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Selective Harmonic Control for Power Converters

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Abstract—**This paper proposes an Internal Model Principle (IMP) based Selective Harmonic Controller (SHC) for power converters. The proposed SHC offers an optimal control solution for power converters to mitigate power harmonics. It makes a good trade-off among cost, complexity and performance. It has high accuracy and fast transient response, and it is cost-effective, easy for real-time implementation, and compatible for design rules-of-thumb. An application on a three-phase PWM converter has confirmed the effectiveness of the proposed control scheme in terms of harmonic mitigation.**

I. INTRODUCTION

Today, a rapid growing amount of current harmonics and voltage harmonics due to non-linear electric loads frequently cause serious power quality problems in the electrical power systems [1], [2]. Power converters demand optimal control strategies for harmonics compensation, which should achieve high control accuracy, fast transient response, good robustness, and easy implementation [1]. In practical applications, harmonics usually concentrate on some particular frequencies [3]. For example, in the *n*-pulse converter systems, $nk\pm1$-order (k = 1, 2, 3, …) harmonics dominate the Total Harmonic Distortion (THD).

Hence, it is possible to selectively compensate the harmonics according to the characteristic of the harmonic distributions in *n*-pulse power converters [4]-[6]. Based on the Internal Model Principle (IMP), classic Repetitive Controller (RC) [7]-[18] and ReSonant Controller (RSC) [19]-[21], which can achieve zero steady-state error in the control of any periodic signal and a sinusoidal signal respectively, and provide very simple but effective harmonic control solutions. However, a compact recursive RC can achieve zero tracking error at all harmonic frequencies, but yields typically slow total convergence rate. Paralleled Multiple ReSonant Controllers (MRSC) at selected harmonic frequencies can render fast transient response, but would increase the computational burden and design complexity in dealing with a large number of harmonics. The $6l\pm1$ RC [10], [11] and the recursive odd harmonic RC [12], [13] offer an accurate, fast, and feasible Selective Harmonic Control (SHC) solution for single-phase power converters and three-phase power converters respectively. However, a universal selective harmonic control strategy is still open for exploration.

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Considering the above issues, in this paper, a universal SHC solution has been proposed in § II for the power converters to mitigate the harmonics selectively. The analysis and synthesis of the SHC systems are also addressed. The SHC solution has been applied to a three-phase Pulse Width Modulation (PWM) converter for case study in § IV. The results have verified the effectiveness of the proposed SHC solution for power converters in terms of harmonic mitigations.

II. SELECTIVE HARMONIC CONTROL

A. Classic Repetitive Control

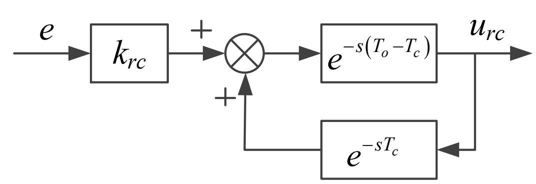


Fig. 1. Repetitive controller $G_{rc}(s)$.

As it is shown in Fig. 1, a classic RC can be written as,

$$G_{rc}(s)=\frac{u_{rc}(s)}{e(s)}=\frac{k_{rc}\cdot e^{-sT_0}}{1-e^{-sT_o}}e^{sT_c} \tag{1}$$

where k_{rc} is the control gain; $T_o=2\pi/\omega_o=1/f_o$ is the fundamental period of signals with f_o being the fundamental frequency; ω_o being the fundamental angular frequency; and T_c is the lead phase compensation time. The classic RC only consumes a little computation in its implementation.

Eq. (1) for the classic RC can be expanded as [11]-[13],

$$G_{rc}(s)=k_{rc}\left[-\frac{1}{2}+\frac{1}{T_o}\sum_{h=0}^{\infty}\frac{s}{s^2+(h\omega_0)^2}\right]e^{sT_c} \tag{2}$$

which indicates that the RC is equivalent to the parallel combination of a proportional gain (i.e. $-k_{rc}/2$), an integrator and infinite resonant controllers (RSCs) (i.e. the internal models of DC and all harmonic signals). These RSC components, which will approach infinity at harmonic frequencies $h\omega_0$, enable the RC to compensate all harmonic frequencies. Since the control gains for all RSC controllers in (2) are identical, i.e. k_{rc}/T_o, it is impossible for the RC to optimize its transient response by tuning control gains independently for selected harmonic frequencies.

