Optimal Design of Composite Structures Under Manufacturing Constraints

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Optimal Design of Composite Structures Under Manufacturing Constraints

Ph.D. Thesis

by

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To the love of my life
Louise

Preface

This thesis has been submitted to the Faculty of Engineering and Science at Aalborg University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering. The underlying work has been carried out at the Department of Mechanical and Manufacturing Engineering, Aalborg University during the period from August 2011 to December 2014. The work is part of the research project entitled *Optimal Design of Composite Structures Under Manufacturing Constraints* which has been funded by the Danish Research Council for Technology and Production Sciences (FTP) Grant. no 10-082695. This support is gratefully acknowledged.

The project has primarily been supervised by Professor Erik Lund at Aalborg University to whom I am grateful for having provided competent guidance, and unquestionable support for pursuing my own ideas. Thank you for always providing a positive and inspiring working environment, and for convincing me to pursue the position as a Ph.D. student. Also, a special thanks goes to my colleagues at DTU, especially my co-supervisor Senior Scientist Mathias Stolpe for his fruitful input to the research during either our project meetings, conferences, or at other memorable gatherings.

I would also like to thank my former office buddy, Ph.D. Søren Nørgaard Sørensen for introducing me to his work, his joyful spirit, and our fruitful discussions which ultimately lead to our joint research on the Discrete Material and Thickness Optimization method. I would also like to take the opportunity to thank my colleagues in the Computational Mechanics group for always being supportive and for providing a friendly working atmosphere. It has been a joy and a privilege to work side by side with such dedicated people, thank you all.

Finally, I would like to express my gratitude towards my parents for their loving support, and especially my partner, Louise, whose endless patience and kindness always makes me feel like I am the luckiest man in the world.

René Sørensen
Aalborg, December 2014

Abstract

The purpose of this work has been to develop methods and parameterizations for gradient based discrete material and thickness optimization of laminated composite structures. Here, special attention has been on increasing the manufacturability of the optimized design by incorporating so-called manufacturing constraints or design rules.

Up until now, researchers have proposed a wide variety of methods and parameterizations for performing discrete material optimization of laminated composite structures. However, the majority of these have focused on laminates with a fixed number of layers i.e., constant thickness laminates. Still, for many high performance structures weight becomes an important design parameter both with respect to performance and cost. Hence, determining both an optimum material composition and thickness variation could further help manufacturers to design even lighter structures, thereby increasing performance while reducing material expenses. It is therefore vital to develop new methods which simultaneously can determine both an optimum material and thickness variation.

Because of the complexity associated with laminated composite structures, engineers typically rely on finite element analysis using equivalent single layer shell formulations in order to validate the integrity of the entire structure. These computationally efficient methods are, however, not capable of predicting out-of-plane stress fields near e.g., a change in the thickness variation which can lead to failure due to delamination. In order to avoid detailed modeling around these critical regions, engineers can apply design rules which effectively reduce the influences from these local effects. By incorporating these design rules into the optimization problem as manufacturing constraints, engineers can avoid these undesired structural designs, and thereby reduce the amount of time required for manual post-processing.

In this work, methods for performing simultaneous material and thickness optimization of laminated composite structures are presented. The methods have been demonstrated for problems considering mass minimization including non-linear structural constraints on linearized buckling load factors, natural eigenfrequencies, limited displacements, together with a series of known manufacturing constraints.

A new type of parameterization is also introduced in the form of thickness filters. These filters replace the previously applied layerwise density variables with continuous through-the-thickness design variables. Besides a substantial reduction in the number of design variables, the filters also eliminate the need for having explicit constraints for preventing intermediate void from appearing in between layers of the laminate. New methods are also presented which makes it possible for engineers to control the minimum size of the geometry and the material continuity while the optimizer is free to determine which material to apply, together with the location, shape, and size of the selected materials.

A total of three scientific journal papers have been produced which document the capabilities of the presented methods and parameterizations.

Dansk Resumé

Formålet med nærværende arbejde har været at udvikle nye metoder og parametriseringer til diskret materiale- og tykkelsesoptimering af laminerede kompositkonstruktioner ved hjælp af gradientbaserede metoder. Her har der været særlig fokus på at gøre det lettere at fremstille de optimerede konstruktioner ved at inkludere såkaldte designregler i optimeringsprocessen.

Hidtil har forskere foreslået en lang række af metoder og parametriseringer til diskret materialeoptimering af laminerede kompositstrukturer. Imidlertid har de fleste af disse fokuseret på laminater med et fast antal lag, dvs. laminater med konstant tykkelse. Med den fortsatte udvikling af stadig mere avancerede og højtydende konstruktioner bliver den samlede vægt dog en kritisk designparameter både med hensyn til ydeevne og omkostninger. Det er derfor afgørende at udvikle nye metoder, der simultant kan fastlægge både en optimal materialesammensætning samt tykkelsesvariation. Dette vil kunne hjælpe producenter til at fremstille endnu lettere konstruktioner og derved øge disse konstruktioners ydeevne samt reducere materialeomkostninger.

På grund af kompleksiteten forbundet med laminerede kompositkonstruktioner benytter ingeniører sig typisk af elementmetoden sammen med homogeniserede skalformuleringer for at analysere og validere integriteten af hele strukturen. Disse beregningsmæssige effektive metoder er imidlertid ikke i stand til at forudsige spændingskoncentrationer nær strukturelle detaljer såsom en lokal ændring i laminattykkelsen. Disse spændingskoncentrationer kan resultere i en delaminering, hvilket kan have fatale konsekvenser til følge for konstruktionen. For at undgå detaljerede modelleringer af disse kritiske områder kan ingeniører anvende såkaldte designregler, som implicit tager højde for disse lokale effekter. Ved at inkludere disse designregler i optimeringsproblemet som bi-betingelser kan ingeniører undgå disse uønskede strukturelle detaljer, og derved reducere mængden af tid, der typisk kræves til manuel efterbehandling af det optimerede design.

I det fortløbende arbejde præsenteres nye metoder, der simultant kan fastlægge både en optimal materialesammensætning samt tykkelsesvariation med henblik på optimalt design af laminerede kompositkonstruktioner. Disse metoder er blevet demonstreret for problemer, hvor den samlede strukturelle masse minimeres, og hvor der ligeledes er inkluderet strukturelle ikkelineære bi-betingelser for lineær buling, naturlige egenfrekvenser, begrænsning af flytninger, samt en række af kendte designregler.

En ny parametrisering bestående af tykkelsesfiltre bliver ligeledes introduceret. Disse filtre erstatter de tidligere anvendte lagvise densitetsvariable med kontinuerte designvariable, som strækker sig ned igennem tykkelsen af laminatet. Foruden en væsentlig reduktion i antallet af designvariable eliminerer filtrene behovet for at have eksplicitte bi-betingelser for at forhindre tomrum i mellemliggende lag. Der præsenteres også nye metoder, som gør det muligt at definere minimumsgrænser for geometri- og materialekontinuitet, alt imens optimeringsalgoritmen frit kan bestemme, hvilke materialer, der skal anvendes, samt disses placering og udformning.

I alt er der blevet udarbejdet tre videnskabelige tidsskriftsartikler, som dokumenterer de udviklede metoder og parametriseringer.

Mandatory pages

This dissertation gives an introduction to the research areas covered by the Ph.D., project and the three papers submitted to refereed scientific journals.

