

## Pitch Estimation and Tracking with Harmonic Emphasis On The Acoustic Spectrum

Karimian-Azari, Sam; Mohammadiha, Nasser; Jensen, Jesper Rindom; Christensen, Mads Græsbøll

*Published in:*

I E E E International Conference on Acoustics, Speech and Signal Processing. Proceedings

*DOI (link to publication from Publisher):*

[10.1109/ICASSP.2015.7178788](https://doi.org/10.1109/ICASSP.2015.7178788)

*Publication date:*

2015

*Document Version*

Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Karimian-Azari, S., Mohammadiha, N., Jensen, J. R., & Christensen, M. G. (2015). Pitch Estimation and Tracking with Harmonic Emphasis On The Acoustic Spectrum. *I E E E International Conference on Acoustics, Speech and Signal Processing. Proceedings*, 4330-4334. <https://doi.org/10.1109/ICASSP.2015.7178788>

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

### Take down policy

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.

# Pitch Estimation and Tracking with Harmonic Emphasis on the Acoustic Spectrum

April 23, 2015

Sam Karimian-Azari<sup>1</sup>, Nasser Mohammadiha<sup>2</sup>,  
Jesper R. Jensen<sup>1</sup>, and Mads G. Christensen<sup>1</sup>

ska@create.aau.dk

<sup>1</sup>Audio Analysis Lab, AD:MT, Aalborg University

<sup>2</sup>University of Oldenburg



AALBORG UNIVERSITY  
DENMARK



# Agenda



- ▶ Introduction
  - ▶ Noisy Harmonic Signal Approximation
  - ▶ ML Pitch Estimate from UFE
- ▶ Bayesian Methods
  - ▶ Motivation
  - ▶ HMM
  - ▶ Kalman Filter
- ▶ Numerical Results
- ▶ Conclusion

Pitch Estimation and  
Tracking

Sam Karimian-Azari  
et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space:  
HMM

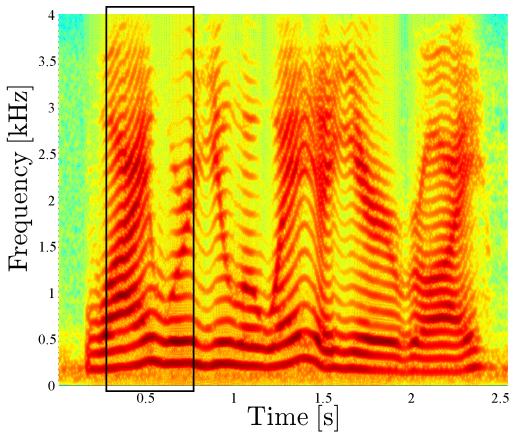
2- Continuous state-space:  
Kalman Filter

Numerical Results

Conclusion

# Introduction

## Harmonic Signal Model



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

2

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space:  
HMM

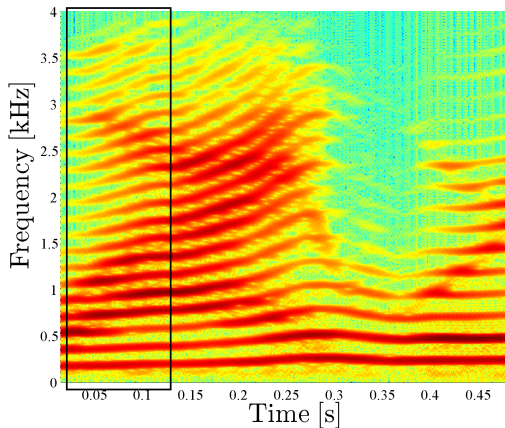
2- Continuous state-space:  
Kalman Filter

Numerical Results

Conclusion

# Introduction

## Harmonic Signal Model



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

3

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space:  
HMM

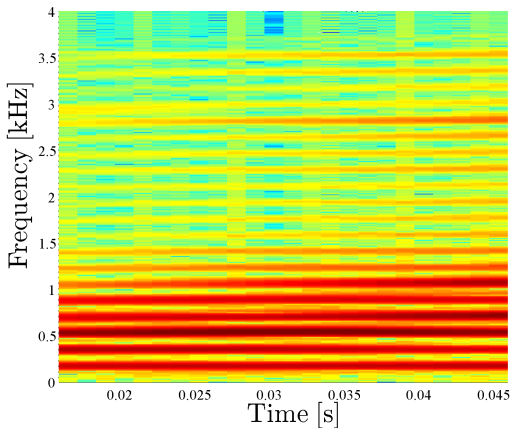
2- Continuous state-space:  
Kalman Filter

