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Stabilization of Multiple Unstable Modes for Small-Scale Inverter-Based Power Systems with Impedance-Based Stability Analysis

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Abstract— This paper investigates the harmonic stability of small-scale inverter-based power systems. A holistic procedure to assess the contribution of each inverter to the system stability is proposed by means of using the impedance-based stability criterion. Multiple unstable modes can be identified step-by-step coming from the interactions among inverters and passive networks. Compared to the conventional system stability analysis, the approach is easy to implement and avoids the effect of potential unstable system dynamics on the impedance ratio derived for the stability analysis. PSCAD/EMTDC simulations of a Cigre LV network Benchmark system with multiple renewable energy sources are carried out. The results confirm the validity of the proposed approach.

Keywords— Harmonic stability; Distribution system; Stability;

I. INTRODUCTION

The Impedance Based Stability Criterion (IBSC) was originally proposed to design input filters for switch-mode power supplies with input filters [1]. Later on, it was expanded to study the stability of DC distributed power systems [2]. A number of stability criteria were proposed thereafter for defining the stability margin of interconnected dc-dc converters [3]. Recently, as the fast growing of renewable energy sources are enabled by power electronics, the IBSC was applied to analyze the harmonic stability in AC distributed system [4].

The impedance-based analysis predicts the system stability based on the equivalent impedances of two connected subsystems, which consequently simplifies the complex impedance connections into two equivalent impedances, namely the source impedance and the load impedance [1]. Fig. 1(a) shows an example for grid-connected inverters, where the interaction between the inverter and grid are modeled by closed-loop output admittance of inverter Y_S and an equivalent load admittance Y_L of the grid. The two admittances can be represented as a closed loop transfer function, which contains stability information about the two interconnected admittances. By extending the basic concept of Fig.1 (a) into multiple inverter connected network case as shown in Fig. 1 (c), the admittance relation of the overall network is simplified and the

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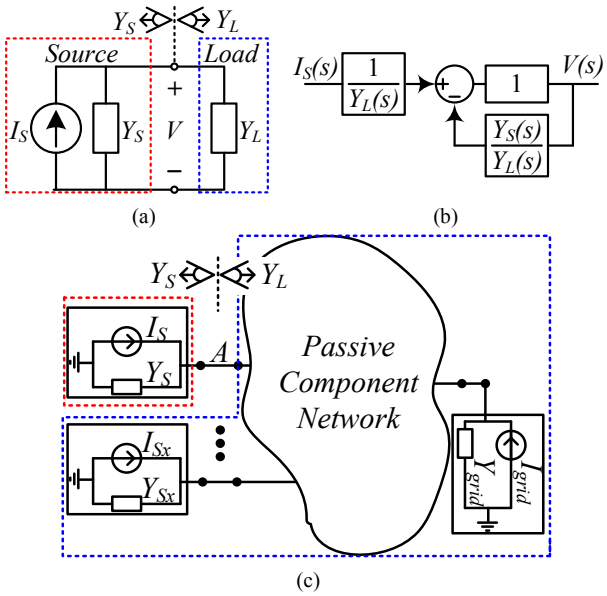


Fig. 1. Small-signal admittance representation of: (a) a current source and load; (b) closed loop transfer function representation; (c) small scale inverter-based power system.

stability information on an arbitrary node A can be analyzed. In this case, the load admittance Y_L contains all network admittances except for the source admittance Y_S. In detail, the load admittance contains all other inverter admittances (Y_{Sx}) on the other nodes, it also contains the impedances in the network passive components such as resistances, inductances and capacitances, which come from the distribution lines and transformers. In addition, the grid admittance from the up-stream network such as medium-voltage network is represented as Y_{grid}. In order to use the Nyquist stability criterion for the IBSC, the stability on the current source I_S and the stability of load the system Y_L needs to be procured individually. The inverter system should be able to operate in stand-alone, so the current source I_S is stable. Also the load system stability can be examined by looking for the right-half-plane poles (RHP) of the inverse load admittance $1/Y_L$ [5]. However, when each of the individually stable inverters only having LHP poles is aggregated into a common output admittance Y_L, the overall

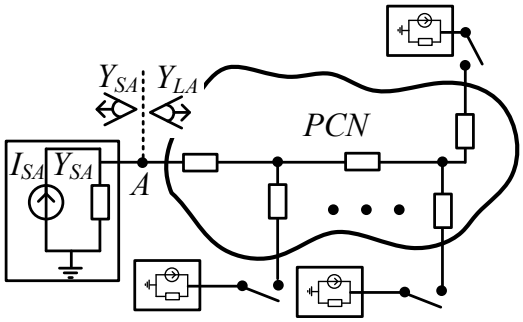


Fig. 2. Passive component network (PCN).