Thesis Title

Optimal Design of Composite Structures Under Manufacturing Constraints

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Publications in refereed journals

- Sørensen S.N, Sørensen R., Lund E. (2014): "DMTO - a method for Discrete Material and Thickness Optimization of laminated composite structures", *Structural and Multidisciplinary Optimization* 50(1):25-47
- Sørensen R., Lund E. (2014): "Thickness Filters for Discrete Material and Thickness Optimization of Laminated Composite Structures", *Structural and Multidisciplinary Optimization*, in review
- Sørensen R., Lund E. (2014): "In-plane material filters for the Discrete Material Optimization method", *Structural and Multidisciplinary Optimization*, submitted

Publications in proceedings and monographs with review

- Sørensen R., Lund E. (2012): "Ply-based Optimization of Laminated Composite Shell Structures under Manufacturing Constraints", *Proceedings of the 15th European Conference on Composite Materials*, Venice, Italy, June, 8 pages
- Sørensen S.N., Sørensen R., Lund E. (2012): "Thickness optimization of laminated composites using the discrete material optimization method", *Proceedings of the 25th Nordic Seminar on Computational Mechanics*, Lund, Sweden, October, pp. 201–204
- Sørensen R., Sørensen S.N, Lund E. (2013): "A Methodology for Material and Thickness Optimization of Laminated Composite Structures", *Book of Abstracts, 10th World Congress on Structural and Multidisciplinary Optimization*, Orlando, Florida, USA, May, 1 page
- Lund E., Sørensen R., Sørensen S.N. (2013): "Multi-Criteria Multi-Material Topology Optimization of Laminated Composite Structures including Local Constraints", *Book of Abstracts, 10th World Congress on Structural and Multidisciplinary* Optimization, Orlando, Florida, USA, May, 1 page
- Sørensen R., Lund E. (2013): "Multi-Material and Thickness Optimization Utilizing Casting Filters for Laminated Composite Structures", *Proceedings of the 26th Nordic Seminar on Computational Mechanics*, Oslo, Norway, October, pp. 38-41
- Lund E., Sørensen R., Niu B. (2014): "Comparison Of Discrete Material Optimization Approaches For Optimization Of Laminated Composites" *Book of Abstracts, 11th World Congress on Computational Mechanics*, Barcelona, Spain, July, 2 pages
- Sørensen R., Lund E. (2014): "On Discrete Material and Thickness Optimization of Laminated Composite Structures using Filters", *Book of Abstracts, 4th International Conference on Engineering Optimization*, Lisbon, Portugal, September, 1 page.

This thesis has been submitted for assessment in partial fulfillment of the PhD degree. The thesis is based on the submitted or published scientific papers which are listed above. Parts of the papers are used directly or indirectly in the extended summary of the thesis. As part of the assessment, co-author statements have been made available to the assessment committee and are also available at the Faculty. The thesis is not in its present form acceptable for open publication but only in limited and closed circulation as copyright may not be ensured

Contents

1 Introduction

In the following, a brief introduction to the main topics and objectives of the Ph.D., project is presented. The topics include laminated composites, their use, production and modeling methods, and subsequently how to optimize the performance of such structures.

1.1 Laminated composites

Composite materials are in short a mixture between two or more different materials where the goal is to utilize the best properties of the applied constituents. A well-known example is concrete which is composed of water, aggregate, and cement. When the cement is hardened it forms a so-called matrix material which binds the aggregate materials together forming a solid composite material. Another example is fiber reinforced materials where strong fibers such as carbon or glass fibers are embedded in a light weight polymer matrix such as epoxy. By stacking and bonding fiber reinforced plies on top of each other it is possible to form a laminated composite structure. Because each ply can have a unique fiber orientation it is possible to *tailor* the material properties for the specified application, typically resulting in a high stiffness to weight ratio. It is also possible to combine fiber reinforced plies with blocks made of foam or wood, so to make a lightweight sandwich structure. This design freedom has made laminated composites a more and more popular alternative to conventional lightweight materials such as aluminium. Consequently, laminated composites are today applied in a wide variety of high performance products ranging from badminton rackets and mountain bikes, to automotive vehicles, aircraft fuselages, and wind turbine blades. An example of a five layered laminated composite plate with a layup made from Carbon and Glass Fiber Reinforced Polymer (CFRP/GFRP) plies separated by a foam core is shown in Fig. 1.1.

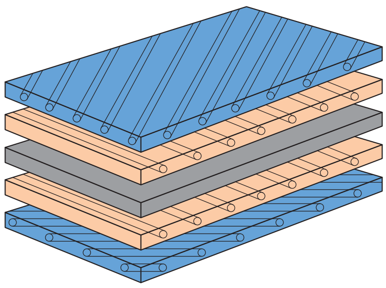


Fig. 1.1 Exploded view of a five layered laminated composite plate. The layup consists of carbon and glass fiber reinforced polymer plies separated by a foam core.

Depending upon the size, shape, applied materials, and cost, different methods can be applied for manufacturing of laminated composite structures. For large structures such as wind turbine blades variations of Vacuum Assisted Resin Transfer Molding (VARTM) can be used. Here, the dry fiber mats and core materials are positioned in a mold by hand and wrapped in a vacuum bag. When the layup process is complete the mold is sealed and liquid resin/epoxy is sucked through the fiber and core materials using a vacuum. When all the reinforcement materials have been wetted by the resin the curing process is initiated. Fig. 1.2(a) shows a wind turbine blade

(a) A cured wind turbine blade manufactured using the VARTM process

(b) Upraised part of a mold used for placing the reinforcement materials

Fig. 1.2 Photographs courtesy of Siemens Wind Power A/S

which has been manufactured using this technique, and Fig. 1.2(b) shows part of a mold used for placing the reinforcement materials.

An essential part of the design process is to determine a suitable layup i.e., which materials to apply, and also where to place them relative to each other. Here, accurate modeling and simulation methods are required in order to determine how the structure responds to the applied loads and boundary conditions. For structures which can be simplified as plates the Classical Laminate Theory (CLT) can be applied, see e.g., Jones (1999). However, for structures which have complex geometries and loading conditions engineers tend to apply Finite Element Analysis (FEA) in order to validate the integrity of the entire structure. For small parts, an accurate analysis can be made by modeling all layers with 3D solid elements, however, this approach quickly become too computational expensive. In order to reduce the computational cost of the analysis, equivalent single layer shell elements are typically applied. Still, for large structures such as wind turbine blades where the number of layers may exceed several hundred at different locations, determining a suitable layup can be an iterative and time consuming process. This has led to an increasing interest within the field of numerical optimization methods as these methods can aid engineers in determining a suitable layup during the design process.

1.2 Design and optimization

When designing laminated composite structures, engineers not only have to determine which materials to apply, but also determine in which order they should be assembled or stacked, so to obtain a suitable layup with the required structural characteristics. However, because of the complex relationship between the layup and the structural response, it can be difficult to grasp the consequences of changing the layup at various locations. This has lead to natural interest in the development of robust numerical optimization methods to aid engineers in the design process.

Numerical optimization methods rely on two essential components, an analysis model and an optimization algorithm. As previously mentioned, the analysis is typically conducted using a finite element model which is capable of simulating the response of several structural criteria. The analysis model is parameterized by assigning a series of design variables which can be adjusted by an optimizer. In the context of laminated composites, these design variables are typically related to the layup e.g., the number of plies, their material composition (GFRP/CFRP/foam), orientation, and thickness.

From a design perspective, applying fiber orientations and ply thicknesses as continuous design variables may lead to designs with excellent performance. However, from a manufacturing perspective these designs may be difficult to realize. This is because some manufacturers only apply a limited set of fiber mats each with a fixed orientation and thickness. The optimized designs may therefore require a substantial amount of manual post-processing in order to determine a layup which complies with both the available materials and the structural requirements. This has lead to an increasing interest in multi-material parametrization and optimization methods which can determine an optimum layup based on discrete sets of material candidates. Multi-material optimization methods are at the center this work. Consequently, the most significant of these methods are detailed in Section 2.1 for constant thickness designs, and in Section 2.2 for variable thickness designs.

Given the applied parametrization i.e., type of design variables, the optimizer can be configured to either maximize or minimize an objective function which could represent a structural property, response, or even cost. Besides the objective function, additional constraints may be also necessary to include in order to satisfy specific performance demands. These constraints can either be related to some structural criteria, such as the maximum displacement of a point, or related to specific requirements regarding the production or design processes.

Generally, structural criteria can be divided into two categories i.e., global and local criteria. Global criteria are related to the response of the entire structure e.g., mass, stiffness, buckling load factors, or eigenfrequencies. In the context of laminated composites, the most commonly applied setup of criteria functions are mass constrained minimization of compliance which has been investigated by numerous authors. For gradient based multi-material optimization, examples can be found in e.g., Stegmann and Lund (2005), Lund and Stegmann (2005) Bruyneel (2011), Hvejsel et al. (2011), Hvejsel and Lund (2011), Gao et al. (2012), Sørensen and Lund (2013), and Kennedy and Martins (2013). For examples regarding buckling load factors and eigenfrequencies see e.g., Lund (2009), Kennedy and Martins (2013), Stegmann and Lund (2005), Niu et al. (2010), and Blasques (2014).

