Intelligent Energy Management System for Virtual Power Plants

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Intelligent Energy Management System for Virtual Power Plants

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Abstract

Wind power is the fastest growing renewable energy source with the highest share of power production in the Danish power system. Increasing wind power production also causes new challenges for the power system.

One possibility to enable higher shares of wind power in the system is to build virtual power plants (VPPs). In this work, VPPs refer to wind power plants (WPPs) connected to an electrical battery energy storage system (BESS) which is in close proximity to the WPP, and both plants are able to participate in the Danish power market (ancillary service markets and day-ahead market). BESSs demonstrated to be suitable storage technologies that have been integrated in power systems worldwide in recent years. Such storage systems are underlying a fast development track and have improved over the past decades considerably. This makes an increase in the number of VPPs more likely in future.

Potential investors in VPPs face several questions before and after an investment decision for a specific BESS is made. This work addresses the following questions:

1. Is a VPP a profitable investment and if so, which technology or combination of different technologies of BESSs and which size should be purchased?

2. Once the BESS is purchased and grid connected: When should the VPP submit bids to which power market and in what quantity?

3. Once the awards on the power market are announced and the latest wind power production forecast is available: How should the VPP be operated in order to face minimum penalty payments for imbalances due to, e.g. changes in wind power production?

This thesis proposes a deterministic mixed-integer-linear-programming (MILP) formulation to address the above stated optimization problems. One generic (MILP) problem is formulated that can be used to address each of the above
stated questions depending on the input parameters provided to the model. The model focuses on the BESS including capacity fade which is a battery specific property. It determines the performance, live-time, and – most important – the annualized costs of the BESS. Modeling capacity fade opens up the possibility to take into account BESS’s annualized costs based on a function of its state-of-health (SoH).

The proposed MILP formulation is verified based on three case studies following the problem formulation. There is one case study for each question described above.
Vindkraft er den vedvarende energikilde, der producerer mest energi til det danske elektricitetsmarked og er samtidig den hurtigst voksende energikilde. Der er dog udfordringer forbundet med denne vækst.

En mulighed for at lade vindkraften udgøre en endnu større andel af elproduktionen, er at opføre virtuelle kraftværker (virtual power plants, VPP). I denne afhandling samarbejder VPP’s med vindkraftværker, altså WPP’s (Wind power plants), som er forbundet med et elektrisk batteri-lagringssystem (BESS, battery energy storage system), i umiddelbar nærhed af WPP. Både vindkraftværket og batteriet kan sende strøm på elmarkedet.

Gennem de seneste år har BESS vist sig at være en velegnet lagringsteknologi, og er derfor blevet integreret i elsystemer over hele verden. Batterilagringssystemet gennemgår en hurtig udvikling og er forbedret gennem de seneste årtier, hvilket sandsynliggør et øget antal VPP’s i fremtiden.

Før potentielle investorer i en VPP overhovedet overvejer at, og reelt kan, investere i en BESS, oplever de dog adskillige udfordringer. Disse forskellige udfordringer er opstillet som problemformulering for denne afhandling:

1. Er en VPP en profitabel investering? Og i så fald, hvilken størrelse, type eller kombinationer af forskellige typer BESS skal der investeres i?

2. Hvilken mængde strøm og i hvilke tidsrum skal VPP’en sende strøm til elmarkedet, når et BESS samtidig er forbundet til elnettet. Og til hvilket elektricitetsmarked skal denne strøm sendes?

3. Når elprisen er kendt og den seneste forudsigelse af vindkraftproduktionen er kendt, hvordan skal VPP’en så drives for at opnå minimale bøder for ubalancer, såsom ændringer i vindkraftproduktionen, som opstår efter den seneste justering?

Den foreslåede MILP-formel er verificeret på baggrund af tre case studies. Der er et case study til hvert punkt i problemformuleringen.
Acknowledgments

This PhD project was full of challenges, it was a journey which strengthened my academic and personal skills. I would like to express thankfulness to all the people who contributed to this thesis and supported me during this exciting period.

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Philipp Hubert Johannes Braun
Fredericia, Denmark,
April 2015
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## Abbreviations

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<tr>
<td>A</td>
<td>Ampere</td>
</tr>
<tr>
<td>Ah</td>
<td>Ampere hour(s)</td>
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<tr>
<td>BESS</td>
<td>Battery energy storage system (including power electronics and other devices in order to safely operate batteries)</td>
</tr>
<tr>
<td>CAPEX</td>
<td>Capital expenditures</td>
</tr>
<tr>
<td>DA</td>
<td>Day-ahead power market, also called electricity spot market</td>
</tr>
<tr>
<td>DoD</td>
<td>Depth-of-discharge</td>
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<tr>
<td>Fig.</td>
<td>Figure</td>
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<tr>
<td>ES</td>
<td>Energy storage</td>
</tr>
<tr>
<td>LP</td>
<td>Linear programming</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed integer linear programming</td>
</tr>
<tr>
<td>MWh</td>
<td>Megawatt hour(s)</td>
</tr>
<tr>
<td>NaS</td>
<td>Sodium-sulfur battery</td>
</tr>
<tr>
<td>NiCd</td>
<td>Nickel cadmium battery</td>
</tr>
<tr>
<td>OTC</td>
<td>Over-the-counter</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
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<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>OPEX</td>
<td>Operation expenditures</td>
</tr>
<tr>
<td>PLA</td>
<td>Piecewise linear approximation</td>
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<tr>
<td>PFR</td>
<td>Primary frequency regulation</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
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<tr>
<td>RP</td>
<td>Regulating power</td>
</tr>
<tr>
<td>SoC</td>
<td>State of charge</td>
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<tr>
<td>SoH</td>
<td>State of health</td>
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<tr>
<td>TSO</td>
<td>Transmission system operator</td>
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<tr>
<td>V</td>
<td>Volt</td>
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<tr>
<td>VPP</td>
<td>Virtual power plant</td>
</tr>
<tr>
<td>VRB</td>
<td>Vanadium redox flow battery</td>
</tr>
<tr>
<td>VRLA</td>
<td>Valve regulated lead-acid battery</td>
</tr>
<tr>
<td>W</td>
<td>Watt</td>
</tr>
<tr>
<td>Wh</td>
<td>Watt hour(s)</td>
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<tr>
<td>WPP</td>
<td>Wind power plant</td>
</tr>
<tr>
<td>Zebra</td>
<td>Sodium nickel chloride battery</td>
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<tr>
<td>ZnBr</td>
<td>Zinc bromine flow battery</td>
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1. Introduction

The first chapter discusses the background of this PhD project and explains the VPP concept. The research scope and project objectives of this PhD thesis are explained.

1.1. Background and motivation

Many countries, in particular in the EU, have shown a strong growth of renewable energies in the past years. Especially Denmark is an example, where wind power production has reached about 33% of the total electricity demand in 2013 while ten years earlier the share was just about 16% (see Fig. 1.1) [1]. Globally, wind power generation accounted for just 2.6% out of the total electricity consumption in 2013 (projected number) [2].

Renewable energy systems have an intermittent power production profile and a forecast of such a production profile has a certain error. The higher the share of renewable power generation in the grid, the higher the impact of any forecast error for renewable generation. One solution to reduce this impact is to build virtual power plants (VPPs). These are a combination of different generation plants usually based on a mix of renewable energy systems and conventional power plants or energy storage (ES), aggregated into a single-acting unit [3]. The combination of conventional generation or ES with renewable energy systems allows the VPP to increase its predictability. Increased predictability would enable even higher shares of renewable energy sources in the power system. This can pave the way for further growth of wind power and other renewable power generation sources in the grid.

The VPP consisting of wind power plants (WPPs) connected to a battery energy storage system (BESS) is chosen in this work because wind power has the highest
Figure 1.1.: Wind power production out of total electricity demand in Denmark (data from [1])

The share of renewable energy generation worldwide (except for hydro-power). In 2011, about 48% of electricity was generated with wind power out of all renewable electricity generation (excluding hydro-power) worldwide [4].

Besides the big share of WPP amongst the renewable power generation, BESS are chosen to be part of VPPs in this work because BESSs underlie a vast technological development and undergo improvements in battery life, cost, safety, and storage capabilities including the development of new battery technologies [5, 6]. The driver behind the development of better BESS, especially li-ion batteries, is coming rather from the automotive and other sectors than from the energy sector where research is done on improved BESSs for electric vehicles or other devices using BESSs [6]. The energy sector might take profit of this development by incorporation of such storage technologies into the power system.

The question is, if BESSs as part of VPPs would be profitable and what the optimum components of such a system would be. And once an investment decision has been made, the question of optimum operation of the VPP arises. This demands for the development of a high-level optimization model for the VPP because BESSs are still costly investments and once an investment is made, the
entire VPP should be operated in an optimum manner to ensure highest possible profits.

These questions are further elaborated in sec. 1.3 below but first the concept of VPPs is discussed in more detail in the next section.

1.2. Virtual power plant concept

The concept of VPPs considered within this thesis is outlined in Fig. 1.2. The focus lies on VPPs consisting of WPPs and BESS(s). The VPP depicted in Fig. 1.2 should indicate that the plant consists of two types of battery ES technologies (not just two batteries) in order to stress that the VPP can consist of any number of different BESS technologies. A combination of different BESS technologies is called hybrid BESS in this work which can be more profitable than a VPP based on a single BESS technology. Further, the BESSs and WPPs are grid connected and power can be sold on the day-ahead (DA) market or on the ancillary service markets. A VPP has many degrees of freedom on how to operate it over a certain time period and the optimum operation might not be obvious. Therefore, a high level control algorithm (indicated with optimization model in Fig. 1.2 ) has to be developed that operates the entire VPP in an optimum manner. The solution of the optimization model is information about the amount of power to bid into each power market as well as set points send to the low-level controllers in each unit of the VPP (indicated by the gray arrows in Fig. 1.2 pointing from the optimization model to the BESSs and WPPs). On the other hand, the optimization model receives plant information like state of charge (SoC) of the BESS or wind power forecasts.

The low-level controller takes care of all dynamics in each unit while the high level controller (optimization model) just considers the main plant dynamics that are relevant for the chosen discretization step. The low level controllers are not separately shown in Fig. 1.2 and it is assumed that they are part of the depicted WPP and BESSs. Such low level controllers would be the battery management system or the wind farm controller. They have to ensure safe plant operations and could finally shut down the plant if an error is detected which the high-level controller (optimization model) is not designed for. Moreover, it is assumed that
the VPP forms its own balancing responsible party.

\[\text{Market prices} \rightarrow \text{Optimization model} \rightarrow \text{Energy Storage} \rightarrow \text{Wind power} \rightarrow \text{Optimization model} \rightarrow \text{Market prices}\]

**Figure 1.2:** Virtual power plant (VPP) concept

### 1.3. Research scope

The scope of this thesis is to determine an:

"Intelligent energy management system for virtual power plants."

The energy management system can be considered as the high-level optimization model discussed in sec. 1.2 above. The following objectives for this PhD project have been identified:

- Optimum scheduling of the VPP
- Optimum dispatch of the VPP
- Optimum sizing of the BESS

Optimum scheduling should find the optimum operation of the VPP based on forecasts of market prices and wind power production. The optimum scheduling algorithm would typically be run before the first deadline for submission of bids to one of the power markets where the VPP participates in. The result of the optimum scheduling algorithm is used to determine the size of power bids into each power market.

The optimum dispatch algorithm is called after bids are awarded and market
prices are revealed as well as a new wind power forecast might be available in order to find the optimum set points for the VPP in each discretization period. The optimum dispatch problem can be considered as a rescheduling problem. As shown in sec. 4.2 on page 48, the optimum rescheduling problem has been previously and successfully addressed in literature for one power market. In this work, the focus is not on a new problem formulation for rescheduling but shows that this concept can also be applied to various power markets in order to reschedule VPPs.

The final objective, optimum sizing of the BESS, is used to support investment decisions in order to find the optimum size and BESS technology (or combination of BESS technologies) for a VPP in a specific location.

The goal of all three objectives is to maximize the profit of the VPP.

The following list describes important properties that should be incorporated in all three objectives:

- The VPP is able to participate in various power markets under a specific market regime.
- The model of the BESS includes aging of the batteries and other relevant battery specific parameters.
- The costs of the BESS are calculated based on its usage which influences the lifetime.
- Any number of different BESS technologies and size and any number of WPPs are able to be addressed by the optimization problem formulation. Especially hybrid BESSs based on a combination of different battery technologies should be able to be handled.

In summary, the research scope of this work is to determine a suitable optimization technique and to develop an optimization problem formulation in order to create an intelligent energy management system for VPPs.

Having discussed the concept of VPPs and the scope of this work, the next chapter discusses BESSs and their applications in power systems. It covers a pre-selection of BESSs and identifies the most important applications under the Danish market rules. Assumptions and limitations of this work are presented in the end of the next chapter in sec. 2.10.
2. Battery energy storage technologies and applications

There are many kinds of different electrical ES technologies and besides electrical ES technologies, there are also thermal ESs which are not considered in this work. Fig. 2.1 provides an overview of various important available electrical ES technologies. The overview is not comprehensive as there exist many different technologies which are of minor relevance. Electrical ES can be divided into mechanical, electromagnetic, and electro-chemical ES. The focus of this work is on batteries which are part of electro-chemical ES. Battery ES are chosen for two reasons:

1. Battery ES technologies are portable and can be set up in close proximity to WPPs.

2. Battery ES undergo a fast development and have improved in recent years (see sec. 1.1).

Having set the focus on battery ESs, this chapter provides an overview of important battery ES technologies and how they can be applied in power systems. Technical specifications of battery technologies are presented, as well as aging mechanisms and applications for battery ES are discussed. Finally, Denmark is picked as an example and applications are identified that are remunerated under the Danish power market regulations. This chapter provides the bases in order to develop an appropriate optimization algorithm in chapter 5.
2.1. Battery ES technologies

Different battery ES technologies have different properties concerning lifetime, efficiency, relation between power rating and battery energy, energy density, costs, safety, response time, etc.. This section and the following subsections describe the functional principle of batteries and give insights into important battery ES technologies.

All battery cells consist of two electrodes, the anode and the cathode which are apart of each other, and the medium between the electrodes is called electrolyte (see Fig. 2.2). The electrolyte has to be able to conduct ions but has to be an insulator prohibiting the flow of electrons. Otherwise, there would be a short-circuit in the battery cell. The difference in chemical potential lets then flow electrons through the terminals of the cell. This is called discharging where electrons pass through an electrical load originating from the negative electrode. Cells that can only be discharged are called primary cells while cells that can also be charged are called secondary cells. While charging a secondary battery, the chemical reaction of discharging is reversed (see Fig. 2.2 right hand side). In this work, only secondary batteries are considered. In general, different battery technologies can be distinguished by the material of the electrodes and the electrolyte. This paragraph is based on [8].
2.1 Battery ES technologies

The following list specifies important parameters that characterize the performance or state of battery ES. All parameters defined within this section are listed in the nomenclature at the end of this chapter on page 31 not to interfere with parameters defined for the problem formulation in chapter 9.3 on page 145.

- **Battery energy**: Is the amount of energy stored in the battery in Wh. It is determined by integrating the instantaneous value of the current (A) times the instantaneous value of the voltage (V) in respect of time during charging or discharging of the battery [8].

- **Capacity**: Amount of charge in ampere-hours “that can be withdrawn from a fully-charged battery under specified conditions”. [8]

- **C-rate**: This is the rate at which the battery is charged or discharged relative to the rated capacity and the unit is ampere [8]. The following example helps to understand this concept. A battery with a rated capacity of 50Ah is charged with $0.5Cr$ which is a charging current of $0.5 \cdot 50 = 25A$. The battery is fully charged within $1/Cr$ hours which is in this example 2 hours. The efficiency is not considered in the C-rate and it is only related to the nominal capacity.

- **Cycle-life**: “The number of cycles that can be obtained from a battery before it fails to meet selected performance criteria” [8].

![Functional principle of battery ES](image-url)
• Depth-of-discharge (DoD): Is the ratio “of the ampere-hours discharged from a battery at a given rate to the available capacity under the same specified conditions” [8]. The DoD is the complement of the state-of-charge (SoC) with \( DoD = 1 - SoC \).

• Efficiency: Usually refers to the round-trip efficiency \( \eta \) defined as the ratio of energy delivered by the battery when completely discharged from fully charged state and energy needed to completely charge the battery ES again [9].

• Maximum DoD: Maximum value of DoD that the battery cannot exceed. This influences the maximum usable capacity of the battery. [10]

• Power rating: The maximum power capability as specified by the manufacturer [8].

• Power to energy ratio: Batteries, except of flow batteries, have a fixed power to energy ratio which depends on the technology and the specific battery model considered [11]. It means that one cannot double the power rating of a specific battery without increasing the battery energy by two times.

• Self-consumption of BESS: Is defined in this work as the energy consumed by the BESS in order to operate the batteries safely and within specified conditions. Self-consumption results from devices such as chiller units to maintain the temperature of the BESS or the battery management system, for instance.

• Self-discharge: Is the energy dissipation of the battery [9] under open-circuit conditions due to internal chemical reactions or short-circuits. It results in a reduction of the SoC over time but it does not reduce the maximum storable amount of energy in the BESS. Self-discharge depends on battery temperature, technology, SoC, or SoH, for instance [12].

• SoC: Is the remaining capacity of a battery ES in comparison to the available capacity of a fully charged battery ES [13]. The SoC is the complement of the DoD with \( SoC = 1 - DoD \).

• SoH: Is defined in [14] as “the ability of a cell to store energy, source and sink high currents, and retain charge over extended periods, relative to its
2.1 Battery ES technologies

initial or nominal capabilities”. It is expected that the more a battery is
cycled and thus ages, the lower is its capability to store energy.

After the discussion of the basic technical background of batteries, the following
sections provide an overview of important battery technologies.

2.1.1. LiIon

**Principle:** Redox reaction in electrochemical cell. The electrodes are made of
different compositions, e.g. lithiated metal oxide and layered graphitic carbone
while the electrolyte can be lithium salts dissolved in organic carbonates. There
are many sub-types of li-ion battery technologies with different materials in the
electrodes and electrolyte. Every sub-type would need to be treated as its own
battery technology due to varying parameters. [9]

**Pros:** High energy density, low weight, low self-discharge, high cycle-life, high
efficiency, many multi-megawatt utility scale installations [15]. Low maintenance
requirements [16].

**Cons:** Not as mature as lead-acid batteries, high cost [9].

2.1.2. LeadAcid

**Principle:** Redox reaction in electrochemical cell [9]. Electrodes are made of
lead metal and lead oxide, the electrolyte is a sulphuric acid [15].

**Pros:** Most mature battery technology, low cost, high number of installed capac-
ity, availability in different sizes [15].

**Cons:** Low specific power and energy density, short cycle life, toxicity, high main-
tenance requirements, low efficiency, self-discharge, temperature sensitivity [15].

Moreover, lead-acid batteries have a low maximum DoD which means that not
all the energy stored in the battery can be used (at certain currents). The lower
the discharge current, the more energy can be withdrawn but for higher discharge
currents (e.g. $> 0.5Cr$) the maximum DoD is low (or it can be said that the
maximum usable energy in the battery is low) compared to other battery tech-
nologies (see [16] fig 2.3, for example). This relation is called Peukert effect and is
dominant for lead-acid batteries. It also exists for other battery technologies but
shows a smaller effect (see [17] for li-ion batteries, for instance). For this work it is assumed that the minimum SoC of a lead-acid battery is 40%.

Another negative characteristic of lead-acid batteries is the charge- and discharge time which may vary widely. Standard lead-acid batteries are charged with a much smaller current over a longer period than they can be discharged. An example for a standard lead-acid battery data whose suggested usage is in solar- and wind energy applications (amongst others) is found in the data sheet of [18]. The recommended discharging current is about 40 times higher than the recommended charging current which results in a long time to charge the battery. The maximum discharge current for the capacity measured at 1h and 1.75 volt per cell is stated in the range of 10 to 30Cr. Opposed to the exemplarily referenced data sheet, literature shows successful tests of fast charging lead-acid batteries but which may - on the other hand - shorten the lifetime of lead-acid batteries. [19] charged lead-acid batteries with up to 4.6Cr and [20] reported charging currents of up to 1Cr. At this point an assumption has to be made regarding the ratio of charging and discharging rate. It is assumed that a maximum charge rate is 30% of the maximum discharge rate.

2.1.3. Sodium-sulfur battery (NaS)

Principle: Redox reaction in electrochemical cell under high temperature. At the positive electrode is liquid (molten) sulphur and liquid (molten) sodium is at the negative electrode and the two active materials are separated by a solid beta alumina ceramic electrolyte. The battery has to be operated at around 300-350 °C. [9]

Pros: Medium to high efficiency, pulse power capability, medium cycle life, high energy density.

Cons: Few manufacturers [21]. High cost, heat source required to keep operating temperature if battery is not enough cycled causing a lower efficiency, high self-discharge [9].
2.1 Battery ES technologies

2.1.4. Vanadium redox flow battery (VRB)

**Principle:** VRBs belong to the type of flow batteries and the positive and negative electrolytes are separated from each other and stored in two tanks. The two contain the vanadium ions and flow from the tanks to the electrochemical cell. If the battery is discharged, the two electrolytes are the same and thus, if they are mixed, they will not contaminate each other but simply discharge the battery. The cell contains an exchange membrane which keeps the two electrolytes apart when pumped through the cell. [22]

**Pros:** Power rating and battery energy are not coupled. The power rating is determined by the cell while the battery energy is defined by the amount of electrolyte stored in the tanks and the concentration of vanadium ions in the electrolyte. VRBs have long lifetimes mainly limited by membrane and are easy to handle compared to other flow batteries. In addition, over discharge will not damage the battery and the costs for energy applications are favorable. [15] Additionally, this type of battery has a low self-discharge [22].

**Cons:** Large space requirements, low energy density, low efficiency [15, 22].

2.1.5. Zinc bromine battery (ZnBr)

**Principle:** The ZnBr is also a flow battery. Two different electrolytes flow in separated tanks around carbon-plastic composite electrodes. The electrolytes are separated by a non-selective micro-porous membrane. Only the concentration of the dissolved elemental bromine makes the two electrolytes different. The life-limiting factor is the corrosion that takes place when the system operates. Therefore, the lifetime of the system depends rather on the hours operated than on the cycle life. [15]

**Pros:** Battery energy and power rating can be selected apart from each other and are not coupled, high energy density, low cost in terms of battery energy. [15]

**Cons:** Low efficiency, only two major manufacturers, considered less mature than VRBs [9]. Low lifetime of ca- 6000 hours in operation, and the electrolyte is an environmental hazard [15].
2.1.6. Nickel cadmium battery (NiCd)

Principle: Redox reaction in electrochemical cell. The electrodes contain nickel hydroxide and cadmium hydroxide, respectively and the cell has an alkaline electrolyte. [9].
Pros: Mature technology, low maintenance requirements, robust reliability, high energy density. [9].
Cons: Prohibited in EU (with exceptions) [23]. Toxicity, low cycle life, high cost [9].

2.1.7. Sodium nickel chloride battery (Zebra)

Principle: Similar to NAS batteries, Zebra batteries are molten sodium based batteries. The negative electrode is based on molten sodium and the positive electrode contains nickel chloride. The operating temperature is around 270 °C. [15]
Pros: High efficiency, medium to high cycle life, limited over- or under charge can be tolerated and better safety characteristics compared to NaS batteries. [15]
Cons: High self-discharge, low energy- and power density compared to NaS batteries, one manufacturer. [9]

2.2. Pre-selection of battery technologies

In order to limit the scope of the simulations in chapter 6 to chapter 8, the two most promising BESS technologies are selected in this sub-chapter. This selection does not mean that the developed MILP formulation in chapter 5 is limited to only these two technologies. The MILP formulation itself remains to be generic for various kinds of BESS technologies as long as the necessary input data are available. However, to be able to run the optimization reasonable fast on a standard laptop computer, the number of battery technologies need to be limited. If a more powerful computer is available, the optimization can be run for a larger set of battery technologies. For this work, it is sufficient to demonstrate the optimization problem formulation on two different battery technologies.
2.2 Pre-selection of battery technologies

Having described the most important battery energy storage technologies above, Tab. 2.1 provides a summary of their properties which serves as a foundation to select the two most important battery ES technologies for the use in VPPs. It has to be annotated that data in Tab. 2.1 serves as an indication because they can only provide a certain range of properties that exist on the market at a certain point of time. Often, there are many subcategories of one ES available and the exact specification might differ between manufacturers and products even though they belong to the same technology category. This means that users of the proposed algorithm need to find the exact data of their battery model before running the algorithm.
<table>
<thead>
<tr>
<th>Technology</th>
<th>Energy density</th>
<th>Power density</th>
<th>Round-trip efficiency</th>
<th>Cycle life</th>
<th>Self-discharge</th>
<th>Power-to-energy ratio</th>
<th>Cost¹</th>
<th>Cost²</th>
</tr>
</thead>
</table>

Table 2.1.: Comparison of different battery ES technologies. ¹: 1EUR = 1.35 USD, ²: Variable power to energy ratio
2.3 Battery aging

One important battery ES that is found more commonly also in power system application are li-ion batteries. They promise a high round-trip efficiency combined with a high cycle life (see Tab. 2.1). However, they tend to be an expensive ES technology. Complementary to li-ion batteries are lead-acid batteries with a low cycle life and a low to medium round-trip efficiency and relatively low costs. Lead-acid batteries are a mature technology existing in industrial applications for longer time than li-ion batteries. Their complementary properties make these two battery ES technologies interesting for further considerations and they are used for the case studies in chapter 6 to chapter 8.

Other battery technologies listed in Tab. 2.1 would also be interesting for consideration, however, the high temperature batteries NaS and Zebra are not as commercialized as li-ion and lead-acid batteries and only a very limited number of manufacturers exist. Concerning NiCd batteries, the EU has banned widely the use of this technology due to toxicity making it unfavorable to be part of the comparison. Finally, flow batteries (VRB and ZnBr) are excluded because of their low efficiencies and limited number of manufacturers.

Having made the pre-selection of battery ES technologies, the next section discusses aging of batteries as being an important driver of costs.

2.3. Battery aging

Aging is an important property when discussing battery ES. Generally, the more a battery is used the lower becomes its available capacity and its power rating until it finally has to be replaced. This means that parameters influencing aging have a direct impact on the battery lifetime and hence on the costs of the battery ES. This sections discusses aging of batteries in order to provide the bases for the development of a proper battery ES model in chapter 5.

Aging of a battery ES means its “permanent loss of capacity [and power [26]] as a result of battery use and/or the passage of time” [8]. This permanent loss of capacity and power is called hereafter capacity- and power fade. Due to the fact that capacity fade is the factor determining the lifetime of the battery (see below), the focus of this work is on capacity fade and power fade is not considered.
Capacity fade can be divided into calendric capacity fade and exercised capacity fade [27].

Calendric capacity fade is independent of the energy throughput through the BESS and is an on-going process over time and depends on parameters like SoC or temperature, for instance. On the contrary, exercised capacity fade is due to energy throughput through the battery (exercising the battery) and depends mainly on cycle depth or Ah/energy throughput that depends on the model used [28].

A measure of capacity fade is the state-of-health (SoH) defined as the ratio of the capacity available at a certain point of time versus the originally available capacity of the battery ES [29]. The SoH at which the battery ES needs to be replaced depends on the application of the battery ES and what is perceived as acceptable. A commonly found value for the SoH limit in literature is 80% (see e.g. [30, 31, 27, 28]) at which replacement of the battery is required and when the battery has reached its end of life. These assumptions are chosen for this work.

However, it can also be assumed that the BESS can still be used beyond the 80% SoH limit and that the BESS still has a certain value. Thus, the replacement decision needs not necessarily take place at 80% SoH, or any other predefined SoH. For future work, it is recommended to extend the proposed algorithm and to incorporate methods of industrial maintenance and replacement as they are discussed in [32] page 31ff, for instance.

2.3.1. Calendric capacity fade

Batteries that are stored over a certain period of time irreversible lose parts of their maximum capacity caused by side reactions due to instability of the battery materials [7, 33]. Factors that accelerate calendric capacity fade are increased temperature and high SoC (high cell voltage) [34].

Data on calendric capacity fade have to be obtained experimentally through lab tests or data from manufacturers or other available sources have to be used but they might be difficult to find for a specific battery technology. An example of calendric capacity fade data for li-ion batteries can be found in [33]. If data are not available and cannot be obtained by own lab tests for the desired battery
2.3 Battery aging

technology, it might become necessary to assume a certain calendric capacity fade for a certain period.

2.3.2. Exercised capacity fade

Considering exercised capacity fade, the general relation is observed: The higher the energy throughput, the higher the capacity fade [27]. A standard parameter that indicates lifetime of a battery ES regarding exercised capacity fade is to state the number of full cycles that a battery ES can perform before it has to be replaced [27]. A full cycle is from 0% SoC to 100% SoC and back to 0% SoC, for instance.

There are different methods how to account for exercised capacity fade. One method is to simply count the energy or Ah throughput through the battery (amount of energy or Ah charged and discharged into/from the battery) [28]. These are called throughput models [28]. Another method would be to count the number of cycles the battery ES performs. This method is called cycle counting [28]. Cycle counting does not consider Ah or energy throughput but is based on the SoC profile which means the amplitude of the charge cycle affects the amount of lifetime reduction [28]. Details of a cycle counting method for lead-acid batteries and li-ion batteries can be found in [28] and [7], respectively.

Generally, it is possible to combine these two methods and also to link them to other factors [28].

In this work, it is assumed that an energy throughput model is sufficient to describe exercised capacity fade. Due to the fact that manufacturers usually specify the number of full-cycles rather than a maximum Ah- or energy throughput, the number of full cycles have to be translated into maximum Ah- or energy throughput. Formula 2.1 describes this conversion [28].

\[
Throughput = \sum_{i} \left( \frac{C_{Nom} \cdot DoD_{i}}{N_{i}} \right) N_{i}
\]

Throughput = \sum_{i} \left( \frac{C_{Nom} \cdot DoD_{i}}{N_{i}} \right) N_{i} \quad (2.1)

\( C_{Nom} \) describes the nominal battery capacity and \( DoD_{i} \) is the specific depth-of-discharge for the \( i^{th} \) measurement out of \( i \) measurements. \( N_{i} \) is the number of cycles until failure of the battery at point \( i \). The lower the DoD the more cycles
can a battery withstand [28]. The Throughput can be calculated as an average over all $i$ or a specific range of points can be picked that represents the most dominant DoD range [28].

When calculating Throughput, [28] does not distinguish between $Ah$ or $Wh$ to use for $C_{Nom}$ in order to compute the throughput of the battery, even though both units are not the same as explained on page 11. In this work, it is also assumed that the life of the battery can be determined based on energy throughput in $Wh$ and that this provides sufficient accuracy.