Numerical Results

Conclusion

# Introduction

## Harmonic Signal Model



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

4

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space:  
HMM

2- Continuous state-space:  
Kalman Filter

Numerical Results

Conclusion

### Harmonic Signal Model:

$$s(n) = \sum_{l=1}^{L(n)} \alpha_l e^{j(\omega_l(n)n + \varphi_l)},$$

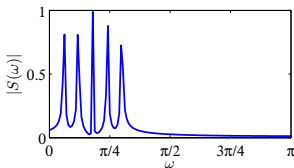
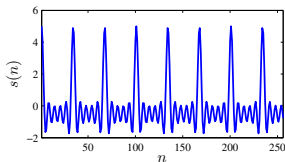
where  $\omega_l(n) = l\omega_0(n)$  for  $l = 1, \dots, L(n)$ ,

$L(n)$  : number of sinusoids

$\alpha_l$  : real magnitudes

$\omega_0$  : fundamental frequency

$\varphi_l$  : phases of harmonics



(1)

5

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

# Signal Model

Additive noise



The observed signal can be written as a sum of a desired signal  $s(n)$  and a noise signal  $v(n)$ , i.e.,

$$\begin{aligned} x(n) &= s(n) + v(n) \\ &= \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n). \end{aligned} \tag{2}$$

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

6

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion



At a high narrowband SNR, the harmonic frequency  $\omega_l$  is perturbed with a real-valued phase-noise [S.Tretter 1985], which has a normal distribution with zero mean and the variance

$$E\{\Delta\omega_l^2(n)\} = \frac{\sigma^2}{2\alpha_l^2} \quad (3)$$

We can approximate  $x(n) = \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \varphi_l)} + v(n)$  like

$$x(n) \approx \sum_{l=1}^L \alpha_l e^{j(\omega_l n + \Delta\omega_l(n) + \varphi_l)} \quad (4)$$

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

7

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

# Signal Model

## Unconstrained frequency estimates (UFE)



Unconstrained frequency estimates (UFE) of the constrained frequencies:

$$\hat{\Omega}(n) = [\hat{\omega}_1(n), \hat{\omega}_2(n), \dots, \hat{\omega}_L(n)]^T \quad (5)$$

$$= \mathbf{d}_L(n) \omega_0(n) + \Delta\Omega(n), \quad (6)$$

where

$$\mathbf{d}_L(n) = [1, 2, \dots, L(n)]^T \quad (7)$$

$$\Delta\Omega(n) = [\Delta\omega_1(n), \Delta\omega_2(n), \dots, \Delta\omega_L(n)]^T, \quad (8)$$

and

$$\mathbf{R}_{\Delta\Omega}(n) = E\{\Delta\Omega(n)\Delta\Omega^T(n)\} \quad (9)$$

$$= \frac{\sigma^2}{2} \text{diag}\left\{\frac{1}{\alpha_1^2}, \frac{1}{\alpha_2^2}, \dots, \frac{1}{\alpha_L^2}\right\}.$$

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

8

19

# Max. Likelihood (ML) Pitch Estimator



For the time-frame  $\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-M-1)]^T$ , the PDF of the UFE is

$$P(\hat{\Omega}(n)|\omega_0(n)) \sim \mathcal{N}(\mathbf{d}_L(n)\omega_0(n), \mathbf{R}_{\Delta\Omega}(n)). \quad (10)$$

The ML pitch estimator:

$$\hat{\omega}_0(n) = \arg \max_{\omega_0(n)} \log P(\hat{\Omega}(n)|\omega_0(n)) \quad (11)$$

$$= [\mathbf{d}_L^T(n)\mathbf{R}_{\Delta\Omega}^{-1}(n)\mathbf{d}_L(n)]^{-1} \mathbf{d}_L^T(n)\mathbf{R}_{\Delta\Omega}^{-1}(n)\hat{\Omega}(n) \quad (12)$$

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

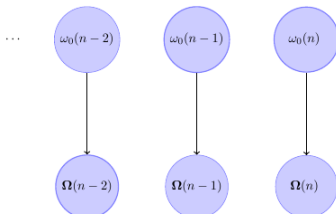
9

19

# Bayesian Pitch Estimator

## Motivation

- ▶ The ML Estimators are statistically efficient, e.g., the non-linear least-squares (NLS), and the weighted least squares (WLS) [H.Li, et al. 2000], but the minimum variance is limited by the number of samples.
- ▶ Consecutive pitch values are estimated independently



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

10

19

# Bayesian Pitch Estimator

## Motivation



- Pitch values are usually correlated in a sequence, i.e.,

$$P(\omega_0(n) | \omega_0(n-1), \omega_0(n-2), \dots), \quad (13)$$

that motivate Bayesian methods to minimize an error incorporating prior distributions.