Local criteria effect local quantities related to specific points on the structure e.g., displacement constraints, fatigue limits, or in-plane failure criteria such as max strain, max stress, or the Tsai-Wu criterium, see e.g., Tsai and Wu (1971), Kim et al. (1994), and Groenwold and Haftka (2006). In the context of gradient based topology optimization with isotropic materials stress constraints are rather well established, see e.g., Duysinx and Bendsøe (1998), Le et al. (2010), París et al. (2010), Bruggi and Duysinx (2012), or Holmberg et al. (2013). Recently, Holmberg et al. (2014) published a method for including high-cycle fatigue constraints for isotropic topology optimization. In the context of laminated composite structures details regarding stress constraints for multi-material optimization can be found in the ongoing work by Lund et al. (2013)

Besides structural requirements, constraints can also be specified to take other considerations into account such as the applied production methods and other design rules or guidelines. This category of constraints is generally referred to as manufacturing constraints. By including such manufacturing constraints in the design process, engineers may spent less time on manually post-processing the optimized design. Ideally, the optimized design would thus be easier to convey down the development process, and thereby reduce the overall cost associated with the development. This specific category of constraints is one of the main topics of this thesis, and it is therefore given special attention in Section 2.3 which goes through the current state-of-art with respect to manufacturing constraints in the context of laminated composite structures.

1.3 Objectives

The presented Ph.D. project is part of the research project entitled *Optimal Design of Composite Structures Under Manufacturing Constraints*. The research project has been conducted in collaboration between the Department of Mechanical and Manufacturing Engineering, Aalborg University and the Department of Wind Energy, DTU where Dr.techn. Mathias Stolpe and Ph.D. Konstantinos Marmaras were project partners.

The overall objective of this work is to develop parameterizations and methods such that manufacturing constraints and cost can be taken into account when designing and optimizing laminated composite structures. The methods and parameterizations must be able to perform multi-material optimization from a discrete set of candidates using efficient gradient based optimization methods. Here the multi-material optimization approach by Hvejsel and Lund (2011) serves as the foundation for the new parametrization. The ability to vary the thickness distribution of the laminate is essential in order to reduce the weight and thereby the material expenses. Here, Sørensen and Lund (2013) extended the parametrization by Hvejsel and Lund (2011) so to enable simultaneous multi-material and thickness optimization of laminated composite structures. With the above mentioned parameterizations the foundation for the work has been made. The focus of this thesis is therefore to further develop these parameterizations and associated methods, so to obtain an efficient general purpose method that is aimed at increasing the manufacturability of the optimized design. In order to do so, the following sub-objectives need to be addressed.

- The method by Sørensen and Lund (2013) is intended for mass constrained minimization of compliance problems, and the framework thus needs to be generalized in order to include structural criteria functions like e.g., mass, buckling, displacement, and eigenfrequencies.

- The formulation by Sørensen and Lund (2013) introduces a relative large set of constraints in order to prevent intermediate voids from appearing between layers. It is believed that these constraints can be avoided by the use of so-called casting constraints or filters by Gersborg and Andreasen (2011). By formulating these filters such that they operate through the thickness of the laminate, explicit constraints can be eliminated while also reducing the total number of design variables presented to the optimizer.

- Currently, engineers tend to apply a so-called patch formulation in order to increase manufacturability of the optimized design. Here, patches consist of design regions or domains which can be influenced by the optimizer. By manually adjusting the size of these patches, engineers can target the available production equipment and thereby increase the manufacturability of the optimized design. However, by prescribing a fixed patch layout the design domain is effectively reduced. A poor patch layout could thus result in even worse optimized laminate layups. In conventional topology optimization with isotropic materials, engineers can impose a minimum length scale by use of various filtering techniques, see e.g., Sigmund (1997), Sigmund (2007), Wang et al. (2011), or Svanberg and Svärd (2013). By generalizing these methods for multi-material and thickness optimization, the engineers can control the size of the fiber mats by simply prescribing a minimum size for the material and thickness variation, and consequently, avoid the time consuming process of determining a suitable patch layout.

- The current formulation by Sørensen and Lund (2013) can produce external ply drops. However, external ply drops are not desirable to apply as they increase the number of free edges in the laminate. These free edges increase the risk of edge delamination due to out-of-plane stresses near the free surface. It is believed that the constraints proposed by Sørensen and Lund (2013) can be reformulated such that only internal ply drops can appear.

2 State-of-the-art

This chapter describes the current state-of-the-art within the field of gradient based multi-material and thickness optimization of laminated composite structures. It is recognized that other parameterizations exists for optimizing laminated composite structures e.g., fiber orientations, ply thicknesses, lamination parameters, and free material optimization. The interested reader is here referred to the review papers by Ghiasi et al. (2009) and Ghiasi et al. (2010b) for further information.

In this thesis, the research area is divided into three categories. The first category deals with multi-material optimization of laminated composite structures with constant thickness. The focus is thus on determining an optimum layup/stacking sequence for a predefined number of layers through the thickness of the laminate. For the second category, the focus is on determining both an optimum layup and thickness variation. The third and final category reviews commonly applied manufacturing constraints in connection with both constant and variable thickness optimization.

2.1 Constant thickness optimization

Historically, the principles applied in gradient-based multi-material optimization have their origin in the field of classic topology optimization with isotropic materials. Therefore, the main principles from this scientific field are briefly highlighted in the following.

2.1.1 Classic topology optimization

In classic topology optimization with isotropic materials, the goal is to determine an optimum distribution of a single material candidate. It is therefore necessary to distinguish between two material phases, solid and void. In a general finite element environment, using either 2D or 3D solids, the design parameterization can be defined as

$$x_e = \begin{cases} 1 & \text{if there is material in element } e \\ 0 & \text{void} \end{cases}$$

where each design variables, x_e, can be interpreted as the density or influence associated with each element. Consequently, whether or not the material properties associated with each element should contribute or not thus depends upon the value of the design variable. To exemplify this, the constitutive properties and the mass for each element should thus be parameterized as

$$\boldsymbol{E}_e = x_e \, \boldsymbol{E} \tag{2.1}$$

$$m_e = x_e \, \varrho \tag{2.2}$$

$$x_e \in \{0; 1\}, \quad \forall(e)$$

where $\boldsymbol{E}$ is the constitutive matrix and ϱ is the density associated with the applied material candidate. Other problem dependent material parameters should of course also be parameterized with the design variables, if required.

However, in order to apply efficient gradient-based optimization methods, the strict integer definition of the design variables must be relaxed. This relaxation can be obtained by treating the design variables as continuous variables, and thereby expand the design space by allowing intermediate material phases to appear. This type of relaxation is generally referred to as the density approach. However, as these intermediate material phases do not represent neither solid nor void, the final solution should preferably be a discrete 0-1 solution, so a correct representation of the material parameters can be obtained. In the case of mass constrained minimum compliance problems, Bendsøe (1989) introduced the Solid Isotropic Material with Penalization (SIMP) interpolation scheme.

$$\boldsymbol{E}_e = x_e^p \, \boldsymbol{E} \tag{2.3}$$

$$m_e = x_e \, \varrho \tag{2.4}$$

$$x_e \in]0;1], \quad \forall(e)$$

Here, all design variables are raised to some power $p > 1$ when interpolating the constitutive properties. By doing so, intermediate valued design variables effectively reduce the constitutive properties causing an implicit penalization of the compliance objective function. Notice that a linear interpolation is still applied of the mass. Later, Stolpe and Svanberg (2001) argued to apply an alternative to the SIMP method in the form of the Rational Approximation of Material Properties (RAMP) interpolation scheme

$$\boldsymbol{E}_e = \boldsymbol{E}_0 + \frac{x_e}{1 + q\,(1 - x_e)} \Delta \boldsymbol{E} \tag{2.5}$$

$$x_e \in [0;1], \quad \forall(e)$$

Here $\Delta \boldsymbol{E} = \boldsymbol{E} - \boldsymbol{E}_0$ where $\boldsymbol{E}_0$ represents the stiffness of void, and has properties such that $\boldsymbol{E} - \boldsymbol{E}_0 \succ 0$ and $\boldsymbol{E}_0 \succ 0$. Intermediate valued design variables are effectively penalized for $q > 0$. The major difference between the SIMP and RAMP schemes is that the RAMP scheme produces non-zero sensitivities for q when $x_e = 0$. Both interpolation schemes have been applied to a wide variety of problems within the field of topology optimization, see e.g., Bendsøe and Sigmund (2003), Rozvany (2009), and Deaton and Grandhi (2014) for a comprehensive review.

