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Robust DOA Estimation of Harmonic Signals Using Constrained Filters on Phase Estimates



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Introduction

Existing approaches to direction of arrival (DOA) estimation:

- ► Time-difference of arrival (TDOA) based estimators that scale the TDOA of successive microphones.
- Beamforming based methods that steer the array in a range of possible directions, and maximize output power versus the DOA.
- High-resolution estimators based upon spatiospectral correlation matrix estimates.

The TDOA estimators possess an advantage over the two other methods in terms of computational complexity. Conventional TDOA estimators are designed assuming a single-source. However, the harmonic characteristic of audio signals facilitates a remarkable ability to estimate TDOAs of multiple sources which do not have spectral overlap.

DOA Estimate $\hat{\theta}$ (step 2)

While $\hat{\theta}_I = \sin^{-1}(\hat{\psi}_I / I\omega_0 f_s \tau_0)$ for $I = 1, \dots, L$, the DOA of the harmonic source can be estimated from L phase shift estimates:

$$\hat{\boldsymbol{\Psi}} = \omega_0 f_s \tau_0 \sin(\theta) \boldsymbol{\Gamma}_L + \Delta \boldsymbol{\Psi},$$

$$\Delta \Psi = [\Delta \psi_1, \Delta \psi_2, \dots, \Delta \psi_L]^T : \text{ phase shift noise}$$
$$\mathbf{\Gamma}_L = [1, 2, \dots, L]^T$$

Apply a filter $\mathbf{h} \in \mathbb{R}^{L}$:

$$\sin(\hat{\theta}) = \mathbf{h}^T \hat{\mathbf{\Psi}} = \omega_0 f_s \tau_0 \sin(\theta) \mathbf{h}^T \mathbf{\Gamma}_L + \mathbf{h}^T \Delta \mathbf{\Psi}.$$

With the constraint that $\mathbf{h}^T \mathbf{\Gamma}_L = 1/\omega_0 f_s \tau_0$, MSE{sin($\hat{\theta}$)} = $\mathbf{h}^T \mathbf{R}_{\Delta \Psi} \mathbf{h}$. Design: min $\mathbf{h}^T \mathbf{R}_{\Delta \Psi} \mathbf{h}$ subject to $\mathbf{h}^T \mathbf{\Gamma}_L = 1/\omega_0 f_s \tau_0$. $\mathbf{h}_{\text{MVDR}} = \frac{1}{\omega f_{\text{s}} \tau_{0}} \mathbf{R}_{\Delta \Psi}^{-1} \mathbf{\Gamma}_{L} (\mathbf{\Gamma}_{L}^{T} \mathbf{R}_{\Delta \Psi}^{-1} \mathbf{\Gamma}_{L})^{-1},$ DOA estimate:

We design optimal filters based on estimated noise statistics to apply on multi-channel phase estimates.

Formulation

Observed signal in an array (*M* microphones):

$$\mathbf{y}(n) = \sum_{l=1}^{L} \alpha_l \, e^{j \, (l\omega_0 n + \varphi_l)} \, \beta \, \mathbf{d}_{\theta}(l\omega_0) + \mathbf{v}(n) \tag{1}$$

 $\mathbf{d}_{\theta}(\omega)$: a steering vector for DOA of θ at $\omega \in [0, \pi]$ $\beta = \text{diag}\{[\beta_1, \beta_2, \dots, \beta_M]\}$: magnitude attenuations $\mathbf{v}(n) = [v_1(n), v_2(n), \dots, v_M(n)]^T \in \mathbb{C}^M$: Gaussian noise $v_m(n)$ has the real and imaginary uncorrelated parts with the variance of $\sigma_m^2/2$

$$SNR'_m = \frac{(\beta_m \alpha_l)^2}{\sigma_m^2}$$
: narrowband SNR

If $SNR'_m \gg 1$, the additive Gaussian noise can be converted to a normally distributed phase noise $\Delta \varphi_m(I\omega_0)$ with the variance of $E\{[\Delta \varphi_m(I\omega_0)]^2\} =$

$$\hat{\theta} = \sin^{-1}(\mathbf{h}_{\text{MVDR}}^{T}\hat{\mathbf{\Psi}}).$$

Simulation Results

DOA estimates of a synthetic signal, i.e., $\omega_0 = 0.15\pi$, L = 5, and M = 5, in different SNRs of colored noise and using different number of microphones (SNR= 20 dB):