result can show up to be unstable, i.e. having RHP. This is due to the interaction between them, individually stable inverters interacting and creating an unstable system. This complicates the use of the IBSC as the information of each of the individual inverters transfer function is lost in the aggregated transfer function. In other words, even if each of the components in the load admittance is designed stable, the combined load admittance stability still remains unknown [4],[6]–[9]. Up until now, there was no approach to address this uncertainty in the load stability for AC systems. This paper proposes the step-by-step instability elimination procedure. It starts from the secured stable admittances of the passive components, and then expanded by including the inverters one-by-one. Finally, all of the unstable modes are identified and a stable power electronics based power system is obtained. PSCAD/EMTDC simulations of a Cigre low-voltage (LV) distribution network with multiple inverters [10] is carried out to confirm the validity of this method. Experimental result verifies the proposed methodology.

II. Stable Passive Network and Concept Expantion

In order to use IBSC for network stability anlysis, stable load admittance must be ensured [5]. Distribution lines such as cables in a small-scale power system are composed of passive components such as resistors, inductors and capacitors. Passive components do not have negative real parts in the small signal modeling, so it does not provide any unstable right half plane (RHP) poles and zeros into the Passive Component Network (PCN). This passivity in the distribution lines and the passive components gives the basis for absolute stable load admittance of the network [11]. Assume first that there is only one grid inverter connected to PCN and all the other active components are disconnected from the network as shown in Fig. 2. The current source I_{SA} is a standalone stable unit, and the inverse of the load admittance $1/Y_{LA}$ has no RHP pole. Then the IBSC prerequisites are satisfied. Stability on node A can be analyzed based on the Nyquist stability criterion of the admittance ratio T_{mA} in (1).

$$T_{mA} = \frac{Y_{SA}}{Y_{LA}} \tag{1}$$

If (1) satisfies the Nyquist stability criterion, the voltage on node A is stable with respect to the current source I_{SA}. Once its stability is obtained on node A, the PCN will be kept stable at all connected terminals where the other inverters can be connected further. Hence, the following statements are made:

1. The network with only passive components is always stable;
2. The arbitrary node in the PCN is stable when all active components connected nodes are stable.

Statement 2 provides the IBSC to expand its usability to all the nodes in the power system. However, the use of IBSC to assess the contribution of each active component is not straightforward, since the load system seen from one stable inverter may be already destabilized by the interactions of the other inverters [12].

III. Sequential Stabilizing Procedure

The concept of a sequential stabilizing procedure is to structure the stable load admittance that contains all the inverters in the network. Starting from the absolutely stable load admittance of PCN, which is not having any unidentified active devices, the load admittance is expanded to include the whole network by adding the inverter one by one. In every step, the system stability is evaluated by the Nyquist stability criterion. If the criterion turns out to be unstable, then the lastly adopted inverter's source admittance should have more stabilizing functions such as damping resistors or active damping functions. Again, this inverter is reinvestigated iteratively until the system becomes stable. When all the

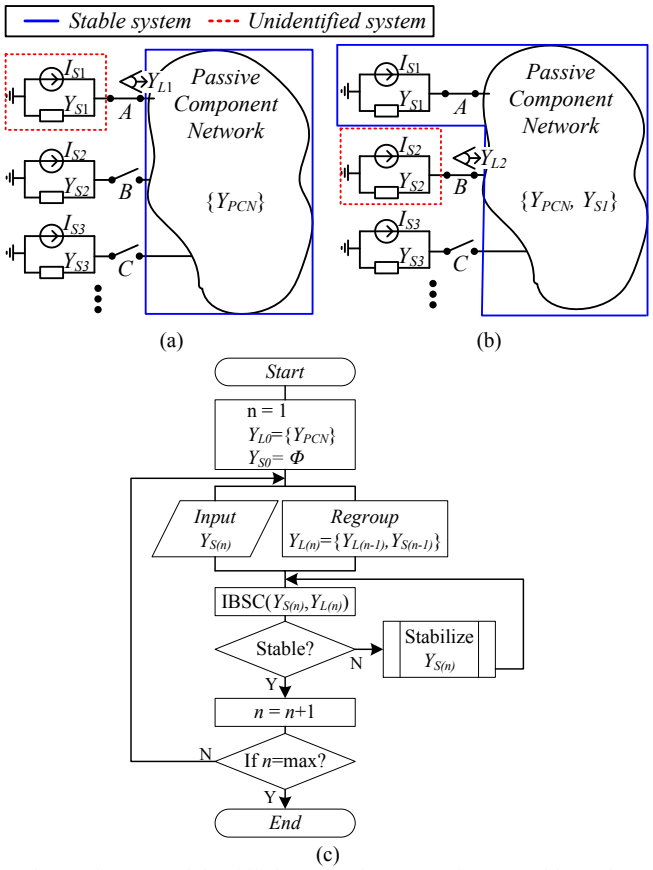


Fig. 3. The sequential stabilizing procedure: a) An inverter with passive component network; b) the second inverter with a stable admittance network; c) The proposed sequential stabilizing procedure.