B. Proposed Selective Harmonic Control

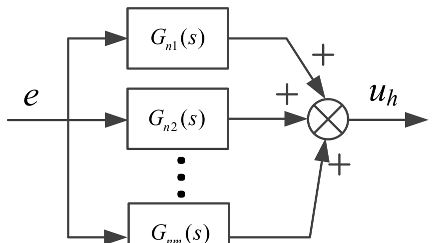


Fig. 2. Proposed selective harmonic controller $G_{nm}(s)$.

It is known that (nk±1)-order (k=1, 2, …) harmonics often dominate the THD in n-pulse converter systems. In order to mitigate these harmonics selectively for n-pulse converters, an SHC is proposed as it is shown in Fig. 2. This control solution is a combination of multiple SHC modules ($G_{nm}(s)$), which can be generated as follows [11], [12]:

$$
\begin{aligned}
G_{nm}(s) &= \frac{k_m}{2}\left(\frac{e^{-sT_0/n+j(2\pi m/n)}}{1-e^{-sT_0/n+j(2\pi m/n)}}+\frac{e^{-sT_0/n-j(2\pi m/n)}}{1-e^{-sT_0/n-j(2\pi m/n)}}\right)e^{sT_1} \\
&= k_m \frac{\cos(2\pi m/n)e^{sT_o/n}-1}{e^{2sT_o/n}-2\cos(2\pi m/n)e^{sT_o/n}+1}e^{sT_c}
\end{aligned} \tag{3}
$$

where k_m is the control gain for the corresponding SHC module $G_{nm}(s)$; T_o has been defined previously; T_c is the phase-lead compensation time; n and m are integers with $n > m \geq 0$. Moreover, since

$$
\begin{aligned}
&\frac{e^{-2\pi\left(\frac{s}{n\omega_o}\pm\frac{jm}{n}\right)}}{1-e^{-2\pi\left(\frac{s}{n\omega_o}\pm\frac{jm}{n}\right)}} \\
&= -\frac{1}{2}+\frac{n}{T_o}\frac{1}{(s\pm jm\omega_o)}+\frac{n}{T_o}\sum_{h=1}^{+\infty}\frac{2(s\pm jm\omega_o)}{(s\pm jm\omega_o)^2+(nh\omega_0)^2}
\end{aligned} \tag{4}
$$

in which h = 1, 2, … and m = 0, 1, 2, …, n-1. It ca be seen that Eq. (4) specifically includes the RSC at the (nk±m)-order harmonic frequencies. Compared with the RC of (1), since the convergence rate of any RSC is proportional to its gain, the error convergence rate at the (nk±m)-order harmonic frequencies of the SHC module of (3) can be n/2 times faster if k_m=k_{rc}.

In practical applications, the modified selected SHC modules $G_{nm}(s)$ will be employed as,

$$
G_{nm}(s)=\frac{k_m G_f(s)\left[\cos(2\pi m/n)e^{sT_o/n}Q(s)-Q^2(s)\right]}{e^{2sT_o/n}-2\cos(2\pi m/n)e^{sT_o/n}Q(s)+Q^2(s)} \tag{5}
$$

where $G_f(s)$ is a phase-lead compensation filter to stabilize the overall closed-loop system; the low-pass filter $Q(s)$ is employed to make a good trade-off between the tracking accuracy and the system robustness.

