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Katerina Smiljkovikj, Petar Popovski and Liljana Gavrilovska

Abstract—**Wireless cellular networks evolve towards a hetero-geneous infrastructure, featuring multiple types of Base Stations (BSs), such as Femto BSs (FBSs) and Macro BSs (MBSs). A wireless device observes multiple points (BSs) through which it can access the infrastructure and it may choose to receive the downlink (DL) traffic from one BS and send uplink (UL) traffic through another BS. Such a situation is referred to as *decoupled DL/UL access*. Using the framework of stochastic geometry, we derive the association probability for DL/UL. In order to maximize the average received power, as the relative density of FBSs initially increases, a large fraction of devices chooses decoupled access, i.e. receive from a MBS in DL and transmit through a FBS in UL. We analyze the impact that this type of association has on the average throughput in the system.**

Index Terms—**Heterogeneous networks, decoupled down-link/uplink, average throughput.**

I. INTRODUCTION

In the quest for better wireless connectivity and higher data rates, the cellular network is becoming heterogeneous, featuring multiple types of Base Stations (BSs) with different cell size. Heterogeneity implies that the traditional strategies in cell planning, deployment and communication should be significantly revised [1]. Since the number of BSs becomes comparable to the number of devices [2] and the deployment pattern of the BSs is rather irregular, there are multiple BSs from which a device can select one to associate with.

The key issue in a wireless heterogeneous setting is the way in which a device selects an Access Point (AP). The authors in [1] and [3] indicate that the AP selected for downlink (DL), termed Downlink AP (DLAP), is not necessarily the same as the Uplink AP (ULAP). The current cellular networks use a criterion applicable to the DL for association in both directions, i.e. a device selects the BS that offers maximal Signal-to-Interference-plus-Noise Ratio (SINR) in the DL and then uses the same BS for UL transmission. When DLAP≠ULAP, we say that the device has a *decoupled access*. There are two main drivers for decoupled access: (1) the difference in signal power and interference in DL as compared to UL [1]; and (2) the difference in congestion between BSs [3]. Decoupled DL/UL access has been considered in [4], where the authors devise separate criteria for selection of DLAP and ULAP, respectively, and demonstrate the throughput benefits by using real-world data from planning tools of a mobile operator. An-other related work that considers different associations in UL

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and DL is [5], where coverage probability and throughput are analyzed for dynamic TDD networks enhanced with Device-to-Device (D2D) links.

This letter focuses on the analytical characterization of the decoupled access by using the framework of stochastic geometry [6]. We use the same association criteria as in [4]. We perform a joint analysis of the DL and UL association, using the same realization of the random process that describes spatial deployment of the BSs and devices. The analysis is performed for a two-tier cellular network, consisting of Macro BSs (MBSs) and Femto BSs (FBSs). This is used to obtain the central result of the paper, which is the set of association probabilities for different DL/UL configurations. The analytical results are closely matching the simulations and provide interesting insights about the decoupled access in terms of e.g. fairness regarding the UL throughput. Combining novel results from this letter with already available results in the literature, we provide an analytical justification of the phenomenon of decoupled access compared to current DL-based association in heterogeneous networks.

The letter is organized as follows. Section II describes the system model. In Section III, we derive the association probabilities and the average throughput. Section IV gives the numerical results and Section V concludes the paper.

II. SYSTEM MODEL

We model a two-tier heterogeneous cellular network. The locations of BSs are modeled with independent homogeneous Poisson Point Processes (PPPs). We use Φ_v to denote the set of points obtained through a PPP with intensity λ_v, where $v = M$ for MBSs, $v = F$ for FBSs and $v = d$ for the devices. Similarly, we use P_v with $v \in \{M, F, d\}$ to denote the transmission power of the node v. The variables $x_M, x_F \in \mathbb{R}^2$ denote the two-dimensional coordinate of MBS and FBS, respectively. The analysis is performed for a typical device located at the origin, which is the spatial point $x_d = (0,0)$. By Slivnyak's theorem [7], the distribution of a point process in $\mathbb{R}^2$ is unaffected by addition of a node at the origin. The power received by a typical device in DL from a BS located at $x_v \in \Phi_v$, where $v \in \{F, M\}$ is denoted by $\mathrm{S}_{v,D}$. The power received by a BS from the typical device in UL is denoted by $\mathrm{S}_{v,U}$. These powers are given by:

$$\mathrm{S}_{v,D} = P_v h_{x_v} \|x_v\|^{-\alpha} ; \mathrm{S}_{v,U} = P_d h_{x_v} \|x_v\|^{-\alpha} \qquad (1)$$

where $\|x_M\|$ and $\|x_F\|$ are distances from the points $x_M \in \Phi_M$ and $x_F \in \Phi_F$ to the origin, respectively, and α is the path loss exponent ($\alpha > 2$). h_{x_v} is independent exponentially distributed random variable with unit mean, representing

