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# Robust DOA Estimation of Harmonic Signals Using Constrained Filters on Phase Estimates

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## Introduction

Existing approaches to direction of arrival (DOA) estimation:

- Time-difference of arrival (TDOA) based estimators that scale the TDOA of successive microphones.
- Beamforming based methods that steer the array in a range of possible directions, and maximize output power versus the DOA.
- High-resolution estimators based upon spatiospectral correlation matrix estimates.

The TDOA estimators possess an advantage over the two other methods in terms of computational complexity. Conventional TDOA estimators are designed assuming a single-source. However, the harmonic characteristic of audio signals facilitates a remarkable ability to estimate TDOAs of multiple sources which do not have spectral overlap.

We design optimal filters based on estimated noise statistics to apply on multi-channel phase estimates.

## Formulation

Observed signal in an array ($M$ microphones):

$$\mathbf{y}(n) = \sum_{l=1}^{L} \alpha_l \, e^{j(l\omega_0 n + \varphi_l)} \, \boldsymbol{\beta} \, \mathbf{d}_\theta(l\omega_0) + \mathbf{v}(n) \qquad (1)$$

$\mathbf{d}_\theta(\omega)$ : a steering vector for DOA of $\theta$ at $\omega \in [0, \pi]$

$\boldsymbol{\beta} = \mathrm{diag}\{[\,\beta_1, \beta_2, \dots, \beta_M]\}$ : magnitude attenuations

$\mathbf{v}(n) = [\, v_1(n), v_2(n), \dots, v_M(n)\,]^T \in \mathbb{C}^M$ : Gaussian noise

$v_m(n)$ has the real and imaginary uncorrelated parts with the variance of $\sigma_m^2/2$

$$\mathrm{SNR}_m^l = \frac{(\beta_m \alpha_l)^2}{\sigma_m^2} \ : \ \text{narrowband SNR}$$

If $\mathrm{SNR}_m^l \gg 1$, the additive Gaussian noise can be converted to a normally distributed phase noise $\Delta\varphi_m(l\omega_0)$ with the variance of $\mathrm{E}\{[\Delta\varphi_m(l\omega_0)]^2\} = \frac{1}{2\,\mathrm{SNR}_m^l}$ [1].

$$\Delta\boldsymbol{\Phi}_l = [\Delta\varphi_1(l\omega_0), \Delta\varphi_2(l\omega_0), \dots, \Delta\varphi_M(l\omega_0)]^T \ : \ \text{phase noise vector} \qquad (2)$$

$$\mathbf{R}_{\Delta\boldsymbol{\Phi}_l} = \mathrm{E}\{\Delta\boldsymbol{\Phi}_l \Delta\boldsymbol{\Phi}_l^T\} = \mathrm{diag}\left\{\left[\,\frac{1}{2\,\mathrm{SNR}_1^l}, \frac{1}{2\,\mathrm{SNR}_2^l}, \dots, \frac{1}{2\,\mathrm{SNR}_M^l}\,\right]\right\} \qquad (3)$$

Approximate noisy signal model:

$$\mathbf{y}(n) \approx \sum_{l=1}^{L} \alpha_l \, e^{j(l\omega_0 n + \varphi_l)} \, \mathbf{D}_v(l\omega_0) \, \boldsymbol{\beta} \, \mathbf{d}_\theta(l\omega_0), \qquad (4)$$

with $\mathbf{D}_v(l\omega_0) = \mathrm{diag}\{\exp(j\Delta\boldsymbol{\Phi}_l)\}$.

## Phase Shift Estimate $\hat{\psi}_l$ (step 1)

Multi-channel phase estimates:

$$\hat{\boldsymbol{\Phi}}_l = \boldsymbol{\Pi}_M \begin{bmatrix} \varphi_l \\ \psi_l \end{bmatrix} + \Delta\boldsymbol{\Phi}_l \qquad (5)$$

$\hat{\boldsymbol{\Phi}}_l = [\hat{\Phi}_{l,1}, \hat{\Phi}_{l,2}, \dots, \hat{\Phi}_{l,M}]^T$ : collection of phase estimates

$\Phi_{l,m} = \varphi_l - (m-1) l\omega_0 f_s \tau_0 \sin(\theta)$ in a uniform linear array (ULA)

$\boldsymbol{\Pi}_M \in \mathbb{R}^{M\times 2}$ is a known matrix based on the number of microphones and the linear relationship between phases.

