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Communication Strategies for Two Models of Discrete Energy Harvesting

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***Abstract*—Energy harvesting is becoming a viable option for powering small wireless devices. Energy for data transmission is supplied by the nature, such that when a transmission is about to take place in an arbitrary instant, the amount of available energy is a random quantity. The arrived energy is stored in a battery and transmissions are interrupted if the battery runs out of energy. We address communication in slot-based energy harvesting systems, where the transmitter communicates with ON-OFF signaling: in each slot it can either choose to transmit (ON) or stay silent (OFF). Two different models of harvesting and communication are addressed. In the first model an energy quantum can arrive, with a certain probability, in each slot. The second model is based on a frame of size F: energy arrives periodically over F slots, in batches containing a random number of energy quanta. We devise achievable strategies and compare the slot- with the frame-based model in the case of an errorless transmission channel. Additionally, for the slot-based model and channel with errors, we provide a new proof of the capacity achieved by the save-and-transmit scheme.**

I. Introduction

Energy harvesting is an emerging field that enables wireless devices to harvest energy from the nature. This concept is particularly important for small low-complexity devices with a long expected lifetime, e.g. wireless sensor nodes. Energy sources include vibrations, blood sugar, solar cells and thermoelectricity [1]. Unlike traditional battery-powered devices, future energy levels have a random behavior within time intervals in the order of symbol durations. Arriving energy is stored in a battery for later usage if not used immediately, and transmissions are interrupted if the device runs out of energy. Due to the random behavior of future energy levels, it is difficult to utilize the energy in the most efficient way and there is a trade-off between high transmission rates and a low probability of running out of energy [2].

Wireless systems with energy harvesting have received much research interest in recent years. Many works consider the problem of finding a transmission scheme that is optimal in terms of the achieved throughput. The works [2], [3], [4], [5] consider the problem of communication on a channel that enables a higher instantaneous throughput with higher energy usage in accordance to a monotonic and strictly concave rate function. An example of a channel having such rate function is the AWGN channel. A first attempt to analyze energy harvesting systems from an information-theoretic perspective is done in [6]. A transmission scheme called *save-and-transmit* is introduced and it is proved to be capacity-achieving on the AWGN channel. Recently, [7] considered the binary energy harvesting problem with a unit-size battery.

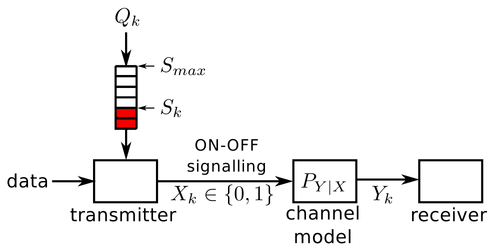

Fig. 1. The slot-based model.

In this paper, we consider slot-based point-to-point communication for wireless systems with energy harvesting and ON-OFF signaling, in which energy is only used when an ON symbol is sent. First, a slot-based energy harvesting model is considered. A naïve achievable scheme and an upper bound are introduced, and the capacity with an infinite-capacity battery is found for arbitrary binary-input memoryless channels. We show that the *save-and-transmit* scheme introduced in [6] is applicable for systems with ON-OFF signaling and can be proved using the Martingale theory. Next, a frame-based model is presented: the harvested energy becomes usable only at the beginning of each frame that consists of F slots, which is modeled through a random number of energy quanta that arrives at the frame start. We consider two cases: when energy quanta may be saved from frame to frame and when energy is discarded in the end of a frame. The latter model captures the effect of battery leakage or the separation between the operations of harvesting and communication. Achievable information rates are found based on the framework of Channel Side Information at the Transmitter (CSIT) [8].

The paper is structured as follows. Section II describes the investigated communication models of a wireless system with energy harvesting. The slot-based and the frame-based model are introduced. Section III addresses the slot-based model. In Section IV, transmission schemes for the frame-based model are considered. Finally, numerical results are presented Section V followed by conclusions in Section VI.

II. Communication Models

We present the two models separately, but as elaborated in Section V, the slot-based model can be used to emulate the frame-based model under suitable assumptions.