Rayleigh fading at the point x_v. Each receiver in the system has a constant noise power of σ^2.

The DL SINR when the device is associated to vBS is:

$$\text{SINR}_v^D = \frac{P_v h_{x_v} \|x_v\|^{-\alpha}}{\sum\limits_{x_j \in \Phi_v \setminus \{x_v\}} P_v h_{x_j} \|x_j\|^{-\alpha} + \sum\limits_{x_i \in \Phi_u} P_u h_{x_i} \|x_i\|^{-\alpha} + \sigma^2}, \tag{2}$$

where $v, u \in \{M, F\}$ and $v \neq u$. With the notion of typical point located at the origin, UL SINR is calculated at the location of ULAP. This involves calculation of distances between the interfering devices and ULAP, which complicates the analysis because none of them is located at the origin. The problem is solved by using the translation-invariance property of stationary point processes, by which the processes $\Phi = \{x_n\}$ and $\Phi_x = \{x_n + x\}$ have the same distribution for all $x \in \mathbb{R}^2$ [7]. Thus, translation of the points for the same value of x preserves the process properties. We use this to shift the points for the distance between the typical device and ULAP such that the ULAP becomes located at the origin.

The interfering devices are modeled by thinning the PPP Φ_d in order to take into account that only one device per BS acts as an interferer, using the same resource as the typical device [8]. By thinning, we randomly select fraction of points from the original point process [7] with probability $p = \frac{\lambda_M + \lambda_F}{\lambda_d}$. The thinned process is denoted as Φ_{I_d} with density $\lambda_{I_d} = p\lambda_d$. The presence of a device in a Voronoi cell of a BS forbids the presence of other devices and introduces dependence among the active devices. However, this dependence is weak, as shown in [8], and it is justified to assume independent PPP for the active devices. The UL SINR at vBS is defined as:

$$\text{SINR}_v^U = \frac{P_d h_{x_v} \|x_v\|^{-\alpha}}{\sum\limits_{x_j \in \Phi_{I_d}} P_d h_{x_j} \|x_j\|^{-\alpha} + \sigma^2} \tag{3}$$

III. ANALYSIS

The analysis is divided into two mutually related parts. We first derive the association probabilities for DL/UL and afterward use them to evaluate the average throughput.

A. Association Probability

In DL, the device is associated to the BS from which it receives the highest average power. In UL it is associated to BS to which it transmits with the highest average power. The average power is obtained by averaging over the received signals given by (1) with respect to the fading. This is justified as the fading-induced variations can lead to ping-pong effects in the association process. The average received signal powers in DL and UL are:

$$\mathbb{E}_h \left[\mathbf{S}_{v,D} \right] = P_v \|x_v\|^{-\alpha}; \mathbb{E}_h \left[\mathbf{S}_{v,U} \right] = P_d \|x_v\|^{-\alpha} \tag{4}$$

ULAP will always be the closest BS at distance $\|x_v\|$, where $v \in \{M, F\}$. The DL case is more complicated since P_v is also variable. Let x_v^0 be the closest point to the origin from the set Φ_v, with $v \in \{M, F\}$. The device is associated to

$$\text{MBS in DL if } P_M \left\| x_M^0 \right\|^{-\alpha} > P_F \left\| x_F^0 \right\|^{-\alpha} \tag{5}$$

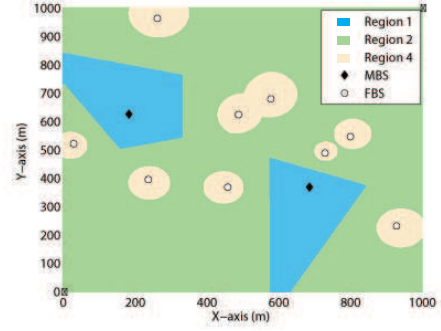$$\text{MBS in UL if } P_d \left\| x_M^0 \right\|^{-\alpha} > P_d \left\| x_F^0 \right\|^{-\alpha} \tag{6}$$

Fig. 1: Association regions (P_M=46 dBm, P_F=20 dBm, α=4).