2.1.2 Multi-material optimization

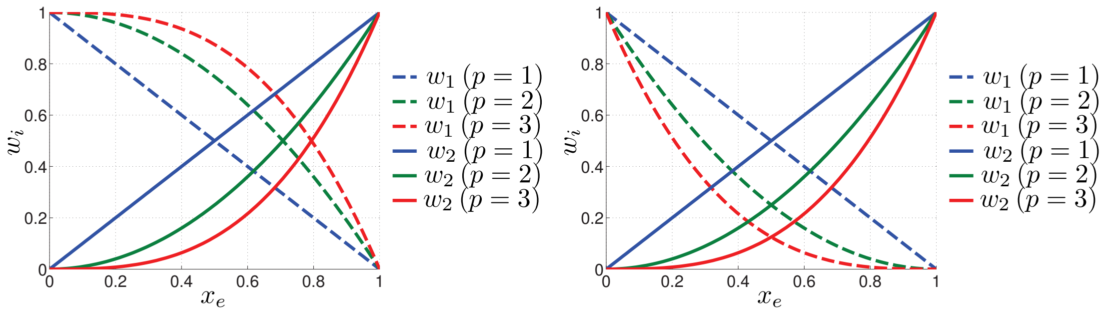
From topology optimization with a single material candidate, Sigmund and Torquato (1997) and Gibiansky and Sigmund (2000) proposed multi-material optimization for two material candidates using the following material interpolation scheme.

$$\boldsymbol{E}_e = (1 - x_e^p)\, \boldsymbol{E}_1 + x_e^p \boldsymbol{E}_2 \tag{2.6a}$$

$$= w_1 \boldsymbol{E}_1 + w_2 \boldsymbol{E}_2 \tag{2.6b}$$

$$x_e \in [0;1], \quad \forall(e)$$

Here, the selection between the two material candidates is determined by the value of a single variable i.e., x_e. The authors applied a SIMP like interpolation scheme in order to penalize intermediate valued design variables. However, the proposed interpolation scheme is biased towards the first material candidate when $p > 1$. This is illustrated in Fig. 2.1(a) which shows the two weight factors, w_1 and w_2, as a function of the design variable, x_e. For $p = 1$ the scheme is unbiased as the weight factors are evenly distributed around $x_e = 0.5$. However, for increasing values of p the intersection point, where the two weight factors are equal in magnitude, is moved towards the right i.e., favoring the first weight factor over a larger interval. In order to make the



(a) Bias material interpolation (b) Unbias material interpolation

Fig. 2.1 Distribution of weight factors associated with expressions (2.6) and (2.7).

scheme unbias for $p > 1$, the following modification can be applied.

$$\boldsymbol{E}_e = (1 - x_e)^p \, \boldsymbol{E}_1 + x_e^p \boldsymbol{E}_2 \tag{2.7a}$$

$$= w_1 \boldsymbol{E}_1 + w_2 \boldsymbol{E}_2 \tag{2.7b}$$

$$x_e \in [0; 1], \quad \forall(e)$$

With the above modification, the penalization scheme becomes unbiased as the two weight factors are evenly distributed around $x_e = 0.5$, for all $p \geq 1$, see Fig. 2.1(b).

Later, Stegmann and Lund (2005) proposed the Discrete Material Optimization (DMO) method which can interpolate between an arbitrary number of material candidates using one design variable for each candidate. Developed for multi-material optimization of laminated composite structures, the constitutive properties for a given layer in a given shell element are interpolated as

$$\boldsymbol{E}_{el} = \sum_{c=1}^{n^c} \left[x_{elc}^p \prod_{j=1}^{n^c} \left[1 - \left(x_{el(j \neq c)} \right)^p \right] \right] \boldsymbol{E}_c \tag{2.8a}$$

$$= \sum_{c=1}^{n^c} w_c \boldsymbol{E}_c \tag{2.8b}$$

$$x_{elc} \in]0; 1], \quad \forall(e, l, c)$$

Here w_c is the weight factor associated with the c'th material candidate. Similar to (2.6), the DMO interpolation scheme, also labeled scheme 4 and DMO4, utilizes a self-balancing principle where an increase in one design variable reduces the weight factors associated with other material candidates. This dependency between the design variables is necessary in order to force the optimizer to select just one material candidate. For $p = 1$, the dependency is linear i.e., an increase in one design variable is followed by equal decrease in the remaining weight factors. The DMO4 scheme has successfully been applied for maximizing the stiffness of laminated composite structures using a discrete set of plies with different fiber orientations as material candidates, see e.g., Stegmann and Lund (2005), Lund and Stegmann (2005).

When considering criteria functions such as buckling load factors, eigenfrequencies, or mass, it is important that the weight factors sum to unity, so to have physically valid results for intermediate valued design variables. When applying the DMO4 scheme the sum of weight factors will not necessarily equal unity for intermediate valued design variables i.e., $\sum_{c=1}^{n^c} w_c \neq 1$. Consequently, a normalized version of the DMO4 scheme, labeled scheme 5 or DMO5, was also proposed by Stegmann and Lund (2005)

$$\boldsymbol{E}_{el} = \sum_{c=1}^{n^c} \frac{1}{\sum_{c=1}^{n^c} w_c} \left[x_{elc}^p \prod_{j=1}^{n^c} \left[1 - \left(x_{el(j\neq c)}\right)^p\right] \right] \boldsymbol{E}_c \tag{2.9a}$$

$$= \sum_{c=1}^{n^c} \frac{w_c}{\sum_{c=1}^{n^c} w_c} \boldsymbol{E}_c \tag{2.9b}$$

$$x_{elc} \in]0;1[, \quad \forall(e,l,c)$$

A drawback of the normalization is that the influence of the penalization is reduced. The scheme may, therefore, not be able to push the design variables to a discrete solution. Another approach could be to only apply scheme 5 for the stress stiffness and mass matrix whereas the stiffness matrix could be interpolated using scheme 4. Nevertheless, the overall problem is unavoidably non-convex when applying either scheme 4 or 5, and the best combination of interpolation schemes will thus be problem dependent. Lund (2009) successfully applied scheme 5 in the context of maximizing minimum buckling load factors for laminated composite plates applying orthotropic plies with different orientations as design variables. Niu et al. (2010) applied the same setup but for minimizing the sound radiation of laminated composite plates. Later, Lindgaard and Dahl (2012) investigated different interpolation schemes in the context of topology optimization with linear and non-linear buckling objective functions.

Alternatively to the original DMO formulations, Hvejsel et al. (2011) applied a linear material interpolation scheme combined with a series of sparse linear equality constraints.

$$\boldsymbol{E}_{el} = \boldsymbol{E}_0 + \sum_{c=1}^{n^c} x_{elc} \Delta \boldsymbol{E}_c \tag{2.10a}$$

$$\sum_{c=1}^{n^c} x_{elc} = 1, \quad \forall(e,l) \tag{2.10b}$$

$$x_{elc} \in [0;1], \quad \forall(e,l,c)$$

By explicitly requiring that the sum of candidate material variables must equal one, an increase in one design variable must be followed by an equal reduction in the others, thus creating a mutual dependency between the design variables. Still, nothing prevents the optimizer for settling on a non-discrete design. Based on the work by Borrvall and Petersson (2001), the authors applied a non-linear in-equality constraint to gradually push the design variables towards a discrete solution.

$$g\left(x\right) = \sum_{e=1}^{n^e} \sum_{l=1}^{n^l} \sum_{c=1}^{n^c} x_{elc} \left(1 - x_{elc}\right) \leq \varepsilon \tag{2.11}$$

Here ε is a parameter prescribing the upper limit for the constraint, and n^e, n^l, n^c is the number of elements, layers, and material candidates, respectively. Initially, the parameter should be large enough to ensure that the constraint is feasible, and afterwards gradually reduced towards zero using a continuation strategy. Because continuation strategies naturally are dependent upon the specific problem at hand, some amount of problem dependent fine-tuning may be necessary. Nevertheless, the authors successfully demonstrated the method on a series of minimum compliance problems applying orthotropic plies with different orientations as material candidates.