A. Slot-based model

The slot-based model for wireless systems with energy harvesting is illustrated in Fig. 1, and has already been used in [7]. Time is divided into slots of equal duration, enumerated $k \in \mathbb{N}$, where $\mathbb{N}$ denotes the natural numbers. Energies are discretized in quanta, where one quantum corresponds to the energy consumed by transmitting a single ON symbol. In each slot, one energy quantum is either harvested with probability a or not with probability $1-a$. Thus the arrived energy in slot k, Q_k, is independently distributed as

$$Q_k \sim \text{Bern}(a), \tag{1}$$

where $\sim$ denotes "distributed as". In the $k-$th slot, the transmitter can either choose to transmit (ON), setting $X_k = 1$ and consume one energy quantum, or stay silent (OFF), setting $X_k = 0$ and consume no energy.

Energy is buffered in a battery, such that S_k energy quanta are stored in the beginning of slot k. S_k is termed the energy level, or the state, in slot k. As the transmitter can not transmit if no energy is available, the following energy constraints have to be satisfied for all $k \in \mathbb{N}$

$$X_k \leq S_k. \tag{2}$$

A recursive formula for the energy level S_k can be written as

$$S_k = \min(S_{\text{max}}, S_{k-1} - X_{k-1} + Q_k) \tag{3}$$

where $S_{\text{max}} \in \mathbb{N}$ denotes the positive battery capacity and $S_1 = 0$. Note that if energy arrives when $S_k = S_{\text{max}}$, then the energy quantum is discarded.

B. Frame-based model

The slot-based model models systems in which energy is available immediately upon arrival. However, we can think of a system in which harvesting and communication do not take place simultaneously. The system enters in a harvesting mode and collects a random number energy quanta, which is supplied to be used for communication in the coming F slots. In the best case, the transmitter will use only as many as it needs of the quanta, and the other will remain in the energy buffer. However, if the harvesting periods are separated in time, then the other extreme occurs - due to energy leakage, the energy buffer is empty when the new harvesting period starts. This latter model, in which all buffered energy must be used/wasted in a frame of size F has been introduced in [9].

Energy arriving in the slots $(i-1)F+1, \ldots, iF$, for $i \in \mathbb{N}$, is first harvested to the battery in slot $iF+1$. An interpretation of this is that there is one energy harvesting round after every F slots, where energy is harvested from energy sources and buffered in the battery.

A frame i is then defined by the slots $iF+1, \ldots, (i+1)F$. The number of energy quanta harvested in the beginning of frame i, $\widetilde{Q}_i$, is then given as

$$\widetilde{Q}_i = \sum_{j=1}^{F} Q_{(i-1)F+j}, \tag{4}$$

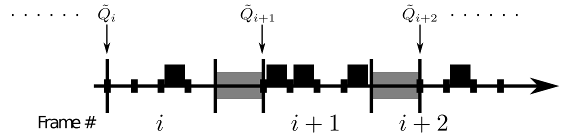

Fig. 2. Frame-based model: The time is divided into enumerated frames. Each frame is divided into F slots. In the beginning of each frame $\widetilde{Q}_n$ energy quanta arrives and can only be used within the next frame. The black boxes designates ON symbols in each frame. For this example $F = 4$, and $\widetilde{Q}_i \geq 1$, $\widetilde{Q}_{i+1} \geq 3$ and $\widetilde{Q}_{i+2} \geq 1$. The periods between the frames, marked by grey boxes, depicts energy harvesting rounds.

and hence $\widetilde{Q}_i$ is Binomial distributed with parameters F and a. The energy level, or frame state, in the beginning of frame i is then denoted by $\widetilde{S}_i$. In frame i, the transmitter sends a binary vector $\mathbf{X}_i = \{X_{(i-1)F+1}, \ldots, X_{iF}\} \in \{0,1\}^F$, which defines the actions taken by the transmitter in frame i. Denote the weight of $\mathbf{x}_i$ by $|\mathbf{x}_i| = \sum_{j=1}^{F} \mathbf{x}_{i,j}$, where $\mathbf{x}_{i,j}$ is the j-th entry of $\mathbf{x}_i$. The transmitter then has to satisfy the following constraint

$$|\mathbf{x}_i| \leq \widetilde{S}_i. \tag{5}$$

As for the slot-based model, the energy level in the beginning of frame k is recursively given by

$$\widetilde{S}_i = \min(\widetilde{S}_{\text{max}}, \widetilde{S}_{i-1} - |\mathbf{X}_{i-1}| + \widetilde{Q}_i), \tag{6}$$

where $k \in \mathbb{N}$ and $|\mathbf{x}|$ denotes the weight of the vector $\mathbf{x}$.