Otherwise, the device is associated to FBS. Let $X_v \equiv \left\| x_v^0 \right\|$. The distribution of X_v follows from the null probability of 2D PPP [7], the probability that there is no point in the circle with radius x, i.e. $\Pr(X_v > x) = e^{-\pi \lambda_v x^2}$. The pdf of X_v is:

$$f_{X_v}(x) = 2\pi \lambda_v x e^{-\pi \lambda_v x^2}, x \geq 0 \tag{7}$$

For two-tier heterogeneous network, there are four possible combinations for choosing DLAP and ULAP:

1) Case 1: DLAP=ULAP=MBS: The probability that a device will be associated to MBS both in DL and UL is:

$$\Pr\left(X_M^{-\alpha} > \frac{P_F}{P_M} X_F^{-\alpha}; X_M^{-\alpha} > X_F^{-\alpha}\right) \tag{8}$$

Assuming $P_F < P_M$, it follows that $P_F/P_M < 1$. Therefore, the intersection of the events is the region defined by $X_M^{-\alpha} > X_F^{-\alpha}$, denoted as Region 1 on Fig. 1. The association probability of Case 1 is calculated as:

$$\Pr(\text{Case 1}) = \Pr(X_M^{-\alpha} > X_F^{-\alpha}) =$$
$$= \int_0^\infty (1 - F_{X_F}(x_M)) f_{X_M}(x_M) \mathrm{d}x_M = \frac{\lambda_M}{\lambda_M + \lambda_F} \tag{9}$$

The derivation of the remaining cases follows the same procedure and we thus only provide the final results.

2) Case 2: DLAP=MBS and ULAP=FBS: Case 2 defines decoupled access since DLAP$\neq$ULAP. The association probability is defined as:

$$\Pr\left(X_M^{-\alpha} > \frac{P_F}{P_M} X_F^{-\alpha}; X_M^{-\alpha} \leq X_F^{-\alpha}\right) \tag{10}$$

The domain that satisfies both events is $\frac{P_F}{P_M} X_F^{-\alpha} < X_M^{-\alpha} \leq X_F^{-\alpha}$ and is denoted as Region 2 on Fig. 1. The association probability for Case 2 is equal to:

$$\Pr(\text{Case 2}) = \frac{\lambda_F}{\lambda_F + \lambda_M} - \frac{\lambda_F}{\lambda_F + \left(\frac{P_M}{P_F}\right)^{2/\alpha} \lambda_M} \tag{11}$$

3) Case 3: DLAP=FBS and ULAP=MBS: The association probability for Case 3 should satisfy the following conditions:

$$X_M^{-\alpha} \leq \frac{P_F}{P_M} X_F^{-\alpha} \cap X_M^{-\alpha} > X_F^{-\alpha} \tag{12}$$

The intersection (12) is an empty set and therefore the probability of DLAP=FBS and ULAP=MBS is $\Pr(\text{Case 3}) = 0$.

4) Case 4: DLAP=ULAP=FBS: The probability for associating to FBS in both DL and UL is defined as:

$$\Pr\left(X_F^{-\alpha} \geq \frac{P_M}{P_F} X_M^{-\alpha}; X_F^{-\alpha} \geq X_M^{-\alpha}\right) \tag{13}$$

Since $P_M/P_F > 1$, the intersection of the events is $X_F^{-\alpha} \geq \frac{P_M}{P_F} X_M^{-\alpha}$, denoted as Region 4 on Fig. 1. The association probability for Case 4 is equal to:

$$\Pr(\text{Case } 4) = \frac{\lambda_F}{\lambda_F + \left(\frac{P_M}{P_F}\right)^{2/\alpha} \lambda_M} \quad (14)$$