Instead of applying a linear interpolation scheme combined with a non-linear constraint, Hvejsel and Lund (2011) later introduced multi-material variations of the SIMP and RAMP interpolation schemes.

$$\boldsymbol{E}_{el} = \boldsymbol{E}_0 + \sum_{c=1}^{n^c} x_{elc}^p \Delta \boldsymbol{E}_c, \qquad p \geq 1 \quad \forall(e,l,c) \quad \text{(SIMP)} \tag{2.12a}$$

$$\boldsymbol{E}_{el} = \boldsymbol{E}_0 + \sum_{c=1}^{n^c} \frac{x_{elc}}{1 + q\,(1 - x_{elc})} \Delta \boldsymbol{E}_c, \qquad q \geq 0 \quad \forall(e,l,c) \quad \text{(RAMP)} \tag{2.12b}$$

$$\sum_{c=1}^{n^c} x_{elc} = 1, \quad \forall(e,l) \tag{2.12c}$$

$$x_{elc} \in [0;1], \quad \forall(e,l,c)$$

Equivalent to the original SIMP and RAMP interpolation schemes, the proposed method makes intermediate valued design variables unattractive by implicitly penalizing the constitutive properties. Again, the resource constraints in (2.12c) ensure that an increase in one design variables must be followed by an equal reduction amongst the remaining design variables, thus creating a dependency between the design variables. In order to gradually penalize intermediate valued design variables, the authors recommended to apply a continuation strategy for the penalization powers, p or q. Again, the authors demonstrated the method on minimum compliance problems applying orthotropic plies with different orientations as design variables.

Similar to the non-linear constraint equation proposed by Hvejsel et al. (2011), Kennedy and Martins (2013) proposed another explicit approach to make intermediate valued design variables unattractive. The authors applied the same linear interpolation scheme for the constitutive properties as proposed by Hvejsel et al. (2011), see (2.10a), however, they added a series of non-linear equality constraints as a penalty term to the objective function. These non-linear constraints are formulated with respect to the sum of material candidates for all layers in each element.

$$\sum_{c=1}^{n^c} x_{elc}^2 = 1, \quad \forall(e,l) \tag{2.13}$$

Using vector notation, the constraints can be expressed as $\boldsymbol{c}_s(x) = \boldsymbol{e}$, where $\boldsymbol{c}_s(x)$ is a vector containing the current values of all the constrains, and $\boldsymbol{e}$ is a vector which represents all the right hand sides of (2.13). Through a series of careful manipulations, it is possible to formulate a penalty term on the basis of the quadratic resource constraints in (2.13). By adding the explicit penalty term to the objective function, it is possible to reduce the non-discreteness of the design variables through an augmented objective function $\Phi(x)$.

$$\text{minimize} \quad \Phi(x) = f(x) + \gamma \boldsymbol{e}^T \left(\boldsymbol{e} - \boldsymbol{c}_s(x)\right) \tag{2.14}$$

$$\text{s.t.} \quad \sum_{c=1}^{n^c} x_{elc} = 1, \quad \forall(e,l) \tag{2.15}$$

$$x_{elc} \in [0;1], \quad \forall(e,l,c)$$

In the above, $f(x)$ is the original objective function, and γ is a factor applied for scaling the penalty term which describes the non-discreteness of the design variables. Similar to multi-objective optimization, the specific weight factor applied for each objective dictates how important each objective is relative to the others. The specific choice of γ thus dictates how important non-discrete design variables are relative to the original objective function. Applying a too high penalization factor, the optimizer may converge to a poor, but discrete solution, and vice versa if the value is too low. In Kennedy and Martins (2013), the authors recommended to apply a continuation strategy where the value of γ is increased gradually. The method was successfully demonstrated for minimum compliance, and for maximizing buckling load factors. In both cases, the design variables consisted of orthotropic plies with different orientations.

As an alternative to the DMO parameterization, which requires one design variable for each material candidate, Bruyneel (2011) introduced the Shape Function with Penalization (SFP) method. Here, four node bi-linear finite element shape functions are applied to interpolate between four material candidates by use of just two design variables i.e., the natural coordinates (R, S) of the element, see Fig. 2.2. In order to drive the design towards discrete selection of a

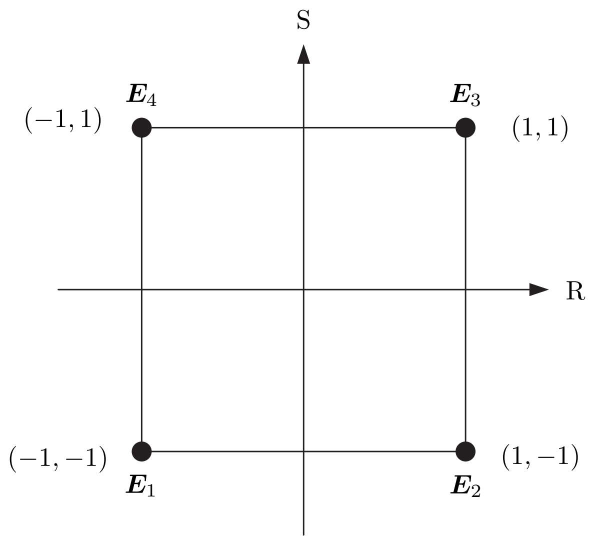


Fig. 2.2 Four node finite element applied for interpolation four constitutive properties E_1-E_4

single candidate material, the authors propose to apply a SIMP like penalization scheme for the weight factors associated with each candidate material. The constitutive properties for a given layer in a given element are thus determined as

$$\boldsymbol{E}_{el} = \boldsymbol{E}_0 + \sum_{c=1}^{4} w_c^p \Delta \boldsymbol{E}_c \tag{2.16}$$

$$w_1 = \frac{1}{4}(1-R)(1-S) \quad w_3 = \frac{1}{4}(1+R)(1+S)$$

$$w_2 = \frac{1}{4}(1+R)(1-S) \quad w_4 = \frac{1}{4}(1-R)(1+S) \tag{2.17}$$

In Bruyneel et al. (2011), the authors extended the method by applying three node plane and eight node brick elements to interpolate between three and eight material candidates, by use of two and three design variables, respectively.

Later Gao et al. (2012) introduced the Bi-valued Coding Parameterization (BCP). The BCP scheme is a generalization of the SFP method as it can handle an arbitrary number of material candidates. As the name suggests, the BCP scheme applies design variables that can attain bi-values of either -1 or 1. For the BCP scheme, the number of design variables, n^x, required to interpolate n^c material candidates is defined as an integer of the calling function of the binary logarithm to the number of material candidates

$$n^x = \lceil \log_2 (n^c) \rceil \tag{2.18}$$

The relationship between the number of material candidates which can be interpolated for a given number of design variables is

$$n^c = [2^{(n^x-1)} + 1, 2^{(n^x)}] \tag{2.19}$$

Consequently, with four design variables, $n^x = 4$, the number of candidates which can be interpolated is between $9 \leq n^c \leq 16$. The BCP scheme thus provides a substantial reduction in the number of design variables when compared to the DMO method that has a one-to-one relationship between the number of material candidates and design variables. Similar to the SFP, the BCP method also applied a SIMP like penalization scheme on the individual weight functions, see Gao et al. (2012) for details regarding the computation of the weight function using the bi-valued design variables.

However, the author note that rearranging the material candidates between two otherwise identical optimization problems can lead to two unique solutions. This sensitivity to the ordering of the material candidates is generally not desirable. Because the BCP method is a generalization of the SFP method, the SFP method will also be sensitive to the ordering of the material candidates. Furthermore, for the example with four design variables, the BCP method can interpolation between 9 and 16 material candidates. However, if only 9 candidates are applied, the remaining 7 "empty slots" must be occupied with some of the 9 candidates such that no "slots" are empty. This repeating of candidates will also generate some bias in the results, something that again is not desirable. Despite these undesirable effects, the BCP method is indeed a great contribution to the field of gradient based multi-material optimization.

2.2 Variable thickness optimization

Thickness optimization of laminated composite structures is typically motivated by a desire to reduce either the cost or weight of the structure. Throughout the literature, several approaches has been suggested. Schmit and Farshi (1973), Mateus et al. (1991), Costin and Wang (1993), and Mateus et al. (1997) applied the thickness of each ply as a continuous design variable, however, as mentioned in the introduction, the application of continuous thickness variables may require considerable amount of time for manual post-processing. Later, Manne and Tsai (1998) applied a similar approach, however, for the optimized design the authors applied a stepwise rounding scheme which monitored if any constraints were violated during the rounding procedure.

For discrete design variables, consisting of ply orientations and the number of plies, direct search or zero-order methods such as Genetic Algorithms (GA)'s are widely applied through out the literature. These methods do not rely on gradient information from the objective and constraints functions, but instead operate solely on their values and stochastic operators for changing the design variables. These methods and algorithms are often referred to as "global" optimization methods which is quite a misleading label as no guarantees can be made with respect to the quality of the solution i.e., the results may not be optimal at all. Despite this, zero-order algorithms are quite useful when e.g., a non-differentiable or a so-called fuzzy/noizy function is applied. In Toropov et al. (2005) the authors applied a genetic algorithm for mass minimization of composite wing panels using discrete ply orientations and the number of plies as design variables.