The frame-based model is depicted in Fig. 2. Note that a simple way to model leakage in the battery is use the energy level update $\widetilde{S}_i = \widetilde{Q}_i$.

In the main body of the paper we will always assume that the ON-OFF communication channel is error-free, such that the channel output Y_i is equal to the channel input $X_i = Y_i$. For the proof presented in the Appendix, we will introduce a channel model with errors.

III. Achievable schemes for the Slot-based model

In this section, communication schemes for the slot-based model are addressed. A naïve scheme and an upper bound are introduced for comparison, and a save-and-transmit strategy is shown to achieve the capacity on arbitrary binary-input channel models. The main contribution, compared to [6], is a different proof technique and the extension to arbitrary binary-input channel models.

A. Naïve achievable scheme

A naïve achievable scheme for the slot-based model can be derived using the Shannon strategies $(0,0)$ and $(0,1)$ as in [7]. The first and the second entry of the Shannon strategies denote the channel inputs, X, when the energy level is $S = 0$ and $S \geq 1$, respectively. The Shannon strategies $(0,0), (0,1)$ are denoted by $U = 0, 1$ and U is i.i.d. with probability $P_U(1) = q$. This communication strategy forms a Markov chain with $S_{\text{max}} + 1$ states, i.e. one for each energy level. The energy level is hence ergodic and the stationary state probability $\pi_0 = \Pr[S = 0]$ is given by

$$\pi_0(q) = \left(1 + \frac{1}{1-q}\sum_{s=1}^{S_{\max}}\left(\frac{a(1-q)}{q(1-a)}\right)^s\right)^{-1}, \qquad (7)$$

and the achievable rate is then given by

$$R_{\text{NIID},S_{\max}} = \max_{q\in[0,1]} H_b((1-\pi_0(q))q) - qH_b(\pi_0(q)), \qquad (8)$$

where the expression to be maximized is the information rate achievable by the Z-channel with crossover probability $\pi_0(q)$ and $H_b(\cdot)$ denotes the binary entropy function [10]. In the special case where $S_{\max} = 1$, the upper bound corresponds to the upper bound in [7].

B. Upper bound

Suppose the receiver knows the state of the transmitter and the transmitter chooses the Shannon strategy $U = 1$ with probability $\Pr[U = 1|S = s] = q_s$ for $s \in \{0, \dots, S_{\max}\}$. As for the naïve achievable scheme, this strategy forms a Markov chain for which the stationary probabilities π_s can be computed. Since the state information is perfectly known at both transmitter and receiver, the capacity of the communication scheme with receiver state information is given by

$$C_{\text{upper},S_{\max}} = \max_{\forall s: q_s\in[0,1]} \sum_{s=1}^{S_{\max}} \pi_s H_b(q_s). \qquad (9)$$

C. Battery with infinite capacity

In the case where the battery has infinite capacity, i.e. $S_{\max} \to \infty$, [7] found that the capacity of the slot-based model is given by

$$C_{S_{\max}\to\infty} = \begin{cases} H_b(a), & a \le \frac{1}{2} \\ 1, & a > \frac{1}{2} \end{cases} \qquad (10)$$

This is shown using the save-and-transmit strategy presented in [6]. In Appendix A, we derive the capacity in (10) using a different proof based on Martingale theory, which simplifies the analysis compared to [6]. Moreover, the result is extended to arbitrary binary-input channel models.

IV. Achievable schemes for the frame-based model

This section introduces achievable schemes for the frame-based model. We first consider the simplified case in which energy that is not used in a frame is discarded at the end of the frame. Next, we elaborate on the case in which unused energy quanta in one frame can be used in the following frame.

A. Without memory

When the remaining energy quanta in one frame are discarded before the next frame, the communication model acts as a memoryless channel. In particular, the state of the channel is i.i.d. as $\widetilde{S} = \widetilde{Q} \sim \text{Binomial}(F, a)$, and hence at most F energy quanta can be stored in the battery. The resulting communication channel can be seen as a channel with channel input constraints. The capacity of such a channel can be found using the framework of Channel Side Information at the Transmitter (CSIT), where the frame state $\widetilde{S}$ is the state of the channel $P_{\mathbf{Y}|\mathbf{X},\widetilde{S}}$, which is defined shortly. Shannon showed that the capacity of such a channel can be achieved by an equivalent channel $T - \mathbf{X} - \mathbf{Y}$, where $T \in \mathcal{T}$ denotes a random Shannon strategy which is i.i.d. according to P_T [8]. A Shannon strategy $t \in \mathcal{T}$ is mapping from a frame state $\widetilde{s} \in \widetilde{\mathcal{S}} = \{0, \dots, F\}$ to a channel input $\mathbf{x} \in \{0,1\}^F$ such that $t(\widetilde{s}) = \mathbf{x} \in \{0,1\}^F$.