B. Average throughput

The average throughput for devices associated to vBS with $v \in \{M, F\}$ in mL direction using nL association rules, with $m, n = D$ for DL and $m, n = U$ for UL, is calculated as:

$$\text{R}_{v,m,n}(\gamma_{th}) = \frac{1}{N_{v,n}}\log_2(1 + \gamma_{th})\text{P}_{c,v,m}(\gamma_{th}) \quad (15)$$

where γ_{th} is target SINR, $\text{P}_{c,v,m}$ is the probability that the instantaneous SINR is greater than γ_{th} and $N_{v,n}$ is the average number of associated devices on vBS and is equal to $N_{v,n} = \lambda_d A_{v,n}/\lambda_v$, with $A_{v,n}$ being the association probability for vBS using nL association rules. Using the association probabilities derived in Section III-A, $A_{v,n}$ is expressed as:

$$\begin{aligned}
A_{M,D} &= \Pr(\text{Case } 1) + \Pr(\text{Case } 2) \\
A_{F,D} &= \Pr(\text{Case } 3) + \Pr(\text{Case } 4) \\
A_{M,U} &= \Pr(\text{Case } 1) + \Pr(\text{Case } 3) \\
A_{F,U} &= \Pr(\text{Case } 2) + \Pr(\text{Case } 4) \quad (16)
\end{aligned}$$

In order to calculate the average throughput in a two-tier network, we first need to calculate the distribution of the distance to the serving BS, which depends on the association process. For DL association rules, given by (5), the pdf of the distance to the serving BS is derived in [9] and is given by:

$$f_{X_{v,D}}(x) = \frac{2\pi\lambda_v}{A_{v,D}} x e^{-\left(\lambda_v + \lambda_u \left(\frac{P_u}{P_v}\right)^{2/\alpha}\right)\pi x^2} \quad (17)$$

For UL association rules, given by (6), the pdf of the distance to the serving BS is given by:

$$f_{X_{v,U}}(x) = \frac{2\pi\lambda_v}{A_{v,U}} x e^{-(\lambda_v + \lambda_u)\pi x^2} \quad (18)$$

The average throughput in the DL is calculated as:

$$\text{R}_D(\gamma_{th}) = \text{R}_{M,D,D}(\gamma_{th})A_{M,D} + \text{R}_{F,D,D}(\gamma_{th})A_{F,D} \quad (19)$$

where $\text{R}_{M,D,D}(\gamma_{th})$ and $\text{R}_{F,D,D}(\gamma_{th})$ are expressed by the general formula given by (15). Using the approach derived in [9], we derive the final expression for the average throughput in DL on vBS:

$$\text{R}_{v,D,D}(\gamma_{th}) = \frac{1}{N_{v,D}}\log_2(1 + \gamma_{th})\frac{2\pi\lambda_v}{A_{v,D}} x \times$$
$$\int_0^\infty e^{-\frac{\gamma_{th}\sigma^2 x^\alpha}{P_v}} e^{-\left(\lambda_v + \lambda_u \left(\frac{P_u}{P_v}\right)^{2/\alpha}\right)(1+\kappa(\alpha,\gamma_{th}))\pi x^2} dx \quad (20)$$

where $\kappa(\alpha, \gamma_{th}) = \gamma_{th}^{2/\alpha}\int_{\gamma_{th}^{-2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}}du$. The key point in the evaluation is the following observation: if the device is associated to MBS located at x the interfering MBSs are at a distance greater than x and the interfering FBSs are at a distance greater than $(P_F/P_M)^{1/\alpha} x$; if the device is

associated to FBS located at x, the interfering FBSs are at a distance greater that x and the interfering MBSs are at a distance greater than $(P_M/P_F)^{1/\alpha} x$.

The average downlink throughput can be also calculated in a more elegant way by using the following:

Lemma 1. *(Equivalent downlink model)* A two-tier heterogeneous model is equivalent to a novel homogeneous model with BSs deployed by PPP $\widetilde{\Phi}_{MF}$ with intensity $\widetilde{\lambda}_{MF} = (\lambda_M + \lambda_F)\left(P_F^{2/\alpha}\frac{\lambda_F}{\lambda_M + \lambda_F} + P_M^{2/\alpha}\frac{\lambda_M}{\lambda_M + \lambda_F}\right)$. Then, the average throughput in DL can be calculated as:

$$\widetilde{R}_D(\gamma_{th}) = \frac{1}{\widetilde{N}_{MF}}\log_2(1 + \gamma_{th})\int_0^\infty e^{-\gamma_{th}\sigma^2 x^\alpha} \times$$
$$2\pi\widetilde{\lambda}_{MF} x e^{-(\kappa(\gamma_{th},\alpha)+1)\pi\widetilde{\lambda}_{MF}x^2} dx \quad (21)$$

where $\widetilde{N}_{MF} = \lambda_d/\widetilde{\lambda}_{MF}$.