- State-of-the-art methods mostly track pitch estimates in a sequential process without concerning noise statistics.



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

11 Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

# Bayesian Pitch Estimator

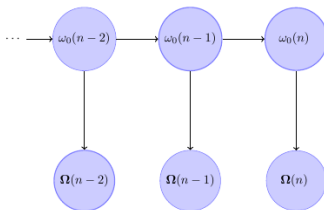
## Hypothesis



- 1- Jointly estimate and track pitch incorporating both the harmonic constraints and noise characteristics.
- 2- Estimate the state  $\omega_0(n)$  through a series of noisy observations:

$$P(\omega_0(n) | \hat{\Omega}(n), \hat{\Omega}(n-1), \dots) \quad (14)$$

- 3- Recursively update the prior distribution of the pitch value.



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

12

Motivation-Hypothesis

1- Discrete state-space:

HMM

2- Continuous state-space:

Kalman Filter

Numerical Results

Conclusion

# Bayesian Pitch Estimator

Discrete state-space (HMM)



Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

13

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

$\omega_0(n)$  : Discrete random variable (Hidden states)

$P(\omega_0(n)|\omega_0(n-1))$  : Transition probability in a 1st-order Markov model,

$$\text{i.e., } \sum_{\omega_0(n)} P(\omega_0(n)|\omega_0(n-1)) = 1$$

$$\hat{\omega}_0(n) = \arg \max_{\omega_0(n)} \log P(\omega_0(n)|\hat{\Omega}(n), \hat{\Omega}(n-1), \dots) \quad (15)$$

$$= \arg \max_{\omega_0(n)} \log P(\hat{\Omega}(n)|\omega_0(n)) + \log P(\omega_0(n)|\hat{\Omega}(n-1), \dots).$$

The priori distribution is defined recursively like

$$P(\omega_0(n)|\hat{\Omega}(n-1), \hat{\Omega}(n-2), \dots) = \sum_{\omega_0(n-1)} P(\omega_0(n)|\omega_0(n-1)) P(\omega_0(n-1)|\hat{\Omega}(n-1), \dots), \quad (16)$$

where  $P(\omega_0(n-1)|\hat{\Omega}(n-1), \dots)$  is the past estimate.

19

# Bayesian Pitch Estimator

state-space representation of the pitch continuity



Continuous state-space:

$$\omega_0(n) = \omega_0(n-1) + \delta(n)$$

$$\hat{\Omega}(n) = \mathbf{d}_L(n) \omega_0(n) + \Delta\Omega(n),$$

where  $\delta(n) \sim \mathcal{N}(0, \sigma_t^2)$  and  $\Delta\Omega(n) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\Delta\Omega}(n))$  are the state evolution and observation noise, respectively.

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

14

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion



# Bayesian Pitch Estimator

Continuous state-space (Kalman filter)



**First**, a pitch estimate is predicted using the past estimates as

$$\hat{\omega}_0(n|n-1) = \hat{\omega}_0(n-1|n-1) \quad (17)$$

with the variance

$$\sigma_K^2(n|n-1) = \sigma_K^2(n-1|n-1) + \sigma_t^2. \quad (18)$$

**Second**, the pitch estimate is updated with the error of

$$\mathbf{e}(n) = \hat{\Omega}(n) - \mathbf{d}_L(n) \hat{\omega}_0(n|n-1). \quad (19)$$

**Then**, the predicted estimate is updated:

$$\hat{\omega}_0(n|n) = \hat{\omega}_0(n|n-1) + \mathbf{h}_K(n) \mathbf{e}(n) \quad (20)$$

$$\mathbf{h}_K(n) = \sigma_K^2(n|n-1) \mathbf{d}_L^T(n) \left[ \mathbf{\Pi}_L(n) \sigma_K^2(n|n-1) + \mathbf{R}_{\Delta\Omega}(n) \right]^{-1}, \quad (21)$$

where  $\mathbf{\Pi}_L(n) = \mathbf{d}_L(n) \mathbf{d}_L^T(n)$ , and update

$$\sigma_K^2(n|n) = \left[ 1 - \mathbf{h}_K(n) \mathbf{d}_L(n) \right] \sigma_K^2(n|n-1). \quad (22)$$