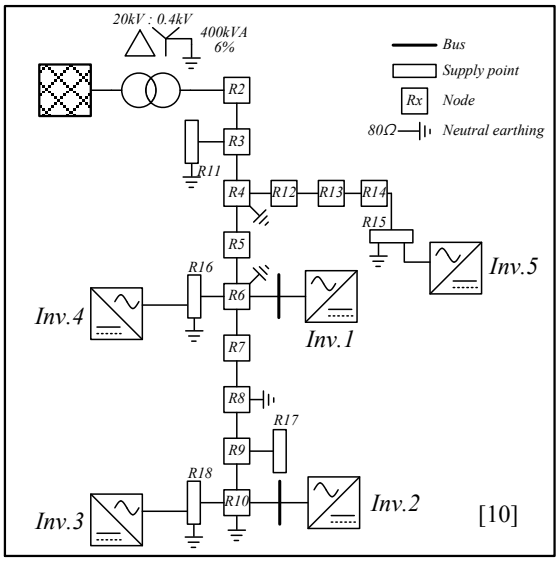


Fig. 4. Benchmark of European LV distribution network (*Inv.1* ~ *Inv.5* are five voltage source inverters)

inverters are evaluated and stable results are obtained from the Nyquist stability criterion, then the procedure ends.

Sequential procedure in a flow chart

Fig. 3 (c) shows the details of the sequential stabilizing procedure for the multiple inverter connected to the power system. The Nyquist stability criteria can start from the initial combination of unknown inverter output admittance Y_{S1} and the PCN admittance Y_{L1} seen from the node A, as shown in Fig. 3 (a). One thing to note is that the equivalent admittance Y_{PCN} depends on the point, where it is measured. In other words, Y_{PCN} has different values for each of the nodes A, B and so on. Once the stability between the connected system Y_{S1} and Y_{L1} is ensured on node *A*, according to the statement 2, then it can be expanded to the next node *B* as shown in Fig. 3 (b). The new stable load admittance Y_{L2} seen from the node *B* contains Y_{PCN} and Y_{S1}. The Nyquist stability criterion evaluates the stability on node B for unidentified source admittance Y_{S2} with respect to the new stable load admittance Y_{L2} obtained from the previous step. If the analysis turns out to be unstable, the newly connected inverter Y_{S2} can be the reason for the instability. In this case, the new inverter can have more damping capabilities, which can stabilize the network so the criterion shows stable result. In this way, the inverters are added one by one. This procedure obtains stable load admittance step by step and finally, it is able to have all inverters to operate in a stable network.

IV. TEST SYSTEM AND INVERTER MODEL

This section describes a benchmark system for small-scale power distribution system, which is used as a test bed for the proposed stabilizing procedure. The model inherently contains passive components such as line impedances and a transformer. Therefore, the overall stability is determined by how the inverters are designed and connected individually. Firstly, it shows detailed data of a benchmark model and also its specification. Secondly, the inverter information in the benchmark network is specified. Lastly, the Norton equivalent model of the inverter for the proposed analysis is shown.

TABLE I. POSITIVE SEQUENCE IMPEDANCE FOR UNDERGROUND CABLE

Node (From-To)	Length [m]	Resistance [$m\Omega$]	Inductance [uH]
R1-R2	35	10.045	18.6052
R2-R3	35	10.045	18.6052
R3-R4	35	10.045	18.6052
R4-R6	70	20.09	37.2104
R6-R9	105	30.135	55.8156
R9-R10	35	10.045	18.6052
R4-R15	135	155.52	196.811
R6-R16	30	34.56	43.7358
R9-R17	30	34.56	43.7358
R10-R18	30	34.56	43.7358
Transformer		3.2	40.7437