The conversion of cycle life into energy throughput is convenient for further model development because the cycle-life can be found throughout various battery ES technologies which allows to calculate energy throughput and to develop an optimization algorithm for different battery ES technologies that is based on energy throughput.

2.4. Self-discharge

Self-discharge is the reversible loss of capacity of a battery occurring over time [27]. It is assumed that self-discharge only depends on the SoC and the type of ES technology which is according to [16], where self-discharge is expressed for some battery technologies as a percentage of the SoC that is internally discharged over a certain time period. Self-discharge is due to internal chemical reactions [16]. It is assumed that self-discharge can be modeled for all battery technologies as described in [16]. Further, it is assumed that the temperature of the BESS is kept constant and thus temperature as an effect on self-discharge can be neglected as well as the influence of the SoH.

2.5. Battery efficiency

As discussed above, when stating the efficiency for a battery it usually refers to the round trip efficiency. In practice, the efficiency depends on various parameters and important ones are temperature, SoC, aging, and C-rate or charging-
2.5 Battery efficiency

and discharging currents, respectively. It is worth to have a closer look at parameters that influence the efficiency because efficiency is one of the cost drivers of the battery technology when using the battery. The higher the efficiency, the lower the wasted energy and the better is the battery ES in economic terms. It has to be noted that the efficiency depends on the selected battery technology and on the specific battery model chosen. Drawing conclusions from one battery model to others of the same kind of battery technology might be difficult. However, the problem for a complete discussion of this issue is data availability. For this work, data from the authors of [35] were provided based on laboratory measurements. The charging- and discharging efficiencies versus DoD are depicted in Fig. 2.3 for the Kokam “Superior Lithium Polymer Battery (SLPB)” (model no. SLPB120216216). The parametric identification of this battery is described in [35] under II B. The efficiency is determined based on the internal resistance $R_{i,soc}$ of the battery that depends on the SoC and is calculated based on

$$R_{i,soc} = \frac{V_{ocv,soc} - V_{peak,soc}}{I} \quad (2.2)$$

where $V_{ocv,soc}$ is the open circuit voltage measured 2h after current pulse has been applied and $V_{peak,soc}$ is the highest (lowest) voltage reached during the pulse current for charging (discharging). $I$ is the charge- or discharge pulse current which is kept constant for each step. The power losses while charging- or discharging the battery can be calculated according to:

$$P_{loss,soc} = I^2 R_{i,soc} \quad (2.3)$$

The efficiency $\eta_{soc}$ can then be determined by

$$\eta_{soc} = 1 - \frac{P_{loss,soc}}{P_{total,soc}} \quad (2.4)$$

where $P_{total,soc}$ is the total charging- or discharging power at the battery terminals.

The above equations in this section are based on or provided by the authors of [35].
Data for different charging- or discharging currents could not be provided by the authors of [35]. Fig. 2.3 reveals that the efficiencies for charging- and discharging differ slightly but considering only charging or discharging, the efficiency can be assumed to be constant for 10%<DoD<90%. Within this range, the average charging efficiency is 95.5% and the average discharging efficiency is 94.5%. This results in a round-trip efficiency of 90.25%. This lies well in the range of 85 to 95% stated in Tab. 2.1. All measured efficiencies do not include losses of power electronics like the inverter or the transformer. It needs also to be stressed that the measured data are only valid for the specifically tested batteries under the specific test conditions. Due to a lack of other data, the results based on [35] are assumed to be representative for this work.

That means that the efficiency is constant for 10% which is also considered as the operating range (see also sec. 2.10) due to oversizing of the BESS. Due to missing data for other parameters, it is further assumed that the efficiency of the battery does neither depend on the charging- nor on the discharging current or the aging of the battery. It is also assumed that the test temperature is the same in practice as well as that this is applicable for other battery technologies. In practice, a major influence on the battery efficiency could be expected based on the charging- or discharging current because $P_{loss,soc}$ depends on its square values. This means that a better assumption would be to say that $R_{i,soc}$ is constant but due to the lack of additional data, it is assumed that the charging- and discharging efficiency is constant.

### 2.6. Battery energy storage systems (BESSs)

Single battery cells have to be combined in order to deliver sufficient power and energy for a desired application. Therefore, batteries are combined in modules which are then stacked together to a bigger unit which is usually housed in containers. Combined with a battery management system that controls the entire unit as well as an inverter and transformer, a BESS is built. Such a BESS is able to provide active power and may also provide reactive power for voltage regulation. Set points can be sent to the battery management system which tries to achieve the desired output. The battery management system also has to take care of each individual cell that operational parameters (like temperature) are not
2.7 Applications for battery energy storages

![Graph: Efficiency for Kokam SLPB battery depending on DoD (based on data of [35])](image)

**Figure 2.3:** Efficiency for Kokam SLPB battery depending on DoD (based on data of [35])

exceeded in order to guarantee a safe operation without unnecessary wear out of the system. (Compare [36])

A disadvantage of a BESS is that it has internal power consumption for its ancillary systems needed to operate the BESS and which is referred to as self-consumption in this work. It is caused, for example, by the battery management system, fans, chiller systems, heating system, etc. Manufacturers of BESSs tend not to specify this parameter. One reason might be that self-consumption depends on the geographic location of the BESS concerning the required amount for cooling the BESS, for instance. Self-consumption is not caused by inefficiency of the battery ES and is independent of the operating pattern of the battery ES. It is assumed in this work that the self-consumption of the lead-acid and li-ion battery is 0.0125MW per installed MW discharger rating.

### 2.7. Applications for battery energy storages

This section discusses important applications that can be provided by the BESS which is based in close proximity to the WPPs. Some applications can only be provided by the combination of BESSs with WPPs (called VPPs here) while other applications can be provided by a BESS alone.
Ramp rate reduction means to reduce the fluctuations of the WPP output power. Technically, the output power gradient $\frac{dP}{dt}$ is smoothed with this application and the smaller the fluctuations, the better it is for the power system stability [7]. If a certain ramp rate may not be exceeded with a WPP, it might need to be down-ramped in certain periods and the WPP generates less energy than it technically could. BESSs can be used in such situations to store the excess energy and this energy can later be released when ramp rates allow it.

Energy arbitrage is an application where energy is bought and stored in the BESS when prices are low and the energy is sold when prices are higher [37]. Energy arbitrage makes thus use of different power prices in different time periods. This application is also called time shift because energy is stored in times when demand is low (normally causing low prices) and the discharge period is shifted to a period of high demand (which usually means higher prices) [37].

Frequency regulation is the ability of a WPP or BESS to provide short durations of power in order to help reducing system imbalances and thus stabilizing the system frequency [37]. The technical requirements for frequency regulation are usually published by the transmission system operator (TSO) or regulator. Frequency regulation can be further divided into primary-, secondary-, and tertiary frequency regulation. Primary frequency regulation has to take immediate response in order to restore the system frequency in case of imbalances until secondary regulation takes over while tertiary frequency regulation has the slowest response time and is usually activated for imbalances that last longer.[38]

Forecast accuracy improvement Selling wind power on an energy market requires to have a forecast of the power production of the next day(s). The actual output of the WPPs normally deviates from its forecast due to forecast errors. In case of a mismatch between sold and delivered energy, WPP operators need to pay a penalty for this deviation. BESSs can be used to reduce imbalances by charging or discharging the BESS and thus penalty payments can be reduced. [37]
2.8 Relevant applications in Denmark (DK1)

**Transmission curtailment**  WPPs might be installed at remote sites where not sufficient transmission capacities are available to transport all generated power and the WPPs would need to be down-ramped in order to not exceed transmission capacities. BESSs can be used in that case to store excess power production when the transmission capacities would otherwise be exceeded. [37]

**Transmission line deferral**  Similar to the above application, BESSs can be used to defer investments in new transmission capacities until a later point of time [15]. This can mean a financial saving and if the costs for the installation of a BESSs are less than the saving, transmission line deferral can be a profitable application.

### 2.8. Relevant applications in Denmark (DK1)

The Danish power system is divided into two parts, DK1 (Jytland and Fyn) and DK2 (Zealand). A brief overview of remunerated applications and the related market regulations for DK1 is presented in this section. The regulatory framework of Denmark is exemplarily chosen because Denmark is a country with a high share of wind power production [39]. The most promising applications are selected and serve as a guideline in developing an appropriate MILP problem formulation in chapter 5.

**Energy arbitrage on DA market**  The Danish DA market is called Elspot and is traded on Nordpool. BESSs and WPPs can both participate in this market and the BESSs can be used to store energy when prices are low and energy can be discharged when prices are higher. In this market, bids have to be submitted until 12am before the day of delivery and the last accepted bid sets the price for all other bids. The lowest possible price is -200€/MWh and the highest possible price is 2000€/MWh. There is no minimum amount stated for a bid. The reference for this paragraph is [40].

**Energy arbitrage on intraday market**  Besides the Elspot market, there also exists the Elbas market for intraday trading up to one hour before the delivery hour [40]. Energy arbitrage is also possible in this market.
Providing regulating power (RP)  Also in the RP market, both - WPPs and BESSs - can participate. The application is to provide power to balance the system which is in the broader sense also to regulate the grid frequency. The demand of RP is defined by the total grid imbalances within one hour. Bids can be send out up to 45 minutes before the upcoming hour of operation. If a generation unit fails to provide RP, there is no penalty meaning that the operator has to pay a penalty equal to the income for the awarded bid in RP and the final balance is zero Euros. Further, it is possible to combine several generation units in order to submit aggregated bids. However, if WPPs want to participate, a separate bid has to be delivered for these plants. Additionally, a bonus that might be paid to WPPs is not affected if the WPP provides RP up. The bonus is paid according to the reading of the WPP’s meter. Thus, it is only affected if providing RP down when the WPP is down-ramped and less electricity is produced. This lowers the bonus paid to the WPP operator. The minimum price of RP up is the DA market price and the maximum price of RP down is also the DA market price. The maximum price for upward regulation is DKK 37,500/MWh. Moreover, the last activated bid sets the price for all other bids. Prices for RP down can be negative. Further details on RP can be found in [41].

Providing primary frequency regulation (PFR)  The PFR market is aimed to provide short term frequency regulation of maximum up to 15 consecutive minutes. And payments to providers of PFR are only based on power provided for PFR and no payment is made for the actual energy delivered. The application is obviously frequency regulation. In this market, bids must be submitted until 3pm the day of delivery and the auction is categorized in blocks of four hours each. Meaning, there is the same price for each block but upwards and downwards regulation is priced differently and the last accepted bid sets the price for all other bids. Moreover, it is possible to combine units to provide PFR; however, WPPs cannot provide PFR alone but need to be backed up by another unit eligible to participate in the PFR market. Generally, BESS are eligible to provide PFR if they can at least provide the bid power for up to 15min. References and further details for PFR are found in [42].
Providing secondary reserve  Secondary reserve should be able to be activated within 15 minutes in order to take over from PFR. The TSO Energinet.dk buys secondary reserve on a monthly bases which makes it unattractive for WPPs due to their intermittent power output which is unrealistic to forecast over such a long period. It is also unattractive for BESSs because they cannot guarantee to maintain regulation continuously which is required by regulations. Due to these limitations, the secondary reserve market is not further considered. For reference of this paragraph, see [42].

Forecast accuracy improvement  This application is not directly addressed by market regulations. Here, revenues are generated by avoided costs for imbalances. Details on the calculation of costs for imbalances are described in [41]. Forecast improvement is an application that can only be provided by VPPs and not by the BESS alone which makes it stand out from the other applications and is therefore chosen to be addressed in the problem formulation in chapter 5 under dispatch function.

Ramp rate limitation  Danish regulations impose certain limits on the wind power ramp rate that may not be exceeded [43]. It is assumed that this means in practice only minor losses for the WPP operator and therefore ramp rate limitation is not selected as an application for the MILP problem formulation.

2.9. Selected applications

As already discussed in the previous section, forecast accuracy improvement is an important application and is included in the optimization problem formulation.

The remaining applications out of the previously discussed applications which can be served with a BESS on the DK1 market are energy arbitrage for the DA- and intraday market as well as the BESS can participate in the PFR- and RP market. Due to the fact that the Elbas market is smaller then the Elspot market, the focus of energy arbitrage is on the DA market and the Elbas market is neglected [44]. Hence, the focus of this work is on the following applications:
• Energy arbitrage on the DA market (WPP and BESS)
• PFR (BESS only)
• RP (WPP and BESS)
• Forecast accuracy improvement (WPP and BESS combined only).

Chapter 5 explains how these four selected applications are addressed in the optimization problem formulation.

2.10. Limitations and assumptions

Having discussed the scope of this work in chapter 1, followed by the description of battery ES technologies and their applications in this chapter, certain assumptions have to be made in order to refine the scope of this work. The following is a list of assumptions and limitations of this work:

• The VPP is connected to a wind farm in close proximity and builds its own balancing responsible party.

• Only battery ESs are considered with a fixed power to energy ratio, no other electrical energy storage systems are part of this study.

• The model of the battery ES system is specifically developed to reflect aging of the BESS. Aging of BESSs are complex processes and most likely nonlinear. Certain assumptions regarding linearization are to be made in order to accommodate such effects.

• The WPP is operated based on a bonus remuneration scheme.

• Bilateral contracts like over-the-counter (OTC) trading are not considered, only the power market prices on the stock exchange are considered.

• The VPP is assumed to be a price-taker.

• There exists a dual pricing system for the PFR- and RP market meaning that prices for upward- and downward regulation can be different.

• The BESS is oversized which allows to assume that maximum charging- and discharging power does not depend on the SoC and can be constant and that the usable 0 to 100% SoC does not reflect the physical possible SoC range.
• The battery model regarding aging accounts for capacity fade, however, power fade is not considered because the factor determining lifetime of the battery is related to capacity fade.

2.11. Nomenclature of chapter 2

The nomenclature related to this chapter is shown in Tab. 2.2. A separate nomenclature for this chapter is used because the introduced variables do not relate to the other chapters.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Nom}$</td>
<td>Ah</td>
<td>Nominal capacity</td>
</tr>
<tr>
<td>$Cr$</td>
<td>A</td>
<td>C-rate</td>
</tr>
<tr>
<td>$DoD_i$</td>
<td>Ah/Ah or Wh/Wh</td>
<td>Depth-of-discharge for $i^{th}$ point where $i$ is the set of $i = 1, \ldots, i$ points of measurements</td>
</tr>
<tr>
<td>$I$</td>
<td>Amperes</td>
<td>Charging or discharging current pulse</td>
</tr>
<tr>
<td>$N_i$</td>
<td>cycles</td>
<td>Maximum number of cycles for DoD $i$ until battery needs replacement</td>
</tr>
<tr>
<td>$P_{loss,soc}$</td>
<td>W</td>
<td>Power loss while charging or discharging depending on SoC step</td>
</tr>
<tr>
<td>$P_{total,soc}$</td>
<td>W</td>
<td>Total power for charging or discharging depending on SoC step</td>
</tr>
<tr>
<td>$R_{i,soc}$</td>
<td>Ohm</td>
<td>Internal resistance battery depending on SoC step</td>
</tr>
<tr>
<td>$T$</td>
<td>°C</td>
<td>Degree centigrade</td>
</tr>
<tr>
<td>$V_{ocv,soc}$</td>
<td>Volt</td>
<td>Open circuit voltage for specific SoC step</td>
</tr>
<tr>
<td>$V_{peak,soc}$</td>
<td>Volt</td>
<td>Highest (lowest) voltage reached during the pulse current for charging (discharging)</td>
</tr>
<tr>
<td>$\eta_{soc}$</td>
<td>W/W</td>
<td>Charging or discharging efficiency for specific SoC step</td>
</tr>
</tbody>
</table>

*Table 2.2.*: Nomenclature of chapter 2
3. Background on optimization techniques

3.1. Selection of optimization technique

This section provides an overview of different optimization techniques and a suitable technique for the problem formulation in this work is chosen. The selected technique is described in more detail in order to understand the problem formulation in chapter 5.

First of all, it has to be noted that optimization problems are present everywhere in our life and there is a wide range of how such problems can be tackled. Over the last decades, numerous techniques and tools have been developed to solve optimization problems. Optimization techniques can be divided into deterministic and heuristic approaches [45]. Deterministic optimization techniques apply analytical properties in order to converge to the global optimum. Important examples of deterministic approaches are:

- Linear Programming (LP)
- Mixed Integer Linear Programming (MILP)
- Stochastic programming
- Quadratic programming
- Convex optimization,
- Etc.

The other class of optimization techniques is based on heuristic approaches where random inputs are used in the search process to find the global optimum. Examples of such techniques are:
Chapter 3 Background on optimization techniques

- Genetic algorithm
- Simulated annealing
- Evolutionary algorithms
- Swarm algorithms
- Etc.

The advantage of heuristic algorithms is that they can handle non-linear and non-convex objective functions and constraints easily. However, due to the random nature of the search methods, optimality cannot be guaranteed and conclusions on the quality of the solution cannot be made, as well as they might become inefficient for large problems [45]. Such large problems can easily arise when the optimization includes discretized time periods which is the case when the optimum operation of the VPP should be found over a certain time period.

Amongst all different optimization techniques, MILP has proven to be successful for two reasons: There are several very effective solvers that can solve large problems and a wide range of problems can be addressed via MILP and LP [46]. Due to the advantages of MILP, this is the optimization technique of choice to develop an intelligent energy management system for VPPs in this work. One efficient solver is CPLEX [47] which is part of the iLog Optimisation Studio that is used to solve the formulated problem.

Further, it has to be stated that this thesis is focusing on the MILP problem formulation and it is not focusing on how to solve the problem by certain algorithms or on how to increase their performance e.g. by tuning. This would exceed the scope of this work but is recommended for future work.

The next sections of this chapter will provide the reader with the basic understanding of the MILP technique and related techniques part of MILP.

3.2. Mixed integer linear programming

This brief introduction into MILP is by no means meant to be complete. It should give the reader a basic understanding of the general form of the MILP problem formulation in order to understand the problem formulation in chapter 5.
3.2 Mixed integer linear programming

For a more detailed discussion of this topic, the reader is referred to [48, 49], for instance.

MILP problems are linear optimization problems where some of the decision variables can only have integer values and other decision variables are continuous. Thus, it is a mixed problem. The general formulation of a MILP is defined by equation 3.1 (compare [50]):

\[
\begin{align*}
\text{Maximize } z &= cx + hy \\
\text{s.t. } &\quad Ax + Gy \leq b \\
&\quad x \in \mathbb{Z}^n_+ \\
&\quad y \in \mathbb{R}^p_+
\end{align*}
\] (3.1)

The integer variables are \( x = (x_1, \ldots, x_n) \) with \( \mathbb{Z}^n_+ \) as the set of non-negative integer n-dimensional vectors. The continuous variables are \( y = (y_1, \ldots, y_p) \) with \( \mathbb{R}^p_+ \) as the set of non-negative real p-dimensional vectors. An optimization problem is defined by specifying the following data with: \( c \) as a \( n \)-vector, \( A \) as a \( m \times n \) matrix, \( G \) as a \( m \times p \) matrix and \( b \) as a \( m \)-vector. The problem formulation in 3.1 only contains inequality constraints but can be converted to equality constraints by adding slack variables. This is also called the conversion of the problem from canonical form to standard form (see [51] for more details). The nomenclature of this chapter is chosen to explain the concept of MILP and does not relate to other chapters in this work. Therefore, variables and parameters are not listed in the Nomenclature on page 145 of this thesis but can be found at the end of this chapter on page 40.

A problem where \( x = 0 \) is called a LP formulation where all variables are continuous.

The advantage of MILP is that it enables to model logical conditions by restricting integer variables to be binary decision variables. This enables e.g. modeling of states of a power plant like on or off, or sequencing where one operation must be finished before another starts. A thorough discussion on the use of discrete variables can be found in [49], chapter 9.
When formulating a MILP problem, it has to be kept in mind that integer variables cause a MILP problem to be more difficult to solve than a problem formulation with purely continuous variables (LP problem) [52]. Therefore, when formulating a problem, as few integer variables as possible should be used.

### 3.2.1. Network flow programming

Network flow programming is a technique which is usually based on an LP formulation that represents a network structure. A network is built of arcs and nodes where the arcs connect the nodes in a logical manner. The circles in Fig. 3.1 are nodes and the arrows indicate arcs that connect the nodes. There are source-, sink-, and transit nodes in a network. Source- and sink nodes are nodes where the flow is entering or leaving the network. In Fig. 3.1 the source node would be “Node 1” and the sink node would be “Node 4”. Transit nodes are “Node 2” and “Node 3”. Nodes could represent for example cities, stop lights, junctions in power systems, etc. In this sense, arcs would correlate to roads for cities and stop lights, and transmission lines would correlate to the junctions in power systems. Flow would be vehicles and electrical energy. This paragraph is based on [53] which the author recommends for a more depth discussion on network flow programming.

![Network flow example](adapted from [53])

If a network structure of a problem can be identified, network flow programming helps to formulate the optimization problem. A network is defined by a set of nodes $V$ and a set of arcs $W$. The direct connections in the network are expressed by arcs from node $k$ to node $l$. The general formulation of a network flow problem (minimum cost) is defined by equation 3.2 where $y_{kl}$ is the flow on arc$(k,l)$ and each arc$(k,l)$ has a maximum flow capacity $u_{kl}$ and associated unit cost $k_{kl}$ (compare [50]). The arc gain is denoted by $g_{kl}$. It is a factor to obtain a different flow at the end of the arc than at the beginning. Gains that are $< 1$ model losses in the network. If all gains are equal 1, the network is said to be a pure network [48].
3.2 Mixed integer linear programming

Whether node \( k \) is a source node, sink node or a transit node depends if \( b_k \) is positive, negative or zero, respectively. In order that a pure network flow problem has a feasible solution, the sum of \( b_k \) has to equal zero [48]. Equation 3.2 is the formulation of the general minimum linear cost network flow problem [50, 48]:

\[
\begin{align*}
\text{Minimize} & \quad z = \sum_{(k,l) \in W} k_{kl} y_{kl} \\
\text{s.t.} & \quad y_{kl} \leq u_{kl} \text{ for } (k, l) \in W \\
& \quad \sum_{l \in V} y_{lk} - \sum_{l \in V} g_{kl} y_{kl} = b_k \text{ for } k \in V \\
& \quad y \in \mathbb{R}^{|W|}_+
\end{align*}
\]

The equality constraint in equation 3.2 is known as the flow conservation constraint meaning that all flows into a node have to equal the outflows of this node except for sink- and source nodes.

There are many sub-problems that belong to the class of network flow problems like the transportation, transshipment, maximum flow, shortest route problem, etc. and the objective function does not always need to be minimized. More information can be found in [48], for instance.

If a network model has constraints that cannot be depicted with arcs and nodes, the network model is said to have side constraints [48]. Such side constraints may include integer decision variables. In fact, there is no strict border line between network problems and MILP problems because there is a MILP formulation for many network problems [51].

### 3.2.2. Piecewise linear approximation (PLA)

Any continuous function of one variable can be approximated with the technique of piecewise linear approximation to be accommodated in the framework of MILP. This technique enables the use of non-linear functions in a MILP optimization problem if they are separable functions in the form \( f(y_1, \ldots, y_j) = \sum_{q=1}^j f_q(y_q) \).
Chapter 3  Background on optimization techniques

The quality of the approximation is defined by the number of breakpoints \( r \) of the piecewise linear function \( f(y) \) with the breakpoints of \( (a_i, f(a_i)) \) for \( i = 1, \ldots, r \). Then, the linear approximation can be written as (compare [50, 49]):

\[
y = \sum_{i=1}^{r} \lambda_i a_i
\]

\[
f(y) = \sum_{i=1}^{r} \lambda_i f(a_i)
\]

\[
\sum_{i=1}^{r} \lambda_i = 1
\]

\[
\lambda = (\lambda_1, \ldots, \lambda_r) \in \mathbb{R}_+^r
\]

\( \lambda \) can be considered as weights attached to the breakpoints [49]. In addition to equation 3.3, one extra condition has to be met that at most two adjacent \( \lambda \) can be non-zero. This condition is also called adjacency condition and means that the weights can only be attached to neighboring points of the PLA. If the PLA is in the objective function, the adjacency condition is met if the piecewise linear functions are convex (concave) in a minimization (maximization) problem and a LP formulation is sufficient. Moreover, an LP formulation is also sufficient if the PLA is on the left hand side of an \( \leq \) constraint and the approximated function is convex (concave) or if the PLA is on the left hand side of an \( \geq \) constraint and the approximated function is convex (concave) in a minimization (maximization) problem. For a complete discussion see [54].

Otherwise, extra constraints based on binary variable \( B \) have to be introduced to enforce the adjacency condition (see [54] or [50], for instance):

\[
\lambda_1 \leq B_1
\]

\[
\lambda_i \leq B_{i-1} + B_i \text{ for } i = 2, \ldots, r - 1
\]

\[
\lambda_r \leq B_{r-1}
\]

\[
\sum_{i=1}^{r-1} B_i = 1
\]

\[
B \in \mathbb{R}^{r-1}
\]
3.2 Mixed integer linear programming

Another way to address the adjacency condition is by the so called “restricted entry rule” which is described in [54], for instance.

This section is based on [50, 49, 54].

3.2.3. Separable programming

The restriction in sec.3.2.2 that PLA can only be applied on separable functions can mean quite a limitation in practice. One way to overcome this problem is that it is often possible to convert non-separable functions into separable functions. Very commonly is the non-separable function which is a product of two variables $y_1y_2$ that can be converted to separable form by the following steps (see [49]).

1. Introduction of the two new variables $u_1$ and $u_2$

2. Establishing the following relation between $u_1$ and $u_2$, and $y_1$ and $y_2$:

$$u_1 = 0.5(y_1 + y_2)$$  \hspace{1cm} (3.5)

$$u_2 = 0.5(y_1 - y_2)$$  \hspace{1cm} (3.6)

3. The term $y_1y_2$ can finally be replace by $u_1^2 - u_2^2$ which is in separable form.

When implementing the conversion to separable form, it has to be considered that $u_2$ can take negative values.

Once there are only separable functions, they can be approximated by PLA according to sec.3.2.2. In this case a PLA for $u_1^2$ and $-u_2^2$ is necessary. In order to find an appropriate PLA of a function, starting break point $a_1$ and finishing point $a_r$ (assuming $a_i < a_{i+1}$) has to be found as the first and last point for approximation of the piecewise linear function. Oftentimes there are minimum and maximum values that $x_1$ and $x_2$ can possibly take implied by constraints. In this case for $u_1$, the smallest value for the PLA is

$$a_1 = ([\text{min value of } y_1] + [\text{min value of } y_2])/2$$  \hspace{1cm} (3.7)
and the biggest value is

\[ a_r = \left( \max \text{ value of } y_1 \right) + \left( \max \text{ value of } y_2 \right) / 2. \] (3.8)

For \( u_2 \),

\[ a_1 = \left( \min \text{ value of } y_1 \right) - \left( \max \text{ value of } y_2 \right) / 2 \] (3.9)

and the biggest value is

\[ a_r = \left( \max \text{ value of } y_1 \right) - \left( \min \text{ value of } y_2 \right) / 2. \] (3.10)

Besides the above demonstrated procedure to convert a non-separable function into separable form, there are other conversions possible that depend on the non-separable function. See e.g. [49] for more information.

In general, separable programming increases the complexity of the problem to be solved and thus should be applied with care.

### 3.3. Nomenclature of chapter 3

The nomenclature related to this chapter is shown in Tab. 3.1. A separate nomenclature for this chapter is used because the introduced variables do not relate to the other chapters.
### 3.3 Nomenclature of chapter 3

**Table 3.1.** Nomenclature chapter 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>Matrix with coefficients of continuous variables for constraints</td>
</tr>
<tr>
<td>( G )</td>
<td>Matrix with coefficients of integer variables for constraints</td>
</tr>
<tr>
<td>((a_i, f(a_i)))</td>
<td>Breakpoints for piecewise linear approximation</td>
</tr>
<tr>
<td>( b_k )</td>
<td>Parameter determining source-, sink-, and transit nodes</td>
</tr>
<tr>
<td>( c )</td>
<td>Matrix with coefficients of integer variables for objective function</td>
</tr>
<tr>
<td>( g_{k,l} )</td>
<td>Gain on arc((k,l))</td>
</tr>
<tr>
<td>( h )</td>
<td>Matrix with coefficients of continuous variables for objective function</td>
</tr>
<tr>
<td>( k_{k,l} )</td>
<td>Unit costs for flow on arc((k,l))</td>
</tr>
<tr>
<td>( n )</td>
<td>Dimension ( x )-vector</td>
</tr>
<tr>
<td>( p )</td>
<td>Dimension ( y )-vector</td>
</tr>
<tr>
<td>( u_{k,l} )</td>
<td>Maximum flow capacity on arc((k,l))</td>
</tr>
<tr>
<td><strong>Decision variables</strong></td>
<td><strong>Description</strong></td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$B$</td>
<td>Binary variable to enforce adjacency condition</td>
</tr>
<tr>
<td>$u_1, u_2$</td>
<td>Variables used for separable programming</td>
</tr>
<tr>
<td>$x$</td>
<td>Integer decision variable</td>
</tr>
<tr>
<td>$y$</td>
<td>Continuous decision variable</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Weights attached to the breakpoints of the piecewise linear approximation</td>
</tr>
</tbody>
</table>

| **Functions** | **Description** |
|---------------|-----------------
| $f(y)$        | Piecewise linear function |
| $z$           | Objective function |

<table>
<thead>
<tr>
<th><strong>Indices</strong></th>
<th><strong>Description</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Indicates breakpoint $i = 1, \ldots, r$</td>
</tr>
<tr>
<td>$k$</td>
<td>Indicates the arc from node $k$</td>
</tr>
<tr>
<td>$l$</td>
<td>Indicates the arc to node $l$</td>
</tr>
</tbody>
</table>
3.3 Nomenclature of chapter 3

<table>
<thead>
<tr>
<th>Sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Set of nodes</td>
</tr>
<tr>
<td>$W$</td>
<td>Set of arcs</td>
</tr>
</tbody>
</table>

Having discussed the essentials of MILP, the next chapter presents the state of the art literature which is relevant for this work in order to understand what has been published in the past and what contributions of this work are.
4. State of the art literature review

This chapter discusses the state of the art literature relevant for this thesis. It is structured according to the three functions of the proposed algorithm which is optimum scheduling, -sizing, and -dispatch. The literature review is focused on LP/MILP problem formulations for various ES technologies applied in different contexts. Literature on other optimization techniques is discussed if relevant for this work.

4.1. Optimum scheduling

The literature review includes references based on deterministic formulations of the optimum scheduling problem as well as on publications on stochastic formulations in order to give a broad overview.

