generated by the BRBES are better than the results generated by the manual system, which uses traditional rule without taking account of uncertainty.
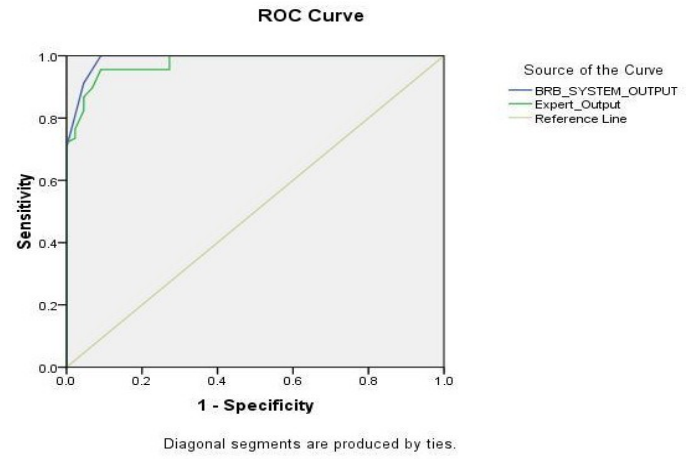

Fig. 4. Roc Curves of Influenza Diagnosis between BRBES and Expert Opinion

VI. CONCLUSION

This paper presented the development and application of a BRBES for influenza diagnosis using signs and symptoms. This BRBES employed RIMER methodology, allowed the handling of various types of uncertainty found in the clinical domain knowledge as well as in the signs and symptoms. Hence, it might be considered as a robust tool and can be utilized in the diagnosis of influenza. Most importantly, the system will play an important role in reducing the cost of lab investigations.

This influenza diagnosis BRBES can provide a percentage of suspicion recommendation, which is more reliable and informative than from the traditional physician's opinion. This system performance is better than traditional (Manual) system. However, this BRBES has been developed only to assess diagnose influenza from its signs and symptoms. Due to demographic differences associated with signs and symptoms the system's expertise can be improved by providing more context/demography-appropriate data, and there is a scope to improve the system by including context-adaptability and context-independence as the features to compare the results of ES with expert opinions from different contexts of the world and not only from a specific context. That way, the ES have potential for further contribution in the clinical domain knowledge. Furthermore, the system can be improved by including a sub-system to compute ROC and AUC to show the system performance in relation to suspicion tests.

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