However, given the frame state $\widetilde{s}$, the energy constraints imply that only a subset of the possible channel inputs can be send. Now, define the alphabet $\mathcal{X}^{\widetilde{s}}$ for each $\widetilde{s} \in \widetilde{\mathcal{S}}$ as the set of all subsets of $\{0,1\}^F$ with weight less than or equal to $\widetilde{s}$. The channel input then have to satisfy $\mathbf{X} \in \mathcal{X}^{\widetilde{S}}$. Note that the model, with the additional constraint that all energy quanta have to be used, have been considered in [9]. We then define the channel $P_{\mathbf{Y}|\mathbf{X},\widetilde{S}}(\mathbf{y}|\mathbf{x},\widetilde{s})$ as following

$$P_{\mathbf{Y}|\mathbf{X},\widetilde{S}}(\mathbf{y}|\mathbf{x},\widetilde{s}) = \begin{cases} P_{\mathbf{Y}|\mathbf{X},\widetilde{S}}(\mathbf{y}|g(\widetilde{s}),\widetilde{s}), & \mathbf{x} \notin \mathcal{X}^{\widetilde{s}} \\ \mathbb{1}_{\{\mathbf{y}=\mathbf{x}\}}, & \mathbf{x} \in \mathcal{X}^{\widetilde{s}} \end{cases}, \qquad (11)$$

where $g(\widetilde{s})$ chooses an arbitrary letter in the alphabet $\mathcal{X}^{\widetilde{s}}$ for each $\widetilde{s} \in \widetilde{\mathcal{S}}$. Note that the channel $P_{\mathbf{Y}|\mathbf{X},\widetilde{S}}$ only takes the values 0 and 1 since we consider errorless channels. Due to this definition, the channel input constraints are satisfied for any Shannon strategies, i.e. picking a Shannon strategy $t \in \mathcal{T}$ such that $t(\widetilde{s}) \notin \mathcal{X}^{\widetilde{s}}$ for some $\widetilde{s} \in \widetilde{\mathcal{S}}$ is equivalent to picking another Shannon strategy $t' \in \mathcal{T}$ with $t'(\widetilde{s}) \in \mathcal{X}^{\widetilde{s}}$. The capacity of the communication model is hence given by

$$C_{\text{frame,no-mem}} = \max_{P_T} I(T; \mathbf{X}). \qquad (12)$$

This is similar to the technique used by [11] in which the capacity of a time-varying amplitude constrained AWGN channel was considered. As consequence of (11), it is only necessary to search for strategies $t \in \mathcal{T}$ satisfying $t(\widetilde{s}) \in \mathcal{X}^{\widetilde{s}}$ for $\widetilde{s} \in \widetilde{\mathcal{S}}$. The cardinality of this set of Shannon strategies is given by

$$|\mathcal{T}| = \prod_{\widetilde{s}=0}^{F}\sum_{j=0}^{\widetilde{s}}\binom{F}{j}. \qquad (13)$$

As the cardinality of $\mathcal{T}$ grows very fast with F, we have only succeeded in computing capacities using (12) for $F \le 4$.

B. With memory

In the case where energy quanta are saved from one frame to next according to (6), the frame state depends on both previous channel inputs and the random energy arrivals. This renders the problem of computing the capacity challenging. The main problem is to find the pmf of the Shannon strategies P_T such that $I(T; \mathbf{X})$ is maximized. Recall that the stationary pmf of frame state $P_{\widetilde{S}}$ can be computed from the Shannon strategies P_T, and conversely, the optimal pmf P_T depends on the stationary pmf of the frame state $P_{\widetilde{S}}$. Due to this dependency, it necessary to optimize jointly over $P_{\widetilde{S}}$ and P_T to obtain the capacity. We propose a suboptimal heuristic approach for the optimization of the achievable information rate $I(T;\mathbf{X})$ in which we alternate between optimization over P_T and computation of P_S.