Apply a filter $\mathbf{W} \in \mathbb{R}^{M\times 2}$:

$$\begin{bmatrix} \hat{\varphi}_l \\ \hat{\psi}_l \end{bmatrix} = \mathbf{W}^T \hat{\boldsymbol{\Phi}}_l = \mathbf{W}^T \boldsymbol{\Pi}_M \begin{bmatrix} \varphi_l \\ \psi_l \end{bmatrix} + \mathbf{W}^T \Delta\boldsymbol{\Phi}_l. \qquad (6)$$

With the constraint that $\mathbf{W}^T \boldsymbol{\Pi}_M = \mathbf{I}_{2\times 2}$, $\mathrm{MSE}\left\{\begin{bmatrix} \hat{\varphi}_l \\ \hat{\psi}_l \end{bmatrix}\right\} = \mathrm{tr}\{\mathbf{W}^T \mathbf{R}_{\Delta\boldsymbol{\Phi}_l} \mathbf{W}\}$.

Design:

$$\min_{\mathbf{W}} \ \mathrm{tr}\{\mathbf{W}^T \mathbf{R}_{\Delta\boldsymbol{\Phi}_l} \mathbf{W}\} \ \text{subject to} \ \mathbf{W}^T \boldsymbol{\Pi}_M = \mathbf{I}_{2\times 2}.$$

$$\Downarrow$$

$$\mathbf{W}_{\mathrm{MVDR}} = \mathbf{R}_{\Delta\boldsymbol{\Phi}_l}^{-1} \boldsymbol{\Pi}_M (\boldsymbol{\Pi}_M^T \mathbf{R}_{\Delta\boldsymbol{\Phi}_l}^{-1} \boldsymbol{\Pi}_M)^{-1}. \qquad (7)$$

## DOA Estimate $\hat{\theta}$ (step 2)

While $\hat{\theta}_l = \sin^{-1}(\hat{\psi}_l \,/\, l\omega_0 f_s \tau_0)$ for $l = 1, \cdots, L$, the DOA of the harmonic source can be estimated from $L$ phase shift estimates:

$$\hat{\boldsymbol{\Psi}} = \omega_0 f_s \tau_0 \sin(\theta) \, \boldsymbol{\Gamma}_L + \Delta\boldsymbol{\Psi}, \qquad (8)$$

$\Delta\boldsymbol{\Psi} = [\Delta\psi_1, \Delta\psi_2, \dots, \Delta\psi_L]^T$ : phase shift noise

$\boldsymbol{\Gamma}_L = [1, 2, \cdots, L]^T$

Apply a filter $\mathbf{h} \in \mathbb{R}^L$:

$$\sin(\hat{\theta}) = \mathbf{h}^T \hat{\boldsymbol{\Psi}} = \omega_0 f_s \tau_0 \sin(\theta) \, \mathbf{h}^T \boldsymbol{\Gamma}_L + \mathbf{h}^T \Delta\boldsymbol{\Psi}. \qquad (9)$$

With the constraint that $\mathbf{h}^T \boldsymbol{\Gamma}_L = 1/\omega_0 f_s \tau_0$, $\mathrm{MSE}\{\sin(\hat{\theta})\} = \mathbf{h}^T \mathbf{R}_{\Delta\boldsymbol{\Psi}} \mathbf{h}$.

Design:

$$\min_{\mathbf{h}} \ \mathbf{h}^T \mathbf{R}_{\Delta\boldsymbol{\Psi}} \mathbf{h} \ \text{subject to} \ \mathbf{h}^T \boldsymbol{\Gamma}_L = 1/\omega_0 f_s \tau_0.$$