References for deterministic optimum scheduling problems  Catalão et al. proposes a non-linear optimum scheduling formulation for hydro-power plants [55] where non-linear head effects of hydro-power stations have been taken into account and a study is conducted for hydro-power plants in Portugal. The problem formulation is based on network flow programming. [56] presents a non-linear model for scheduling of hydro power plants taking also into account non-linear efficiency curves of such plants. However, both studies do not consider the ancillary service markets. Another study of the optimum scheduling problem for cascaded hydro-power storages that is based on network flow programming is found in [57]. The authors also take spinning-reserves into account but no energy markets are considered. [58] also uses network flow programming to schedule hydro- and thermal power plants including spinning reserve. The authors do not consider energy markets, either.
Castronuovo and Lopes [59] formulate a LP problem for a wind farm connected to a hydro-power storage. They assume that the hydro-storage can only be recharged by wind power and not by purchases from the DA market. Moreover, their formulation does not take any ancillary services into account. Another detailed deterministic MILP formulation for a cascaded hydro-power system is found in [60] which is also only targeting the DA market. In addition, the authors of [61] propose an optimum scheduling formulation based on the genetic algorithm for WPPs and BESSs. The authors use a general formulation for the BESS without taking into account battery specific factors.

In contrary to the above mentioned publication, the model in [61] is solved with the genetic algorithm which might become time consuming if the time horizon is extended and, due to the stochastic nature of the genetic algorithm, finding the global optimum cannot be guaranteed. F. Bourry et al. describe a similar optimization problem in [62] focusing on VPPs consisting of WPPs and a pumped-hydro storage. The formulation aims to minimize the penalty risk through imbalances and is based on a rolling-window approach. Besides the DA market, the intraday market is also included in this study. Two strong assumptions in this publication are that the energy storage can only be charged through the WPPs and not through both, the DA market and the WPPs and that the energy storage has an efficiency of 100%.

Another deterministic optimum scheduling problem proposed in [63] is also focusing on pumped-hydro power storages but the authors do not consider wind power in their formulation. However, sales to the spinning and non-spinning reserves are included in addition to the DA market. The authors present a multistage-algorithm to solve the problem. [64] discusses the optimum scheduling problem of an ES located in the Danish power market for energy arbitrage. The optimization method applied is sequential quadratic programming.

**References addressing SoH of batteries** Besides LP / MILP formulations, the optimum scheduling problem can also be formulated as a dynamic programming problem. Examples can be be found in [65, 66, 67, 68]. Dynamic programming is mentioned at this point because [68] introduces an approach to consider the SoH (aging) of a lead-acid BESS based on its capacity fade in the optimization. The system considered is a grid connect photo-voltaic (PV) plant with BESS. The
same concept is applied by [69] for optimum scheduling of a PV-battery system in the framework of a MILP formulation. It is further picked up by [70] in an MILP optimum sizing problem (see sec. 4.3 on page 51).

The capacity fade is implemented in [68, 69, 70] in such a way that it is a constant factor multiplied with the discharging power of the ES. Capacity fade for charging power is not considered. This might be unrealistic as it penalizes discharging over charging. The above sources do not discuss an approach where the capacity fade is proportional to the charging and discharging power. Additionally, the SoC also affects the capacity fade of a battery ES which is not accounted for in [68, 69, 70].

The relation of the SoC and the capacity fade is the following: A battery ES that is kept at high SoC levels has to be replaced faster than the same storage kept at lower levels (without actually charging or discharging the battery ES). Also see battery aging on page 19.

**Stochastic programming formulations**  Stochastic programming formulations of WPPs connected to pumped-hydro storages are discussed in [3] and [65]. There, the authors propose a stochastic programming model based on a MILP formulation to take into account the stochastic nature of wind power as well as the fact that future market prices are unknown. However, their formulation includes only the DA market and ancillary service markets are not addressed. Also the authors of [71] and [72] present a stochastic programming formulation based on MILP focusing on the optimum operation of hydro-power storages in the DA market; however, wind power is not taken into account.

**Summary**  As shown above, the optimum scheduling problem of hydro-power storage is widely discussed in literature and many formulations are based on LP or MILP; however, a MILP formulation specifically for a BESS combined with grid connected WPPs operated on the DA- and ancillary service market is not discussed in literature to the best of the authors knowledge. In particular, all of the above cited publications consider only the DA- or intraday market except [63, 58, 57] that also take spinning- and/or non-spinning reserves into account. Though, the work of [63] does not consider the combination of the pumped-hydro plant with other types of plants like WPPs and it is not based on a MILP formulation. [58] uses network flow programming to schedule hydro- and thermal
power plants taking into account spinning reserve but renewable energy sources
other than hydro-power or other ESs are not part of their work. Moreover, energy
markets are not considered. [57] takes spinning reserve in the problem formulation
into account but energy markets are not part of this study, either.

4.2. Optimum dispatch

The optimum dispatch problem can be considered as a rescheduling problem of
the original schedule after e.g. a change in the wind power forecast has occurred.
When rescheduling the original schedule, the new schedule has to be linked to the
original schedule because any deviations in delivery to the power market other
than the amount awarded (imbalances) might get penalized.

[73] addresses this problem in a simple and straightforward manner. The difference
between the generated power (or updated schedule for generation) \( P_g \) and
the scheduled power \( P_{sch} \) has to be balanced by either upward \((BalanceUp > 0)\)
or downward \((BalanceDown > 0)\) power according to equation 10 in [73]:

\[
BalanceDown - BalanceUp = P_g - P_{sch}.
\]  

Equation 4.1 can also be considered as a node in a network flow programming
framework where \( P_g \) and \( BalanceUp \) are the inflow into the node and \( P_{sch} \) and
\( BalanceDown \) are the outflow out of the node (see Fig. 4.1).

[73] applies this concept successfully on the DA market. This work demonstrates
in chapter 7 that the concept of [73] is applicable also when several power mar-
kets like the DA- and ancillary service markets are addressed simultaneously.
Thus, the dispatch function cannot be claimed as a new contribution to research
but can demonstrate that this concept works also for several power markets (see
chapter 7). The formulation of the dispatch function is found in sec. 5.2.7 on page
92. No further literature is discussed due to the fact that the problem of optimum
dispatch is sufficiently addressed based on equation 4.1.

Summary It is shown that there exists a concept in [73] to address balancing of
a plant for MILP but the concept is restricted to only one power market. Other
4.3 Optimum sizing

The issue of sizing ESs has been addressed in literature for various ES technologies and based on numerous methods that also include methods which are not based on optimization techniques. This section discusses the state-of-the-art literature on sizing ES with regard on MILP formulations. The focus is on battery ESs but there is also important literature on sizing other ES technologies or literature not based on MILP which is discussed if appropriate.

In the following, literature is grouped into methods that focus on sizing hydro power storages, sizing microgrids including ES, sizing battery ES including capacity fade, sizing ES based on stochastic programming, and other sizing techniques which are of relevance.

**Hydro power optimization** [74] discusses a sizing approach for a pumped hydro storage unit in an isolated power system. The storage unit is part of a diesel system including renewable energy generation. The optimization is based on a LP formulation that optimizes the power rating and battery energy of the pumped hydro power station. The power rating is not coupled to the battery energy, which
reflects such a plant correctly, but would not be applicable for most battery ES technologies. No other ES technology besides pumped hydro is considered. In [71], a MILP optimization model is also discussed for a pumped hydro storage unit participating in an electricity market (DA market) combined with wind power generation. The outcome of the optimization is the optimum schedule for the pumped storage plant. The authors also indicate the need to find the optimum pumped hydro storage size by comparing the results for pumped hydro storages with different sizes. However, no MILP based approach is applied to find the optimum power rating or optimum battery energy of the storage device. Moreover, only the DA market is included in the formulation.

**Microgrid models with ES** Another sizing approach for ESs (not specifically pumped hydro storages) is proposed in [75] for a microgrid system. The approach chosen is based on a MILP formulation and the optimum size of the ES is identified by an iterative approach (and not by decision variables), either minimizing the total cost or maximizing the total benefit. The optimum ES is defined by meeting the stated objective. This approach may become cumbersome when more than one ES is used and the optimum of a combination of different ES needs to be found. [76] describes a LP model for a wind-diesel power system connected to a hydrogen storage. The model is used to discuss the sizing question of the hydrogen storage. A simplified model of the hydrogen storage is used for optimization with the objective to minimize costs. The optimum size of the storage is indicated by analyzing the results of the model and not directly through a decision variable. Another example of a MILP battery model can be found in [77] as part of a biomass, photovoltaic, biogas, small-hydro and diesel system. There, the combination of all system components is being optimized including the small-hydro plant. Also [78] discusses a MILP optimum sizing approach for an ES located in a microgrid. The ES model is simplistic and no efficiency losses of the storage are considered as well as the authors assume that the power rating and battery energy can be sized independently of each other.

**References addressing SoH** The article in [79] discusses a MILP optimization approach for sizing battery capacity in a grid-connected PV system. Their battery model incorporates the cost of battery aging (capacity fade). The capacity fade
4.3 Optimum sizing

process is implemented as proposed in [68] and the capacity fade only occurs when discharging the ES. A disadvantage of [79] is that the objective of the optimization is to minimize all costs while the selected battery is compared on the basis of annual net profit. If the cost function would reflect maximizing the annual net profit of the ES, a different size of battery could have been identified. Another disadvantage is that the authors have decoupled optimizing the dispatch schedule from optimizing the capacity of the battery storage. Decoupling may have the consequence that a non-optimum storage for the system is found because the size of the battery ES has an influence on the optimum dispatch schedule. If the dispatch schedule changes, another capacity value of the battery ES might be optimum. Hence, the optimum dispatch schedule and optimum (capacity of the) ES should not be determined independent of each other. Moreover, the authors state in the literature review that “Recently, optimization of storages in grid-connected PV systems attracted increasing interest. However, there is little in the way of guidance on battery sizing with respect to optimal scheduling of the battery.” Despite the fact that this statement is in respect to ES combined with PV systems, the author of this thesis agrees that there is little literature on optimum sizing of ESs based on (MI)LP.

[70] introduces a sizing algorithm for a battery ES connected to a residential PV system which is also grid connected. The optimization is based on a MILP formulation. Capacity fade is also accounted for in their model and it is based again on [68] and the same drawbacks apply as discussed in sec. 4.1 on page 47. The applications served in the formulation of [70] are power arbitrage and peak shaving. The cost function minimizes the sum of the net power purchase costs plus the costs of the capacity fade.

In contrast to [79] and [70], one reference based on a heuristic optimization method needs to be mentioned besides the disadvantages described in chapter 3. The authors in [80] size an ES system based on the genetic algorithm and account for capacity fade of a lead-acid battery via lifetime assessment based on the rainflow cycle counting method. It has the advantage that it can better describe capacity fade than an energy throughput model. A deeper discussion on lifetime models based on the rainflow counting algorithm is found in [7].

In summary, it can be concluded that capacity fade has been addressed based on a MILP formulation in only two references focusing on PV systems and no
literature of capacity fade in ES connected to WPPs is available to the best of the authors knowledge. Both references refer to the concept introduced in [68] with the drawbacks as discussed in sec. 4.1 on page 47.

**Sizing hybrid ES formulations** [81] discusses a mixed integer quadratic programming optimization approach on sizing hybrid ES systems. It can be applied to find the optimum ES combination out of different ES technologies. A model for demonstration is provided in which the electricity is generated by a WPP to serve a residential load. The selected ES technologies are batteries and a hydrogen ES. The authors demonstrate that the optimum hybrid combination shows a 30% improvement regarding cost effectiveness. The authors also point out that it is important to find the optimum size of each component of a hybrid ES system. [82] proposes a sizing method for hybrid ES based on MILP. Their model is an adjustment of the formulation presented in [83]. The authors consider a li-ion and lead-acid battery energy storage for their study. The hybrid ES is sized for a microgrid based on WPPs and a load. The authors assume that the battery energy can be sized independent of the power rating which is incorrect for lead-acid and li-ion batteries. There might be different power to energy ratios available within one battery ES technology. However, each sub-technology of a battery should be treated as its own technology. Moreover, [82] does not consider any restrictions on minimum SoC. This feature becomes necessary when battery ESs are considered like lead-acid which cannot be discharged below a certain threshold at a certain current (see sec. 2.1.2). Further drawbacks of this study are the energy markets not being part of the model and no battery lifetime determining factors like capacity fade being considered. Moreover, the initial SoC is a decision variable which is a rather unrealistic assumption because the initial SoC is usually known as a parameter when the optimization period is short. This can mean that one ES is biased over the other.

The authors of [84] indicate in their overview paper that there exist not many (MI)LP models for hybrid energy systems in literature compared to heuristic optimization techniques (not all references include ESs in their models). It has to be acknowledged that finding a LP or MILP representation of a specific system might be not as straight forward as applying a heuristic optimization method. However, the advantage is that even relatively large (MI-)LP problems
can be solved reliable and fast and also the global optimum can be guaranteed. The authors of [81] also mention that “very few papers have addressed this issue” of optimizing a hybrid ES system based on a LP or MILP formulation. This can be confirmed by the author of this thesis to the best of his knowledge as the literature review reveals only two references, one based on MILP and one based on quadratic programming.

**Stochastic programming models** A stochastic optimum sizing approach for a general type of ES without specification of the technology can be found in [85]. The developed model focuses on an isolated grid based on WPPs and diesel generators. The objective is to find the optimum ES rating that minimizes the energy coming from the wind diesel plant. Their model allows to optimize the power rating and battery energy independent of each other which is not a realistic assumption for most battery ES technologies (see page 12). Due to the fact that an isolated grid is considered, power markets are not part of the formulation.

An optimum sizing method based on stochastic LP for a single ES is proposed in [83]. The model is based on WPPs connected to a load and the objective is to minimize under-generation or energy deviations. A drawback of the model is that the power rating of the ES can also be optimized independently of the battery energy. This assumption cannot be confirmed for most of the ES technologies as described on page 12. A case study with a one week hourly horizon is part of the study where parameters of a NaS battery are used.

**Other relevant concepts:** [86] describes a model approach for thermal ES systems in industrial applications based on a MILP approach. The model of the thermal ES is not of relevance for this work where the focus is on BESSs. However, the author uses the concept of optimizing the size of the ES with a step function. This approach is adapted in this work (see sec.5.2.4 Charger commitment and Discharger commitment).

**Summary** Summarizing the state-of-the-art literature review for sizing BESSs, a comprehensive MILP formulation for sizing BESSs, which are part of VPPs, does not exist in literature to the best of the authors knowledge. A MILP formulation,
that covers the following properties, has so far not been proposed in literature. These properties are part of the proposed formulation of this thesis and are major contributions to research:

- A BESS is part of a VPP that is connected to the power system and can participate in the DA market as well as in the markets for ancillary services.

- Aging of the BESS is based on calendric- and exercised capacity fade where the exercised capacity fade occurs while charging- and discharging the BESS.

- Self-consumption of power for ancillary devices in order to operate the BESS is considered as well as self-discharge of the batteries over time.

- Annualized costs of the BESS are related to the SoH of the BESS where a replacement of the BESS is required at 80% SoH (or any other predefined value).

- The profit achievable on the various power markets is maximized.

- There is a dependency of the optimum schedule and the selected BESS.

- Any number of different battery technologies can be addressed and it can be specified how many different battery ES technologies should maximum be part of the optimum solution and what the maximum power rating of each technology should be.

This concludes the state of the art literature review and the next chapter presents the proposed MILP formulation.
5. Optimization problem formulation

This section discusses the proposed MILP formulation of the optimization problem. Detailed information regarding sets, parameters, and decision variables can be found in the nomenclature on page 145. The problem formulation is kept generic and can be used for all three functions: Optimum scheduling, optimum dispatch, and optimum sizing of the BESS (see Fig. 5.1). The input data provided to the model determines which function is called.

Moreover, in order to allow the evaluation of different country markets, the problem formulation is kept as general as possible in the sense that it can be applied to other markets that are similar to the exemplarily chosen regulatory framework of Denmark (DK1).

Fig. 5.1 provides an overview on the structure of the optimization problem formulation regarding all three functions. It shows that the same MILP formulation is used for all three functions and that the specific function depends on the input data provided into the model.

The main purpose of the scheduling function is to find the optimum amount of bids to be submitted into each power market. In addition, information about the optimum charge- and discharge rate of the BESS as well as the best WPP production set point for each time step is obtained in order to maximize profit. Input data for this function are all relevant technical data of the BESS (which is the same for all three functions) as well as the forecasted wind power production and power prices (for the different markets). The algorithm includes the DA-, PFR-, and RP markets (which is the same for all functions). Hence, all selected applications from sec. 2.9 are considered within the optimum scheduling- (and optimum sizing) function except forecast improvement.

Optimum dispatch addresses how to maximize the profit based on adjustments of input data of the optimum scheduling problem. Updates on the previous optimum
schedule can also be considered a rescheduling problem. The dispatch function is called e.g. when a new wind power production forecast becomes available or market prices differ from the forecast. The main information which is used from the results obtained of this function are the charging- and discharging power of the BESS as well as the power output of the WPP. This function includes the application of forecast improvement because any error on the forecast becomes obvious during actual operation and the BESS helps to reduce penalty costs for imbalances. Generally speaking, this function ensures the most profitable operation of the VPP considering that:

- the wind power production does not match the forecast,
- not all bids were awarded on the market,
- the actual SoC of the BESS differs from the model,
- or market prices differ from the forecast.

Additional input data necessary for this function are (a forecast of) penalty costs which have to be paid for imbalances. Compared to the optimum scheduling function, this function indicates the necessary adjustments of the results obtained from the optimum scheduling function.

The third function, optimum sizing of the BESS, is about finding the optimum set of BESSs and their optimum power rating and battery energy for a given study period. The chosen period should be representative to reflect assumed future prices and wind power production.

In order to increase readability of the problem formulation below, all decision variables begin with a capital letter and all parameters begin with a lower-case character. Additionally, sets begin also with a capital letter but sets do neither appear directly in the constraints nor in the cost function but only in connection with the universal quantifier (\(\forall\)) before the actual constraint and can thus be distinguished from decision variables.

Equations within gray shaded boxes are not part of the MILP formulation and are to be calculated (or assigned) in advance before running the optimization or they are only used for illustrative purposes.
5.1 Objective function

The objective function $OF$ is to be maximized expressing the profit (revenues minus costs) of the VPP which can be achieved on the different power markets over the discretized time periods $\tilde{t}$ (see equation 5.2). In order to account correctly for the profit of the VPP, annualized costs are used to determine the costs of the BESS and WPP over the specific time steps $\tilde{t}$ due to the fact that the WPP and BESS may have different investment horizons. In case the study period $\tilde{t}$ is shorter than one year, the corresponding fraction $\frac{\tilde{t} \cdot d_t}{8760}$ of the annualized costs is considered.

The formula to determine annualized costs ($\text{annualizedCost}_b$) is shown in equation 5.1 where $pv$ is the present value at year zero, $i$ is the interest rate, and $ih$ is the investment horizon (see e.g. [87]). This formula is not part of the MILP formulation.

$$\text{annualizedCost}_b = \frac{pv}{(i+1)^{ih} - 1} \cdot \frac{1}{i(1+i)^{ih}} \quad (5.1)$$

Revenues are generated by sales of power on the DA market, by selling RP up,
and by selling PFR up and down. Costs occur by purchasing power on the DA market, by selling RP down, and by the operation of the WPPs and the BESS(s). Regarding power purchases, the constant $purchaseCostSupplement$ is added to the market price in order to avoid selling the purchased amount within the same time period (see equation 5.4). RP down sales are added to the cost function with positive sign (and not with negative sign), because prices for RP down power have (mostly) negative values as provided by [1]. The following describes the objective function.

Maximize:

$$OF = \left( marketSales - marketPurchase + bonusWpp - costsWppOf - costsBess \\
+ regUpPowerSales + regDownPowerSales + primaryUpSales \\
+ primaryDownSales - buDexpr - bdDexpr \\
- buRegUpDexpr - bdRegUpDexpr \\
- buRegDownDexpr - bdRegDownDexpr \\
- buPrimaryUpDexpr - bdPrimaryUpDexpr \\
- buPrimaryDownDexpr - bdPrimaryDownDexpr \right) td_t$$

(5.2)

Revenues of power sales to the DA market and costs of power purchases are calculated straightforward in equations 5.3 and 5.4.

$$marketSales = \sum_t marketPrice_t \cdot Sale_t$$

(5.3)

$$marketPurchase = \sum_t (marketPrice_t + purchaseCostSupplement) Purchase_t$$

(5.4)

Concerning the costs of the WPP ($costsWppOf$ in equation 5.5), they are split into capital expenditures (CAPEX) and operational expenditures (OPEX) (see sec. A.1 on page 171 for more information on WPP costs). It is assumed that capital expenditures $costWppCapex$ occur independent of the use of the WPP
5.1 Objective function

while the operational expenditures $\text{costWppOpex}$ only depend on the operation of the WPP based on produced MWh electricity. This enables to make the right operating decision between the DA market, the RP upwards, and the RP downwards market based on OPEX and not considering CAPEX which always occurs also when not operating the WPP.

\[
\text{costsWppOf} = \text{costWppCapex} \cdot \frac{t}{8760} + \sum_t \left( \text{WindSpot}_t \cdot \text{costWppOpex} + \text{WindRegUp}_t \cdot \text{regUpEnergyFlowFactor} \cdot \text{costWppOpex} \right) - \sum_t \left( \text{WindRegDown}_t \cdot \text{regDownEnergyFlowFactor} \cdot \text{costWppOpex} \right) 
\] (5.5)

Besides the costs of the WPP, it is assumed that the WPP is operated under a bonus regime which guarantees an additional payment for each produced MWh of electricity. The revenues related to the bonus are defined by $\text{bonusWppOf}$ in equation 5.6.

\[
\text{bonusWppOf} = \sum_t \left( \text{WindSpot}_t \cdot \text{bonusWpp} + \text{WindRegUp}_t \cdot \text{regUpEnergyFlowFactor} \cdot \text{bonusWpp} \right) - \sum_t \left( \text{WindRegDown}_t \cdot \text{regDownEnergyFlowFactor} \cdot \text{bonusWpp} \right) 
\] (5.6)

The costs of the BESS are accounted for in equation 5.7. Two cases need to be distinguished: In case the sizing function is called ($\text{esInvestSwitch} = 1$), the annualized costs $\text{EsCostOF}_b$ are a function of the capacity fade of BESS $b$ which is discussed further below. If the scheduling or dispatch function are called ($\text{esInvestSwitch} = 0$), the annualized BESS costs are calculated in advance as
Chapter 5

Optimization problem formulation

\( esCostPerMw_b \) assuming a certain lifetime of the BESS \( b \).

\[
\text{costsBess} = \sum_b EsCostOF_b \cdot \text{dischargerMax}_b \cdot \text{esInvestSwitch} \cdot \frac{t}{8760} \tag{5.7}
\]

\[ + \sum_b esCostPerMw_b \cdot \text{dischargerMax}_b (1 - \text{esInvestSwitch}) \frac{t}{8760} \]

The costs and revenues of the ancillary service markets are defined in equations 5.8 to 5.11.

\[
\text{regUpPowerSales} = \sum_t \text{RegUp}_t \cdot \text{marketPriceRegUp}_t \tag{5.8}
\]

\[
\text{regDownPowerSales} = \sum_t \text{RegDown}_t \cdot \text{marketPriceRegDown}_t \tag{5.9}
\]

\[
\text{primaryUpSales} = \sum_t \text{PrimaryUp}_t \cdot \text{marketPricePrimaryUp}_t \tag{5.10}
\]

\[
\text{primaryDownSales} = \sum_t \text{PrimaryDown}_t \cdot \text{marketPricePrimaryDown}_t \tag{5.11}
\]

And last but not least, the costs for imbalances are defined in equations 5.12 to 5.21.

\[
\text{buDexpr} = \sum_t \text{BuPower}_t \cdot \text{buCost}_t \tag{5.12}
\]

\[
\text{bdDexpr} = \sum_t \text{BdPower}_t \cdot \text{bdCost}_t \tag{5.13}
\]
5.2 Constraints

\[ buRegUpDexpr = \sum_t buRegUpCost_t \cdot BuRegUp_t \] (5.14)

\[ bdRegUpDexpr = \sum_t bdRegUpCost_t \cdot BdRegUp_t \] (5.15)

\[ buRegDownDexpr = \sum_t buRegDownCost_t \cdot BuRegDown_t \] (5.16)

\[ bdRegDownDexpr = \sum_t bdRegDownCost_t \cdot BdRegDown_t \] (5.17)

\[ buPrimaryUpDexpr = \sum_t buPrimaryUpCost_t \cdot BuPrimaryUp_t \] (5.18)

\[ bdPrimaryUpDexpr = \sum_t bdPrimaryUpCost_t \cdot BdPrimaryUp_t \] (5.19)

\[ buPrimaryDownDexpr = \sum_t buPrimaryDownCost_t \cdot BuPrimaryDown_t \] (5.20)

\[ bdPrimaryDownDexpr = \sum_t bdPrimaryDownCost_t \cdot BdPrimaryDown_t \] (5.21)

5.2. Constraints

A high-level outline of the optimization problem is provided in Fig. 5.2. It shows the basic structure of the problem formulation which can be considered as a network flow programming problem with many side constraints (see sec. 3.2.1 on page 36). The idea to use network flow programming for the proposed model is based on [57], for instance. Fig. 5.2 depicts important nodes and arcs within the optimization problem formulation. The constraints of the proposed MILP formulation are grouped according to this overview in the following, namely BESS
source- and sink node, BESS cost, converter, WPP, and DA market as well as ancillary service markets.

The meaning of the converter in Fig. 5.2 does not refer to power electronics. It simply means accounting of the efficiency of the BESS in order to determine the correct flow to the BESS.

Fig. 5.2 also shows how the battery specific parameters are taken into account: “Calendric capacity fade” \((\text{capacityFadeRate}_t)\), “Exercised capacity fade” \((\text{capacityFadeRateThroughput}_b)\), “Self-consumption” \((\text{selfConsumption}_b)\), and “Self-discharge” \((\text{selfDischargerRate}_b)\).

![Figure 5.2: Overview optimization model](image)

### 5.2.1. BESS: Source storage

The basic model of the BESS is adapted from the hydro-power storage model proposed in [71]. Their concept of a lower and upper reservoir that is linked to each other by the in- and outflow considering an efficiency factor is also part of the proposed problem formulation in this thesis. However, certain parts are amended in order to have a BESS specific model. It can be assumed that the upper reservoir stores the available chemical energy of the BESS while the lower level of the reservoir can be regarded as the chargeable amount of energy, both, before the efficiency factor is applied for charging or discharging. The formulation of the unit commitment of the BESS is based on integer variables but it is written in a different way compared to [71] in order to accommodate decision variables for the sizing function and is adapted from [88], see sec. 5.2.4.
5.2 Constraints

In this thesis, the upper reservoir is called source storage node ($S_{osNode_{b,t}}$) and the lower reservoir is called sink storage node ($S_{isNode_{b,t}}$) in BESS $b$ out of $\bar{b}$ different BESS technologies. An efficiency factor is part of the converter that transforms the power input from MW to stored energy in MWh when multiplied with $t_{d,t}$ in the source- and sink node (see Fig. 5.2). The optimization model is written for VPPs that consist of any number of different BESSs. The general $S_{osNode_{b,t}}$ is formulated in constraint 5.22 for all time steps $t > 0$. For the first time step, the formulation differs. Either an initial SoC $initSoc_b$ for each $b$ can be specified in constraint 5.24, where the SoC (equal to the value of $S_{osNode_{b,t}}$) of the first time step ($t = 0$) is taken as a parameter. Alternatively, a constraint that forces the SoC to be the same for the first and last time step can be included (see constraint 5.26). This feature is especially valuable for longer study periods (e.g. finding optimum BESS) where the SoC of the beginning does not matter but where the SoC of the beginning ($t = 0$) should equal to the SoC at the end of the study period ($t = \bar{t}$).

Moreover, capacity fades and the self-discharge rate as well as self-consumption are accounted for in constraints 5.22, 5.24 and 5.26. Both capacity fades are modeled as a loss of the energy stored in the $S_{osNode_{b,t}}$ whereby the amount of reduced energy is proportional to $S_{osNode_{b,t}}$ (which can be considered as the SoC if divided by the capacity of BESS $b$) for the calendric capacity fade while the exercised capacity fade is proportional to the charging- and discharging power of the BESS (also see page 20f). The formulation of capacity is inspired by [68, 69, 70].

In addition to capacity fade, both – self-discharge and self-consumption – are modeled as an arc from the $S_{osNode_{b,t}}$ to the $S_{isNode_{b,t}}$ causing the BESS to discharge. The self-consumption ($selfConsumption_b$) is a constant while the self-discharge ($selfDischargeRate_b$) is assumed to depend proportionally on the $S_{osNode_{b,t}}$ value.