In order to allow any $\widetilde{S}_{\max}$ and to keep the complexity reasonable low for $\widetilde{S}_{\max} > F$, the Shannon strategies $t \in \mathcal{T}$ are extended as

$$t(\widetilde{s}) = t(F) \qquad \text{for } \widetilde{s} > F. \tag{14}$$

1) Computation of $P_{\widetilde{S}}$ given P_T: With P_T given, the pmf of the energy quanta remaining after frame $k-1$, E_{k-1}, is computed as

$$P_{E_{k-1}}(e) = \sum_{s=0}^{S_{\max}} \sum_{t \in \mathcal{T}} P_{S_{k-1}}(s) P_T(t) \mathbb{1}_{\{s - w(t(s)) = e\}}, \tag{15}$$

for $e \in \{0, \ldots, S_{\max}\}$. The pmf of the frame state in slot k, S_k, is given by

$$P_{\widetilde{S}_k}(\widetilde{s}) = \sum_{i=0}^{\min(F,\widetilde{s})} P_{\widetilde{Q}}(i) P_{E_{k-1}}(\widetilde{s} - i) \tag{16}$$

where $\widetilde{s} \in \{0, \ldots, \widetilde{S}_{\max} - 1\}$ and $P_{\widetilde{S}_k}(\widetilde{S}_{\max})$ is given by $1 - \sum_{\widetilde{s}=0}^{\widetilde{S}_{\max}} P_{\widetilde{S}_k}(\widetilde{s})$. The stationary pmf of the energy level, denoted $P_{\widetilde{S}}$, can then be found by solving the system of linear equations

$$P_{\widetilde{S}_k}(\widetilde{s}) = P_{\widetilde{S}_{k-1}}(\widetilde{s}), \tag{17}$$

for $\widetilde{s} \in \{0, \ldots, \widetilde{S}_{\max}\}$.

2) Optimization of $I(T;\mathbf{X})$ given $P_{\widetilde{S}}$: With $P_{\widetilde{S}}$ given, the highest achievable information rate can be found as in (12). However, optimization in this way does not take into account that T affects $P_{\widetilde{S}}$, i.e. optimal Shannon strategies according (12) may use a high amount of the energy available, and hence decrease the amount of energy saved for the next frame. To accommodate for that, we use a regularized optimization approach in which Shannon strategies are penalized according to their average weight, i.e. Shannon strategies with higher weights are penalized more. This optimization problem can be stated as

$$\begin{aligned} R_{\text{frame}}(P_{\widetilde{S}}) = \beta \sum_{t \in \mathcal{T}} P_T^*(t) w(t) \\ + \max_{P_T} I(T;\mathbf{X}) - \beta \sum_{t \in \mathcal{T}} P_T(t) w(t), \end{aligned} \tag{18}$$

where $\beta > 0$, P_T^* is the pmf maximizing (18) and the average weight of a Shannon strategy $t \in \mathcal{T}$ is defined by $w(t) = \sum_{\widetilde{s} \in \widetilde{\mathcal{S}}} P_{\widetilde{S}}(\widetilde{s}) w(t(\widetilde{s}))$.

By alternating between computation of P_T^* in (18) and computing $P_{\widetilde{S}}$, we arrive at a stationary point in which P_T^* and P_S are consistent with (15)-(18).

V. Numerical Results

We have assessed the performance of both communication models in terms of achievable rates for the proposed schemes.

We first note that, although the slot-based and the frame-based models are fundamentally different, the frame-based model can be emulated in the slot-based model. That is, the frame-based model represents a communication scheme for the slot-based model in which the transmitter decides which symbols to send in the coming F slots every F slot. One problem is that in the slot-based model, the battery may overflow if $S_{\max} = \widetilde{S}_{\max}$. This is avoided by setting the frame state $\widetilde{S}_{\max} = S_{\max} - \lfloor F/2 \rfloor$, i.e. the energy level update is given by

$$\widetilde{S}_k = \min\left(S_{\max} - \lfloor F/2 \rfloor, \widetilde{S}_{k-1} - |\mathbf{X}_{k-1}| + \widetilde{Q}_k\right) \tag{19}$$

with $S_{\max} - \lfloor F/2 \rfloor \geq F$. An example in which this is necessary is the following; let $\widetilde{S}_{\max} = 6$, $F = 4$ and $\widetilde{S}_k = 6$. Suppose that the transmitter sends $\mathbf{x}_k = 0011$ and the sequence of harvested energy during the duration of the frame is 1100. In this case, the battery in the slot-based model overflows such that $\widetilde{S}_{k+1} = 4$, whereas the same scenario in the frame-based model would yield $\widetilde{S}_{k+1} = 6$.