$$\Downarrow$$

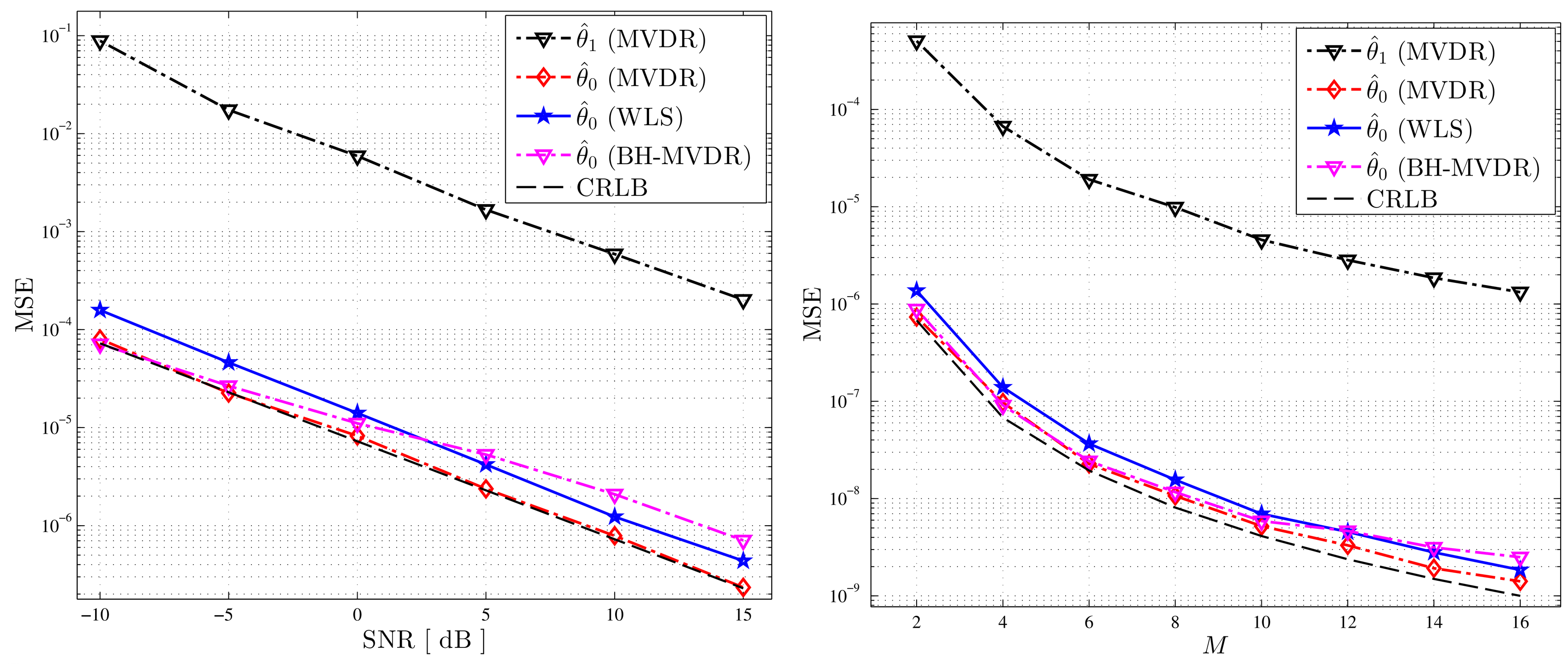
$$\mathbf{h}_{\mathrm{MVDR}} = \frac{1}{\omega f_s \tau_0} \mathbf{R}_{\Delta\boldsymbol{\Psi}}^{-1} \boldsymbol{\Gamma}_L (\boldsymbol{\Gamma}_L^T \mathbf{R}_{\Delta\boldsymbol{\Psi}}^{-1} \boldsymbol{\Gamma}_L)^{-1}, \qquad (10)$$

DOA estimate:

$$\hat{\theta} = \sin^{-1}(\mathbf{h}_{\mathrm{MVDR}}^T \hat{\boldsymbol{\Psi}}). \qquad (11)$$

## Simulation Results

DOA estimates of a synthetic signal, i.e., $\omega_0 = 0.15\pi$, $L = 5$, and $M = 5$, in different SNRs of colored noise and using different number of microphones (SNR= 20 dB):




Compared to:
- Weighted least-squares (WLS) DOA estimator [2]
- MVDR beamforming with harmonic emphasis (BH-MVDR) [3]

Covariance matrix:

$$\mathbf{R}_{\Delta\boldsymbol{\Phi}_l} = \mathrm{E}\{(\hat{\boldsymbol{\Phi}}_l - \mathrm{E}\{\hat{\boldsymbol{\Phi}}_l\})(\hat{\boldsymbol{\Phi}}_l - \mathrm{E}\{\hat{\boldsymbol{\Phi}}_l\})^T\} \qquad (12)$$

$$\mathrm{E}\{\hat{\boldsymbol{\Phi}}_l\} \approx \frac{1}{B} \sum_{b=0}^{B-1} \hat{\boldsymbol{\Phi}}_l(b) - b l\omega_0 \mathbf{1}_M \qquad (13)$$

where $\hat{\boldsymbol{\Phi}}_l(b)$ are estimated from $\mathbf{Y}(b) = [\,\mathbf{y}(b), \mathbf{y}(b+1), \dots, \mathbf{y}(b+N-1)\,]$.

## Conclusion

We have estimated the DOA of a harmonic signal source from multichannel phase estimates.

- We designed optimal filters based on spatial and spectral noise statistics.
- The designed filters are robust against different noise scenarios, e.g., colored noise.
- Results of the proposed method approach to the CRLB.

## References

[1] S. Tretter, "Estimating the frequency of a noisy sinusoid by linear regression (corresp.)," *IEEE Trans. Inf. Theory*, vol. 31, no. 6, pp. 832–835, 1985

[2] J. R. Jensen, M. G. Christensen, and S. H. Jensen, "Statistically efficient methods for pitch and DOA estimation," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, pp. 3900–3904, 2013.

[3] S. Karimian-Azari, J. R. Jensen, and M. G. Christensen, "Fast joint DOA and pitch estimation using a broadband MVDR beamformer," in *Proc. European Signal Processing Conf.*, Sept. 2013.

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