It is assumed that self-consumption is independent of the operation of the BESS and it is assumed to be constant. It is further assumed that power for self-consumption is drawn out of the BESS and not from the grid due to security measures ensuring uninterruptable power supply even during potential short outages of the grid. It has to be pointed out that self-consumption does not include power consumption due to inefficiency of the BESS, the converter, or the trans-
The decision variables $ChargerFlow_{b,t}$ and $DischargerFlow_{b,t}$ are explained in sec. 5.2.4.

\[
\forall b \in B \land \forall t \in T : t \geq 1 : \\
SosNode_{b,t-1} + td_t \cdot ChargerFlow_{b,t} = \\
SosNode_{b,t} + td_t \cdot DischargerFlow_{b,t} \\
+ selfDischargerRate_b \cdot td_t \cdot SosNode_{b,t-1} \\
+ selfConsumption_b \cdot td_t \cdot NbSlicesActive_b \cdot dischargerMax_b \\
+ capacityFadeRate_b \cdot td_t \cdot SosNode_{b,t-1} \\
+ capacityFadeRateThroughput_b \cdot td_t \cdot \\
\left( \frac{ChargerFlow_{b,t}}{chargerConversionFactor_b} + \frac{DischargerFlow_{b,t}}{dischargerConversionFactor_b} \right)
\] (5.22)

The following constraint 5.23 as well as constraints 5.25, 5.27, 5.30, 5.32, and 5.34 ensure that the DK1 market requirement for PFR is met. This guarantees that power can be provided for a certain period. Due to the fact that the PFR requirements depend on the source- and sink nodes, they are stated here but the description is provided further below in sec. 5.2.8 on page 97 f. after the PFR market constraints have been discussed for better understanding. Therefore, constraints 5.23, 5.25, 5.27, 5.30, 5.32, and 5.34 are stated here without further discussion.

\[
\forall b \in B \land \forall t \in T : t \geq 1 : \\
SosNode_{b,t-1} \cdot chargerConversionFactor_b \geq primaryUpReserveFactor \cdot DischargerPrimaryUp_{b,t} \cdot td_t \\
\] (5.23)

The following parameter $switch$ allows to select the SoC of the first time step ($switch = 1$) or to enforce that the SoC of the last and first time step are equal ($switch = 0$). The if-then-else statement has to be implemented in the programming language of the optimization package program and needs to be called before the optimization is started in order to build the correct problem formulation.
5.2 Constraints

If \( \text{switch} = 1 \)

Then:

\[
\forall b \in B \land t = 0 : \\
\begin{align*}
\text{NbSlicesActive}_b \cdot \text{sosNodeMax}_b \cdot \text{initSoc}_b \\
+ \text{ChargerFlow}_{b,t} \cdot td_t = \\
\text{SosNode}_{b,t} + \text{DischargerFlow}_{b,t} \cdot td_t \\
+ \text{self DischargerRate}_b \cdot td_t \cdot \text{sosNodeMax}_b \\
\cdot \text{initSoc}_b \cdot \text{NbSlicesActive}_b \\
+ \text{self Consumption}_b \cdot td_t \cdot \text{NbSlicesActive}_b \cdot \text{dischargerMax}_b \\
+ \text{capacityFadeRate}_b \cdot td_t \cdot \text{sosNodeMax}_b \cdot \text{initSoc}_b \cdot \text{NbSlicesActive}_b \\
+ \text{capacityFadeRateThroughput}_b \cdot td_t \\
\cdot \left( \frac{\text{ChargerFlow}_{b,t}}{\text{chargerConversionFactor}_b} + \frac{\text{DischargerFlow}_{b,t}}{\text{dischargerConversionFactor}_b} \right) \\
\end{align*}
\] (5.24)

\[
\forall b \in B \land t = 0 : \\
\text{NbSlicesActive}_b \cdot \text{sosNodeMax}_b \cdot \text{initSoc}_b \\
\cdot \text{chargerConversionFactor}_b \\
\geq \text{primaryUpReserveFactor} \cdot \text{Discharger PrimaryUp}_{b,t} \cdot td_t
\] (5.25)

Else:

\[
\forall b \in B \land t = 0 : \\
\text{SosNode}_{b,\bar{t}} + \text{ChargerFlow}_{b,t} \cdot td_t = \\
\text{SosNode}_{b,\bar{t}} + \text{DischargerFlow}_{b,t} \cdot td_t \\
+ \text{self DischargerRate}_b \cdot td_t \cdot \text{SosNode}_{b,\bar{t}} \\
+ \text{self Consumption}_b \cdot td_t \cdot \text{NbSlicesActive}_b \cdot \text{dischargerMax}_b \\
+ \text{capacityFadeRate}_b \cdot td_t \cdot \text{SosNode}_{b,\bar{t}} \\
+ \text{capacityFadeRateThroughput}_b \cdot td_t \\
\cdot \left( \frac{\text{ChargerFlow}_{b,t}}{\text{chargerConversionFactor}_b} + \frac{\text{DischargerFlow}_{b,t}}{\text{dischargerConversionFactor}_b} \right)
\] (5.26)
∀b ∈ B ∧ t = 0 :

\[ SosNode_{b,t} \cdot \text{chargerConversionFactor}_b \geq \text{primaryUpReserveFactor} \cdot \text{DischargerPrimaryUp}_{b,t} \cdot t_d \] (5.27)

The limit of the \( SosNode_{b,t} \), which can be regarded as the maximum storable battery energy, is specified by constraint 5.28:

∀b ∈ B ∧ ∀t ∈ T : \( SosNode_{b,t} \leq sosNodeMax_b \cdot \text{NbSlicesActive}_b \) (5.28)

### 5.2.2. BESS: Sink storage

Corresponding to the source storage node constraint, the sink storage node constraint in the first time step differs from the other time steps (see constraints 5.29, 5.31, and 5.33):

∀b ∈ B ∧ ∀t ∈ T : t ≥ 1 :

\[ SisNode_{b,t-1} + \text{DischargerFlow}_{b,t} \cdot t_d + \text{selfDischargerRate}_b \cdot t_d \cdot SosNode_{b,t-1} + \text{selfConsumption}_b \cdot t_d \cdot \text{NbSlicesActive}_b \cdot \text{dischargerMax}_b = SisNode_{b,t} + \text{ChargerFlow}_{b,t} \cdot t_d \] (5.29)

Equivalent to the source storage node, the sink storage node has to ensure a certain SoC to be able to provide PFR down and not to violate the minimum required time to provide PFR (see constraints 5.30, 5.32, and 5.34):

∀b ∈ B ∧ ∀t ∈ T : t ≥ 1 :

\[ SisNode_{b,t-1} \cdot \text{dischargerConversionFactor}_b \geq \text{primaryDownReserveFactor} \cdot \text{ChargerPrimaryDown}_{b,t} \cdot t_d \] (5.30)
5.2 Constraints

Again, the \textit{switch} parameter is used to select either the SoC of the first time step or to enforce that the SoC of the last and first time step are equal.

\textbf{If} : \textit{switch} = 1

\textbf{Then} : 

\[
\forall b \in B \land t = 0 : \\
NbSlicesActive_b \cdot sosNodeMax_b \cdot (1 - initSoc_b) \\
+ td_t \cdot DischargerFlow_{b,t} \\
+selfDischargerRate_b \cdot td_t \cdot sosNodeMax_b \\
initSoc_b \cdot NbSlicesActive_b \\
+ selfConsumption_b \cdot td_t \cdot NbSlicesActive_b \cdot dischargerMax_b \\
= SiSNode_{b,t} + td_t \cdot ChargerFlow_{b,t} \\
\]

\textbf{Else} : 

\[
\forall b \in B \land t = 0 : \\
NbSlicesActive_b \cdot sosNodeMax_b \cdot (1 - initSoc_b) \\
dischargerConversionFactor_b \geq primaryDownReserveFactor \cdot ChargerPrimaryDown_{b,t} \cdot td_t \\
\]

\[
\forall b \in B \land t = 0 : \\
SisNode_{b,t} + td_t \cdot DischargerFlow_{b,t} \\
+selfDischargeRate_b \cdot td_t \cdot SosNode_{b,t} \\
+selfConsumption_b \cdot td_t \cdot NbslicesActive_b \cdot dischargerMax_b \\
= SisNode_{b,t} + td_t \cdot ChargerFlow_{b,t} \\
\]

\[
\forall b \in B \land t = 0 : \\
SisNode_{b,t} \cdot dischargerConversionFactor_b \geq primaryDownReserveFactor \cdot ChargerPrimaryDown_{b,t} \cdot td_t \\
\]
Finally, the maximum amount of storable energy in the $SisNode_{b,t}$ is specified in constraint 5.35. This constraint is used to implement the minimum SoC of the BESS through a proper value of $sisNodeMax_b$ which has to be calculated in advance of the optimization with equation 5.36 (which is not part of the MILP problem formulation). Hence, BESS $b$ is not able to reach any SoC lower than specified by $minSoc_b$ because $SosNode_{b,t}$ cannot be discharged with more energy than $SisNode_{b,t}$ is able to absorb.

$$\forall b \in B \land \forall t \in T : SisNode_{b,t} \leq sisNodeMax_b \cdot NbSlicesActive_b \quad (5.35)$$

$$\forall b \in B : \quad sisNodeMax_b = (1 - minSoc_b) \cdot sosNodeMax_b \quad (5.36)$$

### 5.2.3. Accounting for BESS costs

Accounting for the costs of the BESS differs between the optimum sizing function and the optimum dispatch/scheduling function. Concerning the sizing function, the BESS costs are not known in advance compared to the dispatch and scheduling function which consider a pre-defined BESS size and lifetime. Therefore, constraints 5.37 to 5.68 are especially introduced for the MILP formulation of the sizing function. They are not required for the scheduling and dispatch function. In order to keep the algorithm reasonable fast for the dispatch and scheduling function, the parameter $esInvestSwitch$ is introduced which has to be set accordingly to distinguish the different functions. If the sizing function should be used, $esInvestSwitch$ has to be equal to 1, otherwise it has to be equal to 0 for the dispatch and scheduling function:

*If* : $esInvestSwitch = 1$

*Then* : {


5.2 Constraints

SIZING FUNCTION

The objective of the sizing function is to find the optimum power rating and battery energy and optimum BESS technology(ies) which are - as opposed to the scheduling- and dispatch function - not known in advance. Thus, new decision variables and constraints need to be formulated that indicate the power rating and battery energy(ies) and BESS technology(ies) of the optimum BESS (or combination of BESSs, called hybrid BESS). This approach assumes that the considered BESS technologies have a fixed relation between battery energy and power rating and hence, if the optimum power rating is found, the optimum battery energy is also known.

The sizing function makes use of the previously introduced capacity fades as a measure for the BESS lifetime. The higher the capacity fade, the sooner the BESS has to be replaced and the costs of the BESS can be related to the capacity fade. In this work, it is assumed that the BESS has to be replaced when the BESS has reached 80% of its original capacity. The final capacity fade is determined by constraint 5.37 and the decision variable $FinalCapacity_b$ is introduced indicating the remaining capacity at $t$, also named SoH (see [68]). In addition, it has to be specified that the decision variable $FinalCapacity_b$ cannot be lower than 80% (or any other predefined value) which is when the BESS has reached its end of life. See [30] and [69] for a discussion on capacity fade and appropriate limits when to replace a BESS.

Modeling capacity at final time step

The $FinalCapacity_b$ is calculated based on equation 5.37 where the decision variable $NbSlicesActive_b$ indicates how many slices of the BESS $b$ are selected in the optimum solution. This concept is also used in [86] to find the optimum size of a thermal BESS and the authors call it finding the optimum size based on a “step function”. In this thesis, $NbSlicesActive_b$ is an integer decision variable and indicates how many slices (or steps) in BESS $b$ are activated ($NbSlicesActive_b \geq 1$) or if none of the slices is activated ($NbSlicesActive_b = 0$). In this manner, the optimum power rating of the BESS with the resolution of the slice (step) size is indicated. The slice size in MW is defined by the parameters $chargerMax_b$. 
and $\text{dischargerMax}_b$ and the slice size concerning the battery energy in MWh is defined by $\text{sosNodeMax}_b$ and $\text{sisNodeMax}_b$.

$$\forall b \in B : \text{FinalCapacity}_b = \frac{\text{SisNode}_b, t}{\text{sosNodeMax}_b \cdot \text{NbSlicesActive}_b} + \frac{\text{SosNode}_b, t}{\text{sosNodeMax}_b \cdot \text{NbSlicesActive}_b}$$ (5.37)

However, it is not possible to implement equation 5.37 in a MILP formulation for two reasons:

1. the equation is non-linear and not in separable form
2. $\text{FinalCapacity}_b$ is not defined if $\text{NbSlicesActive}_b = 0$

In order to overcome this problematic, separable programming is applied in combination with PLAs to linearize equation 5.37. In the next step, an indicator variable (in this case $\text{BatteryInt}_b$ which is further discussed in sec. 5.2.4) is used to model the following indicator constraint:

$$\forall b \in B : \text{BatteryInt}_b > 0 \rightarrow \text{equation 5.37 (in linearized form).}$$

By this procedure, $\text{FinalCapacity}_b$ is only modeled for $\text{BatteryInt} > 0$ that implies $\text{NbSlicesActive}_b > 0$.

In equations 5.38 to 5.42, the formulation to convert equation 5.37 into separable form is stated. First, equation 5.37 is converted into terms that contain only two decision variables (equation 5.38).

$$\forall b \in B : \text{FinalCapacity}_b = \frac{\text{SisNode}_b, t}{\text{sosNodeMax}_b \cdot \text{NbSlicesActive}_b} + \frac{\text{SosNode}_b, t}{\text{sosNodeMax}_b \cdot \text{NbSlicesActive}_b}$$ (5.38)

Then, each term can be converted into separable form as discussed in sec. 3.2.3 by introducing extra decision variables $S_{a,b}$ with $a = 0, 1, 2, 3$ and $\forall b \in B$ (compare [49]) and relating them to the following equality constraints (5.39 to 5.42).
5.2 Constraints

\[
S_{0,b} = 0.5 \left( S_{\text{isNode}_{b,t}} + \frac{1}{\text{NbSlicesActive}_b} \right) \tag{5.39}
\]

\[
S_{1,b} = 0.5 \left( S_{\text{isNode}_{b,t}} - \frac{1}{\text{NbSlicesActive}_b} \right) \tag{5.40}
\]

\[
S_{2,b} = 0.5 \left( S_{\text{osNode}_{b,t}} + \frac{1}{\text{NbSlicesActive}_b} \right) \tag{5.41}
\]

\[
S_{3,b} = 0.5 \left( S_{\text{osNode}_{b,t}} - \frac{1}{\text{NbSlicesActive}_b} \right) \tag{5.42}
\]

However, the term \( \frac{1}{\text{NbSlicesActive}_b} \) is non-linear and has to be linearized based on a PLA as described in sec. 3.2.2 (compare [49]). The advantage is that \( \frac{1}{\text{NbSlicesActive}_b} \) has to be only once approximated with a PLA and can be used in equation 5.39 to 5.42. Furthermore, this is not the only necessary PLA throughout this work and in order to avoid repetition, the PLA approach based on sec. 3.2.2 is re-written in equations 5.43 to 5.49 and \( a \) denotes which PLA is meant where \( P_a \) is the set of breakpoints \( p_a = 0, ..., p_a - 1 \) used for the \( (a+1)^{th} \) linear-approximation.

One breakpoint is defined by \( (x_{a,p_a,b}, f(x_{a,p_a,b})) \).

\[
\forall a \in A \land \forall b \in B : \quad PlaY_{a,b} = \sum_{p_a} (\lambda_{a,p_a,b} \cdot f(x_{a,p_a,b})) \tag{5.43}
\]

\[
\forall a \in A \land \forall b \in B : \quad PlaX_{a,b} = \sum_{p_a} (\lambda_{a,p_a,b} \cdot x_{a,p_a,b}) \tag{5.44}
\]

\[
\forall a \in A \land \forall b \in B : \quad \sum_{p_a} \lambda_{a,p_a,b} = 1 \tag{5.45}
\]

The variable \( \lambda_{a,p_a,b} \) can be considered as weights attached to each breakpoint of the approximation (see [49]).
The adjacency condition (constraint 5.46 to 5.49, see [49, 54]) needs to be modeled except under certain conditions which are described below. A thorough discussion on the adjacency condition can be found in [54]. In a maximization problem, the adjacency condition does not need to be implemented if:

- The PLA is in the objective function and the approximated function is concave.
- The PLA is on the left hand side of a \( \leq \) constraint and the approximated function is concave.
- The PLA is on the left hand side of a \( \geq \) constraint and the approximated function is concave.

In case an equality constraint contains a PLA approximation, it should be converted to a \( \leq \) and \( \geq \) constraint and thus the adjacency condition needs to be implemented.

In this model, the adjacency condition needs to be implemented at every PLA because the above stated conditions that allow to omit the adjacency conditions are not met. The reason is shown at each PLA below.

\[
\forall a \in A \land p_a = 0 \land \forall b \in B : \lambda_{a,p_a,b} \leq \text{IntPLA}_{a,p_a,b} \tag{5.46}
\]

\[
\forall a \in A \land 1 \leq p_a \leq \overline{p}_a - 2 \land \forall b \in B :
\lambda_{a,p_a,b} \leq \text{IntPLA}_{a,p_a-1,b} + \text{IntPLA}_{a,p_a,b} \tag{5.47}
\]

\[
\forall a \in A \land p_a = \overline{p}_a - 1 \land \forall b \in B : \lambda_{a,p_a,b} \leq \text{IntPLA}_{a,p_a-1,b} \tag{5.48}
\]

\[
\forall a \in A \land \forall b \in B : \sum_{p_a=0}^{\overline{p}_a-2} \text{IntPLA}_{a,p_a,b} = 1 \tag{5.49}
\]

Having written the PLA in a generic manner, the term \( \frac{1}{\text{NbSlicesActive}_b} \) can now be piecewise linear approximated based on constraint 5.43 to 5.49 and \( a = 4 \) because it is the 5\(^{th} \) PLA (because equation 5.39 to 5.42 are converted to separable
5.2 Constraints

form in equation 5.56 and finally also need to be approximated with a PLA and
\( a = 0, 1, 2, 3 \) is reserved for their PLAs).

\[
\forall b \in B : \quad PlaX_{4,b} = NbSliceActive_b \tag{5.50}
\]

\[
\forall b \in B : \quad PlaY_{4,b} = ReciprocalNbSliceActive_b \tag{5.51}
\]

Due to the fact that \( \frac{1}{NbSlicesActive_b} \) appears in equality constraints 5.52 to 5.55,
the adjacency condition needs to be implemented. (In case the adjacency condition
does not need to be implemented, \( PlaY_{4,b} \) could not be assigned to
\( ReciprocalNbSliceActive_b \) in constraint 5.51 for better readability and \( PlaY_{4,b} \)
would need to be directly implemented in constraints 5.52 to 5.55 instead of
\( ReciprocalNbSliceActive_b \). This needs also to be considered for the other PLAs.)

Based on constraints 5.50 and 5.51, \( \frac{1}{NbSlicesActive_b} \) can be replaced with
\( ReciprocalNbSliceActive_b \) in equation 5.39 to 5.42:

\[
S_{0,b} = 0.5 \left( \text{SisNode}_{b,t} + ReciprocalNbSliceActive_b \right) \tag{5.52}
\]

\[
S_{1,b} = 0.5 \left( \text{SisNode}_{b,t} - ReciprocalNbSliceActive_b \right) \tag{5.53}
\]

\[
S_{2,b} = 0.5 \left( \text{SosNode}_{b,t} + ReciprocalNbSliceActive_b \right) \tag{5.54}
\]

\[
S_{3,b} = 0.5 \left( \text{SosNode}_{b,t} - ReciprocalNbSliceActive_b \right) \tag{5.55}
\]

Finally, equation 5.38 can be re-written in separable form as:

\[
\forall b \in B : \quad \text{FinalCapacity}_b = \frac{1}{\text{sosNodeMax}_b} \left( S_{0,b}^2 - S_{1,b}^2 + S_{2,b}^2 - S_{3,b}^2 \right) \tag{5.56}
\]

The non-linear terms in equation 5.56 can also be linearized based on a PLA as
shown above. The breakpoints for the linearization of the non-linear terms \( S_{a,b}^2 \)
are provided by:
(x_{a,p_a,b}, f(x_{a,p_a,b})), \quad a = 0, 1, 2, 3 \land \forall p_a \in P_a \land \forall b \in B \text{ where, again, } P_a \text{ is the set of breakpoints used for the } (a + 1)^{th} \text{ linear-approximation.}

\begin{align*}
a = 0, 1, 2, 3 \land \forall b \in B : & \quad PlaY_{a,b} = Z_{a,b} \quad (5.57) \\
a = 0, 1, 2, 3 \land \forall b \in B : & \quad PlaX_{a,b} = S_{a,b} \quad (5.58)
\end{align*}

Due to the fact that the terms \(S_{a,b}^2\) appear in the equality constraint 5.56, the adjacency condition needs to be modeled. Then, \(S_{a,b}^2\) can be replaced with \(Z_{a,b}\) for \(a = 0, 1, 2, 3\) and \(\forall b \in B\). Equation 5.56 can be rewritten as:

\[
\forall b \in B : \quad FinalCapacity_{b} = \frac{1}{sosNodeMax_b} (Z_{0,b} - Z_{1,b} + Z_{2,b} - Z_{3,b}) \quad (5.59)
\]

Having linearized equation 5.37, the next step is to define equation 5.59 only if \(BatteryInt_b > 0\) which also implies that \(NbSlicesActive_b > 0\). This results in the indicator constraint 5.60. It can be implemented as described in [49] (chapter 9), known as the small-m and big-M constraints. However, an advanced software package is likely to be more efficient if directly implementing indicator constraint 5.60 in a provided function which would probably use a different approach to address constraint 5.60 not using the small-m and big-M constraints:

\[
\forall b \in B : \quad BatteryInt_b > 0 \rightarrow \\
\quad FinalCapacity_{b} = \frac{1}{sosNodeMax_b} (Z_{0,b} - Z_{1,b} + Z_{2,b} - Z_{3,b}) \quad (5.60)
\]

Furthermore, it is necessary to limit the \(FinalCapacity_{b}\) to values greater than or equal to 0.8 in order to not exceed the above stated end-of-life criteria of the BESS \(b\). However, this is only applicable for BESS \(b\) selected in the final solution which is indicated by \(BatteryInt_b\) if equals 1. If this would be enforced for batteries which are not part of the final solution, this would cause a conflict with equation 5.37 concerning the \(SisNode_{b,t}\) and \(SosNode_{b,t}\) decision variable.
5.2 Constraints

Therefore, $\text{BatteryInt}_b$ is used as an indicator variable in order to model the following condition in indicator constraint 5.61:

$$\forall b \in B: \quad \text{BatteryInt}_b = 1 \rightarrow \text{FinalCapacity}_b \geq 0.8 \quad (5.61)$$

This could again be modeled with a small-m constraint as proposed by [49] but using a dedicated function of a software package is recommended instead for better efficiency of the solver.

**Modeling BESS’ costs based on final capacity**

Once the decision variable $\text{FinalCapacity}_b$ is properly modeled, it can be used to determine the costs of a BESS depending on its usage because $\text{FinalCapacity}_b$ contains information of the total BESS lifetime. This relationship between the BESS’s lifetime and $\text{FinalCapacity}_b$ is represented in constraint 5.62 (equations 5.62 to 5.64 are again not part of the MILP formulation as indicated by the gray shaded box and need to be calculated in advance of the optimization):

$$\forall b \in B: \quad \text{yearsEsOperation}_b = \frac{0.2 \cdot \frac{1}{td_t} \cdot \frac{\bar{t}}{8760}}{1 - \text{FinalCapacity}_b} \quad (5.62)$$

Fig. 5.3 illustrates equation 5.62 and indicates the total BESS lifetime in relation to the remaining capacity between 0.8 and 1 at $\bar{t} = 8760$ and $td_t = 1$. It shows that the BESS would need to be replaced after one year if the capacity is at 80% of its original value. If the capacity is at 90%, the BESS would need to be replaced after two years, etc.

In the next step, the total BESS lifetime $\text{yearsEsOperation}_b$ of BESS $b$ is used to calculate the annualized costs of the BESS which has to be accounted for in the objective function $\text{OF}$. The calculation of the annualized costs differs if $\text{yearsEsOperation}_b \geq 1$ year or $< 1$ year. Depending on the capacity fade, $td_t$, and $\bar{t}$, $\text{yearsEsOperation}_b$ of less than one year can be reached within 80% and 100% of the original capacity. This demands that the costs are accounted for correctly if the lifetime of the BESS is less than one year. These are rather
parameters in equation 5.63 are the present value \( pv_b \) which is the initial investment in the BESS \( b \) per MW and the interest rate \( i \). For \( yearsEsOperation_b \) of less than one year, it is assumed that the interest rate for the one year period is to be paid and that the investment can be done in fractions of the initial investment. For instance, if \( yearsEsOperation_b = 0.4 \), it is assumed that the BESS is replaced 2.5 times in one year. If \( yearsEsOperation_b \geq 1 \) year(s), the formula to calculate the annualized costs in equation 5.1 is applied which can be found in [87], for instance.

Using the annualized cost method to account for the BESS costs assumes that the investment is carried out over the least common multiple of the lifetimes of the BESSs if more than one BESS is part of the optimum solution, or that the salvage value of the BESS for periods shorter than the least common multiple of the lifetimes is as high to not alter the investment decision.
5.2 Constraints

∀b ∈ B :  
\[
\text{annualizedCost}_b = \begin{cases} 
\frac{p_v_b (1+i)}{\text{yearsEsOperation}_b} & \text{if } \text{yearsEsOperation}_b < 1 \\
 p_v_b \cdot \frac{i(1+i)^{\text{yearsEsOperation}_b}}{(1+i)^{\text{yearsEsOperation}_b-1}} & \text{if } \text{yearsEsOperation}_b \geq 1
\end{cases} 
\tag{5.63}
\]

The annualized BESS costs for a lifetime of 1 to 20 years are exemplarily depicted in Fig. 5.4 for a BESS \(b\) with \(p_v_b\) of \(10^6\)€/MW and \(i = 0.1\).

![Graph showing annualized costs of ES per MW vs. number of years of operation]

**Figure 5.4.**: BESS lifetime versus annualized BESS costs per MW

In the next step, the final capacity is linked to the annualized costs of the BESS by replacing \(\text{yearsEsOperation}_b\) in equation 5.63 by equation 5.62. The \(\text{annualizedCost}_b\) in equation 5.64 are now a function of the capacity \(\text{FinalCapacity}_b\), the present value \(p_v_b\), the time step \(t_d\), and the interest rate \(i\).
∀ \( b \in B \): \( annualizedCost_b = \)

\[
\begin{align*}
\begin{cases}
pv_b \frac{(1+i)}{0.2} \frac{t}{i} \frac{8760}{(1-FinalCapacity_b)} & \text{if } yearsEsOperation_b < 1 \\
pv_b \cdot \frac{i(1+i)}{0.2} \frac{t}{i} \frac{8760}{(1-FinalCapacity_b)} & \text{if } yearsEsOperation_b \geq 1
\end{cases}
\end{align*}
\] (5.64)

Fig. 5.5 shows exemplarily the relation between capacity and annualized costs per MW for one specific BESS. It can be concluded that this is a non-linear function and thus a PLA has to be applied to account for the non-linearities. The PLA is again based on constraints 5.43 to 5.45 where \( a = 5 \) in combination with constraints 5.65 and 5.66. In addition to the points shown in Fig. 5.5, there is one additional point included for this PLA which is \((1.1, 0)\) for all \( b \). This enables to account for zero BESS costs \((EsCostVsCapacity_b = 0)\) if the BESS \( b \) is not chosen indicated by \( BatteryInt_b = 0 \), then the indicator constraint 5.60 is not enforced meaning that the lowest costs for this specific BESS \( b \) is chosen in the optimization and that is zero for \( FinalCapacity_b = 1.1 \). The value 1.1 is arbitrarily chosen and any point \( > 1 \) will achieve the desired effect (if the algorithm is able to numerically distinguish the chosen point from 1).

In case a BESS \( b \) is in the optimum indicated by \( BatteryInt_b = 1 \), \( FinalCapacity_b \) is restricted to \( 0.8 \leq FinalCapacity_b \leq 1 \) and the BESS costs cannot be zero because of constraints 5.61 and 5.60.

\[
\forall b \in B : \quad PlaY_{5,b} = EsCostVsCapacity_b
\] (5.65)

\[
\forall b \in B : \quad PlaX_{5,b} = FinalCapacity_b
\] (5.66)

Due to the fact that equation 5.64 is neither convex nor concave because of point \((1.1, 0)\), the adjacency condition in this PLA has to be modeled ([49, 54]). This is enforced by constraint 5.46 to 5.49 for \( a = 5 \).
5.2 Constraints

Figure 5.5.: Capacity versus annualized costs of BESS per MW, $i = 10\%$

Modeling of BESS costs in objective function

Besides defining the annualized BESS costs in relation to the capacity fade, the decision variable $NbSlicesActive_b$ is also necessary in order to calculate the costs of the BESS $b$ in relation to the activated slices. This would lead to the term $EsCostVsCapacity_b \cdot NbSlicesActive_b$ in the objective function but this formulation is not possible in MILP. To tackle this problem, the techniques of separable programming and PLA can be applied again. However, due to the fact that $NbSlicesActive_b$ has comparable small numerical values in relation to $EsCostVsCapacity_b$, tests with actual BESS parameters (parameters used in chapter 7) indicated that the error of the PLA can be significant causing the approximation of $EsCostVsCapacity_b \cdot NbSlicesActive_b$ not being meaningful. Therefore, the following formulation in constraints 5.67 and 5.68 is chosen instead of separable programming:

\[
\forall b \in B : NbSlicesActive_b = 0 \rightarrow EsCostOF_b = EsCostVsCapacity_b \quad (5.67)
\]

\[
\forall b \in B, \forall q_b \in Q_b : NbSlicesActive_b = q_b + 1 \rightarrow \quad (5.68)
\]

\[
EsCostOF_b = EsCostVsCapacity_b \cdot (q_b + 1)
\]
Two cases are to be addressed: First, an BESS \( b \) is not chosen \((NbSlicesActive_b = 0)\) and the costs for the BESS should be zero which is formulated in constraint 5.67. As discussed above on page 78, \( EsCostVsCapacity_b \) takes the value zero in this case. In the second case, \( NbSlicesActive_b > 0 \) which is formulated in constraint 5.68. The integer variable \( NbSlicesActive_b \) can be represented by a parameter in each indicator constraint when to be multiplied with \( EsCostVsCapacity_b \) and separable programming becomes unnecessary.

In case constraints 5.67 and 5.68 cannot be implemented in a dedicated function provided by a software package, they can be translated into indicator constraints as described in [49].

The advantage of this formulation is that there is no approximation error incurring by a PLA allowing for a more accurate calculation of \( EsCostOF_b \) (there might still be an error caused by modeling the indicator constraints 5.67 and 5.68 but it is considered as non-relevant). On the other hand, \( maxNbSlices_b \) should be chosen carefully as larger values can cause longer times to solve the model. This depends on the input parameters to the model and on the computer’s specifications that is used to solve the model. This could be overcome be several iterations of running the optimization problem, starting with a relatively large slice size and then refining the slice size around the previously calculated optimum.