TABLE II. GRID INVERTER SPECIFICATIONS AND THEIR PARAMETERS

		Inverter name				
		Inv. 1	*Inv. 2*	*Inv. 3*	*Inv. 4*	*Inv. 5*
Power rating [kVA]		35	25	3	4	5.5
Base Frequency, f_0 [Hz]		50				
Switching Frequency, f_s [kHz] (Sampling Frequency)		10		16		
DC-link voltage, v_{dc} [kV]		0.75				
Harmonic regulations of LCL filters		IEEE519-1992				
Filter values	L_f[mH]	0.87	1.2	5.1	3.8	2.8
	C_f[uF]/R_d[Ω]	22/0	15/1	2/7	3/4.2	4/3.5
	L_g[mH]	0.22	0.3	1.7	1.3	0.9
Parasitics values	r_{Lf}[mΩ]	11.4	15.7	66.8	49.7	36.7
	r_{Cf}[mΩ]	7.5	11	21.5	14.5	11
	r_{Lg}[mΩ]	2.9	3.9	22.3	17	11.8
Controller gain	K_P	5.6	8.05	28.8	16.6	14.4
	K_I	1000	1000	1500	1500	1500

A. *Test system*

To verify the proposed method, the Cigre benchmark of European LV distribution network [10] is chosen and used as a verification model which is shown in Fig. 4. The LV network voltage is three-phase line-to-line rms 400 V with 50 Hz. The distribution system is tied to 20 kV medium voltage grid with a 400 kVA transformer. Unbalanced passive loads are removed for the simplicity. Each node is connected with underground cable which provides passive components to the grid. In addition, this distribution system is small in size and the maximum distance between the nearby nodes do not exceed 35 meters. The parasitic capacitors in the underground cable become small enough to be neglected. Therefore, only the positive sequence resistance and the inductive reactance are used for the analysis and the parameter values are shown in the Table I.

B. *Inverter design and time domain model*

Five LCL-filtered grid inverters are connected to the network. Each inverter filter is designed based on the IEEE 519-1992 and the filter parameters and their parasitic values are given in the Table II. Also the meaning of the values are depicted in Fig. 5. All the inverters are operated in a grid connected mode, and a sinusoidal PWM method is used for

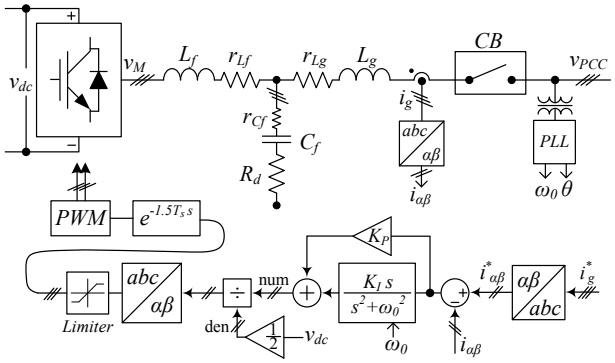


Fig.5. Single line diagram of grid-inverter for time domain simulation.

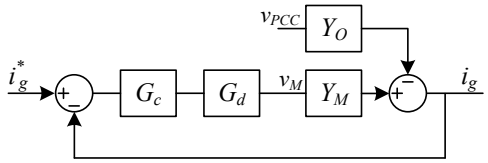


Fig. 6. Norton equivalent model for the grid-inverter with balanced load condition.

the inverters. Also, the connected distribution energy sources are assumed to have constant DC voltage sources. Fig. 5 shows the time domain model of the grid-inverters, which contains a grid current control loop in a stationary reference frame and the parameters given in Table II. This model is used for implementing the PSCAD time domain simulation.

C. *Norton equivalent model for the grid inverter*

In order to perform the IBSC according to the proposed method, the equivalent output admittance of the inverter is required. The output admittance model includes the transfer function of the LCL filter and its control loops. However, the bandwidth of the outer power control and synchronization loops are much slower than the current control loop, so the low frequency oscillations caused by the outer control loops can be neglected. Therefore, the current control loop dominates overall stability of the system and the control loops are simplified into a single current control loop [6]. Fig. 6. shows the Norton equivalent model for the grid inverter. G_C denotes the transfer function of the P+R controller for the fundamental frequency and G_d is the time delay from the digital implementation. Y_O and Y_M denote the open loop output admittance and the control to the output transfer function, respectively.

$$G_C = K_P + \frac{K_I s}{s^2 + \omega_0^2} \tag{2}$$

$$G_d = e^{-1.5T_S s} \tag{3}$$

where K_P and K_I are the controller gains and ω_0 is grid frequency. Also, T_S is the sampling time and the inverse of the switching frequency f_S:

$$\omega_0 = 2\pi f_0 \ , \ T_S = 1/f_S$$

$$Y_M = \left.\frac{i_g}{v_M}\right|_{v_{PCC}=0} = \frac{Z_{Cf}}{Z_{Cf}Z_{Lf} + Z_{Lg}Z_{Lf} + Z_{Cf}Z_{Lg}} \tag{4}$$

$$Y_O = \left.\frac{-i_g}{v_{PCC}}\right|_{v_M=0} = \frac{Z_{Lf} + Z_{Cf}}{Z_{Cf}Z_{Lf} + Z_{Lg}Z_{Lf} + Z_{Cf}Z_{Lg}} \tag{5}$$