The SHC module of (5) provides a universal recursive IMP-based controller, which is tailored for (nk±m)-order harmonic frequencies. For example, let n=1 and m=0, Eq. (5) becomes an RC, and let n=4 and m=1, (5) becomes an odd harmonic RC [10], in which it has been named as "*nk±m order RC*" [13], [14]. In order to compensate more harmonic frequencies for better accuracy while keeping a fast error convergence rate, a universal SHC which includes paralleled SHC modules tailored for the selected harmonics, can be chosen as,

$$
G_{SHC}(s)=\sum_{m\in N_m} G_{nm}(s) \tag{6}
$$

in which m and N_m represent nk±m (k=0, 1, 2, … and m≤n/2) harmonic order and the set of selected harmonic frequencies respectively. It can be observed that the proposed SHC can optimize its transient response by weighting its control gains k_m in accordance with the harmonics distribution.

III. Digital SHC Systems

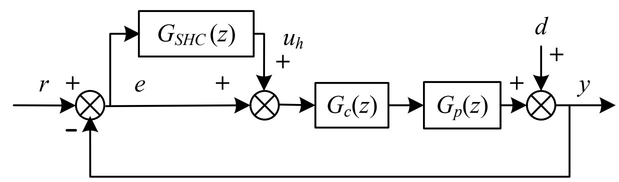


Fig. 3. Digital selective harmonic control (SHC controlled) system.

Fig. 3 shows a typical plug-in SHC system where $G_p(z)$ is the transfer function of the plant; $G_c(z)$ is the feedback controller; $G_{SHC}(z)$ is the z-domain transfer function of the SHC in (6); $r(z)$ is the reference input; $y(z)$ is the output; $e(z)$=$r(z)$-$y(z)$ is the tracking error and the input of $G_{SHC}(z)$; $d(z)$ is the disturbance. And the output $y(z)$ of the plug-in control system can be expressed as follows,

$$
\begin{aligned}
&y(z) \\
&=\frac{\left[1+G_{SHC}(z)\right]H(z)}{1+G_{SHC}(z)H(z)}r(z)+\frac{\left[1+G_c(z)G_p(z)\right]^{-1}}{1+G_{SHC}(z)H(z)}d(z)
\end{aligned} \tag{7}
$$

where $H(z)$ is the transfer function of the conventional feedback control system without the SHC $G_{SHC}(z)$. Besides,

$$
\begin{aligned}
&G_{SHC}(z) \\
&=\sum_{m\in N_m} k_m \frac{\left[\cos(2\pi m/n)z^{N/n}Q(z)-Q^2(z)\right]G_f(z)}{z^{2N/n}-2\cos(2\pi m/n)z^{N/n}Q(z)+Q^2(z)}
\end{aligned} \tag{8}
$$

$$
H(z)=\frac{G_c(z)G_p(z)}{1+G_c(z)G_p(z)} \tag{9}
$$

in which, N=f_s/f_0 with f_0 =1/T_0 being the fundamental frequency and f_s being the sampling frequency; k_m is SHC gain; $G_f(z)$ is the digital form phase compensation filter; $Q(z)$ is the digital low-pass filter with $|Q(e^{j\omega})|\leq 1$. $Q(z)$ is employed to make a good tradeoff between the tracking accuracy and the system robustness, which usually chooses a low-pass filter to remove minor but unexpected high