(\text{This curly brace indicates the end of the sizing function})

**SCHEDULING AND DISPATCH FUNCTION**

For the scheduling and dispatch function, the BESS size is known in advance and the annualized BESS cost can be directly accounted for in the objective function (see constraint 5.7). This leads to the fact that the problem can be tightened because the above introduced constraints 5.43 to 5.68 are not part of the problem formulation for these two functions. However, when implementing the above formulation in a software package, the decision variables introduced in these constraints are still defined in the problem. Therefore, tightening the unused decision variables in the scheduling and dispatch function (see constraints 5.69 to 5.78) allows solving the problem faster. This can be done in the following
manner and is only executed when $esInvestSwitch = 0$ (see the following else statement):

Else : {

\[ \forall a \in A \land 0 \leq p_a \leq p_a - 2 \land \forall b \in B : \quad IntPLA_{a,p_a,b} = 1 \quad (5.69) \]

\[ \forall a \in A \land \forall p_a \in P_a \land \forall b \in B : \quad \lambda_{a,p_a,b} = 0 \quad (5.70) \]

\[ \forall b \in B : \quad NbSlicesActive_b = 1 \quad (5.71) \]

\[ \forall b \in B : \quad EsCostVsCapacity_b = 1 \quad (5.72) \]

\[ \forall b \in B : \quad BatteryInt = 1 \quad (5.73) \]

\[ \forall b \in B : \quad FinalCapacity_b = 1 \quad (5.74) \]

\[ \forall b \in B : \quad ReciprocalNbSliceActive_b = 1 \quad (5.75) \]

\[ a = 0, 1, 2, 3 \land \forall b \in B : \quad Z_{a,b} = 1 \quad (5.76) \]

\[ a = 0, 1, 2, 3 \land \forall b \in B : \quad S_{a,b} = 1 \quad (5.77) \]

\[ \forall b \in B : \quad EsCostOF_b = 1 \quad (5.78) \]

} (This curly brace indicates the end of the constraints used specifically in the scheduling- and dispatch function)

### 5.2.4. Converter

The converter transforms the power input ($Charger_{b,t}$) into the energy stored in the $SosNode_{b,t}$ while removing energy from the $SisNode_{b,t}$ (constraints 5.79 and 5.80). Also compare Fig.5.2. The term converter does not mean a power
Chapter 5 Optimization problem formulation

electronic device. However, in this model, it is the unit responsible for charging and discharging the BESS while accounting for its efficiency. The source storage node $S_{\text{osNode}}_{b,t}$ can be assumed as the upper pond of a reservoir while the $S_{\text{isNode}}_{b,t}$ can be considered as the lower one. While charging, the “medium” in the $S_{\text{isNode}}_{b,t}$ is pumped to the $S_{\text{osNode}}_{b,t}$. When discharging the storage, the “medium” flows from the $S_{\text{osNode}}_{b,t}$ to the $S_{\text{isNode}}_{b,t}$. The decision variables $\text{ChargerFlow}_{b,t}$ and $\text{DischargerFlow}_{b,t}$ are assigned to the flow of the “medium” in the storage. Fig. 5.2 illustrates this by the arcs pointing from the $S_{\text{isNode}}_{b,t}$ through the converter to the $S_{\text{osNode}}_{b,t}$ which can be considered as the $\text{ChargerFlow}_{b,t}$ and vice versa for the $\text{DischargerFlow}_{b,t}$.

The efficiency is considered as a constant in the $\text{chargerConversionFactor}$ and $\text{dischargerConversionFactor}$ which are the reciprocal of the square root of the round-trip efficiency of the BESS, or the square root of the round-trip efficiency, respectively, if no separate values (e.g. based on laboratory test) exist. Further, non-linearities are not considered for charging- or discharging the BESS because it is assumed that the storage is operated below its physical capabilities and that the efficiency can be assumed constant in that region. Physically, the maximum charging- and discharging power rates of BESSs depend (amongst others) on the voltage and are thus related to the SoC. However, in practice, battery energy storage management systems usually operate the BESS below their physical capabilities to ensure a greater lifetime of the BESS and this allows to assume maximum charging- and discharging power rates that are constant from 0% SoC to 100% SoC. This SoC range is not the physical possible range but limited by the battery management system (also see discussion in sec. 2.5 on page 22). Hence, no piecewise approximation of the charging- and discharging efficiency is made at this point in order to take non-linear efficiency curves into account.

Moreover, it is assumed that participation in the RP market requires that some amount of energy bid on the regulating power market will actually be requested by the operator. This is considered by the $\text{regDownEnergyFlowFactor}$ and the $\text{regUpEnergyFlowFactor}$ which has to be set between 0 and 1, with 1 signifying the operator calls for the entire bid.
5.2 Constraints

**Charger converter flow**

The ChargerFlow\(_b,t\) is modeled by 5.79:

$$\forall b \in B \land \forall t \in T :$$

$$Charger_{b,t} + ChargerRegDown_{b,t} \cdot \text{regDownEnergyFlowFactor} = \text{chargerConversionFactor}_b \cdot \text{ChargerFlow}_{b,t} \quad (5.79)$$

5.80 allows to limit ChargerFlow\(_b,t\). This should only be made if such a physical limit exists. Otherwise, a large number should be used for chargerFlowMax\(_b,t\) because the actual charging- and discharging power rating limitations are defined in constraints 5.86 and 5.90.

$$\forall b \in B \land \forall t \in T :$$

$$0 \leq ChargerFlow_{b,t} \leq \text{chargerFlowMax}_b \quad (5.80)$$

If the BESS is discharged, the stored energy in the SosNode\(_b,t\) has to be converted into power which can be sold on the different power markets. When power is requested from the storage, it flows out of the SosNode\(_b,t\) and back into the SisNode\(_b,t\). The efficiency of the discharger (dischargerConversionFactor) is the square root of the round-trip efficiency.

**Discharger converter flow**

The DischargerFlow\(_b,t\) is modeled by constraint 5.81:

$$\forall b \in B \land \forall t \in T :$$

$$Discharger_{b,t} + DischargerRegUp_{b,t} \cdot \text{regUpEnergyFlowFactor} = \text{dischargerConversionFactor}_b \cdot \text{DischargerFlow}_{b,t} \quad (5.81)$$

The parameter dischargerFlowMax\(_b,t\) in the constraint 5.82 should also be set to a large value unless there exists a physical requirement to limit it.

$$\forall b \in B \land \forall t \in T :$$

$$0 \leq DischargerFlow_{b,t} \leq \text{dischargerFlowMax}_b \quad (5.82)$$
**Charger commitment**

The charger commitment is determined through constraints 5.84, 5.93, and 5.94 where $ChargerCommitment_{b,t}$ is a Boolean decision variable and can be either 0 or 1. It ensures that the BESS can only be charged or discharged but not both at the same time within one time step, see sec. 5.2.4. This unit commitment approach is based on [88]. Moreover, the minimum- and maximum charging power rates ($chargerMin$ and $chargerMax$) are specified by constraints 5.83 and 5.86.

The concept of finding the optimum power rating of the BESS by searching for the optimum number of slices $NbSlicesActive_b$ (which can be considered as a step function) has been adapted from [86], constraint 7. The continuous decision variable $NbSlicesActiveTimestepCharger_{b,t}$ is included in constraints 5.83 to 5.86 in order to avoid the term $ChargerCommitment_{b,t} \cdot NbSlicesActive_b$ because a Boolean commitment variable which would be multiplied with an integer decision variable is not feasible in MILP. Another solution would be to tackle the problem again by separable programming and PLA. However, this would lead to a more complicated model. A better solution is to insert two extra constraints (5.84 and 5.85) to overcome this problem. This forces $NbSlicesActiveTimestepCharger_{b,t}$ to be 0 if $ChargerCommitment_{b,t}$ is 0, otherwise it cannot be greater than or equal the optimum slice number $NbSlicesActive_b$ which is a decision variable. The upper bound of $NbSlicesActive_b$ is $maxNbSlices_b$ for all BESSs $b$.

\[
\forall b \in B \wedge \forall t \in T : \\
chargerMin_b \cdot NbSlicesActiveTimestepCharger_{b,t} \leq Charger_{b,t} - ChargerPrimaryUp_{b,t} \tag{5.83}
\]

\[
\forall b \in B \wedge \forall t \in T : \\
NbSlicesActiveTimestepCharger_{b,t} \leq ChargerCommitment_{b,t} \cdot maxNbSlices_b \tag{5.84}
\]

\[
\forall b \in B \wedge \forall t \in T : \\
NbSlicesActiveTimestepCharger_{b,t} \leq NbSlicesActive_b \tag{5.85}
\]
5.2 Constraints

\[
\forall b \in B \land \forall t \in T : \\
chargerMax_b \cdot NbSlicesActiveTimestepCharger_{b,t} \geq Charger_{b,t} + ChargerRegDown_{b,t} + ChargerPrimaryDown_{b,t} \tag{5.86}
\]

Using the possibility to provide PFR up (ChargerPrimaryUp_{b,t}, constraint 5.83) or PFR down (ChargerPrimaryDown_{b,t}, constraint 5.86) based on the current charging level (Charger_{b,t}), is adapted from [58].

**Discharger commitment**

Similarly to the above, discharging of the BESS is formulated through constraints 5.87 to 5.90.

\[
\forall b \in B \land \forall t \in T : \\
dischargerMin_b \cdot NbSlicesActiveTimestepDischarger_{b,t} \leq Discharger_{b,t} - DischargerPrimaryDown_{b,t} \tag{5.87}
\]

\[
\forall b \in B \land \forall t \in T : \\
NbSlicesActiveTimestepDischarger_{b,t} \leq DischargerCommitment_{b,t} \cdot maxNbSlices_b \tag{5.88}
\]

\[
\forall b \in B \land \forall t \in T : \\
NbSlicesActiveTimestepDischarger_{b,t} \leq NbSlicesActive_b \tag{5.89}
\]

\[
\forall b \in B \land \forall t \in T : \\
dischargerMax_b \cdot NbSlicesActiveTimestepDischarger_{b,t} \geq Discharger_{b,t} + DischargerRegUp_{b,t} + DischargerPrimaryUp_{b,t} \tag{5.90}
\]

Similarly to the charger commitment, using the possibility to provide PFR down (DischargerPrimaryDown_{b,t}, constraint 5.87) or PFR up (DischargerPrimaryUp_{b,t}, constraint 5.90) based on the current discharging level (Discharger_{b,t}), is adapted from [58].
Other converter constraints

The constraint 5.91 determines by means of the Boolean decision variable $BatteryInt_b$ whether one BESS technology is included in the optimum solution if $BatteryInt_b = 1$.

$$\forall b \in B : \ NbSlicesActive_b \leq BatteryInt_b \cdot maxNbSlices_b \quad (5.91)$$

The if-then-statement in constraints 5.92 to 5.94 is used to tighten the problem formulation. If the constraint in 5.93 is sufficient, the problem formulation is tighter and the problem can be solved faster than using constraint 5.94. The tighter formulation is only possible if constraint 5.92 is true and in this case the optimum solution does not change for one specific constraint (which both ensure that either the charger or the discharger is activated but not both at the same time).

If : 

$$\forall b \in B : \ chargerMin_b \leq 0.0001 \land dischargerMin_b \leq 0.0001 \quad (5.92)$$

Then : 

$$\forall b \in B \land \forall t \in T : \ ChargerCommitment_{b,t} + DischargerCommitment_{b,t} = BatteryInt_b \quad (5.93)$$

Else : 

$$\forall b \in B \land \forall t \in T : \ ChargerCommitment_{b,t} + DischargerCommitment_{b,t} \leq BatteryInt_b \quad (5.94)$$
The following if-statement is used to specify constraint 5.96 only for the sizing function. The purpose of this constraint is to limit the number of different BESS technologies to less than or equal \( maxNbEs \).

\[
If : \{ \\
\quad esInvestSwitch = 1 \quad \quad (5.95)
\}
\]

\[
Then : \{ \\
\quad \sum_b BatteryInt_b \leq maxNbEs \quad (5.96)
\}
\]

5.2.5. Wind power plant (WPP)

The WPP is modeled in constraints 5.97 to 5.101. The \( WindSpot_t \) decision variable is the amount of power delivered each time step to the DA market from the WPP and the variables \( WindRegUp_t \) and \( WindRegDown_t \) define the amount of wind power delivered to the RP up- and down power market, respectively. The \( regUpEnergyFlowFactor \) and \( regDownEnergyFlowFactor \) are factors between 0 and 1 that reduce the amount of energy delivered to the RP market because a unit may not be called for an entire hour but only for a limited time which is unknown in advance. This factor can be adjusted in order to fit a specific power market and the expectation of the market participant.

According to the DK1 market rules (see sec. 2.8), market participants can simultaneously participate in the RP up and down market and they can bet that the upwards- and downwards RP are both called within the same hour. If this is the
case, the energy delivered for RP up can cancel out the energy delivered to the RP down market while receiving payments for up and down RP. However, it is more likely that only one direction of RP is called and that case is assumed for this model. This is implemented by the Boolean variables $WppRegUpCommitment_t$ and $WppRegDownCommitment_t$ (see constraints 5.99 to 5.101) which ensure that only $WindRegUp_t$ or $WindRegDown_t$ can be provided within the same time step. In case the result of the optimization algorithm is that $WindRegDown_t$ should not be provided in one time step, the plant operator can still manually decide to bid for $WindRegDown_t$ according to his or her judgment.

$$\forall t \in T: \quad \text{WindSpot}_t + \text{WindRegUp}_t \cdot \text{regUpEnergyFlowFactor} \leq \text{windMax}_t$$  \hspace{1cm} (5.97)

$$\forall t \in T: \quad \text{WindRegDown}_t \cdot \text{regDownEnergyFlowFactor} \leq \text{WindSpot}_t$$  \hspace{1cm} (5.98)

$$\forall t \in T: \quad \text{WindRegUp}_t \cdot \text{regUpEnergyFlowFactor} \leq \text{windMax}_t \cdot WppRegUpCommitment_t$$  \hspace{1cm} (5.99)

$$\forall t \in T: \quad \text{WindRegDown}_t \cdot \text{regDownEnergyFlowFactor} \leq \text{windMax}_t \cdot WppRegDownCommitment_t$$  \hspace{1cm} (5.100)

$$\forall t \in T: \quad WppRegDownCommitment_t + WppRegUpCommitment_t = 1$$  \hspace{1cm} (5.101)

Data for the parameter $\text{windMax}_t$ depend on the preferred function of the algorithm, whether the algorithm is used for the scheduling- or dispatch problem or for the sizing problem. It can be either the best available forecast for wind
5.2 Constraints

power production or it can be an actual wind power production time series which is explained below.

**Scheduling- and dispatch function**

For the scheduling- and dispatch function, the forecasted wind power production is to be used. The dispatch function can make use of a better forecast than the scheduling function because it is closer to the actual operation.

\[
\forall t \in T : \quad \text{windMax}_t = \text{windForecast}_t \tag{5.102}
\]

**Sizing function**

In contrast to the scheduling- and dispatch function, the sizing function demands that the actual (historic) WPP production data is used for \( \text{windMax}_t \).

\[
\forall t \in T : \quad \text{windMax}_t = \text{actualWppProduction}_t \tag{5.103}
\]

Constraints 5.102 and 5.103 are not part of the optimization problem but indicate what data should be used for \( \text{windMax}_t \).

**5.2.6. Day-ahead (DA) market node**

The DA market constraints are expressed in constraints 5.104 to 5.108. \( \text{Discharger}_{b,t} \), \( \text{WindSpot}_t \), and \( \text{Purchase}_t \) flow into the market node while \( \text{Charger}_{b,t} \) and \( \text{Sale}_t \) flow out of it.

The decision variables \( \text{BuPower}_t \) and \( \text{BdPower}_t \) are needed for the dispatch
function and are explained in the next subsection.

\[
\forall t \in T : \sum_{b} Discharger_{b,t} + WindSpot_{t} + Purchase_{t} + BuPower_{t} = \sum_{b} Charger_{b,t} + Sale_{t} + BdPower_{t} \tag{5.104}
\]

\[
\forall t \in T : purchaseMin_{t} \leq Purchase_{t} \leq purchaseMax_{t} \tag{5.105}
\]

\[
\forall t \in T : saleMin_{t} \leq Sale_{t} \leq saleMax_{t} \tag{5.106}
\]

\[
\forall t \in T : 0 \leq BuPower_{t} \leq buMax \tag{5.107}
\]

\[
\forall t \in T : 0 \leq BdPower_{t} \leq bdMax \tag{5.108}
\]

At this point, the input data needed for the scheduling- and sizing function are discussed separately from the input data needed for the dispatch function.

**Scheduling- & sizing function**

The input data needed for the scheduling- and sizing function are mentioned here without further explanation because the concept is introduced in sec. 5.2.7 below.

The balance up \( BuPower_{t} \) and balance down \( BdPower_{t} \) variables for the DA market have to be set equal zero (see equations 5.109 and 5.110).

\[
buMax = 0 \tag{5.109}
\]

\[
bdMax = 0 \tag{5.110}
\]
5.2 Constraints

Additionally, the variables $purchaseMax_t$ and $saleMax_t$ are used to define the maximum amount that should be purchased or sold from or to the DA market in each time step $t$. If this is unrestricted, large values are used for these parameters. Moreover, minimum sales and purchases can be specified. This is defined in equations 5.111 and 5.112 and they are assumed to be zero in this study. These minimum values depend on the market participants who can choose other minimum amounts if desired.

\[
\forall t \in T : \quad purchaseMin_t = 0 \quad (5.111)
\]

\[
\forall t \in T : \quad saleMin_t = 0 \quad (5.112)
\]

**Dispatch function**

The parameter $obligationPurchase_t$ is the awarded amount of purchase on the DA market and is further described in sec. 5.2.7 below.

\[
\forall t \in T : \quad purchaseMin_t = obligationPurchase_t \quad (5.113)
\]

\[
\forall t \in T : \quad purchaseMax_t = obligationPurchase_t \quad (5.114)
\]

The parameter $obligationSale_t$ is the amount that got awarded on the DA market for sales and is also described in sec. 5.2.7 below.

\[
\forall t \in T : \quad saleMin_t = obligationSale_t \quad (5.115)
\]
∀ \( t \in T \) : \( saleMax_t = obligationSale_t \)  

The parameters \( buMax \) and \( bdMax \) have to be chosen large enough to allow proper dispatching as they limit the amount of power that is available for upwards- and downwards regulation.

### 5.2.7. Concept of dispatch function

This sub-section discusses the implementation of the dispatch function concept and how input data differs for the different functions. As already explained above, the developed formulation is written in a generic manner in order to suit all three different functions and the user only needs to adjust input data for a specific function. First, input data required for the scheduling- and sizing function is discussed.

**Scheduling- & sizing function**

The dispatch problem is addressed as discussed in sec. 4.2 on page 48 according to [73] with the difference that it is applied on the DA- and ancillary service markets simultaneously. Each market has its own market node as illustrated as “Market” in Fig. 5.6. For simplicity, the inflow into the market node is only coming from the WPP in Fig. 5.6 which is previously called \( P_g \) in Fig. 4.1 but there might also be additional inflows into the node. The outflow of each market for the scheduling- and sizing function is a decision variable that defines how much power is assigned (scheduled) to each market in each time step called “Decision Variable” in Fig. 5.6, left-hand side. In addition to that, there is one more inflow and outflow connected to each market for the purpose of the dispatch function. The inflow is called “Bu” while the outflow is called “Bd” (see Fig. 5.6). These are two decision variables of no use for the scheduling- and the sizing function and are thus set equal to zero for all time steps (left side of Fig. 5.6).

To summarize, there is one node for each power market and each node has an inflow from the source of power generation and an outflow to a power market. In
addition, every market has an in- and outflow for dispatch purpose which is set equal zero for the scheduling- and sizing function. Moreover, the market for PFR and RP -up and -down are each treated as a separate market (one for upwards regulation and one for downwards regulation). In case of RP down, the “Decision Variable” becomes an inflow into the “Market” node and the WPP and BESS are an outflow.

**Figure 5.6.:** Variation of input data between scheduling & sizing function and dispatch function (concept adapted from [73])

**Dispatch function**

The formulation of the dispatch problem makes use of the decision variables called “Bu” and “Bd” in Fig.5.6 (right-hand side) which can, in this case, take any value greater than or equal to zero (indicated by the white fill). Contrary to the scheduling- and sizing function, the “Decision Variable” from the left-hand side of Fig.5.6 becomes now an obligation in the dispatch function and has to take a certain value. This value should equal the amount awarded on that specific market (see e.g. constraints 5.113 to 5.116 for the DA market) based on the result of the scheduling function.

In order to call the dispatch function, a forecast of the penalty costs for imbalances is needed. The forecasted imbalance costs are multiplied with the “Bu” or “Bd”
decision variables accordingly in the objective function (compare equations 5.12 to 5.21).

5.2.8. Ancillary services

This subsection discusses the formulation of the RP- and PFR markets. First, the RP formulation is discussed followed by the formulation of PFR.

Regulation power up

RP upwards regulation (\(RegUp_t\)) can be provided by the BESS (reducing charging power or increasing discharging power: see sec. 5.2.4) or by increasing the WPP output. These three possibilities have to be summed up in constraint 5.117 which is the RP up power market node. Constraint 5.118 specifies the lower and upper limit for \(RegUp_t\) (which can be considered as bounds). Also, the decision variables for the dispatch function are included in constraints 5.119 and 5.120 which can be regarded as lower and upper bound on \(BuRegUp_t\) and \(BdRegUp_t\).

\[
\forall t \in T : \quad RegUp_t + BdRegUp_t = \sum_b DischargerRegUp_{b,t} + WindRegUp_t + BuRegUp_t \tag{5.117}
\]

\[
\forall t \in T : \quad regUpMin_t \leq RegUp_t \leq regUpMax_t \tag{5.118}
\]

\[
\forall t \in T : \quad 0 \leq BuRegUp_t \leq buRegUpMax \tag{5.119}
\]

\[
\forall t \in T : \quad 0 \leq BdRegUp_t \leq bdRegUpMax \tag{5.120}
\]

Scheduling & sizing function  In case the algorithm is used for the scheduling- or sizing function, the constant \(regUpMax_t\) needs to have assigned a proper value for all time steps. If there is a maximum quantity that should not be exceeded, the desired value to \(regUpMax_t\) should be assigned. If not, it should be a large number for all time steps. If the desired minimum is zero, it means that there is
5.2 Constraints

no specific quantity that is to be bid in this market, and in this case $regUpMin_t$ gets assigned the value zero for all time steps.

Moreover, $BuRegUp_t$ and $BdRegUp_t$ have to be set equal to zero in the scheduling- and sizing problem (see equations 5.121 and 5.122). Hence, balancing power will be zero in the optimum solution.

\[
buRegUpMax = 0 \quad (5.121)
\]

\[
bdRegUpMax = 0 \quad (5.122)
\]

**Dispatch function** When the dispatch function is used, the parameters $regUpMin_t$ and $regUpMax_t$ are both equal to the amount awarded on the RP up market in each time step ($obligationRegUp_t$), see equations 5.123 and 5.124. In this way the decision variable $RegUp_t$ equals the amount awarded on the RP up market. The parameters $buRegUpMax$ and $bdRegUpMax$ have to be able to take values greater than or equal zero.

\[
\forall t \in T : \quad regUpMin_t = obligationRegUp_t \quad (5.123)
\]

\[
\forall t \in T : \quad regUpMax_t = obligationRegUp_t \quad (5.124)
\]

**Regulation power down**

Similar to constraint 5.117, the RP down market node is defined by the equality constraint 5.125 but this time $RegDown_t$ is flowing into the node and $ChargerRegDown_{b,t}$ and $WindRegDown_t$ is flowing out of the node. Constraint 5.126 specifies the lower and upper limit for $RegDown_t$ and can be implemented
in a programming package as lower- and upper bound. Moreover, the decision variables for the dispatch function for RP down are also included and constraints 5.127 and 5.120 can be regarded as lower- and upper bound on $BuRegDown_t$ and $BdRegDown_t$.

\[
\forall t \in T : \quad RegDown_t + BuRegDown_t = \sum_b ChargerRegDown_{b,t} + WindRegDown_t + BdRegDown_t \tag{5.125}
\]

\[
\forall t \in T : \quad regDownMin_t \leq RegDown_t \leq regDownMax_t \tag{5.126}
\]

\[
\forall t \in T : \quad 0 \leq BuRegDown_t \leq buRegDownMax \tag{5.127}
\]

\[
\forall t \in T : \quad 0 \leq BdRegDown_t \leq bdRegDownMax \tag{5.128}
\]

**Scheduling & sizing function**  The limits of $BuRegDown_t$ and $BdRegDown_t$ for the sizing function are set in the same manner as for RP up above.

\[
buRegDownMax = 0 \tag{5.129}
\]

\[
bdRegDownMax = 0 \tag{5.130}
\]

**Dispatch function**  In the dispatch function, $regDownMin_t$ and $regDownMax_t$ are equal to the amount awarded on the RP down market ($obligationRegDown_t$).

\[
\forall t \in T : \quad regDownMin_t = obligationRegDown_t \tag{5.131}
\]

\[
\forall t \in T : \quad regDownMax_t = obligationRegDown_t \tag{5.132}
\]
5.2 Constraints

Primary frequency regulation up

As stated on page 28, PFR can only be provided by WPPs in DK1 if they are backed up by conventional generation units. This means that only the BESS can provide PFR when considering VPPs. BESSs count as conventional generation unit in DK1 because of their predictable output and fast enough ramp rates. It would only make sense to let the WPPs provide PFR if the up- and down ramping would be cheaper than with the conventional power generation source. This case is not considered in this study.

PFR upwards can be provided by decreasing charging power or increasing discharging power (see constraints 5.83 and 5.90). The PFR up market node can then be defined in constraint 5.133 which also includes decision variables for the dispatch function $BdPrimaryUp_t$ and $BuPrimaryUp_t$.

\[
\forall t \in T: \quad PrimaryUp_t + BdPrimaryUp_t = \sum_b ChargerPrimaryUp_{b,t} + \sum_b DischargerPrimaryUp_{b,t} + BuPrimaryUp_t
\]  

(5.133)

Again, assigning limits to the decision variables of PFR up in constraints 5.134 to 5.136 is done in the same manner as for RP up on page 94 and can be implemented as lower- and upper bounds.

\[
\forall t \in T: \quad primaryUpMin_t \leq PrimaryUp_t \leq primaryUpMax_t
\]  

(5.134)

\[
\forall t \in T: \quad 0 \leq BuPrimaryUp_t \leq buPrimaryUpMax
\]  

(5.135)

\[
\forall t \in T: \quad 0 \leq BdPrimaryUp_t \leq bdPrimaryUpMax
\]  

(5.136)

Scheduling & sizing function The limits of $BuPrimaryUp_t$ and $BdPrimaryUp_t$ for the scheduling- and sizing function are set in equations 5.137 and 5.138.

\[
\forall t \in T: \quad primaryUpMin_t \leq PrimaryUp_t \leq primaryUpMax_t
\]  

(5.137)

\[
\forall t \in T: \quad 0 \leq BuPrimaryUp_t \leq buPrimaryUpMax
\]  

(5.138)
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\[ buPrimaryUpMax = 0 \] (5.137)

\[ bdPrimaryUpMax = 0 \] (5.138)

**Dispatch function** In the dispatch function, \( primaryUpMin_t \) and \( primaryUpMax_t \) are equal to the amount awarded on the PFR up market \( (obligationPrimaryUp_t) \).

\[ \forall t \in T : \quad primaryUpMin_t = obligationPrimaryUp_t \] (5.139)

\[ \forall t \in T : \quad primaryUpMax_t = obligationPrimaryUp_t \] (5.140)

**Primary frequency regulation down**

Similar to the RP down market, the formulation for the PFR down market is provided by constraints 5.141 to 5.144.

\[ \forall t \in T : \quad PrimaryDown_t + BuPrimaryDown_t = \sum_b ChargerPrimaryDown_{b,t} + \sum_b DischargerPrimaryDown_{b,t} + BdPrimaryDown_t \] (5.141)

Alike for PFR up, constraints 5.142 to 5.144 are the limits for the decision variables of PFR down.

\[ \forall t \in T : \quad primaryDownMin_t \leq PrimaryDown_t \leq primaryDownMax_t \] (5.142)

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5.2 Constraints

\[ \forall t \in T : \quad 0 \leq BuPrimaryDown_t \leq buPrimaryDownMax \quad (5.143) \]

\[ \forall t \in T : \quad 0 \leq BdPrimaryDown_t \leq bdPrimaryDownMax \quad (5.144) \]

**Scheduling & sizing function** Again, the limits of \( BuPrimaryDown_t \) and \( BdPrimaryDown_t \) for the scheduling- and sizing function are set in the same manner as PFR up above.

\[ buPrimaryDownMax = 0 \quad (5.145) \]

\[ bdPrimaryDownMax = 0 \quad (5.146) \]

**Dispatch function** In the dispatch function, \( primaryDownMin_t \) and \( primaryDownMax_t \) are equal to the amount awarded on the PFR down market (\( obligationPrimaryDown_t \)).

\[ \forall t \in T : \quad primaryDownMin_t = obligationPrimaryDown_t \quad (5.147) \]

\[ \forall t \in T : \quad primaryDownMax_t = obligationPrimaryDown_t \quad (5.148) \]

**Additional primary frequency regulation requirement**

In this section, it is explained why constraints 5.23, 5.25, 5.27, 5.30, 5.32, and 5.34 are required: In the Danish market (DK1), PFR has to be provided for maximum 15 min continuously and afterwards a 15 min break is allowed (see sec. 2.8 on page 28). Therefore, during one hour, the provider of PFR can be asked to deliver maximum 30 min continuous power. If the time step is one hour, the
*primaryUpReserveFactor* and *primaryDownReserveFactor* in sec. 5.2.1 and sec. 5.2.2 on page 62ff have to be equal to 0.5.

In this paragraph, the charging of the BESS is described (upper part of Fig. 5.7): On the left-hand side, the case of charging is illustrated between 0MW of charge and the maximum charging power. The actual charging power has to be between the minimum and the maximum power rating (see constraints 5.83 and 5.86) and the charging power of the BESS can at maximum be so high, that the battery energy will not be violated. This is ensured by constraint 5.28. In case of providing PFR down with the charger, the actual charging power can be increased in case the BESS has to provide PFR down. This means that constraint 5.28 might not be any longer sufficient to ensure that the maximum battery energy is not exceeded and the extra constraints 5.30, 5.32, and 5.34 need to be added. These constraints require that the sink node level is high enough meaning that the source node level is low enough to be potentially charged with the extra energy coming from the PFR downwards market (see a) in Fig. 5.7). Considering the case when the charger provides PFR upwards power, no additional constraints are required because the actual inflow in the source storage node will be equal or less then the amount that is caused by the charging power.

Besides charging the BESS, the same considerations have to be made for discharging the BESS (lower part of Fig. 5.7). In this circumstance, PFR upwards can cause a higher discharge of the BESS than it would be caused without PFR. Constraints 5.23, 5.25, and 5.27 ensure that the source node level is high enough to be able to provide the extra discharge.

**In summary,** this chapter provides the problem formulation for the VPP. It has to be highlighted that one problem formulation can be used in order to provide the three individual functions of optimum scheduling, optimum sizing and optimum dispatch. In the next three chapters, case studies are presented where each function is discussed in more detail.
5.2 Constraints

**Figure 5.7.** Primary frequency regulation requirement
6. Case Study I: Optimum scheduling

This case study demonstrates the optimum scheduling function of the developed algorithm which is used to find the optimum operation schedule that maximizes profit of the VPP. Besides the operation schedule, the results of the scheduling function indicate the amounts of power that should be bid into each power market. This case study is structured as follows: First, a numerical example is discussed that indicates the model output over an 48 hour interval. It is used to proof the model by common sense if results are meaningful. In the second part, actual market data from the years 2010 until 2013 are used as input data into the model and results are presented. Moreover, various configurations of the VPP are discussed which are used to indicate the profitability of the BESS.