Where the impedances Z_{Cf}, Z_{Lf} and Z_{Lg} are as follows;

$$Z_{Cf} = r_{Cf} + \frac{1}{sC_f} + R_d \ , \ Z_{Lf} = r_{Lf} + sL_f \ , \ Z_{Lg} = r_{Lg} + sL_g$$

The loop gain of the negative feedback loop in Fig. 5 is as follows.

$$T_{OL} = G_C G_D Y_M \tag{6}$$

Finally, the closed loop output admittance of *x*'th inverter Y_{Sx} is obtained according to the parameters given in Table. II.

$$Y_{Sx} = \left.\frac{-i_g}{v_{PCC}}\right|_{i_g^*=0} = \frac{Y_O}{1+T_{OL}} \tag{7}$$

V. EXAMPLE OF PROPOSED METHOD

This section gives an example of a sequential stabilizing procedure for a given small scale distribution system shown in Fig. 4. By following the sequence given in Fig. 3 (c), the stabilizing procedure is performed as follows.

Step 1: Firstly, the source admittance Y_{S1} at node *R6*, which is the output admittance of *Inv.1* is obtained from (7) and Table II. The load admittance Y_{L1} at the same node is obtained by calculating the equivalent admittance seen from the node *R6,* which contains the only passive component network and there is no inverter connected. So the minor loop gain for node *R6* is derived as follows.

$$T_{m1} = \frac{Y_{S1}}{Y_{L1}} \tag{8}$$

Fig. 7 shows the IBSC result from (8). It shows that the

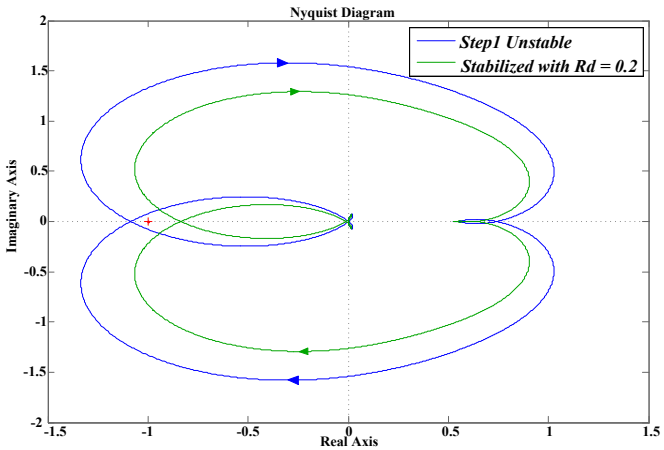


Fig. 7. Step 1: Unstable system with its initial condition (blue); *S*tabilized system with the additional damping resistor R_d in *Inv.1* (green).

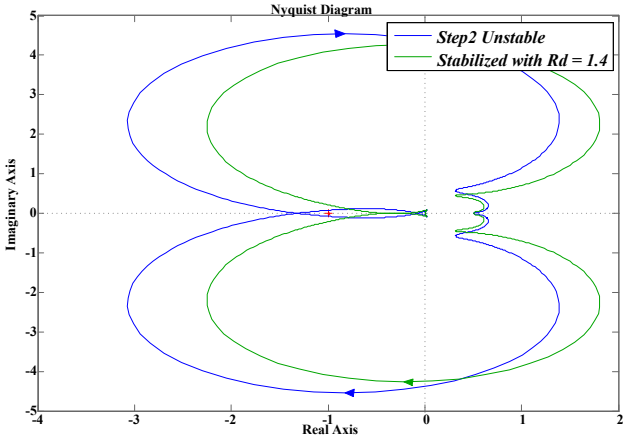


Fig. 8. Step 2 : Unstable system with the initial condition (blue); Stabilized system with the damping resistor in R_d *Inv.2* (green).

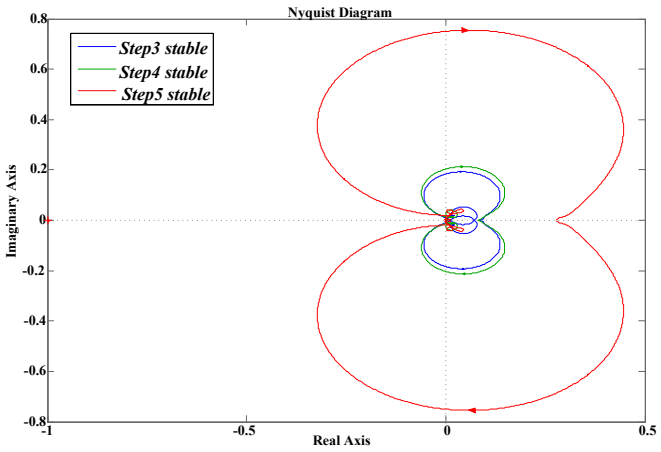


Fig. 9. Step 3~5: Stable systems with different inverters connected to the system.