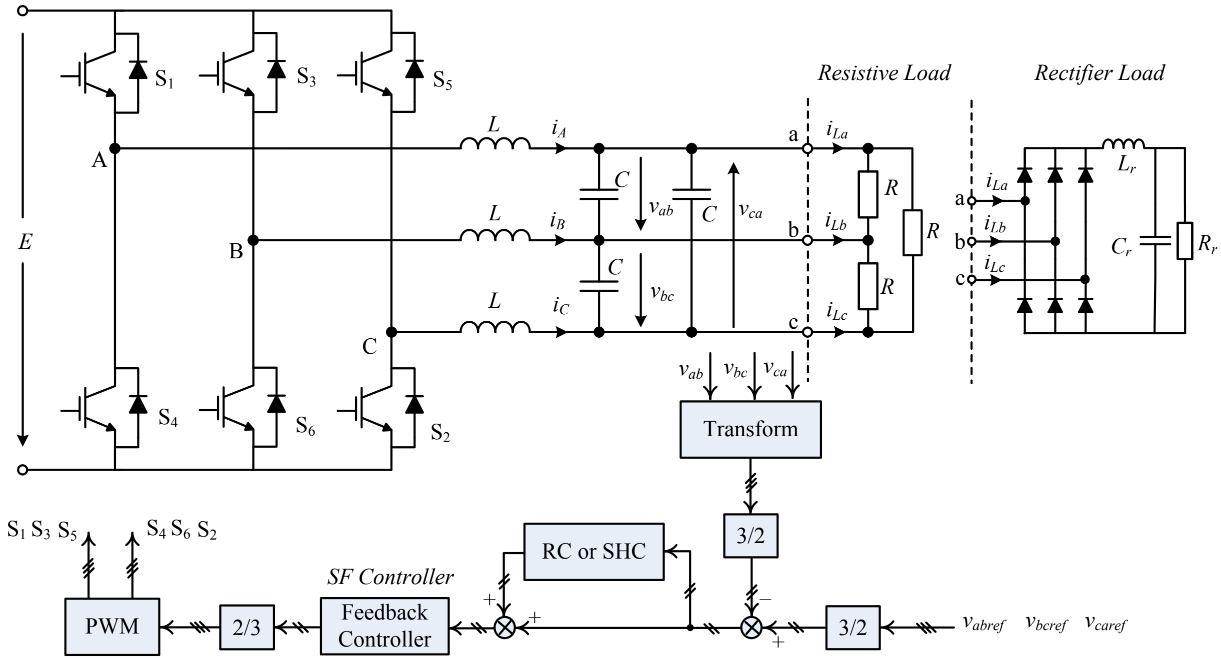


Fig. 4. Repetitive and/or selective harmonic controlled three-phase inverter system.

frequency disturbances (i.e., $|Q(e^{j\omega})|\rightarrow 1$ at low frequencies and $|Q(e^{j\omega})|\rightarrow 0$ at high frequencies, e.g. $Q(z)=\alpha_1 z+\alpha_0+\alpha_1 z^{-1}$ with $2\alpha_1+\alpha_0=1$, $\alpha_0\geq 0$ and $\alpha_1\geq 0$) [14]-[19]. In addition, the filters $G_f(z)$ and $Q(z)$ are also designed to ensure little phase-shift of the harmonics.

The overall system with $Q(z)=1$ in Fig. 3 is asymptotically stable if the following two conditions hold [12], [13]:

- $H(z)$ is asymptotically stable;
- Control gains k_m (≥ 0) satisfy the following inequality

$$0<\sum_{m\in N_m} k_m <2 \qquad (10)$$

Obviously, the above stability criteria for the SHC system can be derived from that for the parallel structure RC system [16]-[18], and is compatible to those for other RC systems [10], [13], [14]. For the larger cluster of harmonic frequencies, larger control gains will be assigned to the corresponding SHC modules. The SHC offers power converters an optimal IMP-based harmonic control solution in terms of high accuracy, fast transient response, cost-effective and easy-implementation. It is compatible with design rules-of-thumb.

IV. Application Case : Three-Phase PWM Inverter

Considering a dSPACE1104 based three-phase PWM inverter experiment system as shown in Fig. 4, where E is the DC-link voltage; v_{ab}, v_{bc}, and v_{ca} are output line-to-line voltages, v_{abref}, v_{bcref}, v_{caref} are the reference output line-to-line voltages; i_A, i_B, and i_C are inductor currents; i_{La}, i_{Lb}, and i_{Lc} are load currents. The control objective of this PWM inverter is to force the voltages v_{ab}, v_{bc}, and v_{ca} to exactly track their reference v_{abref}, v_{bcref}, v_{caref} under various loads. The system parameters are listed in Table I.

TABLE I.
Parameters of the System in Fig. 4.