6.1. Model implementation

The optimization model has been implemented based on the optimization programming language (OPL) of the IBM® iLog® Optimization Studio (version 12.5) and CPLEX® is used to solve the problem. All input data are prepared with Matlab® and the output is stored in MS Excel® files which are read by OPL. When the optimization problem is solved, all results are stored in an MS Excel® file. The optimization is performed for monthly data in order to solve the model reasonably fast. Thereby, the final SoC and the total capacity-fade of the last period of each month are stored and these values are used as input for the next month. The time to solve the optimization for different months for a single BESS varies but takes about 1 to 30 minutes depending on which month is optimized (more time is needed for hybrid BESSs) on a laptop computer with 64-bit Windows®
7 operating system, 8GB memory, and Intel® Core i7 CPU with 2.67GHy. This time is achieved with special CPLEX® tuning parameters which are used for all optimizations performed in this work in order to reduce the computational time. A copy of the input parameters can be found in Appendix B on page 181. These parameters have been found based on trial and error. For future work, it is recommend to use the tuning tool of CPLEX® in order to find even better parameters.

6.2. Results numerical example

The numerical example discussed in this section is solved for 48 time steps assuming a two day period with one hour time steps. The purpose of this example is to demonstrate that the optimization model is correctly formulated and implemented. The parameters used for this example are taken from Tab. A.1 on page 174 for the li-ion BESS except the parameters listed in Tab. 6.1. These parameters differ because the effect of realistic parameters would not be clearly visible in an example of 48 periods with hourly discretization. Therefore, unrealistic high values are chosen to demonstrate their impact over a short optimization period. Moreover, for the sake of simplicity, $\text{chargerMax}_b$, $\text{dischargerMax}_b$, and $\text{sosNodeMax}_b$ are set to 1 and the $\text{initSoc}_b$ is set to 50%.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
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<tbody>
<tr>
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<tr>
<td>$\text{capacityFadeRateThroughput}_b$ [$\frac{\text{MWh}}{\text{MWh}}$]</td>
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<td>$\text{selfDischargeRate}_b$ [$\frac{\text{MW}}{\text{MWh}}$]</td>
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</table>

Table 6.1.: Input parameters for numerical example different from li-ion BESS parameters of Tab. A.1 on page 174.

The numerical example is based on two parts. In the first part (see page 106), the BESS only participates on the DA market without the ancillary service markets. In the second part (see page 108), it is assumed that the VPP can participate in all power markets including the markets for ancillary services.
6.2 Results numerical example

Fig. 6.1 shows the results for the first part. To use the problem formulation described in chapter 5 above for this simplified case, PFR prices and prices for RP up are set equal to 0, and RP down prices are set to large negative values (-500€/MWh) to avoid scheduling power to these markets. Moreover, the WPP output is the same as illustrated in Fig. 6.2 b).

Fig. 6.1 depicts that the BESS is charged with maximum power when the DA market price is low in time step 4 and 18 as well as in time step 29 in order to sell power in time step 1, 8, and 32 and to avoid being recharged when prices are high (e.g. time step 34). Moreover, the exaggerated effect of high self-consumption is shown where the BESS has to be re-charged from time step 12 to 16, just before prices are lowest, in order to provide enough power for the BESS own consumption. During this period, the charging power is just as high that it can cover the self-consumption of the BESS. Fig. 6.1a) also shows that the BESS avoids being recharged when prices are high (e.g. time step 9 to 11 and 33 to 36) and that recharging occurs with maximum power once prices are lowest or favorable (time step 4, 18, and 29). Moreover, the effect of losses due to inefficiency can be seen in the same time steps when the storage is charged with 1MW but the change in SoC shown in Fig. 6.1 b) is less than 100%. Furthermore, the same figure also shows that the SoC is always between 0% and 100%. In addition to the SoC, the impact of capacity fade of the BESS is also illustrated in Fig. 6.1 b) on the right y-axes. The BESS constantly loses capacity during its operation. It has to be remembered that the capacity fade is chosen unrealistically high for this example. The results also indicate that the BESS is completely emptied at the final time step. If the SoC of the last time step would be greater than zero, energy could still be sold to the market and the solution would not be optimal.

Fig. 6.2 shows the results for the second part where the VPP can participate in all power markets including the ancillary service markets and thus results are more complex to be interpreted. The same input data are used as for the first part discussed in Tab. 6.1. Market prices from the second part are depicted in subplot a) in Fig. 6.2 which are positive values except for RP down (negative RP down prices are explained in sec. 2.8 above). If RP down prices are not shown in Fig. 6.2 a), they are set to -500€/MWh in order to avoid sales to the RP down market because there was no demand for RP down in that hour from the TSO. In Fig. 6.2 b), the WPP is assumed to have a maximum output of 3MW and
the power generation of the WPP scheduled to the DA market \((WindSpot_t)\) is shown as a red dashed line while \(windMax_t\) is shown as a gray thin solid line. The same figure also shows sales and purchases to and from the DA market. Moreover, the right y-axes shows the bar chart indicating the difference between the WPP output and sales to the DA market. Fig. 6.2 c) shows the BESS behavior indicating the SoC and charging- and discharging power and Fig. 6.2 d) presents the decision variable values for the ancillary service markets.

This paragraphs explains the results depicted in Fig. 6.2 and the reader is referred to the numbers in blue color in Fig. 6.2 that correspond to the following numbered list:

1. In the first time step the BESS is charged from purchases on the DA market because the WPP production is not high enough that the BESS could be charged from the WPP.

2. During periods of high RP down prices, the WPP and the BESS both sell power to the RP down market.

3. RP up is sold by the WPP and the BESS.

4. When ancillary service market prices are 0€ (or -500€ for RP down), no
6.2 Results numerical example

sales occur to the ancillary service markets.

5. Power is scheduled to the PFR up market for high PFR up prices.

6. Power is scheduled to the PFR down market for high PFR down prices.

7. The SoC changes without the charger or discharger being activated and changes in the SoC are higher than the self-consumption because the BESS provides RP up or -down which affects the SoC.

It has to be noted that the SoC in Fig. 6.2 c) is shifted by minus one time step to match the time-step of charging- and discharging power. This is due to the formulation of the source- and sink storage nodes in sec. 5.2.1 and sec. 5.2.2. In addition, it has to be stressed that the exaggerated input parameters of Tab. 6.1 are also used for Fig. 6.2.

Based on Fig. 6.2, it can be concluded that the BESS is able to schedule power to the various power markets in order to maximize profit of the VPP and that the WPP is able to participate on the DA- and RP markets.

The next section discusses the application of the proposed optimization model on actual market data.
Figure 6.2.: Result numerical example: Includes DA- and ancillary service markets, BESS, and WPPs. a) Market prices; b) DA market related decision variables; c) BESS behavior; d) Ancillary service market results.
6.3. Results case study I

This section applies the developed optimization model on a lead-acid and a li-ion BESS with real market data. All input data for this case study are summarized in Appendix A on page 171. References for the input data, if not otherwise stated, can be found in the previous chapters. If a specific BESS should be used, first the BESS would need to be parametrized and then the developed formulation can be applied on the specific parameters. Regarding the efficiency stated in Appendix A, a remark has to be made that data refer to the ES efficiency and not to the efficiency of the whole BESS including converter or transformer losses, for instance. Due to the fact that efficiency data for entire BESSs were unavailable, efficiency data of only the battery ESs are taken into account which omit, e.g. converter losses, and thus, these data are rather optimistic for the whole BESS.

Market data used for this case study are from the Danish power market west (DK1) and dates from beginning of 2010 until the end of 2013. The Danish TSO (Energinet.dk, see [1]) makes market data freely available. The wind power production data discussed in Appendix A is used for all four years.

Further, the annualized investment costs $esCostPerMw_b$ are calculated based on the BESS’s costs stated in Tab. 2.1 on page 18 and it is assumed that the li-ion BESS has to be replaced after six years while the lead-acid BESS needs to be replaced already after three years. Further, an interest rate $i$ of 10% is used. The optimization is discretized into hourly time steps because market data are only available on hourly bases. Due to the complexity of the model, the optimization is split into months in order to allow solving the problem reasonably fast. Thereby, the final SoC and the final available capacity are used as input data for the first time step of the next month. Moreover, in case that no RP up or -down was demanded in one specific hour, the price is set to -500€/MWh to avoid scheduling power for this market.

Fig. 6.3 illustrates the breakdown of revenues resulting from the different power markets for four different years. The pie-charts in a) (first row) show the results for one MW li-ion BESS regarding the operation on the DA-, RP-, and PRF market including WPPs. The other pie charts in row b) use the same scenario with one MW lead-acid BESS. In the following, Fig. 6.3 is discussed:
• Sales to the DA market have the second biggest share. On the other hand, DA purchases are very low compared to the sales. One reason for a much higher share of sales than purchases is that WPPs can only sell their power generation into the RP up-, RP down- and into the DA market but they have no need to purchase power. In addition to this, purchases on the DA market are slightly more expensive than sales within the same hour. This is due to the introduced purchase cost supplement in order to avoid selling and buying within the same hour. This leads to the fact that the BESS is charged with power from the WPP and only if this is not sufficient, power from the DA market is bought.

• RP up has the biggest share for all years and comparing the size of the share between the two different BESSs, little difference can be seen. The lead-acid BESS has slightly higher shares of RP up sales. One reason for the RP up market sales having a higher share than DA market sales, is the fact that uncertainty of power demand and prices for the RP up market are not present in this optimization due to the usage of historic data. This means that RP up is privileged whenever activated because prices have to be at least as high as the DA market price of the hour in question (see page 28).

  Compared to RP up, RP down contributes on a much smaller level to the share of revenues over all years, considering the lead-acid and li-ion BESS. It has to be remarked that RP down prices have most of the time negative values (sellers of RP down need to pay in order to provide this service) but there are instances when they have positive values. The optimization algorithm makes use of positive RP down values as they indicate that absorbing power is reimbursed. From 2010 until 2013, the accumulated revenues from the RP down market are positive for a) and b) and are displayed in the pie chart for these years but are not visible for some years due to their low values.

• The main difference between both BESSs can be found in the revenues deriving from the PFR up market which has the third biggest share of revenues for both BESS technologies. The li-ion BESS is able to generate a higher share from this market compared to the lead-acid BESS. One interpretation is that it is caused by the low charging power capabilities of lead-acid BESSs
which restrict the amount of sales to the PFR market compared to li-ion BESSs. Comparing the PFR up and down markets, the contribution of the PFR down is low compared to the PFR up market. One reason is that prices for PFR up are usually higher than PFR down prices within the same period.

![Revenue breakdown for a) li-ion battery and b) for lead-acid BESS participating in different power markets](image)

**Figure 6.3.** Revenue breakdown for a) li-ion battery and b) for lead-acid BESS participating in different power markets

The profitability of a VPP based on a li-ion BESS serving different markets is discussed in the following paragraph and results are depicted in Tab. 6.2. The li-ion BESS has been selected exemplarily to highlight the effect of different configurations of the VPP. All four cases have in common that there is wind power generation included and sales as well as purchases can always be made on the DA market. The profit is equal to the objective function value $OF$ of the optimization.

Results of Tab. 6.2 show that the 1MW li-ion BESS is never more profitable than the option without BESS when comparing the first and the second row, or the third and fourth row. Tab. 6.2 also indicates that including the ancillary service markets yields a considerable higher profit than just operating the BESS on the DA market. Without the ancillary markets, the BESS can only be used for energy arbitrage (buying when DA market prices are low and selling when DA market prices are high). In order that li-ion BESSs become a profitable investment, the investment costs in the BESS would need to decrease, the lifetime of the BESS
would need to be extended, or the power- and ancillary service market prices need to be more favorable.

Considering the results listed in Tab. 6.2, it has to be acknowledged that such profit can only be generated with perfect foresight of market prices as well as WPP production, and under the condition that all bids to the markets get awarded. This is unlikely in practice and actual profits are likely to be lower. On the other hand, the assumption of perfect foresight means that imbalances do not occur which is also not true for real operations where the BESS can yield extra benefits by reducing imbalance penalties incurred by the WPP.

<table>
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Table 6.2: Annual profit (OF) in k€ for lead-acid BESS with various market configurations (x indicates included unit or market)

Another aspect of interest is the comparison of the incremental profit for different BESS technologies. Incremental profit is defined as the profit achieved with the BESS and WPP combined minus the profit achieved with WPPs only. This comparison includes different configurations of BESSs. The first two configurations are based on one MW (discharger rating) lead-acid- and li-ion BESS as already discussed in Appendix A. The third and fourth configurations are hybrid BESSs based on the combination of a 1MW and 0.5MW lead-acid- and li-ion BESS (discharger ratings). The charger ratings and battery energy are adjusted accordingly. The other parameters of Appendix A are still valid for the hybrid configurations. Moreover, the ancillary service markets are all taken into account for this comparison. Fig. 6.4 shows the incremental profit of all configurations. The investment in VPPs is financially viable if the incremental profit is positive. Results indicate that the highest profit or smallest loss can be achieved in the year 2010. There,
the VPPs with 1MW lead-acid BESS and 0.5MW hybrid BESS are more profitable than operating WPPs without BESS. In all other years, the VPP is not more profitable than the WPPs alone (except for a small profit of the 0.5MW hybrid configuration in year 2011). Considering all years, the highest losses occur with the 1MW hybrid configuration while the 0.5MW hybrid configuration shows the highest profits (smallest losses). Amongst the non-hybrid configurations, the lead-acid BESS is preferable over the li-ion battery BESS. All in all, Fig. 6.4 indicates that none of the evaluated configurations would allow an attractive business case over the analyzed years.

Figure 6.4.: Incremental annual profit for different BESS configurations (annual profit VPPs – WPPs only)

Taking everything into consideration, it has to be pointed out that the developed algorithm is applied on an exemplary set of BESSs but that the size and the type of BESS technology is not optimized. Consequently, the profit of other sizes of BESSs based on the same configuration could be different and there could be a BESS configuration that achieves a higher profit during all tested years than the WPPs alone. The discussion on optimum sizing can be found in the next chapter (case study II).

Finally, Fig. 6.5 (top) shows the energy throughput of the BESS which is the
sum of energy put in and out of the BESS. In the first two years, the energy throughput of the li-ion BESS was considerably higher than in the other years causing a higher capacity fade due to the higher throughput. Contrary to the li-ion BESS, the lead-acid BESS does not show a higher energy throughput at the first years.

The total energy throughput over the entire period is 4817MWh for the li-ion BESS and 1021MWh for the lead-acid BESS. In total, the li-ion BESS loss is 41% of its capacity over the optimization period and the lead-acid BESS’s loss is 16%, see Fig.6.5 (bottom), respectively. Assuming the BESS’s end of life at 80% of its original capacity, the li-ion BESS has already exceeded its end of life criteria. Concerning the lead-acid BESS, assuming linear extrapolation based on the values of the original capacity and the capacity at the end of month 48, the lead-acid BESS can be operated 4.7 years, which is longer than the assumed lifetime of three years. These findings demand an adjustment of the annual investment costs calculation resulting in lower annualized investments costs for the lead-acid battery, making it more profitable. On the other hand, the lifetime of the li-ion battery is lower than assumed for the cost calculation which makes it less profitable than depicted in Fig.6.4. Taking this into account, the lead-acid BESS will still be more profitable than the li-ion BESS in Fig.6.4 even after adjusting the costs of the BESS. It has to be pointed out that a change in annualized investment costs of the BESS would change the cost function value (profit) of the optimization. Nevertheless, it would not change the result of the decision variables because the BESS’s investment costs do not depend on decision variables in the optimum scheduling and -dispatch problem.

6.4. Conclusions case study I

The optimum scheduling function is applied on a numerical example in order to demonstrate that results are meaningful. Moreover, it is also applied on real market data based on various configurations of the VPP. Results show that none of the tested configurations, which are based on li-ion- and lead-acid BESSs, would allow for an attractive investment decision based on the underlying market data. The presented case study also indicates that the assumed life-times of the BESSs
6.4 Conclusions case study I

Figure 6.5: Energy throughput (top) and capacity (bottom) for li-ion and lead-acid BESSs from 2010 to 2013

are not confirmed by the results of the case study and therefore need to be adjusted for further studies. The lifetime of the li-ion BESS needs to be reduced and the lifetime of the lead-acid BESS needs to be increased. This results in more favorable costs concerning the lead-acid BESS and that can cause the incremental profit of the lead-acid BESS to become zero or greater. This could make investments in VPPs with lead-acid BESS profitable.

Reasons for the lifetime of the li-ion BESS being lower than estimated could be due to the following: The li-ion BESS has a comparable higher energy throughput than the lead-acid BESS, which leads to a much higher capacity fade for the li-ion BESS than for the lead-acid BESS and thus results into a shorter lifetime. This result demands that the values of capacity fade found in literature need to
be verified by laboratory tests in order to verify if such a short lifetime of li-ion BESSs is realistic in practice.

Further conclusions from case study I are:

- Considering capacity fade in the optimization problem is important as it gives valuable information on the BESS’s lifetime and hence it should be used to adjust the annualized investment costs if necessary. Taking the capacity fade not into account would result in unrealistic high profits of the VPP.

- Considerable higher profits can be generated in case the VPP is operated on the ancillary service markets in addition to the DA market. This highlights the importance to consider the ancillary markets in a formulation of the optimum scheduling problem based on Denmark (DK1).

- On the RP up market, the VPP achieves the highest share of revenues followed by the DA market sales. The PFR up market has the third highest share of revenues. This requires a reliable forecast of market prices and when RP up or down is called in order to achieve these revenues in practice.

- The importance of hybrid BESS is shown within this case study where the 0.5MW hybrid configuration yields the highest incremental profit (smallest incremental loss). However, results also underline the importance of proper sizing of the hybrid BESS because the other tested hybrid configuration of 1MW showed the worst performance in terms of profitability.
This chapter discusses the sizing function with a further numerical example in order to illustrate how the algorithm works and to demonstrate that results are meaningful. Due to the long computational time to solve the problem, the demonstration is restricted to the numerical example of 48 time periods from the previous chapter. The CPLEX® tuning parameters presented in Appendix B on page 181 are also used for this example.

This chapter is outlined as followed: First, the calculation of BESS costs is verified by various test cases of a smaller problem formulation. This is followed by the numerical example in sec. 7.2 which shows that the sizing function works correctly.

### 7.1. Testing of BESS cost calculation

The test case described in this section is conducted to verify the correct accounting of the BESS costs described in sec. 5.2.3 on page 68ff. This test gives the opportunity to evaluate the quality of the results from separable programming and PLA of the functions in equation 5.37 (based on decision variable $FinalCapacity_b$) and equation 5.64 (based on decision variable $EsCostVsCapacity_b$). The test is conducted through a smaller problem formulation that includes only the necessary constraints and omits all other constraints. The problem of this test case is formulated in a manner that it can only have a single solution and no objective function is used in the OPL programming language solved by the CPLEX® solver (due to the single possible solution). This testing problem formulation consists of constraints from equation 5.43 on page 71 to equation 5.66 on page 78 (excluding the constraints in the gray shaded boxes). In addition, the following constraints
have been added:

\[
\forall b \in B : SosNode_b = sosNodeTest_b 
\] (7.1)

\[
\forall b \in B : SisNode_b = sisNodeTest_b 
\] (7.2)

\[
\forall b \in B : NbSlicesActive_b = nbSlicesActiveTest_b 
\] (7.3)

\[
BatteryInt_b = 1 
\] (7.4)

The test cases for this problem formulation are only conducted for one battery \((\bar{b} = 1)\). Tab. 7.1 depicts input data used for the test cases as well as the results for each test case. All other input data not shown in Tab. 7.1 are based on the values for the li-ion BESS in sec.6.2 if not otherwise stated. The values of the decision variables \(FinalCapacity_b\) and \(EsCostVsCapacity_b\) in the columns indicated with “MILP” are the results of the reduced MILP problem formulation, while the results of the same variable indicated with “Actual” are the numerically correct calculated values not based on MILP. The column “\(EsCostVsCapacity_b\) Actual” is calculated based on “\(FinalCapacity_b\) Actual” and not based on results of the MILP calculation. The columns “Error1 %” and “Error2 %” indicate thus the total relative error of the MILP results.

The results in Tab. 7.1 verify that the problem formulation proposed in constraints 5.43 on page 71 to constraint 5.66 on page 78 is able to calculate (in most cases) with sufficient accuracy the costs of the BESS in dependence of the capacity fade based on the number of breakpoints chosen. Only five times out of the 17 test cases, the error is greater than \(\pm 5\%\) of \(EsCostsTest\) and in only one case the calculation is considerably wrong with \(-46.63\%\). In this case, the breakpoints are poorly chosen. It is recommended to re-run the optimization problem with adjusted breakpoints that are closer around the first obtained solution in order to further reduce the approximation error.
Table 7.1.: Test cases for BESS cost calculation. Input data: $p_{v_b}=100; \ minSoc = 0, \ maxNbEsSlices_b=5$.

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<td>17,766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10,038</td>
<td>19,071</td>
<td>19,071</td>
<td>19,071</td>
<td>19,071</td>
<td>19,071</td>
<td>19,071</td>
<td>19,071</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>2.40</td>
<td>-1.42</td>
<td>-46.63</td>
<td>4.53</td>
<td>7.37</td>
<td>-1.16</td>
<td>-6.84</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.1 Testing of BESS cost calculation

Not listed in Tab. 7.1 are the breakpoints used for the piecewise linear approximations. Tab. 7.2 gives an overview of the minimum and maximum values for the x-axes values of the breakpoints. The calculation of the minimum and maximum values for separable programming is described in sec. 3.2.3 on page 39f. The breakpoints are linearly spaced except for the points \( a_{5,p_a,b} \). Their calculation is described in Algorithm 7.1.

<table>
<thead>
<tr>
<th>Index; ( a )</th>
<th>( a_{a,1,b} )</th>
<th>( a_{a,p_a,b} )</th>
<th>nb. breakpoints</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( \text{sosNodeMax}_b \cdot \text{maxNbEsSlices}_b )</td>
<td>21</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{1}{\text{maxNbEsSlices}_b} )</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( \text{sosNodeMax}_b \cdot \text{maxNbEsSlices}_b )</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{\text{maxNbEsSlices}_b} )</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>( \text{maxNbEsSlices}_b )</td>
<td>( \text{maxNbEsSlices}_b )</td>
</tr>
<tr>
<td>5</td>
<td>0.8</td>
<td>1.1</td>
<td>see Algorithm 7.1</td>
</tr>
</tbody>
</table>

\textbf{Table 7.2:} Calculation breakpoints for piecewise approximations

Algorithm 7.1 determines the minimum amount of breakpoints necessary for \( a_{5,p_a,b} \) because Fig. 5.5 on page 79 can sufficiently be approximated with a linear function between 0.8 and 0.98 in this example. This interval depends on the input parameters provided into equation 5.64. The interval between 0.98 and 1 cannot be approximated by a linear function (in this example) without a relative big error and a number of breakpoints have to be chosen to approximate this section.
It has to be stressed that this is an important interval of the function in case the capacity fade is small. For example, this is the case, when simulating only a short period as shown in the numerical example below. The determination of the breakpoints can be followed in Algorithm 7.1 which basically calculates the annualized investment costs first and then picks the breakpoints for the non-linear section based on the capacity at which the lifetime of the BESS would be less than a year long. Finally, the breakpoint \((1.1, 0)\) is added for each BESS \(b\) to account for the case of zero BESS costs if \(BatteryInt_b = 0\) which is further discussed on page 78.
Algorithm 7.1 Calculation breakpoints $a_{5,p_5,b}$. Variables that differ from chapter 9.3: $\text{nbPeriodsCapFade} = \tilde{t}$; $(\text{xAxesValues}, \text{yAxesValues}) = (a_{5,p_5,b}, f(a_{5,p_5,b}))$; $\text{nbEs} = \tilde{b}$.

1: $\text{CapacityPoints} = [0.8:0.02:0.98, 0.99, 0.999, 0.9999]$;
2: $\text{yearsEsOperation} = (1/\text{PeriodsPerHour}) \times 0.2 \times (\text{nbPeriodsCapFade} / 8760)./(1 - (\text{CapacityPoints}))$;
3: $\text{for } bt = 1: \text{nbEs} \text{ do}$
4: \hspace{1em} $\text{for } cp = 1: \text{length}(\text{CapacityPoints}) \text{ do}$
5: \hspace{2em} if $\text{yearsEsOperation}(cp) < 1$ then
6: \hspace{3em} $\text{esCostVsCapacity}(cp,bt) = \text{pv}(bt) \times (1 + i) / \text{yearsEsOperation}(cp)$;
7: \hspace{2em} else
8: \hspace{3em} part1 = $((1 + i)^{(\text{yearsEsOperation}(cp))}) - 1$
9: \hspace{3em} part2 = $i \times ((1 + i)^{(\text{yearsEsOperation}(cp))})$
10: \hspace{3em} $\text{esCostVsCapacity}(cp,bt) = \text{pv}(bt) / (\text{part1} / \text{part2})$;
11: \hspace{2em} end if
12: \hspace{2em} $\rightarrow$ calculating es cost for capacity = 1 (infinite investment horizon)
13: \hspace{1em} $\text{end for}$
14: \hspace{1em} $\text{esCostVsCapacity}((\text{length}(\text{CapacityPoints})+1),bt) = \text{pv}(bt) \times i$;
15: \hspace{1em} $\text{end for}$
16: \hspace{1em} $\rightarrow$ Search when $\text{yearsEsOperation}$ becomes $< 1$
17: $\text{absOneYear} = \text{abs}(1-\text{yearsEsOperation})$;
18: $[\text{minValue}, \text{minPoint}] = \text{min}(\text{absOneYear})$;
19: $\text{CapacityPoints} = [\text{CapacityPoints} 1]$;
20: $\text{for } bt = 1: \text{length}(\text{pv}) \text{ do}$
21: \hspace{1em} $\text{for } pp = 1: \text{length}(\text{pointsPLA}) \text{ do}$
22: \hspace{2em} $\text{yAxesValues}(pp,bt) = \text{esCostVsCapacity}(\text{pointsPLA}(pp),bt)$;
23: \hspace{2em} $\text{end for}$
24: \hspace{1em} $\text{yAxesValues}(pp+1,bt) = 0$;
25: \hspace{1em} $\text{end for}$
26: $\text{xAxesValues} = \text{CapacityPoints}(\text{pointsPLA})$;
27: $\text{CapacityPointsPLA} = [\text{CapacityPointsPLA} 1.1] $;
7.2. Numerical example of sizing function

This numerical example is based on the example already discussed previously in sec. 6.2 where $b = 0$ is the li-ion BESS and $b = 1$ is the lead-acid BESS. All input parameters are the same as in Tab. A.1 except for the ones listed in Tab. 7.3 as well as $esInvestSwitch = 1$, and $capacityFadeRate_b$ and $capacityFadeRateThroughput_b$ which are multiplied by $10^2$ in order to make capacity fade more visible over such a short optimization horizon ($\bar{t} = 48$). For the sizing function, the decision variables $NbSlicesActive_b$ and $BatteryInt_b$ are of main interest, because $NbSlicesActive_b$ represents the “size” of the BESS as the factor is multiplied with the parameters $sisNodeMax_b$, $sosNodeMax_b$, $dischargerMax_b$, and $chargerMax_b$ which define the smallest slice size (see chapter 5). The Boolean decision variable $BatteryInt_b$ indicates which BESS technology $b$ is part of the solution if it is equal to one. The sizing function is tested by varying the investment costs $pv_b$ for the BESSs. This is done by multiplying factor $k$ with $pv_b$ (see 2nd column Tab. 7.3). For very low costs of the BESS $b$, it is expected that all slices ($NbSlicesActive_b$) are activated up to $maxNbSlices_b$ in the activated BESSs $b$ indicated by $BatteryInt_b = 1$. Further, the sum of $BatteryInt_b$ is expected to be equal to $maxNbEs$ for very low BESS costs. When increasing $pv_b$, it is expected that less slices are being activated. And finally, no slice should be activated for very high prices.

The results in Tab. 7.3 confirm the expected behavior that no BESS is chosen for $k = 1$ and $k = 10^{-1}$. For $k = 10^{-2}$, one slice of the li-ion BESS and five slices of the lead-acid BESS are chosen, and all slices (not exceeding $maxNbSlices_b$) in both BESSs are activated for $k = 10^{-4}$. It also shows that the objective function value is higher if a BESS is part of the solution compared to the case where no BESS is part of the solution. Also, the lower the BESS’s investment costs, the higher the objective function value. However, no BESS would be selected in the numerical example for $k = 1$ if $capacityFadeRate_b$ and $capacityFadeRateThroughput_b$ were unchanged, as this would mean that the assumed BESS costs in the previous chapter are too expensive to be profitable in this example.

Furthermore, case 2 has to be highlighted where a more expensive BESS is favored over a cheaper BESS concerning the investment costs per MW.

Moreover, the “Error$_2$ %” of Tab. 7.1 is also calculated for this numerical example.
7.2 Numerical example of sizing function

It is not significant in all four test cases shown in Tab. 7.3 and $EsCostOF_b$ are calculated very close to their numerically correct values with an error of less than 10€.
Table 7.3.: Input parameters and results of numerical example of sizing function.

<table>
<thead>
<tr>
<th>Case</th>
<th>Objective function value OF [€]</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>maxNbSlices&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>maxNbEs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NbSlicesActive&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BatteryInt&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FinalCapacity&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EsCostV&lt;br&gt;capacity&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EsCostOF&lt;sub&gt;b&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>costsEs</td>
<td></td>
</tr>
</tbody>
</table>

The lead-acid BESS; <i>b</i> is a factor multiplied with <i>p<sub>b</sub></i> to vary BESS costs.
7.3. Conclusions case study II

The numerical example presented in this chapter shows the application of the sizing function on a short optimization period. Moreover, a testing model shows the correct calculation of the BESS costs in dependence of the capacity fade and the size of the selected BESS. The results of the numerical example cannot be used beyond demonstration of the optimum sizing function. Any “real-world” conclusions cannot be drawn because of the adjusted capacity fade parameters in order to make any capacity fade at all visible for such a short optimization period. The adjusted input parameters make both BESSs look worse than they are and therefore falsely require lower investment costs in order to make the BESSs profitable compared to unchanged input parameters.

The results demonstrate that the proposed optimum sizing algorithm can be used to determine the optimum size and technology, as well as the optimum combination of BESSs in terms of optimizing profit of the VPP. Thus, the model is also able to find an optimum hybrid BESS.