Proof: The DL signal power at the typical point can be represented as $\text{S}_D = Zh\|x_{MF}\|^{-\alpha} = h\left\|Z^{-\frac{1}{\alpha}}x_{MF}\right\|^{-\alpha} = h\|y_{MF}\|^{-\alpha}$, where Z is a discrete random variable with two possible values, P_M and P_F, with probabilities $\Pr(P_M) = \lambda_M/(\lambda_M + \lambda_F)$ and $\Pr(P_F) = \lambda_F/(\lambda_M + \lambda_F)$, respectively. The points x_{MF} are from a PPP Φ_{MF} with density $\lambda_{MF} = \lambda_M + \lambda_F$. By equivalence theorem [10], the spatial points y_{MF} form new PPP with density $\widetilde{\lambda}_{MF} = \lambda_{MF}\mathbb{E}\left[Z^{2/\alpha}\right]$. ∎

The average throughput in UL without decoupled access is calculated using DL association rules given by (5):

$$\text{R}_U(\gamma_{th}) = \text{R}_{M,U,D}(\gamma_{th})A_{M,D} + \text{R}_{F,U,D}(\gamma_{th})A_{F,D} \quad (22)$$

where $\text{R}_{M,U,D}(\gamma_{th})$ and $\text{R}_{F,U,D}(\gamma_{th})$ are evaluated as:

$$\text{R}_{v,U,D}(\gamma_{th}) = \frac{1}{N_{v,D}}\log_2(1 + \gamma_{th})\int_0^\infty e^{-\frac{\gamma_{th}\sigma^2 x^\alpha}{P_d}} \times$$
$$e^{-\pi\lambda_{I_d}\kappa(\alpha,\gamma_{th})x^2} f_{X_{v,D}}(x)dx \quad (23)$$

It can be observed that the distribution to the serving BS and the association probabilities are from DL association rules.

The average throughput in UL with decoupled access is:

$$\text{R}_U^d(\gamma_{th}) = \text{R}_{M,U,U}^d(\gamma_{th})A_{M,U} + \text{R}_{F,U,U}^d(\gamma_{th})A_{F,U} \quad (24)$$

where $\text{R}_{M,U,U}^d(\gamma_{th})$ and $\text{R}_{F,U,U}^d(\gamma_{th})$ are evaluated as:

$$\text{R}_{c,v,U}^d(\gamma_{th}) = \frac{1}{N_{v,U}}\log_2(1 + \gamma_{th})\int_0^\infty e^{-\frac{\gamma_{th}\sigma^2 x^\alpha}{P_d}} \times$$
$$e^{-\pi\lambda_{I_d}\kappa(\alpha,\gamma_{th})x^2} f_{X_{v,U}}(x)dx \quad (25)$$

Remark 1 (Equivalent uplink model) *A two-tier HetNet model with homogeneous devices is represented by an equivalent homogeneous model with BSs deployed by PPP Φ_{MF} with intensity $\lambda_{MF} = \lambda_M + \lambda_F$. This is a consequence of the UL association rule, which is based on path-loss only. Then, the average throughput in UL can be elegantly calculated as:*

$$\widetilde{\text{R}}_U^d = \frac{1}{N_{MF}}\log_2(1 + \gamma_{th})\int_0^\infty e^{-\frac{\gamma_{th}\sigma^2 x^\alpha}{P_d}} \times$$
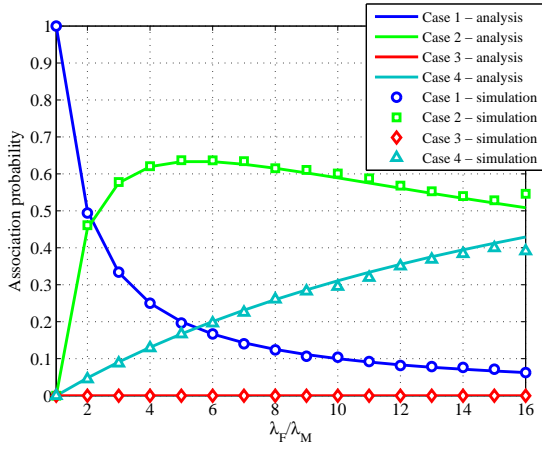$$2\pi\lambda_{MF} x e^{-(\lambda_{MF} + \lambda_{I_d}\kappa(\gamma_{th},\alpha))\pi x^2} dx \quad (26)$$

Fig. 2: Joint association probabilities for Cases 1-4 (P_M=46 dBm; P_F=20 dBm; α=4).