Another popular approach is to decouple the different design variables by optimizing them in two separate phases or levels. In Liu et al. (2000) the authors first optimized the number of plies using a gradient based method followed by a stacking sequence optimization using a genetic algorithm. The two stages were then repeated until the algorithm had converged. Liu et al. (2011) applied a similar approach for mass minimization of composite structures. In the commercial software OptiStruct by Altair Engineering the optimization procedure is split into three distinct phases, see Zhou et al. (2011) and Zhou and Fleury (2012). A disadvantage of this decoupling of the thickness and material problems is that the obtained results may be suboptimal. This is because the structural response is dependent on both parameters simultaneously. In the following section, the focus is therefore on methods which can perform simultaneous material and thickness optimization using gradient based methods.

2.2.1 Simultaneous material and thickness optimization

By simultaneously optimizing both the material candidates and the thickness variations, the optimizer has the full potential to utilize the complexity of the structural response associated with laminated composites. The ability to take advantage of these mechanisms thus holds great potential for further weight and cost savings. In this section, the presented methods are subdivided into two categories. The first category focuses on methods for optimizing single layered composites, and the second category focuses on methods for optimizing multi-layered laminated composites.

Single layered composites

In the original formulation by Sigmund and Torquato (1997), Gibiansky and Sigmund (2000) the proposed multi-material formulation included a topology variable to control the removal of material. The scheme thus makes it possible to conduct simultaneous material and topology optimization with two material candidates.

$$\begin{aligned} &\boldsymbol{E}_e = \rho_e^q \left[(1 - x_e^p) \, \boldsymbol{E}_1 + x_e^p \boldsymbol{E}_2 \right] \\ &x_e \in [0;1], \quad \forall (e) \\ &\rho_e \in]0;1], \quad \forall (e) \end{aligned} \tag{2.20}$$

In order to push the continuous density variables towards a discrete value, a SIMP-like penalization scheme was applied. Although not aimed at laminated composites, the parameterization could be applied for single layered composites.

In the context of composite structures, Bruyneel (2011) presented a similar approach for the SFP parameterization. Here, the author likewise added a so-called topology or density variable to control the removal of material.

$$\begin{aligned} &\boldsymbol{E}_e = \boldsymbol{E}_0 + \rho_e^q \sum_{c=1}^{4} w_c^p \Delta \boldsymbol{E}_c \\ &\rho_e \in [0;1], \quad \forall (e) \end{aligned} \tag{2.21}$$

The added density variable effectively scales the sum of candidate weight factors, making it possible to remove a given layer. This type of parameterization is quite flexible when it comes to the distribution of the two different types of design variables. It is, e.g., possible to have the material variables cover larger areas than the density variables i.e., the material candidate variables may effect the entire span of the structure while the density variables can be distributed on element level. The same can also be done for the opposite case i.e., the material candidate variables could be distributed on element level, while the density variables could cover entire layers.

Hvejsel and Lund (2011) also proposed a means for changing the topology of the structure while simultaneously determining which material to apply. Instead for requiring that the sum of candidate material variables must equal one, the authors proposed to change the equality constraints to inequality constraints.

$$\boldsymbol{E}_e = \boldsymbol{E}_0 + \sum_{c=1}^{n^c} x_{ec}^p \Delta \boldsymbol{E}_c, \tag{2.22a}$$

$$\sum_{c=1}^{n^c} x_{ec} \leq 1, \quad \forall(e) \tag{2.22b}$$

$$x_{ec} \in [0;1], \quad \forall(e,c)$$

By not requiring that the sum of material variables must equal one at all times, the optimizer can collectively set them to zero, and thereby change the topology of the structure without the need of additional design variables. The proposed optimization problem is thus simpler than what was proposed by Bruyneel (2011) which relies on a product between a density variable and the candidate material variables. However, because the topology is dependent upon the material variables alone, the proposed method is in it self incapable of obtain material and topology variations on different scales i.e., having material variables cover larger areas than the variation in topology, and vice versa. Nevertheless, from a computational aspect the method is quite effective as the applied constraints are linear and can be set up in a sparse format.

Multi-layered laminated composites

Simultaneous material and thickness optimization of multi-layered laminated composite structures has recently been investigated by Sørensen and Lund (2013) who proposed a novel extension to the DMO method. Here, the authors applied the multi-material parameterization by Hvejsel and Lund (2011) combined with a layerwise topology or density variable to control the removal of material in each layer. In order to push the design variables towards discrete values, the authors applied the RAMP penalization scheme for both the density and candidate variables.

$$\boldsymbol{E}_{el} = \boldsymbol{E}_0 + \frac{\rho_{el}}{1 + q\,(1-\rho_{el})} \sum_{c=1}^{n^c} \frac{x_{elc}}{1 + p\,(1 - x_{elc})} \Delta \boldsymbol{E}_c \tag{2.23a}$$

$$\sum_{c=1}^{n^c} x_{elc} = 1, \quad \forall(e,l) \tag{2.23b}$$

$$x_{elc} \in [0;1], \quad \forall(e,l,c)$$

$$\rho_{el} \in [0;1], \quad \forall(e,l)$$

The parameterization is thus similar to what was proposed by Bruyneel (2011), however, now in the context of multi-layered structures. In order to obtain manufacturable designs when optimizing the thickness variations of laminated composites, intermediate void through the thickness of the laminate must be avoided, see Fig. 2.3 for an illustration.

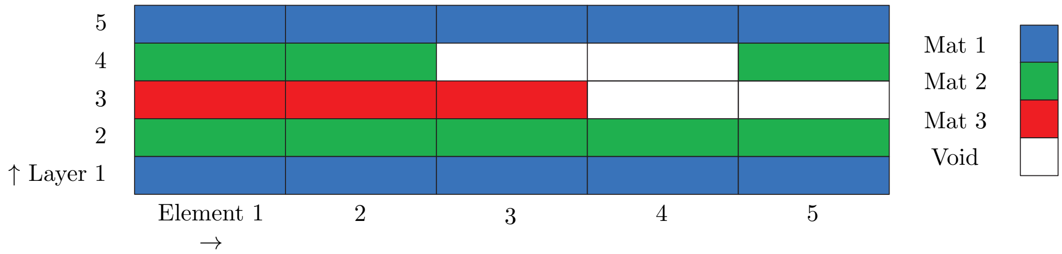


Fig. 2.3 Illustrative example of a laminate with intermediate void. The laminate is modeled with five elements each with five layers. The laminate consists of three different solid materials, and void. The void regions are encapsulated between layers of solid material.

As can be seen from the figure, the presence of intermediate void through the thickness of the laminate is not physically meaningful, and likewise makes it impossible to manufacture.

Consequently, the authors formulated and tested several so-called manufacturing constraints to ensure a continuous presence of material uninterrupted by void. Intuitively, one of these attempts simply prescribed that a layer on top of another layer should have a lower density i.e.,

$$\rho_{e(l+1)} \leq \rho_{el} \tag{2.24}$$

however, this formulation was unsuccessful as so-called density bands could appear. These density bands consisted of non-discrete valued density variables distributed from the top layer to the bottom layer. The constraints in (2.24) were feasible, however, the RAMP penalization scheme could not push the layerwise density variables towards discrete values, despite the applied level of penalization. The phenomenon was predominate for structures loaded in bending where high moments of inertia are desirable.

As previously mentioned, several schemes were tested in order to remedy the problem. Ultimately, the authors settled on following formulation

$$\rho_{e(l+1)} \leq f\left(\rho_{el}, T\right) \tag{2.25}$$

where $f\left(\rho_{el}, T\right)$ is a function that specifies the maximum density a layer can attain based upon the density of the layer below together with a threshold parameter, T. The function is composed of two linear functions and is defined as

$$f(\rho_{el}, T) = \begin{cases} \frac{T}{1-T}\rho_{el} & \text{if} \quad \rho_{el} < (1-T) \\ \frac{1-T}{T}\rho_{el} + \frac{2T-1}{T} & \text{else} \end{cases} \qquad \text{where} \quad 0 < T \leq 0.5 \tag{2.26}$$

The authors concluded that $T = 0.1$ was a good compromise between low measures of non-discreteness and the computational cost. For increasing values e.g., $T = 0.2$ or $T = 0.5$, the undesired density bands began to reappear. For smaller values e.g., $T = 0.05$ the density bands became even less apparent, but with additional computational costs due to an increased number of iterations required for convergence.