network with *Inv.1* is unstable (blue) as it encircles (-1,0j). However, Y_{L1} has always been stable from statement 1, so Y_{S1} could be modified instead. In order to add stabilizing function to the Y_{S1}, the damping resistor R_d is inserted to the *Inv.1,* which can reshape the output admittance Y_{S1}. As it can be seen in Fig.7, the result satisfies the Nyquist criterion (green) so the passive component system becomes stable with *Inv.1*. Fig. 10 (a) and Fig. 10 (b) are representing the analyzed results of the unstable and stable case respectively.

Step 2: Once the stability is obtained from the previous step, the new stabilizing procedure can start with other nodes based on statement 2. In this case, *Inv.2* at node *R10* is added to the stable network obtained from the previous step. Therefore, the Y_{L2} contains Y_{L1} and Y_{S1}

$$T_{m2} = \frac{Y_{S2}}{Y_{L2}} \tag{9}$$

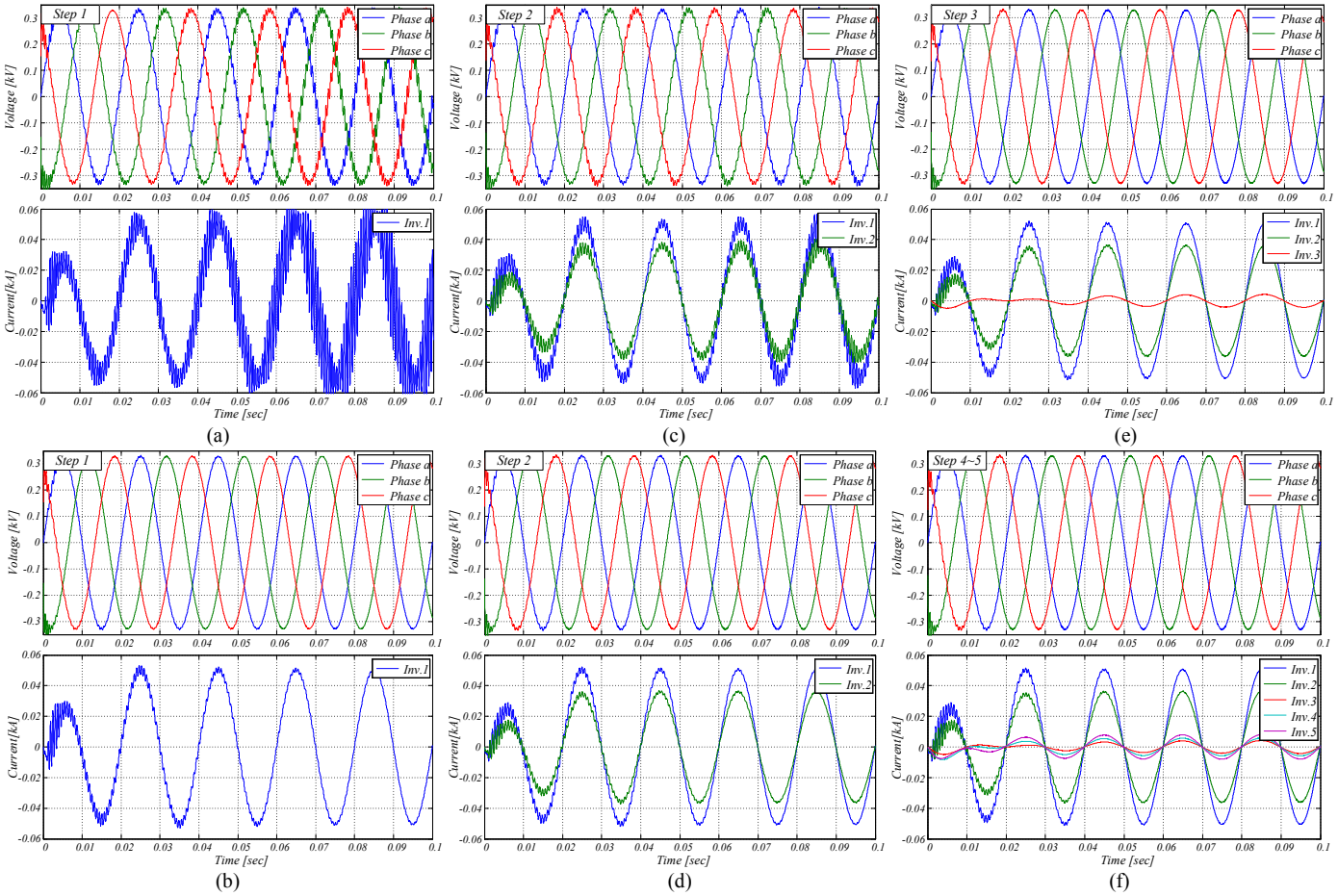
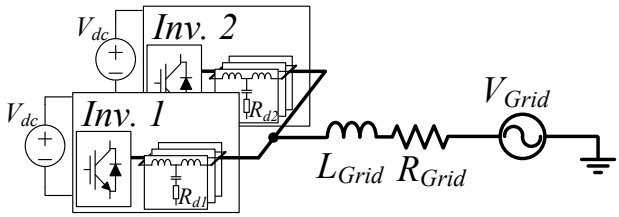