The throughput gain for Case i, $i \in \{1, 2, 3, 4\}$, is defined as the ratio between the throughput achieved with and without decoupling and is denoted as η(Case i). The average throughput gain is calculated as: $\overline{\eta} = \sum_{i=1}^{4} \Pr(\text{Case } i)\eta(\text{Case } i)$.

IV. NUMERICAL RESULTS

The association probabilities for each of the cases are equal to the percentage of devices that will be associated with the particular case. Fig. 2 shows the association probabilities for different densities of FBSs and it gives an important information about DL/UL decoupling. The percentage of devices that choose decoupled access of Case 2 (DL through MBS and UL through FBS) increases rapidly by increasing the density of FBSs. As the density of FBSs increases further, the probability for decoupled access starts to decrease slowly at the expense of increased probability for Case 4. There is a region of interest for λ_F with a high percentage $> 60\%$ of devices for which the decoupled access is optimal. As $\lambda_F / \lambda_M \to \infty$, the probability of decoupled access will go to zero.

Fig. 3 shows the throughput gain for the devices associated to (M/F)BSs and the average gain. There is a difference between, on one side, the accurate simulation of the devices with PPP Φ_d, and, on the other side, its approximation by simulation of independent PPP Φ_{I_d} for the active devices only. While the UL coverage probability with DL/UL decoupling is strictly superior, the congestion of the BSs affects the throughput in a different manner. Basically, FBSs have significantly small DL coverage and therefore associate very small number of devices compared to MBSs, but each device gets higher throughput. It is visible that for γ_{th}=2 dB the throughput achieved on MBSs is 40 times higher with decoupling, while the throughput achieved on FBSs is 5 times lower with decoupling. The average throughput gain is always positive. It can be concluded that by DL/UL decoupling the devices with low SINR (located in Regions 1 and 2) achieve significant improvement, at the expense of marginal decrease in the UL throughput of the devices in Region 4. This suggests that decoupled access can be used as a tool towards achieving fairness among the accessing devices.
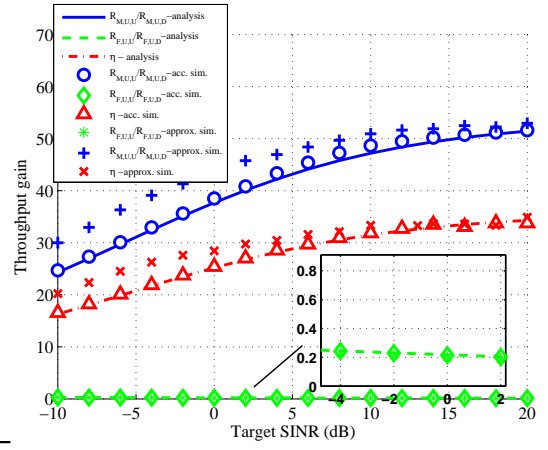

Fig. 3: UL throughput gain analysis, compared with an accurate simulation of devices (acc. sim.) and approximation by independent active devices (approx. sim.) (P_M=46dBm; P_F=20dBm; P_d=20dBm; α=4; λ_F=10λ_M; σ^2=10^{-12}).

V. CONCLUSION

This letter considers the problem of device association in a heterogeneous wireless environment. The analysis is done using models based on stochastic geometry. The main result is that, as the density of the Femto BSs (FBSs) increases compared to the density of the Macro BSs (MBSs), a large fraction of devices chooses to receive from a MBS in the downlink (DL) and transmit to a FBS in the uplink (UL). This is the concept of *decoupled access* and challenges the common approach in which both DL and UL transmission are associated to the same BS. It is shown that the decoupling of DL and UL can be used as a tool to improve the fairness in the UL throughput. Part of our future work refers to the architecture for decoupled access, which includes signaling and radio access protocols.

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