Nominal value	Actual value	Rectifier load
E_n=500 V L_n=3 mH C_n=50 μF R_n=50 Ω	E= 400 V L= 5 mH C=100 μF R= 50 Ω	L_r=5 mH C_r=1100 μF R_r=60 Ω
Reference output voltages		**Others**
v_{abref}=270sin(100πt) V v_{bcref}=270sin(100πt-2π/3) V v_{caref}=270sin(100πt+2π/3) V		f_o=50 Hz f_s=6 kHz

In the discrete-time domain, the state-space equation for the three-phase inverter system with nominal parameter values can be given by two identical independent single-phase systems as follows [15]-[17],

$$\begin{cases}\begin{pmatrix}x_1(k+1)\\ x_2(k+1)\end{pmatrix}=\begin{pmatrix}0.963 & 0.548\\ -0.033 & 0.991\end{pmatrix}\begin{pmatrix}x_1(k)\\ x_2(k)\end{pmatrix}+\begin{pmatrix}3.70\\ 13.3\end{pmatrix}u(k)\\ y(k)=x_1(k)\end{cases}$$

where $x_1=v_\alpha$ or v_β, $x_2=i_\alpha$ or i_β, $u=\Delta T_\alpha/T_s$ or $\Delta T_\beta/T_s$ are values in the two-phase static (α, β) coordinate system.

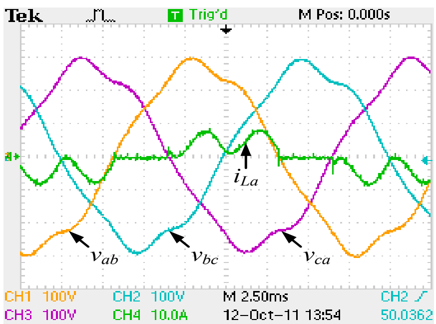
A State Feedback (SF) controller is selected rules of thumb as follows,

$$u(k)=2.7y_{ref}(k)-0.78y(k)-0.89x_2(k).$$

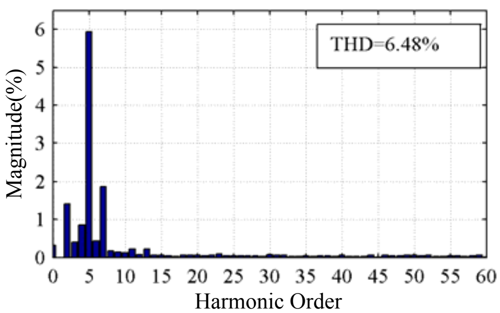
Fig. 5 shows the steady state response of the SF controlled inverter with the rectifier load. From Fig. 5(b), the (6k±1)-order harmonic frequencies occupy about 60 % of the THD, (6k±2)-order harmonic frequencies occupy about 20 % of the

THD. Since the RC control gain is designed as k_{rc}= 0.6, the SHC control gains are consequently designed as k_1= 0.36, k_2= 0.12, k_0=k_3= 0.06 for comparison according to (5) and (6) in order to achieve an accurate but fast control performance.

Fig. 6 and Fig. 7 show the steady state response with the SF plus RC or SHC controller. Fig. 8 shows the dynamic tracking error with RC or SHC controller being plugged into the SF controlled inverter with rectifier load at t =1.5 s respectively. It can be clearly seen that both the RC controller and the proposed SHC controller can achieve good tracking of fundamental voltages and also yield very low THD. Moreover, the convergence rate of the SHC is about three times faster than that of the RC.



(a)

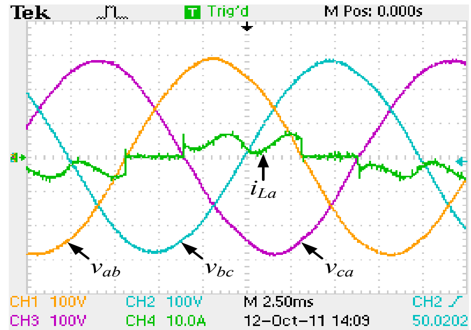


(b)

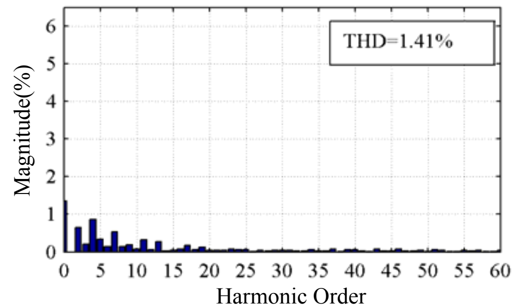
Fig. 5. Steady-state responses with only state-feedback controller: (a) output voltage and load current and (b) harmonic spectrum of tracking error voltage.