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

Conclusion

15

The ML estimator of the covariance matrix among  $N$  estimates:

$$\begin{aligned}\mathbf{R}_{\Delta\Omega}(n) &= \mathbf{E}\{\Delta\Omega(n)\Delta\Omega^T(n)\} \\ &= \frac{1}{N} \sum_{i=n-N+1}^n \Delta\Omega(i)\Delta\Omega^T(i),\end{aligned}\quad (23)$$

where  $\Delta\Omega(n) = \hat{\Omega}(n) - \hat{\mu}(n)$ , and  $\mu(n) = \mathbf{E}\{\hat{\Omega}(n)\}$ .

Exponential moving average:

$$\hat{\mu}(n) = \lambda \hat{\Omega}(n) + (1 - \lambda) \hat{\mu}(n-1) \quad (24)$$

The forgetting factor  $0 < \lambda < 1$  recursively updates the time-varying mean value.

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

16

2- Continuous state-space: Kalman Filter

Numerical Results

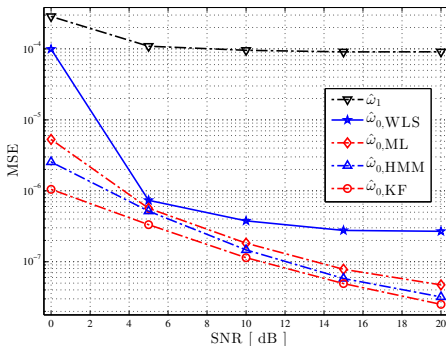
Conclusion

# Numerical Results

## Synthetic signal



A linear chirp signal ( $r = 100$  Hz/s) with  $L = 5$  harmonics, random phases, and identical amplitudes during 0.1 s.



$M = 80$ ,  $\omega_0(1) = 400\pi/f_s$ ,  $f_s = 8.0$  kHz,  $\sigma_t = \sqrt{2\pi r/f_s^2}$ , and for the HMM-based pitch estimator, the frequency range  $\omega \in [150, 280] \times (2\pi/f_s)$  was discretized into  $N_d = 1000$  samples.

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

17 Numerical Results

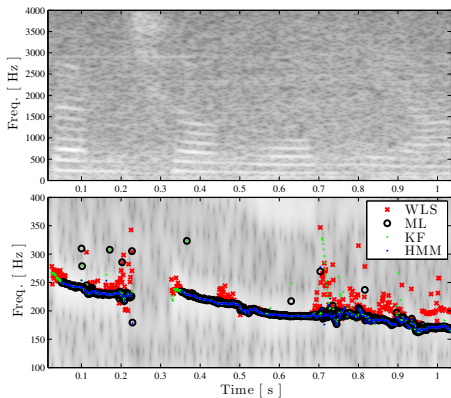
Conclusion

# Numerical Results

Real signal



Speech signal + Car noise at SNR= 5 dB.



The MAP order estimation [Djuric 1998],  $M = 240$ ,  $\lambda = 0.9$ , and  $N = 150$ .

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space:  
HMM

2- Continuous state-space:  
Kalman Filter

18 Numerical Results

Conclusion

19

Audio Analysis Lab, AD:MT,  
Aalborg University, Denmark

# Conclusion



- ▶ For pitch estimation, we have formulated the ML estimate from the UFE.
- ▶ For pitch estimation and tracking, we have proposed HMM- and KF-based methods.
- ▶ Experimental results showed that both HMM- and KF-based methods outperform the corresponding ML pitch estimators.
- ▶ The KF-based method statistically performs better than the HMM-based method, while it tracks pitch changes more accurately than the KF-based method.

Pitch Estimation and Tracking

Sam Karimian-Azari et al.

Introduction

Harmonic Signal Model

Noisy Signal Approx.

ML Pitch Estimation

Proposed Method

Motivation-Hypothesis

1- Discrete state-space: HMM

2- Continuous state-space: Kalman Filter

Numerical Results

19 Conclusion

Thank you!



**AALBORG UNIVERSITY**  
DENMARK