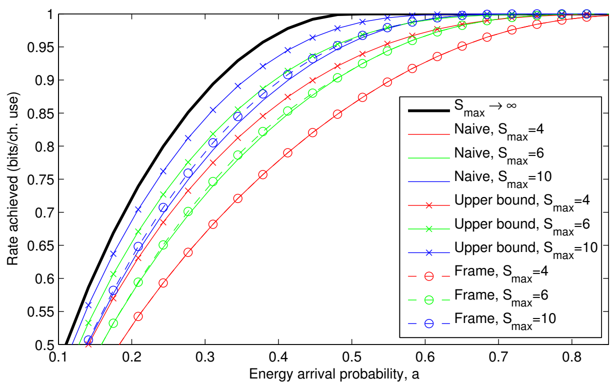

Fig. 3. Information rates achieved by the naïve scheme and the emulated frame-based scheme. The upper bounds are plotted for comparison.

For the slot-based model, the achievable rate of the naïve achievable scheme and the emulated frame-based scheme are plotted in Fig. 3 along with the upper bound in (9). The information rates achieved by the emulated frame-based scheme are found using (19) and by optimizing over the frame length $F \in \{1, \ldots, 4\}$ and the parameter $\beta > 0$ in (18) for each $S_{\max}$. The main observation is that the emulated frame-based achievable scheme achieves slightly higher information rates than the naïve scheme at certain energy arrival probabilities when $S_{\max} \geq 6$.

Achievable information rates for the frame-based model are depicted in Fig. 4. It is seen that saving energy quanta from frame to frame significantly increases the achievable information rates.

We note that the optimization problems in (12) and (18) are efficiently solved using the Blahut-Arimoto algorithm for $F \leq 4$.

VI. Conclusions

In this paper we have considered transmission schemes for wireless systems with energy harvesting and ON-OFF signaling. For the slot-based scheme, we have introduced a

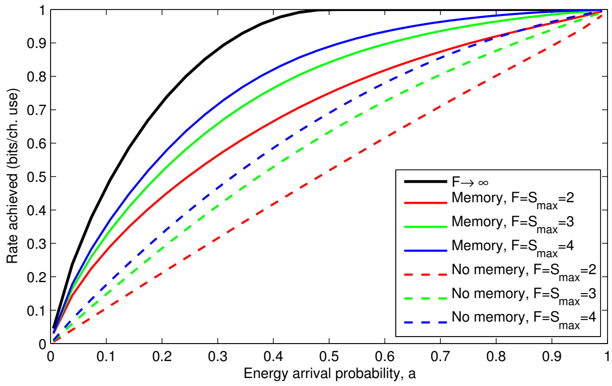

Fig. 4. Information rates achievable (normalized by F) for the frame-based model by the achievable schemes with and without memory. The upper bound, in which $F \to \infty$, is given by the save-and-transmit scheme.

naïve achievable scheme and an upper bound, and we have shown that the capacity of the model with an infinite-capacity battery transmitting on an arbitrary discrete binary-input memoryless channel is achieved by the save-and-transmit scheme using results from Martingale theory. A frame-based model in which energy is only harvested every F slots is presented for systems where energy is only harvested periodically. We have considered the case where the energy quanta are lost in the end of each frame, i.e. before the next energy harvesting round and when energy quanta not used in one frame can be used in the following frame. Achievable schemes are devised using Shannon strategies. It is further shown that the frame-based model can be emulated in the slot-based model, and numerical results shows that the emulated frame-based scheme achieves higher information rates than the naïve for certain energy arrival probabilities.

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Appendix A
Save-and-Transmit Strategy

Ozel et al. [6] established that the save-and-transmit strategy achieves the capacity of a slot-based communication system with infinite battery over the AWGN channel. In [7], it was further noted the the capacity of slot-based energy harvesting systems with ON-OFF signaling is achieved by a similar a strategy. In this appendix, this result is extended to arbitrary binary-input channel models and our proof is based on Martingale theory which simplifies the analysis compared to [6]. In the following, we consider the n slots, and X^n and Y^n denote the sequences $\{X_1, \ldots, X_n\}$ and $\{Y_1, \ldots, Y_n\}$, respectively.