The conditional statement in (2.26) makes the overall function non-smooth, and the authors concluded that a Sequential Linear Programming (SLP) optimizer was more effective than an optimizer based on a Sequential Quadratic Programming (SQP) method. Consequently, the upper bounds of the density variables have to be updated in each iteration, effectively limiting the progression of the thickness variation by guiding the design in a stepwise manner.

Although the parameterization in (2.23) is generic, the proposed method was specialized for mass constrained minimization of compliance using an element-wise parameterization. Generalizing the method so other constraints and objectives can be applied together with a patch parameterization is the main topic of Paper 1, see Section 3.1 for a resume of the work.

Later, Gao et al. (2013) extended the BCP scheme to also include the possibility to terminate individual plies. The authors applied a similar approach to Bruyneel (2011) and Sørensen and Lund (2013) by adding a topology variable to scale the candidate weight factors, however, now using a bi-valued format. However, the method lacks the ability to prevent intermediate void from appearing through the thickness of the optimized laminate. The optimized designs may thus not be manufacturable.

Throughout the literature, several authors have proposed to apply explicit manufacturing constraints so to reduce the amount of time required for manual post-processing of the optimized designs. In the following section, some of the most widely applied design rules within the field of laminated composites are highlighted.

2.3 Manufacturing constraints

Typically, engineers rely on finite element analysis using equivalent single layer shell formulations in order to validate the integrity of the structures. These methods are, however, not capable of predicting out-of-plane stresses which may develop in areas where the structural stiffness changes rapidly. In order to capture these local effects with a sufficient level of detail, higher order shell formulations or 3D solid finite element models have to be applied. However, these methods are generally not suitable in the context of optimization due to the substantial increase in the computational requirements. In order to avoid such detailed models, engineers can apply design rules, also known as ply book rules, so to implicitly considering these effects through explicit mathematically formulated manufacturing constraints. By incorporating these manufacturing constraints into the optimization problem, engineers can avoid these undesired structural designs, and thereby reduce the amount of time required for manual post-processing.

Common types of manufacturing constraints which can be found in the literature are e.g., requirements for symmetric and balanced layups, contiguity constraints, adjacency or so-called blending rules, and constraints limiting the thickness variation rate. Most of these constraints have their origins in the aerospace industry, see e.g., Kassapoglou (2010). In the following sections, the motivation and recent developments are presented for each of the mentioned constraints.

Symmetric and balanced laminates

Symmetric and balanced laminates are typically required if the bend-twist and shear-extension couplings of a laminate plate must be avoided. This is typically exploited in the industry as such layups present little distortion or spring back during manufacturing, see e.g., Jones (1999) and Ghiasi et al. (2010a).

For constant thickness optimization, the requirements are typically enforced simply by mirroring, or linking, the design variables for half of the laminate and thus reducing the total number of design variables by a factor of two. This linking of the design variables is thus computational effective, and ensures that the optimized designs obey the requirements. In Grosset et al. (2001), the authors proposed an approach where the objective function was penalized when the constraints were violated. The authors concluded that the simple linking procedure was considerably more effective than penalizing the objective function.

However, for variable thickness optimization, the implementation becomes more complicated as the material selection now depends upon the change in thickness and vice versa. Consequently, explicit constraints have to be formulated in order to ensure that the requirements are satisfied. Furthermore, because the thickness variation effects the response for the entire structure, simply enforcing symmetry and balance on element level is insufficient i.e., the entire structure has to be considered.

Contiguity

By limiting the number of unidirectional plies with identical fiber orientations that can be stacked on top of each other, the risk of matrix failure through the thickness of the laminate can be reduced. This has lead to the development of contiguity constraints that can limit the number of contiguous layers with the same fiber orientation. Here, Le Riche and Haftka (1993), Liu et al. (1999), Seresta et al. (2007), Toropov et al. (2005), and Liu et al. (2011) investigated such constraints in the context of stacking sequence optimization applying genetic algorithms. Bruyneel et al. (2012) presented an explicit constraint using the SFP parameterization for constant thickness optimization. Later, Sørensen and Lund (2013) proposed a linear constraint formulation applicable for the different DMO methods for both constant and variable thickness optimization. Kennedy and Martins (2013) likewise proposed a non-linear variation applicable for the DMO parameterizations.

Adjacency

The adjacency constraints limit the change in fiber directions between contiguous plies through the thickness of the laminate, and thus reduce abrupt changes for the in-plane stiffness between contiguous plies. Abrupt changes in stiffness can introduce large interlaminar and thus out-of-plane stresses which ultimately can lead to failure due to edge delamination. These constraints have recently been investigated by Bruyneel et al. (2012) and Kennedy and Martins (2013) in the context of gradient based discrete material optimization.

Thickness variation rate

When varying the thickness of laminated composite structures, the number of plies which may be terminated or dropped simultaneously should be limited. If too many plies are terminated at one location, the associated change in stiffness can lead to delamination due to out-of-plane stresses.

For continuous thickness optimization, Costin and Wang (1993) formulated two explicit thickness constraints. The first controls the maximum allowable variation of ply thicknesses across the laminate, and the second ensures that adjacent elements have continuity among the individual ply thicknesses. However, the continuous thicknesses have to undergo manual post-processing in order to translate these smooth transitions into either a termination or an addition of individual plies.

For discrete thickness optimization, Sørensen and Lund (2013) developed a linear constraint which limits the number of plies which can be dropped between adjacent design regions. This method has been proven successful, however, combined with their applied material interpolation scheme it is only capable of performing external ply drops. External ply drops are generally undesirable from a manufacturing point of view as the laminate may have a high tendency for edge delamination.

3 Description of the papers

In the following, the papers which constitute the main contributions of the thesis are briefly summarized.

3.1 Paper 1

In Sørensen and Lund (2013), the authors proposed a novel method for performing discrete material and thickness optimization of laminated composite structures. However, the method was specialized for mass constrained minimum compliance problems using an element-wise parameterization, where all the material candidates were limited to consist of the same orthotropic material but with different fiber orientations. This limitation did, however, facilitate a linear mass constraint which is desirable as it is both simple and computationally efficient. The linearity of the mass constraint was utilized in the setup of the optimization procedures where the constraint was approached in small increments. However, the method could thus not be applied for e.g., sandwich structures as the addition of the core material would result in a bi-linear and thus non-convex mass constraint.

In Paper 1, the optimization strategies developed by the previous authors were generalized such that the proposed parameterization could be applied for other industrially relevant problems. The focus of the work is therefore on the development of efficient gradient based optimization strategies which can be applied for both non-linear and non-convex objective and constraint functions. From a commercial aspect, cost and structural performance are considered as key parameters. To reflect this, the method considers mass minimization with structural constraints on linearized buckling load factors, natural eigenfrequencies, and limited displacements. Furthermore, in order to increase the manufacturability of the optimized design, the manufacturing constraints originally developed by Sørensen and Lund (2013) were included.

When minimizing the total mass of a structure, the ability to include structural non-linear constraints is essential in order to maintain the integrity of the structure. However, as such constraints may be infeasible to begin with, robust methods for handling infeasible non-linear constraints must be included. Here, a so-called *Merit function* or *elastic programming* approach was applied to ensure unconditional feasibility of the linearized optimization problem. The applied approach was inspired by Svanberg's work on the Method of Moving Asymptotes (MMA), see e.g., Svanberg (1987).

The proposed method was demonstrated on a series of different parameterizations of a generic main spar used in some designs of wind turbine blades. The different configurations demonstrated the flexibility of the parameterization. Here, different sizes and shapes of candidate material and geometry domains, also referred to as patches, could easily be configured so to comply with the manufacturers production facilities and the available materials. The results did, however, demonstrate the impact a poor patch layout can have on the performance of the optimized design. Still, all the examples converged to solutions with feasible structural constraints within 28-142 design iterations which shows the effectiveness of the proposed method. In general, the results obtained by the DMTO method can serve as a great source of inspiration for manufacturable thickness variations and stacking sequences throughout the structure.

3.2 Paper 2

In Paper 2, the layerwise density variables introduced by Sørensen and Lund (2013) are replaced with continuous through-the-thickness design variables. This was accomplished by utilizing so-called casting constraints or filters originally proposed by Gersborg and Andreasen (2011). By replacing the layerwise density variables with continuous through-the-thickness design variables, the constraints for preventing intermediate void could be avoided. Similarly, the layerwise constraints limiting the thickness variation rate could instead be formulated on element level with respect to the new variables. Consequently, the number of design variables and constraints presented to the optimizer could be significantly reduced when compared to the previous work, see Table 3.1.