$$\Delta \Phi_{I} = [\Delta \varphi_{1}(I\omega_{0}), \Delta \varphi_{2}(I\omega_{0}), \dots, \Delta \varphi_{M}(I\omega_{0})]^{T} : \text{ phase noise vector} \quad (2)$$
$$\mathbf{R}_{\Delta \Phi_{I}} = \mathsf{E}\{\Delta \Phi_{I} \Delta \Phi_{I}^{T}\} = \mathsf{diag}\left\{\left[\frac{1}{2\,\mathsf{SNR}_{1}^{I}}, \frac{1}{2\,\mathsf{SNR}_{2}^{I}}, \dots, \frac{1}{2\,\mathsf{SNR}_{M}^{I}}\right]\right\} \quad (3)$$

Approximate noisy signal model:

$$\mathbf{y}(n) \approx \sum_{l=1}^{L} \alpha_l \, e^{j \, (l\omega_0 \, n + \varphi_l)} \, \mathbf{D}_{\nu}(l\omega_0) \, \beta \, \mathbf{d}_{\theta}(l\omega_0), \tag{4}$$
with $\mathbf{D}_{\nu}(l\omega_0) = \text{diag}\{\exp(j\Delta \mathbf{\Phi}_l)\}.$

Phase Shift Estimate $\hat{\psi}_l$ (step 1)

Multi-channel phase estimates:

$$\hat{\mathbf{\Phi}}_{I} = \mathbf{\Pi}_{M} \begin{bmatrix} \varphi_{I} \\ \psi_{I} \end{bmatrix} + \Delta \mathbf{\Phi}_{I}$$
(5)

 $\hat{\mathbf{\Phi}}_{l} = [\hat{\Phi}_{l,1}, \hat{\Phi}_{l,2}, \dots, \hat{\Phi}_{l,M}]^{T}$: collection of phase estimates $\Phi_{I,m} = \varphi_I - (m-1)I\omega_0 f_s \tau_0 \sin(\theta)$ in a uniform linear array (ULA) $\Pi_M \in \mathbb{R}^{M \times 2}$ is a known matrix based on the number of

Compared to:

- Weighted least-squares (WLS) DOA estimator [2] - MVDR beamforming with harmonic emphasis (BH-MVDR) [3]

Covariance matrix:

$$\mathbf{R}_{\Delta \Phi_{I}} = \mathsf{E}\{(\hat{\Phi}_{I} - \mathsf{E}\{\hat{\Phi}_{I}\})(\hat{\Phi}_{I} - \mathsf{E}\{\hat{\Phi}_{I}\})^{T}\}$$
(12)

$$\mathsf{E}\{\hat{\mathbf{\Phi}}_I\} \approx \frac{1}{B} \sum_{b=0}^{B-1} \hat{\mathbf{\Phi}}_I(b) - b I \omega_0 \mathbf{1}_M \tag{13}$$

where $\hat{\Phi}_l(b)$ are estimated from $\mathbf{Y}(b) = [\mathbf{y}(b), \mathbf{y}(b+1), \dots, \mathbf{y}(b+N-1)].$

Conclusion

We have estimated the DOA of a harmonic signal source from multichannel phase estimates.

We designed optimal filters based on spatial and spectral noise statis-

microphones and the linear relationship between phases.

Apply a filter $\mathbf{W} \in \mathbb{R}^{M \times 2}$:

$$\begin{bmatrix} \hat{\varphi}_{l} \\ \hat{\psi}_{l} \end{bmatrix} = \mathbf{W}^{T} \hat{\mathbf{\Phi}}_{l} = \mathbf{W}^{T} \mathbf{\Pi}_{M} \begin{bmatrix} \varphi_{l} \\ \psi_{l} \end{bmatrix} + \mathbf{W}^{T} \Delta \mathbf{\Phi}_{l}.$$
(6)
With the constraint that $\mathbf{W}^{T} \mathbf{\Pi}_{M} = \mathbf{I}_{2 \times 2}, \text{ MSE} \left\{ \begin{bmatrix} \hat{\varphi}_{l} \\ \hat{\psi}_{l} \end{bmatrix} \right\} = \text{tr} \{ \mathbf{W}^{T} \mathbf{R}_{\Delta \mathbf{\Phi}_{l}} \mathbf{W} \}.$

Design:

- tics.
- The designed filters are robust against different noise scenarios, e.g., colored noise.
- Results of the proposed method approach to the CRLB.

References

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