As the energy is buffered in a battery with infinite capacity the following energy constraints have to be satisfied for any slot $k \in \{1, \ldots, n\}$

$$\sum_{i=1}^{k} X_i \leq \sum_{i=1}^{k} Q_i. \tag{20}$$

These constraints are necessary to ensure that X_i can not be 1 if no energy is available in the buffer.

The channel $X - Y$ is an arbitrary binary-input memoryless channel. The channel is defined by the function $h(p)$ as following:

$$h(p) = I(X; Y), \tag{21}$$

where $X \sim \text{Bern}(p)$. The function $h(p)$ is concave and attains a maximum in the interval $[0, a]$, which is denoted by a_{opt}. It is noted that $a_{\text{opt}} = a$ for symmetric binary-input channels with $a \leq \frac{1}{2}$. The capacity of the slot-based communication system with ON-OFF signaling for infinite battery is then upper bounded by $h(a_{\text{opt}})$.

Within n slots, the save-and-transmit scheme has two phases. In the first phase, no energy is used, and hence energy is accumulated in the battery for $g(n)$ slots, where $g(n)$ denotes a monotonic increasing function such that $\lim_{n\to\infty} g(n)/n = 0$. In the second phase, on average, a_{opt} energy quanta are used per slot.

The following proposition proves that the upper bound $h(a_{\text{opt}})$ can be achieved using the *save-and-transmit* scheme.

Proposition 1. *The capacity of the channel defined by the slot-based model with an infinite-capacity battery is given by $h(a_{\text{opt}})$ for arbitrary binary-input memoryless channels.*

Proof. The proposition is proved by defining a transmission scheme achieving the upper bound $h(a_{\text{opt}})$. We consider n slots. Let X_k be defined as:

$$X_k \sim \text{Bern}(a_{\text{opt}}) \qquad \text{for } g(n) < k \leq n \tag{22}$$
$$X_k = 0 \qquad \text{for } 1 \leq k \leq g(n). \tag{23}$$

It is noted that X_k are i.i.d. for $g(n) < k \leq n$. This scheme is feasible if the energy constraints (20) are satisfied. We show that these energy constraints are satisfied almost surely for a certain choice of $g(n)$ when n tends to infinity.

Let $Q^{1,g(n)}$ denote the sequence of energy arrivals from time 1 to $g(n)$. The sequence $Q^{1,g(n)}$ becomes ϵ-letter typical with high probability as $g(n)$ tends to infinity [12]:

$$1 - \delta_\epsilon(g(n)) \leq \Pr[Q^{1,g(n)} \in T_\epsilon^{g(n)}(P_Q)] \leq 1, \tag{24}$$

where $\delta_\epsilon(k) = 2|\mathcal{Q}|e^{-k\epsilon^2\mu}$ and $T_\epsilon^{g(n)}(P_Q)$ is the set of ϵ-letter typical sequences of length $g(n)$ with each letter distributed according to the pmf $P_Q(\cdot)$. Thus according to [12, Theorem 1.1] the number of occurrences of the letter 1 is bounded as:

$$g(n)a(1-\epsilon) \leq \sum_{i=1}^{g(n)} q_i \leq g(n)a(1+\epsilon), \tag{25}$$

where q_i is the i-th entry of the ϵ-letter typical sequence $q^{1,g(n)}$.

Define the sequence $Z_k = Q_{g(n)+i} - X_{g(n)+i}$ for $k = 1, \ldots, n - g(n)$. The mean and variance of this sequence are given as $\mathbb{E}[Z_k] = a - a_{\text{opt}}$ and $\text{Var}[Z_k] = a + a_{\text{opt}}$, respectively. Now, define the sequence S_k as

$$S_k = k(a_{\text{opt}} - a) + \sum_{i=1}^{k} Z_i = a_{\text{opt}} - a + Z_k + S_{k-1} \tag{26}$$

for $k = 1, \ldots, n - g(n)$ and $S_0 = 0$. The sequence S_k is a Martingale since $\mathbb{E}[S_k | S_{k-1}, \ldots, S_1] = S_{k-1}$ [13]. It is noted that $\mathbb{E}[S_k] = 0$ and the second moment is given by $\mathbb{E}[S_k^2] = k(a + a_{\text{opt}})$ for $k \in \{1, \ldots, g(n)\}$. Now, if the energy constraints are not satisfied, all $n - g(n)$ slots are counted as errors. Using the defined scheme this happen if, for any $k = g(n)+1, \ldots, n$, $\sum_{i=1}^{k} Z_i < -\sum_{i=1}^{g(n)} Q_i$. We define the following events:

- E_1 is the event that $Q^{1,g(n)}$ is not typical i.e. $Q^{1,g(n)} \notin T_\epsilon^{g(n)}(P_Q)$.
- E_2 is the event that the energy constraints are not satisfied i.e. $\sum_{i=1}^{k} Z_i \leq -\sum_{i=1}^{g(n)} Q_i$ for any $k = 1, \ldots, n - g(n)$.

Thus the event E_2^C and therefore also $(E_1 \cup E_2)^C$ is sufficient to make the scheme feasible. An upper bound for the probability of an infeasible transmission is therefore given by:

$$\Pr[E_2] \leq \Pr[E_1] + \Pr\left[E_2 \mid E_1^C\right] \tag{27}$$

The probability $\Pr[E_1]$ is upper bounded by $\delta_\epsilon(g(n))$ from (24), and hence the probability $\Pr[E_1] \to 0$ as $n \to \infty$.

As S_k is a Martingale, Kolmogorov's martingale inequality [13, Corollary 1] can be used to upper bound $\Pr[E_2 | E_1^C]$ as:

$$\Pr\left[E_2 \mid E_1^C\right] = \Pr\left[\min_{1 \leq k \leq n-g(n)} \sum_{i=1}^{k} Z_i \leq -\sum_{i=1}^{g(n)} Q_i \;\middle|\; \sum_{i=1}^{g(n)} Q_i \geq g(n)a(1-\epsilon)\right] \tag{28}$$

$$\leq \Pr\left[\min_{1 \leq k \leq n-g(n)} \sum_{i=1}^{k} Z_i \leq -g(n)a(1-\epsilon)\right] \tag{29}$$

$$\leq \Pr\left[\min_{1 \leq k \leq n-g(n)} k(a_{\text{opt}} - a) + \sum_{i=1}^{k} Z_i \leq -g(n)a(1-\epsilon)\right] \tag{30}$$

$$\leq \Pr\left[\max_{1 \leq k \leq n-g(n)} |S_k| \geq g(n)a(1-\epsilon)\right] \tag{31}$$

$$\leq \frac{\mathbb{E}[S_{n-g(n)}^2]}{(g(n)a(1-\epsilon))^2}, \tag{32}$$

where the inequality in (30) follows since $k(a_{\text{opt}} - a) \leq 0$ by definition of a_{opt} and (32) follows from Kolmogorov's martingale inequality. By letting n tend to infinity and by defining the function $g(n) = n^p$ for $\frac{1}{2} < p < 1$ yields

$$\lim_{n\to\infty} \Pr[E_2 | E_1^C] \leq \lim_{n\to\infty} \frac{\mathbb{E}[Z_{n-g(n)}^2]}{(g(n)a(1-\epsilon))^2} \tag{33}$$

$$= \frac{a + a_{\text{opt}}}{a^2(1-\epsilon)^2} \lim_{n\to\infty} \frac{n - g(n)}{g(n)^2} \tag{34}$$

$$= \frac{a + a_{\text{opt}}}{a^2(1-\epsilon)^2} \lim_{n\to\infty} \frac{n}{n^{2p}} \tag{35}$$

$$= 0, \tag{36}$$

where the last step follows from the fact that $\frac{1}{2} < p < 1$. Since both $\Pr[E_1]$ and $\Pr[E_2 | E_1^C]$ tend to zero, the scheme is achievable almost surely.

The information rate obtained using this scheme is found as:

$$\frac{1}{n} I(X^n; Y^n) = (1 - \Pr[E_1 \cup E_2]) \frac{1}{n} \sum_{i=g(n)+1}^{n} I(X_i; Y_i) \tag{37}$$

$$= (1 - \Pr[E_1 \cup E_2]) \frac{n - g(n)}{n} h(a_{\text{opt}}) \tag{38}$$

$$\overset{n\to\infty}{\to} h(a_{\text{opt}}). \tag{39}$$

Thus the upper bound is asymptotically achieved, and is therefore the capacity of the channel defined by the communication model. □