Another point to highlight is that the specific investment costs of a BESS regarding its battery energy or power rating are taken into account. This does not mean that a cheap BESS is chosen over a more expensive BESS in the optimum solution because the model considers all relevant properties of the BESS (discussed in chapter 5) and the proposed problem formulation identifies if extra costs for a more expensive technology pay-off.

Having demonstrated the algorithm on a small example, it could be applied on a larger problem that might contain more time steps $t$, more BESS technologies $b$, and/or an increase in the number of slices ($maxNbSlices_b$). Due to the properties of the computer system used in this work (described in sec. 6.1 on page 103), there was no opportunity to test a larger problem of the sizing function due to the long solving time. Thus, future work is to run a sizing problem for a representative time period and with more different BESS technologies. Beyond computer hardware, investigations into better tuning settings of the solver, which is used to solve the MILP problem, are recommended to further improve performance and/or to enable that larger problem sizes can be solved. In addition, a tighter problem formulation would be of advantage to reduce the time needed to solve the problem.
8. Case Study III: Optimum dispatch

The following case study discusses the optimum dispatch function which indicates optimum adjustments of the optimum scheduling function’s results. Adjustments of the original schedule become necessary for example due to deviations of the latest wind power production forecast from a previous forecast or if bids do not get awarded on the market. Another reason for re-scheduling can occur if the modeled SoC of the BESS differs from the actual SoC. The result of the optimum dispatch function is an adjusted schedule that ensures minimum penalty payments for imbalances by maximizing the overall profit.

This chapter is similarly structured as chapter 6: First, a numerical example is run on the optimum dispatch function followed by a small study with actual market data from 2013. In this case study, only the impact of the deviation of the WPP production from its forecast is discussed and it is assumed that all bids get awarded on the power markets and no adjustments of the SoC of the BESS are necessary.

8.1. Wind power forecast data preparation

In order to illustrate a deviation of the WPP production from its forecast, the best case would be to have actual wind power forecast data available. However, this is not the case and an artificial forecast needs to be created. The artificial forecast generation is presented in Algorithm 8.1 which is pseudo-code.

At this point, it needs to be stressed that $windMax$, in chapter 6 would be based on a WPP’s power production forecast because the actual wind power production is unknown in advance. Due to the lack of real forecast data, the available actual wind power production data are already used in case study I in chapter 6 because
Algorithm 8.1 Forecast error generation

1: signR = rand(length(WppData));
2: for s = 1:(length(WppData)) do
3:   if signR ≤ 0.49999 then
4:     signV = -1;
5:   else
6:     signV = 1;
7:   end if
8: end for
9: errorForecast = 0.1;
10: magnitudeScaling = rand(length(WppDataAdjusted)) * errorForecast;
11: WppForecast = WppData + (signV .* magnitudeScaling .* WppData);
12: for s = 1:(length(WppData)) do
13:   if WppForecast(s) > maxWppOutput then
14:     WppForecast(s) = maxWppOutput;
15: end if
16: end for

it is more extended than case study III. For this reason, artificially generated forecast data is used in the dispatch algorithm for windMax_t in order to generate a difference in windMax_t between the optimum scheduling- and dispatch problem.

8.2. Results numerical example optimum dispatch

The input parameters used for the numerical example of the dispatch function are the same as discussed in Tab.6.1 on page 104 and presented in subplot a) in Fig.6.2 on page 108 except for the prices depicted in Fig.8.1. The prices for balance down- and balance up power are chosen in order to have some consecutive periods of over-production as well as under-production in the grid. E.g. period t = 1, 2, ..., 9 shows an over-production in the grid. This can be seen as buCost_t are equal to the DA market price (marketPrice_t) in these time periods. If bdCost_t equals marketPrice_t, there is an under-production in the grid. In the following, the results of the numerical example for the dispatch function are discussed in detail.

Fig.8.2 shows the difference of important decision variables between the optimum dispatch- and optimum scheduling problem, except in the first subplot, where the
8.2 Results numerical example optimum dispatch

Error of the wind power forecast is shown. This error is the only input parameter which is different in regards to the numerical example of the optimum scheduling problem. The forecast error is generated manually in order to better show its impact (and not by Algorithm 8.1). By displaying the difference in decision variable values, the adjustment indicated by the optimum dispatch function becomes obvious. Any difference in $\text{WindGenerator}_t$, $\text{WindRegUp}_t$, and $\text{WindRegDown}_t$ — which is caused by the forecast error — needs to be compensated either through a balance up- or balance down decision variable or through the BESS. If there is a positive forecast error (first subplot Fig.8.2), this indicates that there is more wind power available than originally predicted in the forecast. In such a case, the variables $\text{WindGenerator}_t$, $\text{WindRegUp}_t$, and $\text{WindRegDown}_t$ do not need to take any different values because the scheduled generation can still be provided by operating the WPP below its maximum capabilities. This can be seen in period 41 to 48, for instance. However, if the WPP produces extra power, this can be used if it is profitable, see $t = 18, 19, 20$. In that instance it is more profitable to over-produce and to compensate the overproduction on the balance down ($Bd_t$) market compared to loosing the bonus for wind power production.

Another time period of interest is $t = 9, 10$ where the output of $\text{WindGenerator}_t$ and $\text{WindRegDown}_t$ is less than scheduled (fourth subplot Fig.8.2). The under-production of $\text{WindGenerator}_t$ is compensated by $Bu_t$ while the under-
production of $\text{WindRegDown}_t$ is compensated by $\text{BdRegDown}_t$, which is the expected behavior. This is because $\text{RegDown}_t$ in the RP down node (see equation 5.125) is flowing into the node and not out of it. This means that $\text{BdRegDown}_t$ is required if there is an under-production (not enough power is absorbed from the grid) in the RP down market.

Moreover, the second and third sub-plot indicate that the BESS in the dispatch function is used to reduce penalty payments for imbalances. Last but not least, the difference on balance up and -down decision variables for the PFR up and -down market are for all $t$ equals 0 (not explicitly shown in Fig. 8.2). This result confirms the expectation, because the scheduled output to the PFR up and -down market may not be adjusted and using a large numerical value for imbalance penalties of this market does not alter the decision variable for PFR up and -down in the optimum dispatch problem.

## 8.3. Results monthly dispatch

This section applies the dispatch function on actual market data based on a li-ion and lead-acid BESS for the year 2013 split into single months. Input data are the same as used for case study I in sec. 6.3 (excluding the hybrid BESS). Moreover, results of sec. 6.3 indicating the amount of bids scheduled for each market are taken as new input parameter for the dispatch function. The only input data adjusted for the optimum dispatch function is the wind power production (according to Algorithm 8.1). Due to the fact that the actual wind power production profile is used in sec. 6.3, Algorithm 8.1 is run on the production profile and the artificially generated forecast is taken as input data for the dispatch function. The mean error of the wind power forecast is displayed in the last row of Tab. 8.1 and is 0.13MW on an annual basis. Results show that the mean forecast error is relatively low and varies slightly across different months. A relation between the mean forecast error and the absolute reduction of the objective function value $OF$ is visible for both BESSs in Tab. 8.1 and Tab. 8.2. This is expected because the higher the forecast error, the higher should be the reduction of $OF$ but the actual reduction is also based on market prices for imbalances.

The results of the optimum dispatch problem are depicted in Tab. 8.1 and Tab. 8.2.
8.3 Results monthly dispatch

Figure 8.2.: Difference in decision variables values optimum scheduling vs. optimum dispatch results

for the li-ion and lead-acid battery, respectively. Both tables show the relative and the absolute reduction of the objective function value $OF$ for each month, comparing the result of the optimum scheduling problem with the result of the optimum dispatch problem. At this point, it has to be mentioned, that the scheduling function was run again for each month of 2013 but without transferring the capacity fade of the previous month to the next month (for automation reasons) and the same was done for the dispatch function.

Generally, the lower the reduction of $OF$ the better, because the less penalty payments need to be paid for imbalances. The results indicate that the relative reduction varies heavily between different months but absolute reductions are all
within the same order of magnitude. This relative variation is due to the fact that the \( OF \) values for different months can vary from negative to positive values and show no clear pattern. Moreover, results indicate that the absolute reductions of \( OF \) for both, the li-ion- and lead-acid BESS, are similar for all months and that the two BESSs with the same power rating have a similar contribution on the optimum dispatch result. However, the relative reduction is much more dramatic for the lead-acid BESS with 22% versus 8% for the li-ion BESS.

For further insight into the results of the dispatch function, the objective function terms regarding balance power of the different markets are included in Tab. 8.1 and Tab. 8.2. The sum of all balance up and -down penalties equal the total costs for imbalance penalties. It has to be noted that the absolute reduction of \( OF \) does not necessarily equal the sum of imbalance penalties because the reduction of \( OF \) also includes the adjusted WPP costs.

Comparing the penalty payments for the li-ion- and lead-acid BESS for each month and on average, a similarity is found. Both BESSs have the highest penalty payments for upward regulations because in case the WPP produces less than scheduled, the production of the WPP cannot be up-ramped and balance power needs to be bought. However, in case of over-productions, the WPP can easily be down-ramped. This makes mainly upwards regulation necessary. The highest imbalance penalties are to be paid for \( buRegUpDexpr \) and the second highest penalties are to be paid for \( buDexpr \). This is because higher contributions to the \( OF \) are coming from the RP upwards market than from the DA market.

Finally, there are small amounts of imbalances resulting from \( bdRegDownDexpr \) that is used to balance \( RegDown_t \) power which has not many bids scheduled in the optimum scheduling problem compared to \( RegUp_t \) and \( Sales_t \). No imbalance costs arise by \( buRegDownDexpr \) and \( bdRegUpDexpr \) as this can be avoided through down-ramping of the WPP.
<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Annual average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction OF [€] (optimum scheduling vs. optimum dispatch)</td>
<td>4834</td>
<td>3095</td>
<td>4881</td>
<td>2693</td>
<td>4153</td>
<td>2300</td>
<td>2497</td>
<td>1173</td>
<td>2107</td>
<td>1610</td>
<td>2597</td>
<td>2913</td>
<td>2904.5</td>
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<tr>
<td>Reduction OF [%]</td>
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<td>11.0</td>
<td>13.4</td>
<td>6.8</td>
<td>52.2</td>
<td>3.9</td>
<td>4.7</td>
<td>4.3</td>
<td>1.8</td>
<td>5.2</td>
<td>6.7</td>
<td>8.0</td>
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<tr>
<td>$bdexpr$ [€]</td>
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<td>11</td>
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<td>7</td>
<td>6</td>
<td>11</td>
<td>56</td>
<td>42</td>
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<td>1379</td>
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<td>619</td>
<td>533</td>
<td>722</td>
<td>576</td>
<td>605</td>
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<td>876</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
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<tr>
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<td>$buRegUpExpr$ [€]</td>
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<td>2372</td>
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<td>1221</td>
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<td>1115</td>
<td>610</td>
<td>1265</td>
<td>945</td>
<td>1336</td>
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<tr>
<td>Mean forecast error [MW]</td>
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<td>0.19</td>
<td>0.10</td>
<td>0.17</td>
<td>0.10</td>
<td>0.12</td>
<td>0.08</td>
<td>0.12</td>
<td>0.10</td>
<td>0.12</td>
<td>0.16</td>
<td>0.13</td>
</tr>
</tbody>
</table>

**Table 8.1.:** Monthly results optimum dispatch li-ion BESS (year 2013)
### Case Study III: Optimum dispatch

#### Table 8.2: Monthly results optimum dispatch lead-acid BESS (year 2013)

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>Annual average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduction OF [€] (optimum scheduling vs. optimum dispatch)</td>
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<td>3098</td>
<td>4856</td>
<td>2691</td>
<td>4147</td>
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<td>2501</td>
<td>1171</td>
<td>2106</td>
<td>1605</td>
<td>2596</td>
<td>2892</td>
<td>2895</td>
</tr>
<tr>
<td>Reduction OF [%]</td>
<td>138.7</td>
<td>7.4</td>
<td>22.7</td>
<td>5.0</td>
<td>66.8</td>
<td>3.5</td>
<td>3.8</td>
<td>1.2</td>
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<td>1.5</td>
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<td>1</td>
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<td>7</td>
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<td>2</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.0</td>
</tr>
<tr>
<td>bdRegUpDexpr [€]</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
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<td>1335</td>
<td>2359</td>
<td>1134</td>
<td>2199</td>
<td>1092</td>
<td>1221</td>
<td>600</td>
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<td>609.8</td>
<td>1265.6</td>
<td>946.9</td>
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<td>Mean forecast error</td>
<td>Same as in Tab. 8.1.</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
8.4 Conclusions

The conclusions of case study III are summarized by the following points:

- The optimum dispatch function is capable to properly reschedule the VPP by finding the solution with the minimum penalty payments for imbalances. It is able to take the optimum dispatch decision considering the DA- as well as the ancillary service markets.

- Tab. 8.1 and Tab. 8.2 show that the value of \( OF \) is always less in the optimum dispatch problem compared to the optimum scheduling problem. This confirms the expectation and serves as another indication that the dispatch function is correctly formulated.

- The artificial wind power forecast error calculated by Algorithm 8.1 is low with a mean error of 0.13MW for the underlying wind power production data. It is likely that this error is higher in reality. However, actual forecast data was not available and thus an artificial forecast was generated based on an actual wind power production profile in order to generate input data that deviates from the optimum scheduling problem. For future studies, it is recommended to receive an actual forecast that relates to the wind power production data in question.

- The high share of imbalance penalties through \( buRegUpDexpr \) occur if large bids to the RP up market are allowed in the first place but market participants cannot be sure that RP up is actually demanded in a specific hour. Otherwise, \( buDexpr \) will most likely show the biggest contribution to imbalance penalties.

- Results in Tab. 8.1 and Tab. 8.2 are very similar to each other in regards to the absolute reduction of \( OF \). The power rating is the only property the lithium- and lead-acid BESSs have in common while all other parameters differ from each other. Therefore, this result is unexpected. This demonstrates the big impact of the discharger power rating in the dispatch function on the absolute reduction of \( OF \). Regarding the relative reduction of \( OF \), this impact cannot be seen.
9. Conclusions, contributions, and future work

This chapter presents the conclusions drawn and highlights important contributions of this work. Last but not least, an outlook for future work is presented.

9.1. Conclusions

This work proposes a MILP formulation to address the questions of optimum scheduling, dispatch (rescheduling), and optimum sizing of VPPs. It can be considered as an intelligent energy management system for VPPs. The focus of this work is the MILP formulation which covers a realistic BESS model in order to address specific properties like capacity fade or self-discharge. Moreover, the algorithm incorporates exemplarily Danish (DK1) power market rules where the VPP is able to send bids to the DA- and ancillary service markets. Three case studies demonstrate the suitability of the proposed problem formulation regarding optimum scheduling, -dispatch, and -sizing of VPPs. The conclusions of each case study are summarized below:

Case study I

- This case study is used to demonstrate optimum scheduling of VPPs based on the DA- and ancillary service power markets. It also illustrates that the optimum scheduling problem can address hybrid BESS systems based on different battery technologies.

- Moreover, it demonstrates that the proposed MILP formulation can be used to model capacity fade of the BESS. However, the parameters used to model capacity fade based on literature values result in a relatively high capacity
Chapter 9 Conclusions, contributions, and future work

fade of the li-ion BESS. Results indicate that battery parameters are to be determined and verified by laboratory tests.

• This case study also highlights that the tested configurations of BESSs (li-ion, lead-acid, and hybrid BESSs) are not yet a profitable investment. In order that the tested BESSs become profitable, DA- and ancillary service market prices need to be more attractive, BESS costs need to decrease, or the lifetime of the BESSs need to be increased.

• It also reveals that monthly time periods with one hour resolution can be solved relative quickly with standard laptop equipment (<30 min for a single BESS). However, the actual time to solve the optimization problem depends on the input data provided.

Case study II

• The numerical example in case study II shows that the annualized costs of the BESS are correctly calculated in relation to the capacity fade. This means that costs of the BESS are accounted for based on its usage (SoH). The more a BESS is cycled, the shorter is its lifetime and thus the higher are its annualized costs. On the other hand, revenues from markets will most likely increase the more the BESS is cycled. These trends are considered in the objective function by maximizing the profit (revenues minus costs) of the VPP.

• The proposed optimum sizing algorithm is able to determine the optimum size(s) and BESS technolog(y/ies) independent of each other by choosing the optimum operating profile(s) of the BESS(s) and WPPs as well as taking the costs of the BESS(s) for the related operating pattern(s) into account. The proposed algorithm can explicitly handle hybrid BESSs. An extra investment in BESSs beyond the optimum cannot be justified.

• The optimum sizing problem is more difficult to solve compared to the optimum scheduling- and dispatch problem and powerful computer equipment is recommended to solve this problem in reasonable time. The proposed formulation can be considered as a first step to address the problem of relating the costs of the BESS to its SoH in a MILP formulation. Further improvements would target a faster run of the model, e.g. by tightening
9.1 Conclusions

the proposed MILP formulation.

Case study III

- The optimum dispatch function is able to reschedule the VPP based on input parameters that differ from the optimum scheduling problem. Input parameters that can become different in respect to the scheduling problem need to be adjusted. These can be the wind power production forecast, market prices, the amount of power scheduled to each market in order to match the awarded amount of bids, or the latest available actual SoC. It is demonstrated that bids on the DA- and RP market are correctly adjusted with balance up and -down power as allowed by DK1 market rules. Also, bids submitted to the PFR market are not adjusted as required by market regulations in DK1. Choosing very high prices for balancing PFR up and -down prohibits any adjustment of the awarded bids. This shows that the concept introduced by [73] to balance a plant can be extended to multiple energy markets.

- The case study indicates that the VPP is mainly balanced through $Bu_t$ and $BuRegUp_t$ in case of an under-production of the WPPs and that potential over-production are mostly handled through down-ramping the WPPs.

- The optimum dispatch problem takes slightly less time to be solved than the optimum scheduling problem described in case study I.

Overall, the three case studies verify that the proposed intelligent energy management system is capable to adequately handle the optimum scheduling-, dispatch-, and sizing problem of VPPs by providing the appropriate input parameters to the algorithm.

General conclusions

The proposed optimization problem formulation can become computationally heavy to solve depending on the length of the time period and the number of BESSs considered for optimization. Due to the lack of a powerful workstation computer or even a cluster computer that could be used for this work, the optimum sizing problem is restricted to a numerical example which could be solved with the available laptop computer (based on the adjusted CPLEX parameters).
In practice, it is recommended to invest in a powerful workstation computer or even to use a cluster computer. This will allow to solve the optimum sizing problem over a longer, more representative, time period.

9.2. Major contributions

Major contributions of this work to research are as follows:

- The proposed MILP algorithm can be used for optimum sizing, -scheduling, and -dispatch specifically for BESSs connected to WPPs, called VPPs, and operating them on different power markets. This enables the operator or investor of VPPs to solve three different questions with one algorithm by choosing appropriate input data.

- As part of the proposed MILP algorithm, the model of the BESS is able to take important battery specific parameters into account such as capacity fade, self-discharge, minimum SoC, and self-consumption.

- Considering the optimum sizing function for BESSs, the proposed MILP formulation takes into account the capacity fade (SoH) of the BESS that occurs over the optimized operating pattern of the VPP. The capacity fade is then related to the annualized costs of the BESS. Capacity fade can also be considered as aging of the BESS: The faster it ages, the sooner it needs to be replaced. The algorithm considers that the annualized costs of the BESS(s) need to be justified by the revenues the BESS(s) can generate over the same time period. The BESS that yields maximum profit is selected as the optimum or no BESS is chosen if not profitable.

- In the problem formulation, it is possible to specify any number of \( b \) different BESSs that should be considered when finding the optimum solution. Further, the maximum number of different BESSs (\( max.NbEs \)), which can be part of the optimum solution, needs to be specified. Thus, the model is able to indicate optimum hybrid BESSs that consist of different BESS technologies. Also, the maximum battery energy and power rating of each BESS \( b \) can be specified by \( max.NbSlices_b \). Each battery technology out of the optimum hybrid BESS can therefore have its specific battery energy.
and power rating. The optimum result is easy to read and is indicated by the decision variables $\text{BatteryInt}_b$ and $\text{NbSlicesActive}_b$.

- An optimum scheduling-, dispatch-, and sizing problem is formulated based on MILP that addresses the DA- and two ancillary service markets (RP and PFR markets) for WPPs connected to BESSs.

- The proposed formulation targets at the Danish power market (west) and their market regulations are translated into a MILP formulation. However, the algorithm can easily be adjusted for other power market regulations by changing input parameters of the model. This may be sufficient to adjust the model for another power market regulation.

- The proposed formulation can be used modularly meaning that the algorithm can be applied on BESSs or WPPs alone without interaction. Moreover, it is possible to choose a specific market to participate in. This can be specified by the input parameters provided to the model.

## 9.3. Future work

Having discussed the conclusions and contributions of this work, the following list is a proposal for future work to improve the proposed algorithm even further.

- Additionally to the work carried out in this thesis, a sensitivity study is recommend for a better understanding how changes of input parameters affect results. Due to the issue of the long computational time of some functions, it is recommended to specify certain input parameters for testing sensitivities and to re-run the problem in order to observe changes in the results. A starting point could be the optimum dispatch problem in order to identify how changes in the wind power forecast influence the optimum schedule.

- Furthermore, future work can include an example over a short optimization period (e.g. 24h to 48h) to demonstrate the gain in profit for the VPP based on the proposed algorithm versus a manually chosen schedule.

- Also, the proposed MILP formulation makes use of text book formulations
Chapter 9 Conclusions, contributions, and future work

to tackle PLAs. In order to achieve a tighter MILP formulation, this can be improved as proposed by [89] or [46], for instance.

• Further considerations on the time required to solve the model should be undertaken. The solver used to solve the problem formulation in this work is CPLEX® and the model implementation is done in IBM® iLog® CPLEX® Optimization Studio. One possibility is to run the automated tuning tool of this software package to find improved settings of the CPLEX® solver.

• Another obvious next step is to take uncertainties into account such as the wind power forecast or market prices. The proposed MILP formulation can be converted into a robust optimization or stochastic optimization algorithm.

• Moreover, the intra-day market can be incorporate in the optimization problem for an even more complete picture of the DK1 power market.

• Currently, the model is restricted to WPPs and BESSs and future work is to incorporate other power generation technologies like solar- or biomass power plants. In addition, other ES technologies like e.g. flywheels can be future work for implementation in the model.

• Further improvements can be made concerning the calculation of the capacity fade which is in this model based on the SoC and energy throughput through the BESS. A more accurate method is the so called rain-flow cycle counting algorithm for BESSs that is discussed e.g. in [7].

• In addition, the proposed algorithm can be adapted to incorporate methods of industrial maintenance and replacement to find the optimum replacement SoH of the BESS that may be different from the chosen 80% SoH limit in this work.
Nomenclature

Sets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Set Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>As the set of $\bar{a}$ linear approximations: $a = 0, 1, \ldots, \bar{a} - 1$</td>
</tr>
<tr>
<td>$B$</td>
<td>As the set of $\bar{b}$ BESSs: $b = 0, 1, \ldots, \bar{b} - 1$</td>
</tr>
<tr>
<td>$P_a$</td>
<td>As the sets of $\bar{p}_a$ breakpoints used for the $(a + 1)^{th}$ linear approximation: $p_a = 0, 1, \ldots, \bar{p}_a - 1$</td>
</tr>
<tr>
<td>$Q_b$</td>
<td>As the set of $maxNbSlices_b$ possible slices that the BESS $b$ can consist of: $q_b = 0, 1, \ldots, maxNbSlices_b - 1$</td>
</tr>
<tr>
<td>$T$</td>
<td>As the set of $\bar{t}$ time periods, $t = 0, 1, \ldots, \bar{t} - 1$</td>
</tr>
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</table>

Table 9.1.: Sets

Table 9.2.: Parameters used in the MILP formulation

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<th>Parameters</th>
<th>Unit</th>
<th>Comments</th>
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<td>Breakpoints used for piecewise linear approximations</td>
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<td>Upper bound $bdPower$</td>
</tr>
<tr>
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<td>PFR down balancing down costs</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>Unit</td>
<td>Description</td>
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<td>--------------</td>
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<td>Primary frequency regulation down maximum balance down power</td>
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<td>PFR up balancing down costs</td>
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<td>$bdPrimaryUpMax$</td>
<td>MW</td>
<td>Primary frequency regulation up maximum balance down power</td>
</tr>
<tr>
<td>$bdRegDownCost_t$</td>
<td>€</td>
<td>RP down balancing down costs</td>
</tr>
<tr>
<td>$bdRegDownMax$</td>
<td>MW</td>
<td>Regulation down maximum balance down power</td>
</tr>
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<td>$bdRegUpCost_t$</td>
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<td>RP up balancing down costs</td>
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<td>MW</td>
<td>Regulation up maximum balance down power</td>
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<td>€/MWh</td>
<td>Bonus paid to WPPs</td>
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<td>Upper bound $BuPower$</td>
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<td>Primary frequency regulation down maximum balance up power</td>
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<tr>
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<td>Primary frequency regulation up maximum balance up power</td>
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### Nomenclature

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<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
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<td>RP down balancing up costs</td>
</tr>
<tr>
<td>$buRegDownMax$</td>
<td>MW</td>
<td>Regulation down maximum balance up power</td>
</tr>
<tr>
<td>$buRegUpCost_t$</td>
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<td>Balance cost for regulation power (which is -1 x regulating power price for the DK1 market)</td>
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<td>$capacityFadeRate_b$</td>
<td>$\frac{MW}{MW\cdot h}$</td>
<td>Calendric capacity fade (see sec. 2.3.1 ond page 20 and sec. A.1 on page 171)</td>
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<td>$capacityFadeRateThroughput_b$</td>
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<td>$\frac{1}{\sqrt{\text{round trip efficiency}}}$</td>
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<td>Max energy flow charger</td>
</tr>
<tr>
<td>$chargerMax_b$</td>
<td>MW</td>
<td>Max charging power out of market node</td>
</tr>
<tr>
<td>$chargerMin_b$</td>
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<td>Min charging power out of market node</td>
</tr>
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<td>$\frac{€}{MW\cdot a}$</td>
<td>Annualized CAPEX of WPP</td>
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<td>OPEX of WPP</td>
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<td>Nomenclature</td>
<td>Description</td>
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</tr>
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<td>ES costs $\frac{\text{€}}{\text{MW}}$</td>
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</tr>
<tr>
<td>$\text{esInvestSwitch}$</td>
<td>1 for sizing function, else 0</td>
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</tr>
<tr>
<td>$\text{initSoc}_b$</td>
<td>State of charge of storage at the beginning (period 0) $\frac{\text{MWh}}{\text{MWh}}$</td>
<td></td>
</tr>
<tr>
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<td>DA market price €</td>
<td></td>
</tr>
<tr>
<td>$\text{marketPricePrimaryDown}_t$</td>
<td>Price PFR down €</td>
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</tr>
<tr>
<td>$\text{marketPricePrimaryUp}_t$</td>
<td>Price PFR up €</td>
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</tr>
<tr>
<td>$\text{marketPriceRegDown}_t$</td>
<td>Price RP down €</td>
<td></td>
</tr>
<tr>
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<td>Price RP up €</td>
<td></td>
</tr>
<tr>
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<td>Limits the maximum number of ES technologies to be part of the final solution. $(\text{maxNbEs} \leq B)$</td>
<td></td>
</tr>
<tr>
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<td>Max number of slices (increments) of which BESS $b$ can consist of.</td>
<td></td>
</tr>
<tr>
<td>$\text{primaryDownReserveFactor}$</td>
<td>Factor to determine the minimum SisNode level to be able to provide PFR up $\frac{\text{MWh}}{\text{MWh}}$</td>
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<tr>
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<td>Maximum power for primary frequency regulation down MW</td>
<td></td>
</tr>
<tr>
<td>Nomenclature</td>
<td>Unit</td>
<td>Description</td>
</tr>
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<td>------------------------------------</td>
<td>------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>primaryDownMin_t</td>
<td>MW</td>
<td>Minimum power for primary frequency regulation down</td>
</tr>
<tr>
<td>primaryUpMax_t</td>
<td>MW</td>
<td>Maximum power for primary frequency regulation up</td>
</tr>
<tr>
<td>primaryUpMin_t</td>
<td>MW</td>
<td>Minimum power for primary frequency regulation up</td>
</tr>
<tr>
<td>primaryUpReserveFactor</td>
<td>MWh/MWh</td>
<td>Factor to determine the minimum SosNode level to be able to provide PFR up</td>
</tr>
<tr>
<td>purchaseCostPerturbation</td>
<td>€</td>
<td>Difference between buying and selling power on the DA market where buying is more expensive (avoids that large quantities are bought and sold within the same hour)</td>
</tr>
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<td>purchaseMax_t</td>
<td>MW</td>
<td>Maximum purchase from DA market in one period</td>
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<tr>
<td>purchaseMin_t</td>
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<td>Maximum purchase from DA market in one period</td>
</tr>
<tr>
<td>regDownEnergyFlowFactor</td>
<td>MWh/MWh</td>
<td>Factor to determine actual energy dispatch for RP down</td>
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<tr>
<td>regDownMax_t</td>
<td>MW</td>
<td>Maximum power for RP down in one period</td>
</tr>
<tr>
<td>regDownMin_t</td>
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<td>regUpEnergyFlowFactor</td>
<td>MWh/MWh</td>
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</tbody>
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### Nomenclature

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<thead>
<tr>
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<tr>
<td>$regUpMax_t$</td>
<td>MW</td>
<td>Maximum power for RP up in one period</td>
</tr>
<tr>
<td>$regUpMin_t$</td>
<td>MW</td>
<td>Minimum power for RP up in one period</td>
</tr>
<tr>
<td>$saleMax_t$</td>
<td>MW</td>
<td>Maximum sales to DA market in one period</td>
</tr>
<tr>
<td>$saleMin_t$</td>
<td>MW</td>
<td>Minimum sales to DA market in one period</td>
</tr>
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<td>$\frac{MW}{h}$</td>
<td>Self-consumption of ES in one period</td>
</tr>
<tr>
<td>$selfDischargeRate_b$</td>
<td>$\frac{MW}{h}$</td>
<td>Loss of SoC of storage per hour</td>
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<td>Source storage energy</td>
</tr>
<tr>
<td>$sosNodeMax_b$</td>
<td>MWh</td>
<td>Sink storage energy</td>
</tr>
<tr>
<td>$\text{switch}$</td>
<td>—</td>
<td>$If \ switch = 1 :$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$SoC_0 = \text{initSoc}_b$;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$If \ switch = 0 :$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$SoC_0 = SoC_{\text{nbPeriods}-1}$</td>
</tr>
<tr>
<td>$td_t$</td>
<td>h</td>
<td>Time step</td>
</tr>
<tr>
<td>$windMax_t$</td>
<td>MW</td>
<td>Wind power production of time period $t$</td>
</tr>
</tbody>
</table>
Table 9.3: Terms of cost function