Fig. 10. Time domain simulation of *R4* node voltages (upper) and the inverter currents (lower): a) Step 1 unstable case; b) Step1 stabilized case; c) Step 2 unstable case; d) Step 2 stabilized case; e) Step 3 stable case; f) Step 4~5 stable case.



- V_{Grid} : 3-phase, 400V rms line to line, grid simulator
- L_{Grid} : 150uH, R_{Grid} : 0.1 Ohm

Fig. 11. Experimental setup for the two paralleled inverters to validate the method.

TABLE III. GRID INVERTER PARAMETERS

	Power Rating	Switching Frequency	DC Link	Filter Topology	Filter Value	Controller Gain
Inv. 1	5 kVA	10 kHz	750 V	LCL	L_f = 1.8 mH C_f = 9.4 uF L_g = 1.8 mH	K_P = 6 K_I = 1000
Inv. 2	5 kVA	10 kHz	750 V	LCL	L_f = 3 mH C_f = 4.7 uF L_g = 3 mH	K_P = 15.5 K_I = 1000

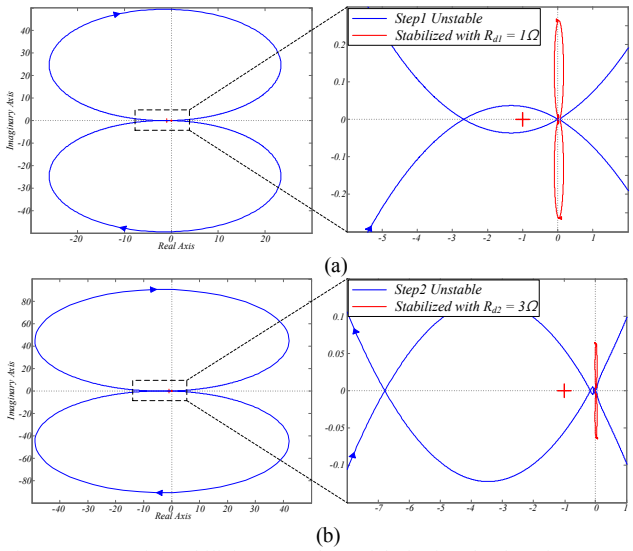


Fig. 13. Sequential stabilizing procedure of designing the damping resistor R_d to obtain stable two-inverter system in Fig. 11.

Fig. 8 shows the IBSC result by (9) representing the unstable network (blue). In the same manner as the previous step, a damping resistor R_d in the *Inv.2* is changed from the initial value 1.2 to 1.4 [Ω]. In that case the system becomes stable again (green). It is verified in Fig. 10 (c) and (d) respectively in the time domain.

Step 3 ~ 5: The same as the previous steps, inverters are added to the stable network step by step. In these cases, the initial conditions are enough to make the system stable. Finally, by summing up the all inverters step by step, the stable system with the five inverters is obtained. The related time domain simulations are shown in Fig.10 (e) and (f). One thing to note is that this example is just one of many different sequential stabilizing pathways and it may not be the optimal solution. However, it demonstrates that the method to obtain a stable network by using the IBSC is valid. Most important is that the inverter has to be checked once as a source admittance,