	Paper 1	Paper 2
Topology variables	$n^e \cdot n^l$	n^e
Intermediate void constraints	$n^e \cdot \left(n^l - 1\right)$	0
Thickness variation rate constraints	$n^e \cdot \left(n^l - 1\right)$	n^e

Table 3.1: Comparison between the number of topology variables and constraints for the original DMTO method (Paper 1) and the proposed method in Paper 2. Number of elements: n^e, Number of layers: n^l.

In order to further increase the manufacturability, and also avoid mesh dependent designs, a standard in-plane density filter by Bruns and Tortorelli (2001) and Bourdin (2001) was applied. By use of the filter radius, manufacturers can manipulate the size of the fiber mats, and ultimately target the available materials and the production facilities. Because the in-plane density filter is formulated with respect to the filtered through-the-thickness design variables, the "physical" layerwise density variables still have a high degree of discreteness; something generally not possible in classical topology optimization when applying a linear density filter. Because the density filter relies on a linear formulation, the applied manufacturing constraints limiting the thickness variation rate remain linear as well making them simple and computationally inexpensive.

Two different casting filters, or thickness filters in this context, were presented. The first is a direct application of the casting constraint presented by Gersborg and Andreasen (2011), whereas the second was developed on the basis of the threshold projection filter by Wang et al. (2011). The proposed filters were tested on a series of numerical examples consisting of a clamped plate under uniform pressure. For all examples, the objective was to minimize the total mass with a non-linear structural constraint limiting the displacement of the plate center. The examples were subdivided into two categories. For the first category steel was the only available material candidate, and for the second category four GFRP plies with fiber orientations at $\{-45°, 0°, 45°, 90°\}$ were considered.

The results clearly demonstrated that the material interpolation scheme combined with the thickness filters were able to provide near discrete designs for both material and density variables. However, because the two thickness filters are non-linear functions, variations did appear in the fiber layup and consequently also the thickness variation. Here, the second thickness filter generally obtained the lowest mass while being within the prescribed infeasibility limit. For both filters, the specified radius for the in-plane density filter was successfully transferred to the underlying physical design.

In conclusion, the proposed method provides a substantial reduction in the number of design variables and constraints when compared to the previous work. By combining the proposed through-the-thickness design variables with a linear density filter, near discrete designs could be obtained while still preserving the minimum length scale specified by the in-plane density filter.

This approach thus makes it possible for manufacturers to manipulate the geometric outcome by specifying a minimum length scale through the in-plane density filter.

3.3 Paper 3

With the established methods from Paper 2, the geometry of the individual plies can be manipulated by specifying a minimum length scale through the in-plane density filter. However, the material distribution still relies on a patch approach. These patches effectively enforce a specific length scale by grouping elements together and thereby avoid mesh dependent designs. Because the shape and size of a patch are statically fixed they cannot be influenced by the optimizer. Consequently, great insight and ingenuity are required in order to determine a good patch design. It was, however, demonstrated in Paper 1 that what might seem as a good patch design could still give unsatisfactory results with respect to the performance of the optimized layup.

In Paper 3, a method is presented for providing mesh independent designs without the use of a restrictive patch layout. The proposed method combines the linear density filter by Bruns and Tortorelli (2001) and Bourdin (2001) together with the threshold projection filter by Wang et al. (2011). Here, the density filter is used to establish a dependence between neighboring design variables within the specified filter radius, and the projection filter is used to push the design variables towards their discrete bounds. However, because the projection filter is a non-linear function in the design variables, the optimizer can potentially generate superior, but non-physical pseudo-materials. In order to overcome this problem, the projected design variables are re-scaled in a so-called normalization filter. By adjusting the filter radius, engineers can effectively specify the smallest size an area must have with constant material continuity, while the optimizer is free to determine which material to apply, together with the location, shape, and size of the selected materials.

The method is demonstrated for a series of examples where the underlying problem consists of a clamped plate under uniform pressure. The examples considered fiber angle optimization for a single layered plate with respect to minimum compliance, and multi-material optimization of a laminated sandwich plate with respect to minimum mass subject to a constraint on compliance. The results show that the method is indeed capable of imposing the pre-specified material continuity onto the optimized layup, while still providing near discrete designs.

4 Concluding remarks

This chapter highlights the main contributions of the thesis and comments on related areas which could be addressed in future work.

4.1 Conclusions and contributions

The overall objective of this work has been to develop parameterizations and methods such that manufacturing constraints and cost could be taken into account when designing and optimizing laminated composite structures. This lead to four sub-objectives from which three were addressed in the attached journal papers. The proposed methods and parameterizations from these papers stand as novel contributions to the field of multi-material and thickness optimization of laminated composite structures. In the following, the main contributions from each paper are highlighted.

- In Paper 1, the proposed method by Sørensen and Lund (2013) was generalized such that simultaneous material and thickness optimization can be conducted when including non-linear and non-convex structural criteria. In order to test the capabilities of the method, results from a series of numerical examples were presented. The examples consisted of a generic main spar used in some designs of wind turbine blades. The results show that the method is indeed capable of efficiently handling complicated problems with many non-linear constraints.

- In Paper 2, the concept of thickness filters was introduced. These filters effectively replace the previously applied layerwise density variables with continuous through-the-thickness design variables. Consequently, the filters eliminate the need for having explicit constraints for preventing intermediate void from appearing between layers of the laminate. The number of design variables and constraints is thus significantly reduced when compared to the previously applied methods. A quite elegant improvement over the previous methods is made by combining the new through-the-thickness design variables with a standard linear density filter. This combination makes it possible for engineers to control the size of the geometry while still obtaining near discrete designs. Furthermore, because the applied density filter is a linear function of the design variables, the constraints governing the thickness variation rate also remain linear.

- In Paper 3, in-plane material filters are introduced for the discrete material optimization method. Generally, when optimizing laminated composite structures the design variables are typically defined on element level. However, doing so makes the problem mesh dependent and decreases manufacturability of the optimized design. To overcome this problem, the design variables can be assigned to groups of elements also referred to as patches. Still, as the patch layout is fixed it cannot be influenced by the optimizer, and they therefore effectively restrict the design possibilities. With the proposed in-plane material filters, engineers can now specify a minimum length scale which governs the size of areas with constant material continuity, thereby being able to target the available materials and production facilities while the optimizer is free to determine which material to apply, together with the location, shape, and size of the selected materials.

4.2 Future work

In accordance with the project objectives, one problem still stands unsolved i.e., the support of handling internal ply drops when conducting simultaneous material and thickness optimization. This problem could be addressed by two different approaches. The first approach would be to formulate explicit manufacturing constraints to ensure a continuous transition of material candidates between adjacent layers where ply drops appear. However, this naturally leads to dependence between the material selection in one area to the material and thickness variation in adjacent areas. Because of this dependency between the thickness and material variables, it is considered unlikely that such manufacturing constraints could be formulated as linear constraints. Another approach could be to formulate a new type of density filter which operates across layers between adjacent finite elements, using some of the same principles applied in Paper 3.

In Paper 2 and Paper 3, the applied density filters utilize a linear weight function based on the Euclidian norm of the vector between the considered shell elements. This approach is perfectly valid for flat geometries i.e., plates, however, for curved shell structures this approach is inadequate. Here, the distance between adjacent elements must follow a path along the contours of the structure. Determining these paths effectively in a general finite element framework still needs to be investigated.

In Paper 3, the applied projection and normalization filters make the parameterization highly non-linear. It could be of interest to investigate other formulations which could do the same, but without making the design variables inevitably non-linear. This nonlinearity of the design variables is not desirable when manufacturing constraints related to the choice of material are applied. Another approach could be to generalize the method of Borrvall and Petersson (2001) such that is could be applied for multi-material topology optimization. This was partially done by Hvejsel et al. (2011), but the authors did not include a density filter to impose regularization onto the problem. By formulating the non-linear penalty constraint with respect to the filtered variables, defined through the density filter, the design variables would remain linear. Consequently, linear manufacturing constraints would also remain linear, while still being able to control the material continuity and obtaining near discrete designs.

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Paper 1

DMTO - a method for Discrete Material and Thickness Optimization of laminated composite structures

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Paper 2

Thickness Filters for Discrete Material and Thickness Optimization of Laminated Composite Structures

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Paper 3

In-plane material filters for the Discrete Material Optimization method

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