V. CONCLUSIONS

An IMP-based SHC method has been proposed to provide a tailor-made optimal control solution to the tracking or elimination of selective harmonic frequencies for power converters. The hybrid structure enables it to achieve a high accuracy due to the removal of most of harmonics, a fast transient response due to parallel combination of optimally weighted SHC modules, cost-effective and easy real-time implementation due to the universal recursive SHC modules, and compatible design rules-of-thumb. The analysis and synthesis of the optimal selective harmonic control system have been addressed. It also provides a universal framework for housing various RC schemes. The application example of a three-phase PWM converter have demonstrated the advantages of the proposed SHC scheme.

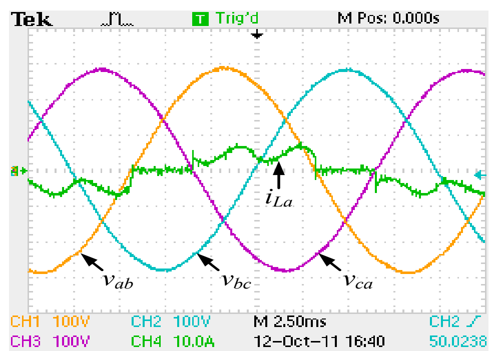


(a)

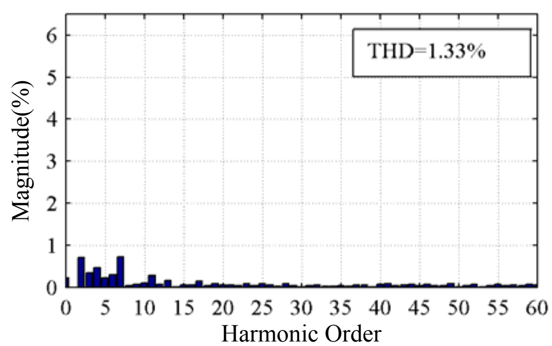


(b)

Fig. 6. Steady-state responses with the state-feedback control plus repetitive control: (a) output voltage and load current and (b) harmonic spectrum of tracking error voltage.

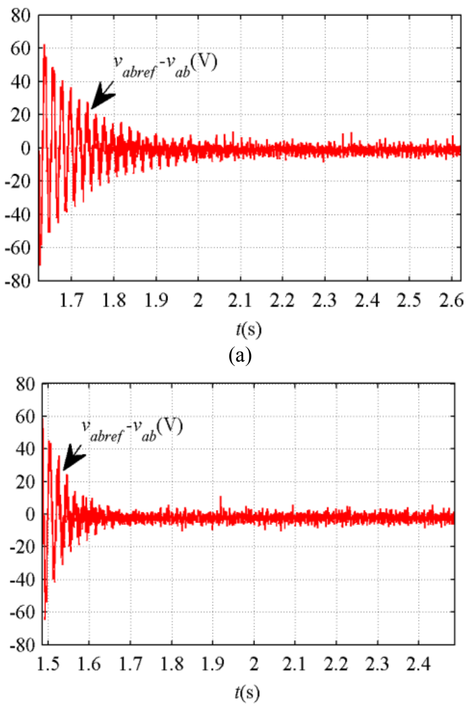


(a)



(b)

Fig. 7. Steady-state responses with the state-feedback control plus the proposed selective harmonic control: (a) output voltage and load current and (b) harmonic spectrum of tracking error voltage.



(b)

Fig. 8. Dynamic tracking errors with RC and SHC being plugged into the SF controlled system: (a) state-feedback control plus the classic repetitive control and (b) state-feedback control plus the proposed selective harmonic control.

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