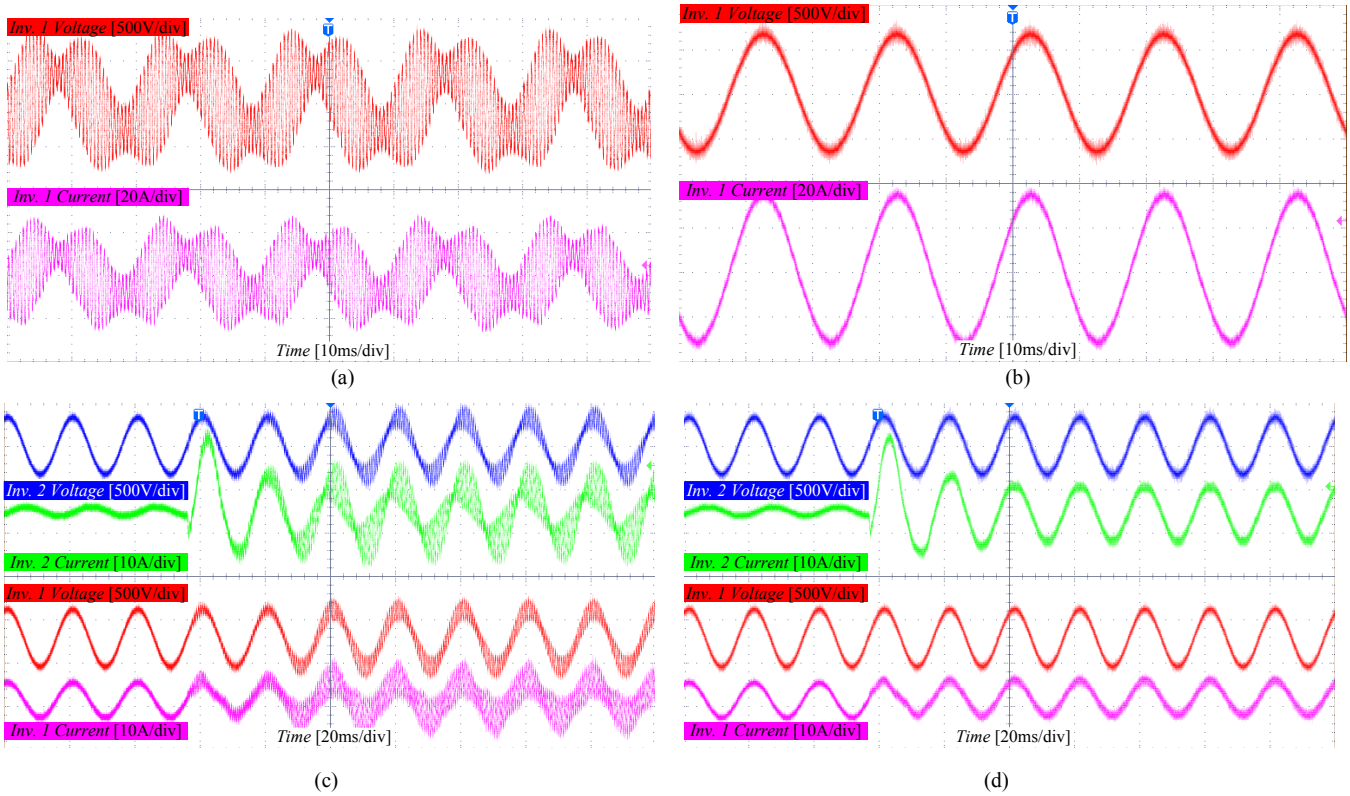


Fig. 13. Experimental waveforms using two paralleled inverters: a) Step 1 unstable case voltage and current;
b) stabilized with passive damping of *Inv.1* with R_{d1} = 1 Ω; c) Step 2 unstable with the two inverters; d) stabilized with passive damping of *Inv.2* R_{d2} = 3 Ω

and the load admittance must be expanded step by step.

VI. Experimental results

In order to verify the proposed method, two paralleled inverter systems are organized as shown in Fig.11 and step by step stabilizing procedure is performed. The grid-inverter filter values and controller gains are given in Table III. In Fig. 12, the Nyquist stabilility analysis results obtained from the proposed procedure. When *Inv. 1* is connected to the grid simulator without having a damping resistor R_{d1}, the system is unstable as shown in Fig. 12 (a). So the damping resistor is added to the *Inv. 1* and the system becomes stable. Afterwards, the *Inv. 2* is connected to the stable system made by step 1 as shown in Fig.12 (b). It is also unstable without having a damping resistor. So the damping resistor R_{d2} is added to *Inv. 2* and the system becomes stable. Fig. 13 shows the experimental stability result of each case and demonstrates.

VII. Conclusions

This paper proposes a method to address the harmonic instability problems in a small-scale power electronics based power system. The limitation in using the conventional IBSC is the presence of unidentified lumped load admittance. A holistic procedure to assess the contribution of each inverter to the system stability is proposed by means of the step by step stabilizing procedure. This can eliminate the multiple unstable conditions resulting from interactions among inverters in a passive network. However, this method can have many degrees of freedoms to obtain the stable network by changing the stabilizing sequence. It can make a stable system with all inverters, but it may not be able to provide an optimal solution. Simulation results and experimental results support the validity of the method.

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