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bdDexpr$</td>
<td>€</td>
<td>Costs for balance down DA market node</td>
</tr>
<tr>
<td>$bdPrimaryDownDexpr$</td>
<td>€</td>
<td>Costs for balance down PFR down market node</td>
</tr>
<tr>
<td>$bdPrimaryUpDexpr$</td>
<td>€</td>
<td>Costs for balance down PFR up market node</td>
</tr>
<tr>
<td>$bdRegDownDexpr$</td>
<td>€</td>
<td>Costs for balance down RP down market node</td>
</tr>
<tr>
<td>$bdRegUpDexpr$</td>
<td>€</td>
<td>Costs for balance down RP up market node</td>
</tr>
<tr>
<td>$bonusWppOf$</td>
<td>€</td>
<td>Revenues of bonus from WPP</td>
</tr>
<tr>
<td>$buDexpr$</td>
<td>€</td>
<td>Costs for balance up DA market node</td>
</tr>
<tr>
<td>$buPrimaryDownDexpr$</td>
<td>€</td>
<td>Costs for balance up PFR down market node</td>
</tr>
<tr>
<td>$buPrimaryUpDexpr$</td>
<td>€</td>
<td>Costs for balance up PFR up market node</td>
</tr>
<tr>
<td>$buRegDownDexpr$</td>
<td>€</td>
<td>Costs for balance up RP down market node</td>
</tr>
<tr>
<td>$buRegUpDexpr$</td>
<td>€</td>
<td>Costs for balance up RP up market node</td>
</tr>
<tr>
<td>$costsBess$</td>
<td>€</td>
<td>Costs of BESS</td>
</tr>
<tr>
<td>$costsWppOf$</td>
<td>€</td>
<td>Costs WPP</td>
</tr>
</tbody>
</table>
Table 9.4.: Decision variables of MILP formulation (unless otherwise stated, all decision variables are continuous and non-negative):

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Unit</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{a,p_a,b}$</td>
<td>—</td>
<td>Weights used for piecewise linear approximation</td>
</tr>
<tr>
<td>$BatteryInt_b$</td>
<td>—</td>
<td>Boolean: BESS investment decision variable</td>
</tr>
<tr>
<td>$BdPower_t$</td>
<td>MW</td>
<td>Power for balancing down</td>
</tr>
<tr>
<td>$BdPrimaryDown_t$</td>
<td>MW</td>
<td>Balance power down for primary frequency regulation down</td>
</tr>
<tr>
<td>$BdPrimaryUp_t$</td>
<td>MW</td>
<td>Balance power down for primary frequency regulation up</td>
</tr>
<tr>
<td>$BdRegDown_t$</td>
<td>MW</td>
<td>Balance power down for regulation down</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>Unit</td>
<td>Description</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$BdRegUp_t$</td>
<td>MW</td>
<td>Balance power down for regulation up</td>
</tr>
<tr>
<td>$BuPower_t$</td>
<td>MW</td>
<td>Power for balancing up</td>
</tr>
<tr>
<td>$BuPrimaryDown_t$</td>
<td>MW</td>
<td>Balance power up for primary frequency regulation down</td>
</tr>
<tr>
<td>$BuPrimaryUp_t$</td>
<td>MW</td>
<td>Balance power up for primary frequency regulation up</td>
</tr>
<tr>
<td>$BuRegDown_t$</td>
<td>MW</td>
<td>Balance power up for regulation down</td>
</tr>
<tr>
<td>$BuRegUp_t$</td>
<td>MW</td>
<td>Balance power up for regulation up</td>
</tr>
<tr>
<td>$Charger_{b,t}$</td>
<td>MW</td>
<td>Charging power out of market node</td>
</tr>
<tr>
<td>$ChargerCommitment_{b,t}$</td>
<td>—</td>
<td>Boolean: Charger commitment</td>
</tr>
<tr>
<td>$ChargerFlow_{b,t}$</td>
<td>MW</td>
<td>Charging power inside battery after efficiency loss</td>
</tr>
<tr>
<td>$ChargerPrimaryDown_{b,t}$</td>
<td>MW</td>
<td>Charging power from PFR down market</td>
</tr>
<tr>
<td>$ChargerPrimaryUp_{b,t}$</td>
<td>MW</td>
<td>Charging power from PFR up market</td>
</tr>
<tr>
<td>$ChargerRegDown_{b,t}$</td>
<td>MW</td>
<td>Charging power from RP down</td>
</tr>
<tr>
<td>$Discharger_{b,t}$</td>
<td>MW</td>
<td>Discharging power out of market node</td>
</tr>
<tr>
<td>Variable</td>
<td>Units</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------------------------</td>
<td>------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$\text{DischargerCommitment}_{b,t}$</td>
<td>—</td>
<td>Boolean: Discharger commitment</td>
</tr>
<tr>
<td>$\text{DischargerFlow}_{b,t}$</td>
<td>MW</td>
<td>Discharging power</td>
</tr>
<tr>
<td>$\text{DischargerPrimaryDown}_{b,t}$</td>
<td>MW</td>
<td>Discharging power into PFR down market</td>
</tr>
<tr>
<td>$\text{DischargerPrimaryUp}_{b,t}$</td>
<td>MW</td>
<td>Discharging power into PFR up market</td>
</tr>
<tr>
<td>$\text{DischargerRegUp}_{b,t}$</td>
<td>MW</td>
<td>Discharging power into RP up market</td>
</tr>
<tr>
<td>$\text{EsCostOF}_{b}$</td>
<td>$\frac{\varepsilon}{\text{MW} \cdot a}$</td>
<td>Annualized costs of BESS $b$ accounted for in the objective function</td>
</tr>
<tr>
<td>$\text{EsCostVsCapacity}_{b}$</td>
<td>$\frac{\varepsilon}{\text{MW} \cdot a}$</td>
<td>Annualized BESS costs per MW depending on capacity fade</td>
</tr>
<tr>
<td>$\text{FinalCapacity}_{b}$</td>
<td>$\frac{\text{MW} \cdot h}{\text{MW} \cdot h}$</td>
<td>Capacity at final time step of BESS $b$</td>
</tr>
<tr>
<td>$\text{IntPLA}_{a,p,a,b}$</td>
<td>—</td>
<td>Boolean: Enforcing adjacency condition in piecewise linear approximation</td>
</tr>
<tr>
<td>$\text{NbSlicesActive}_{b}$</td>
<td>—</td>
<td>Integer variable: Number of active slices in BESS $b$</td>
</tr>
<tr>
<td>$\text{NbSlicesActiveTimestepCharger}_{b,t}$</td>
<td>—</td>
<td>Number of slices active in specific time step $t$ for charger</td>
</tr>
<tr>
<td>$\text{NbSlicesActiveTimestepDischarger}_{b,t}$</td>
<td>—</td>
<td>Number of slices active in specific time step $t$ for discharger</td>
</tr>
</tbody>
</table>
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OF</td>
<td>€</td>
<td>Objective function value</td>
</tr>
<tr>
<td>( PlaX_{a,b} )</td>
<td>—</td>
<td>X-axes value of piecewise linear approximation</td>
</tr>
<tr>
<td>( PlaY_{a,b} )</td>
<td>—</td>
<td>Y-axes value of piecewise linear approximation</td>
</tr>
<tr>
<td>( PrimaryDown_t )</td>
<td>MW</td>
<td>Sales on PFR down market</td>
</tr>
<tr>
<td>( PrimaryUp_t )</td>
<td>MW</td>
<td>Sales on PFR up market</td>
</tr>
<tr>
<td>( Purchase_t )</td>
<td>MW</td>
<td>Amount of purchase from DA market</td>
</tr>
<tr>
<td>( ReciprocalNbSliceActive_b )</td>
<td>—</td>
<td>Approximation of ( NbSlicesActive_b )</td>
</tr>
<tr>
<td>( RegDown_t )</td>
<td>MW</td>
<td>Sales on RP down market</td>
</tr>
<tr>
<td>( RegUp_t )</td>
<td>MW</td>
<td>Sales on RP up market</td>
</tr>
<tr>
<td>( S_{a,b} )</td>
<td>( \sqrt{MWh} )</td>
<td>Decision variable used for separable programming. ( S_{a,b} ) has to allow negative values for ( a = 1, 3 ).</td>
</tr>
<tr>
<td>( Sale_t )</td>
<td>MW</td>
<td>Amount of sales on DA market</td>
</tr>
<tr>
<td>( SisNode_{b,t} )</td>
<td>MWh</td>
<td>Energy level in sink storage node</td>
</tr>
<tr>
<td>( SosNode_{b,t} )</td>
<td>MWh</td>
<td>Energy level in source storage node</td>
</tr>
<tr>
<td>( WindSpot_t )</td>
<td>MW</td>
<td>Power delivered to the DA market by the WPP</td>
</tr>
</tbody>
</table>
Nomenclature

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$WindRegUp_t$</td>
<td>MW</td>
<td>RP up provided by the WPP</td>
</tr>
<tr>
<td>$WindRegDown_t$</td>
<td>MW</td>
<td>RP down provided by the WPP</td>
</tr>
<tr>
<td>$WppRegDownCommitment_t$</td>
<td>MW</td>
<td>Boolean: Commitment variable WPP RP up</td>
</tr>
<tr>
<td>$WppRegUpCommitment_t$</td>
<td>MW</td>
<td>Boolean: Commitment variable WPP RP down</td>
</tr>
<tr>
<td>$Z_{a,b}$</td>
<td>$\sqrt{MWh}$</td>
<td>Variable used for linear approximation of $S_{a,b}^2$</td>
</tr>
</tbody>
</table>

Table 9.5.: Parameters not part of MILP formulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$actualWppProduction_t$</td>
<td>MW</td>
<td>Wind power production of turbines</td>
</tr>
<tr>
<td>$annualizedCost_b$</td>
<td>€</td>
<td>Annualized investment cost of ES</td>
</tr>
<tr>
<td>$costWppCapex$</td>
<td>€</td>
<td>Capital expenditures (CAPEX) for WPP</td>
</tr>
<tr>
<td>$costWppOpex$</td>
<td>€</td>
<td>Operational expenditures (OPEX) for WPP</td>
</tr>
<tr>
<td>$finalCapacityFade_b$</td>
<td>$\frac{MWb}{MWh}$</td>
<td>Total irreversible capacity fade at final time step</td>
</tr>
<tr>
<td>$i$</td>
<td>%</td>
<td>Interest rate</td>
</tr>
</tbody>
</table>
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ih)</td>
<td>Investment horizon (number of years of an investment)</td>
</tr>
<tr>
<td>(k)</td>
<td>Factor that is multiplied with (pv_b) to vary BESS costs for case study II.</td>
</tr>
<tr>
<td>(\text{min}Soc_b)</td>
<td>Minimum SoC of BESS (b)</td>
</tr>
<tr>
<td>(obligationPrimaryDown_t)</td>
<td>MW  Obligation primary down is the result of the scheduling function for sales on the PFR down market.</td>
</tr>
<tr>
<td>(obligationPrimaryUp_t)</td>
<td>MW  Obligation primary up is the result of the scheduling function for sales on the PFR up market.</td>
</tr>
<tr>
<td>(obligationPurchase_t)</td>
<td>MW  Obligation purchase is the result of the scheduling function for purchases on the DA market.</td>
</tr>
<tr>
<td>(obligationRegDown_t)</td>
<td>MW  Obligation reg down is the result of the scheduling function for sales on the RP down market.</td>
</tr>
<tr>
<td>(obligationRegUp_t)</td>
<td>MW  Obligation reg up is the result of the scheduling function for sales on the RP up market.</td>
</tr>
<tr>
<td>(obligationSale_t)</td>
<td>MW  Obligation sales is the result of the scheduling function for sales on the DA market.</td>
</tr>
</tbody>
</table>
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_g$</td>
<td>MW</td>
<td>Power generated (used for illustration in Fig. 4.1)</td>
</tr>
<tr>
<td>$P_{sch}$</td>
<td>MW</td>
<td>Power scheduled (used for illustration in Fig. 4.1)</td>
</tr>
<tr>
<td>$pv_b$</td>
<td>€/a</td>
<td>Present value of BESS investment costs (at year 0)</td>
</tr>
<tr>
<td>$pv$</td>
<td>€/a</td>
<td>Present value</td>
</tr>
<tr>
<td>$switch$</td>
<td></td>
<td>Switch to distinguish constraint of first time step of sisNode and sosNode.</td>
</tr>
<tr>
<td>$windForecast_t$</td>
<td>MW</td>
<td>Wind power forecast of time period t</td>
</tr>
<tr>
<td>$yearsEsOperation_b$</td>
<td>a</td>
<td>Number of years until end of life of ES is reached</td>
</tr>
</tbody>
</table>
Bibliography


[43] “Technical regulation 3.2.5 for wind power plants with a power output greater than 11 kW,” 2010. [Online]. Avail-
able: http://www.energinet.dk/SiteCollectionDocuments/Engelske%20dokumenter/El/55986-10_v1_Grid%20Code%202010.pdf


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A. Input data

This appendix discusses all input parameters used in chapter 6 to chapter 8. If a referenced document provides a range instead of a specific value for one parameter, for this work the average of the stated minimum and maximum value is taken.

A.1. Calculation of input parameters

**Exercised capacity fade**  Exercised capacity fade is represented by the parameter $capacityFadeRateThroughput_b$ in the problem formulation (and has the unit $\frac{MWh}{MWh}$). As modeled in sec. 5.2.1, the exercised capacity fade occurs while charging and discharging the BESS. In Tab. 2.1 on page 18 the storage life is indicated based on the cycle life which needs to be converted to a storage life based on energy throughput according to equation 2.1 on page 21.

For li-ion batteries, Tab. 2.1 indicates a cycle life of 2,000 to 10,000 cycles. For this study, it is assumed that the cycle life of a li-ion battery is 6,000 cycles at 100% DoD. Considering a 1MW li-ion BESS specified in Tab. A.1, this converts (based on equation 2.1) to 1500MWh throughput and a loss of $1.66 \times 10^{-5}$ MWh per 1MWh throughput through the charger and discharger. If the exercised capacity fade would only occur at the charger or discharger, the amount would needed to be multiplied by two.

Regarding lead-acid batteries, a cycle life of 500 to 1,000 cycles is indicated (see Tab. 2.1) and 750 cycles are assumed for this study. For a 1MW lead-acid BESS, this means a total throughput until end of life of 3000MWh and a capacity fade rate of $1.33 \times 10^{-4}$ MWh per 1MWh throughput.

**Calendric capacity fade**  Data of the calendric capacity fade $capacityFadeRate_b$ (formulated in sec. 5.2.1) of li-ion BESSs is used from [33] Fig. 1. The results
obtained are for a specific li-ion battery chemistry and might not be applicable to other types of li-ion battery technologies. However, due to limited data availability, these data are taken as a reference number. In practical applications of the algorithm, the author recommends to perform necessary lab tests beforehand in order to obtain accurate and reliable values of a specific BESS in question. In Fig. 1 in [33], the calendric capacity fade is shown for cells at four different temperatures up to 440 days. The cell at 15°C is chosen as reference because it shows the lowest calendric capacity fade indicating that this temperature is the optimum operating temperature out of the four tested cells. In an actual BESS, the temperature would be controllable which justifies to pick the cell with the lowest capacity fade. Fig. 1. in [33] indicates a capacity fade of 6.4% after 440 days. This means that the BESS end of life (80% of the original capacity) would be reached after 1375 days or 3.8 years based on linear interpolation. This results in a $6.06 \cdot 10^{-6} \text{MW/MWh}$ calendric capacity fade.

Regarding data for the lead-acid BESS, reference [90] indicates a calendar life of value regulated lead-acid batteries (VRLA) of greater than 3 years but no actual test data proofs this number. For this work, a three year lifetime is considered at 100% SoC. This is equal to $7.61 \cdot 10^{-6} \text{MW/MWh}$.

Further, it has to be annotated that the problem formulation takes the SoC into account for the calendric capacity fade. A SoC of 100% means that the calendric capacity fade occurs as specified by the variable $\text{capacityFadeRate}_b$ and at 0% SoC no calendric capacity fade is happening. This might be incorrect as shown in [33] where capacity fade is also happening at lower SoC but this relationship would be non-linear. Due to the problem of implementing such non-linear effects, which would further increase the already large model and due to the issue of data availability, the chosen approach is considered to be sufficient. This means for this model that the stated 3.8 and 3 years, respectively, are likely to be exceeded regarding the calendric capacity fade.

However, on the other hand, data for the calendric- and exercised capacity fade are from different sources that have not studied the correlated effect of exercised- and calendric capacity fade. In the developed model, both capacity fades are summed up which can lead to a shorter lifetime of the BESS than achievable in practice. This highlights the demand for laboratory tests of both capacity fades related to each other for the BESSs in question.
A.1 Calculation of input parameters

**Wind power costs** All cost data concerning wind power production are from reference [91] where costs for a 2.5MW WPP are discussed. The investment costs are stated with 3.235 Mio € including foundation and erection. Operational costs are 125,000 €/a including e.g. insurance, reserve assets, and full maintenance contract. Based on available wind power data of a 12MW wind farm for one year with unknown location, wind speed, and time stamp, the full load hours are taken into account with 2032h per year (which is a capacity factor of 23.2%). It is not possible to state the IEC wind class for this wind farm because of unknown wind speed data. Assuming an investment horizon of 20 years, the annuity of the investment costs is 379,982€ at an interest rate of 10%. Adding the operational costs to the annuity, the total annual costs are 504,982€ which, divided by the full load hours, amount to 99.39€/MWh.

Besides the costs of wind power generation, there is also a bonus available for wind power production in Denmark at around 33€/MWh [92]. Considering the average DA market price in Denmark from 2009 until 2013 which was 41.16€/MWh [93], the total income for this period would be 74.16€/MWh for wind power (onshore). Comparing this number with the above calculated costs of 99.39€/MWh, wind power would not be a profitable business based on the underlying wind production data with a capacity factor of 23.2%. Moreover, by averaging the DA market data, the total income may be assumed higher than actually achievable because it is likely that in periods with high wind power generation, the DA market price is lower than in the other periods with a lower level of wind power in the grid.

One problem in the calculation of the capacity factor is the unknown location of the wind farm from which the wind data is used. [94] shows an overview of the capacity factor for wind turbines in different locations. For Denmark, [94] indicates a capacity factor of 35.7% which is higher than from the wind farm where the wind production data are from. Considering the capacity factor of 35.7% and the same assumptions as above, the total wind power production costs are 64.59€/MWh which lets wind power be an attractive business case in Denmark. This number is also based on investment costs of 3.235 Mio € for the 2.5MW plant which results in annualized costs of 1.823 Mio €/a for the 12MW wind farm under the same assumptions as above. The investment costs are stated under costsWppCapex in the table below. Furthermore, the same operational costs are assumed as above with 125,000€/a for the 2.5MW plant which equals
600,000€/a for the 12MW wind farm. Considering the capacity factor of 35.7%, the operational costs are 15.99€/MWh which are stated under costsWppOpex in the table below.

**Charger- and discharger flow** The parameters \(\text{dischargerFlowMax}_b\) and \(\text{chargerFlowMax}_b\) are used in the problem formulation if the energy flow from the sink- to the source node and vice versa should be limited. In this work, no limit is applied and a high number relative to the \(\text{chargerMax}_b\) and \(\text{dischargerMax}_b\) value is used. In this case the value 100MW is chosen for both battery technologies.

### A.2. Input parameters case study I

Tab. A.1 provides an overview of all input parameters of the case study I. Case study II and III are also based on this table except input parameters that differ which are stated separately. If a specific case study requires additional input data not listed below, they are also stated separately. References for each value are provided in the annex above or in chapter 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
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<th>Lead-acid</th>
</tr>
</thead>
<tbody>
<tr>
<td>(bdCost_t)</td>
<td>€</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdMax)</td>
<td>MW</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdPrimaryDownCost_t)</td>
<td>€</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdPrimaryDownMax)</td>
<td>MW</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdPrimaryUpCost_t)</td>
<td>€</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdPrimaryUpMax)</td>
<td>MW</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(bdRegDownCost_t)</td>
<td>€</td>
<td>0</td>
<td></td>
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</tbody>
</table>
### A.2 Input parameters case study I

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Value</th>
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<tbody>
<tr>
<td><code>bdRegDownMax</code></td>
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</tr>
<tr>
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</tr>
<tr>
<td><code>bdRegUpMax</code></td>
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</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td><code>buMax</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>buPrimaryDownCost_t</code></td>
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<td>0</td>
</tr>
<tr>
<td><code>buPrimaryDownMax</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>buPrimaryUpCost_t</code></td>
<td>€</td>
<td>0</td>
</tr>
<tr>
<td><code>buPrimaryUpMax</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>buRegDownCost_t</code></td>
<td>€</td>
<td>0</td>
</tr>
<tr>
<td><code>buRegDownMax</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>buRegUpCost_t</code></td>
<td>€</td>
<td>0</td>
</tr>
<tr>
<td><code>buRegUpMax</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>bonusWpp</code></td>
<td>€/MWh</td>
<td>33</td>
</tr>
<tr>
<td><code>capacityFadeRate_b</code></td>
<td>MW/MWh</td>
<td>6.06·10^{-6}</td>
</tr>
<tr>
<td><code>capacityFadeRateThroughput_b</code></td>
<td>MW/MWh</td>
<td>1.66·10^{-5}</td>
</tr>
<tr>
<td><code>chargerConversionFactor_b</code></td>
<td>MW/MW</td>
<td>1.065</td>
</tr>
<tr>
<td><code>chargerFlowMax_b</code></td>
<td>MW</td>
<td>100</td>
</tr>
<tr>
<td><code>chargerMax_b</code></td>
<td>MW</td>
<td>1</td>
</tr>
<tr>
<td><code>chargerMin_b</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>costWppCapex</code></td>
<td>€/MWa</td>
<td>151,917</td>
</tr>
<tr>
<td><code>costWppOpex</code></td>
<td>€/MWh</td>
<td>15.99</td>
</tr>
</tbody>
</table>
## Chapter A Input data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>dischargerConversionFactor$_b$</td>
<td>MW $/ MW$</td>
<td>0.955 $\sqrt{0.8}$</td>
</tr>
<tr>
<td>dischargerFlowMax$_b$</td>
<td>MW</td>
<td>100</td>
</tr>
<tr>
<td>dischargerMax$_b$</td>
<td>MW</td>
<td>1</td>
</tr>
<tr>
<td>dischargerMin$_b$</td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td>esInvestSwitch</td>
<td>—</td>
<td>0</td>
</tr>
<tr>
<td>$i$</td>
<td>%</td>
<td>10</td>
</tr>
<tr>
<td>$ih$</td>
<td>$a$</td>
<td>6</td>
</tr>
<tr>
<td>initSoc$_b$</td>
<td>MWh $/ MWh$</td>
<td>0.5 $/ 0.5$</td>
</tr>
<tr>
<td>marketPrice$_t$</td>
<td>€</td>
<td>[1]</td>
</tr>
<tr>
<td>marketPricePrimaryDown$_t$</td>
<td>€</td>
<td>[1]</td>
</tr>
<tr>
<td>marketPricePrimaryUp$_t$</td>
<td>€</td>
<td>[1]</td>
</tr>
<tr>
<td>marketPriceRegDown$_t$</td>
<td>€</td>
<td>From [1] (if price not available, $marketPriceRegDown$_t = -1000$)</td>
</tr>
<tr>
<td>marketPriceRegUp$_t$</td>
<td>€</td>
<td>From [1] (if price not available, $marketPriceRegUp$_t = -1000$)</td>
</tr>
<tr>
<td>minSoc$_t$</td>
<td>MWh $/ MWh$</td>
<td>0 $/ 0.4$</td>
</tr>
<tr>
<td>$l$</td>
<td>—</td>
<td>Depends on month</td>
</tr>
<tr>
<td>primaryDownReserveFactor$_b$</td>
<td>MWh $/ MWh$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
### A.2 Input parameters case study I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>primaryDownMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>primaryDownMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>primaryUpMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>primaryUpMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>primaryUpReserveFactor</code></td>
<td>MWh</td>
<td>0.5</td>
</tr>
<tr>
<td><code>purchaseCostSupplement</code></td>
<td>€</td>
<td>0.01</td>
</tr>
<tr>
<td><code>purchaseMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>purchaseMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>pv_b</code> (see Tab. 2.1)</td>
<td>€/MW</td>
<td>975,000</td>
</tr>
<tr>
<td><code>regDownEnergyFlowFactor</code></td>
<td>MWh/MWh</td>
<td>0.9</td>
</tr>
<tr>
<td><code>regDownMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>regDownMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>regUpEnergyFlowFactor</code></td>
<td>MWh/MWh</td>
<td>0.9</td>
</tr>
<tr>
<td><code>regUpMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>regUpMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>saleMax_t</code></td>
<td>MW</td>
<td>10000</td>
</tr>
<tr>
<td><code>saleMin_t</code></td>
<td>MW</td>
<td>0</td>
</tr>
<tr>
<td><code>selfConsumption_b</code></td>
<td>MWh/MWh</td>
<td>0.0125</td>
</tr>
<tr>
<td><code>selfDischargeRate_b</code></td>
<td>MW/MWh</td>
<td>8.3 \cdot 10^{-5}</td>
</tr>
<tr>
<td><code>sisNodeMax_b</code></td>
<td>MWh</td>
<td>((1-\text{minSoc}) \cdot \text{sosNodeMax})</td>
</tr>
<tr>
<td><code>sosNodeMax_b</code></td>
<td>MWh</td>
<td>0.25</td>
</tr>
</tbody>
</table>
A.3. Input parameters case study III

The input parameters for case study III that differ from case study I (Tab. A.1) are listed in Tab. A.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{switch})</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>(td_t)</td>
<td>(h)</td>
<td>(\forall t \in T : \ td_t = 1)</td>
</tr>
<tr>
<td>(\text{windMax}_t)</td>
<td>(MW)</td>
<td>Differs hourly</td>
</tr>
</tbody>
</table>

Table A.2.: Input parameters case study III

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(bdCost_t)</td>
<td>(€)</td>
<td>From: [1]</td>
</tr>
<tr>
<td>(bdMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
<tr>
<td>(bdPrimaryDownCost_t)</td>
<td>(€)</td>
<td>100,000</td>
</tr>
<tr>
<td>(bdPrimaryDownMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
<tr>
<td>(bdPrimaryUpCost_t)</td>
<td>(€)</td>
<td>100,000</td>
</tr>
<tr>
<td>(bdPrimaryUpMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
<tr>
<td>(bdRegDownCost_t)</td>
<td>(€)</td>
<td>(bdCost_t)</td>
</tr>
<tr>
<td>(bdRegDownMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
<tr>
<td>(bdRegUpCost_t)</td>
<td>(€)</td>
<td>(bdCost_t)</td>
</tr>
<tr>
<td>(bdRegUpMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
<tr>
<td>(buCost_t)</td>
<td>(€)</td>
<td>From: [1]</td>
</tr>
<tr>
<td>(buMax)</td>
<td>(MW)</td>
<td>1000</td>
</tr>
</tbody>
</table>
### A.3 Input parameters case study III

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bu_{PrimaryDownCost_t}$</td>
<td>€</td>
<td>100,000</td>
</tr>
<tr>
<td>$bu_{PrimaryDownMax}$</td>
<td>MW</td>
<td>1000</td>
</tr>
<tr>
<td>$bu_{PrimaryUpCost_t}$</td>
<td>€</td>
<td>100,000</td>
</tr>
<tr>
<td>$bu_{PrimaryUpMax}$</td>
<td>MW</td>
<td>1000</td>
</tr>
<tr>
<td>$bu_{RegDownCost_t}$</td>
<td>€</td>
<td>$bu_{Cost_t}$</td>
</tr>
<tr>
<td>$bu_{RegDownMax}$</td>
<td>MW</td>
<td>1000</td>
</tr>
<tr>
<td>$bu_{RegUpCost_t}$</td>
<td>€</td>
<td>$bu_{Cost_t}$</td>
</tr>
<tr>
<td>$bu_{RegUpMax}$</td>
<td>MW</td>
<td>1000</td>
</tr>
<tr>
<td>esInvestSwitch</td>
<td>—</td>
<td>1</td>
</tr>
<tr>
<td>i</td>
<td>%</td>
<td>10</td>
</tr>
<tr>
<td>ih</td>
<td>a</td>
<td>Not needed</td>
</tr>
<tr>
<td>$primary_{DownMax_t}$</td>
<td>MW</td>
<td>$obligation_{PrimaryDown_t}$</td>
</tr>
<tr>
<td>$primary_{DownMin_t}$</td>
<td>MW</td>
<td>$obligation_{PrimaryDown_t}$</td>
</tr>
<tr>
<td>$primary_{UpMax_t}$</td>
<td>MW</td>
<td>$obligation_{PrimaryUp_t}$</td>
</tr>
<tr>
<td>$primary_{UpMin_t}$</td>
<td>MW</td>
<td>$obligation_{PrimaryUp_t}$</td>
</tr>
<tr>
<td>$purchase_{Max_t}$</td>
<td>MW</td>
<td>$obligation_{Purchase_t}$</td>
</tr>
<tr>
<td>$purchase_{Min_t}$</td>
<td>MW</td>
<td>$obligation_{Purchase_t}$</td>
</tr>
<tr>
<td>$regDownMax_t$</td>
<td>MW</td>
<td>$obligation_{RegDown_t}$</td>
</tr>
<tr>
<td>$regDownMin_t$</td>
<td>MW</td>
<td>$obligation_{RegDown_t}$</td>
</tr>
<tr>
<td>$regUpMax_t$</td>
<td>MW</td>
<td>$obligation_{RegUp_t}$</td>
</tr>
<tr>
<td>$regUpMin_t$</td>
<td>MW</td>
<td>$obligation_{RegUp_t}$</td>
</tr>
<tr>
<td>$saleMax_t$</td>
<td>MW</td>
<td>$obligation_{Sale_t}$</td>
</tr>
<tr>
<td>$saleMin_t$</td>
<td>$MW$</td>
<td>$obligationSale_t$</td>
</tr>
</tbody>
</table>
B. CPLEX Parameters (OPS file)

Algorithm B.1 CPLEX tuning parameters (OPS file)

```xml
<?xml version="1.0" encoding="UTF-8"?>
<settings version="2">
  <category name="cplex">
    <setting name="brdir" value="1"/>
    <setting name="preslvnd" value="2"/>
    <setting name="probe" value="2"/>
    <setting name="cutsfactor" value="30.0"/>
    <setting name="nodelim" value="200000"/>
    <setting name="epgap" value="0.0010"/>
  </category>
</settings>
```